Implicit Regularization: NNLO developments

Adriano Cherchiglia[2], Brigitte Hiller[1], Marcos Sampaio[2]

WorkStop/ThinkStart 2.0: paving the way to alternative NNLO strategies
GGI, Florence

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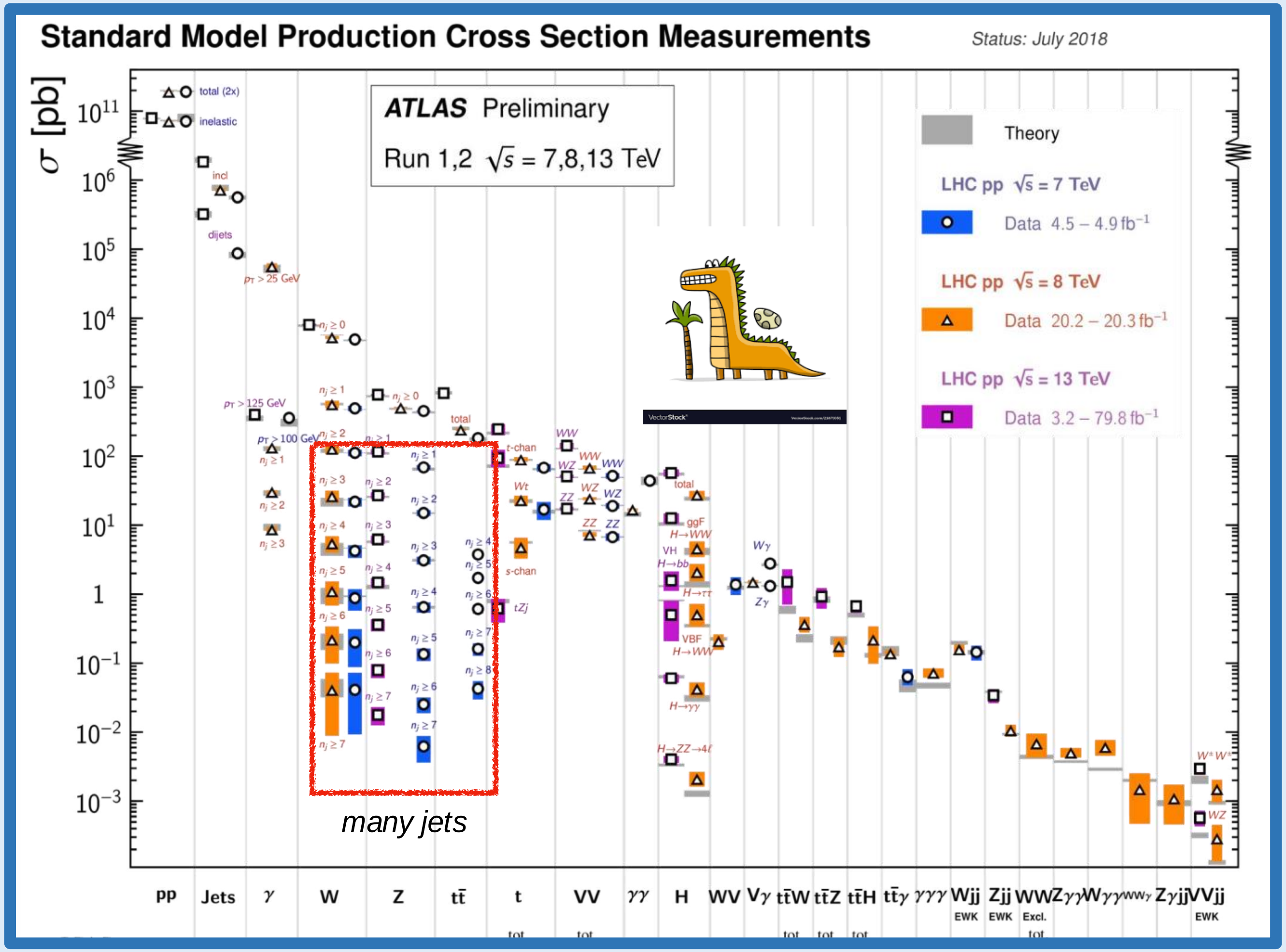
4-6 November 2019

Novel Strategies for N^2LO and N^3LO and precision observables in the SM and beyond

MOTIVATIONS



Up-to-date Review,
Higgs Physics at the
HL-LHC and HE-LHC,
arXiv:1902.00134v2

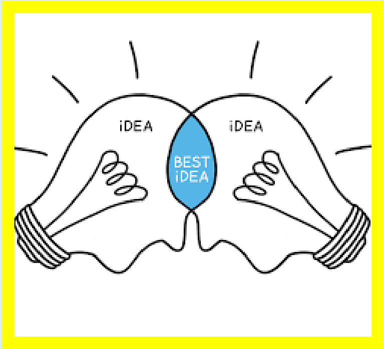
Some SM open questions

- **Dark Matter**
- **Quantum Gravity and Dark Energy**
- **CP violation and Baryonic asymmetry**
- **Neutrino oscillations and masses**
- **Why 3 families? Flavor Puzzle Problem**
- **Why low value of m_H despite large quantum corrections?**
- **Why 26 free parameters (18 of which related to Higgs physics)?**
- **Strong CP problem, etc.**

Two ways to explore BSM physics

1. **Increasing Energy/Luminosity**
   (LHC expected to run at 27 TeV)

2. **Rely on very high precision**
   (tiny discrepancies between theory and experiment)



High Precision@ Large Hard Calculations

T.Binoth

Example: NLO calculation in QED

$$S_{fi} = \langle f \,|\, S \,|\, i \rangle = \delta_{fi} + i\,T_{fi}$$

S Matrix

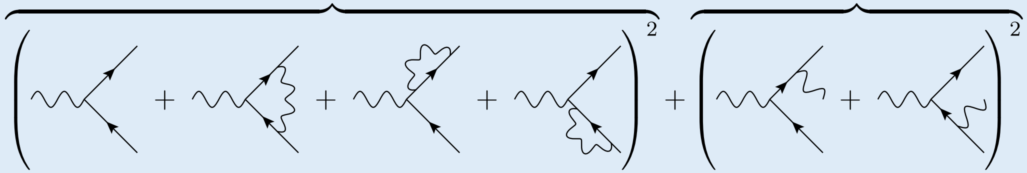
$$i\,T_{fi} = i(2\pi)^4 \delta\left(\sum_j p_{f_j} - \sum_k p_{i_k}\right) \mathcal{M}(i \to f)$$

$$d\sigma = \frac{|\mathcal{M}(i \to f)|^2}{F}\, d\Omega$$

Cross-section

Perturbation Theory

$\sigma^{\rm NLO}_{{\rm V}(\gamma^* \to e^+ e^-)}$ $\sigma^{\rm NLO}_{{\rm R}(\gamma^* \to e^+ e^- \gamma)}$



IR poles

$q^2_{\rm inc.\,\gamma} \equiv s_{12}$

$$\sigma^{\rm NLO}_{{\rm V}(\gamma^* \to e^+ e^-)} = \sigma^{\rm LO}\left(1 + \frac{2\alpha}{3\pi}\left(\frac{s_{12}}{\mu}\right)^{-\epsilon} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(4\pi)^{-\epsilon}\Gamma(1-2\epsilon)}\left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} - 8 + \mathcal{O}(\epsilon^1)\right)\right)$$

$$\sigma^{\rm NLO}_{{\rm R}(\gamma^* \to e^+ e^- \gamma)} = \sigma^{\rm LO}\left(1 + \frac{2\alpha}{3\pi}\left(\frac{s_{12}}{\mu}\right)^{-\epsilon} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(4\pi)^{-\epsilon}\Gamma(1-2\epsilon)}\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon^1)\right)\right)$$

$$\sigma^{\rm NLO}_{\gamma^* \to e^+ e^- (\gamma)} = \sigma_0\left(1 + \frac{\alpha}{\pi}\right)$$

Bloch-Nordsieck Theorem

QCD: Kinoshita-Lee-Nauenberg (**KNL**) theorem

Any QFT with massless field is free of IR divergences after sum over virtual and degenerate initial- and final-state real contributions

∞ in QFT

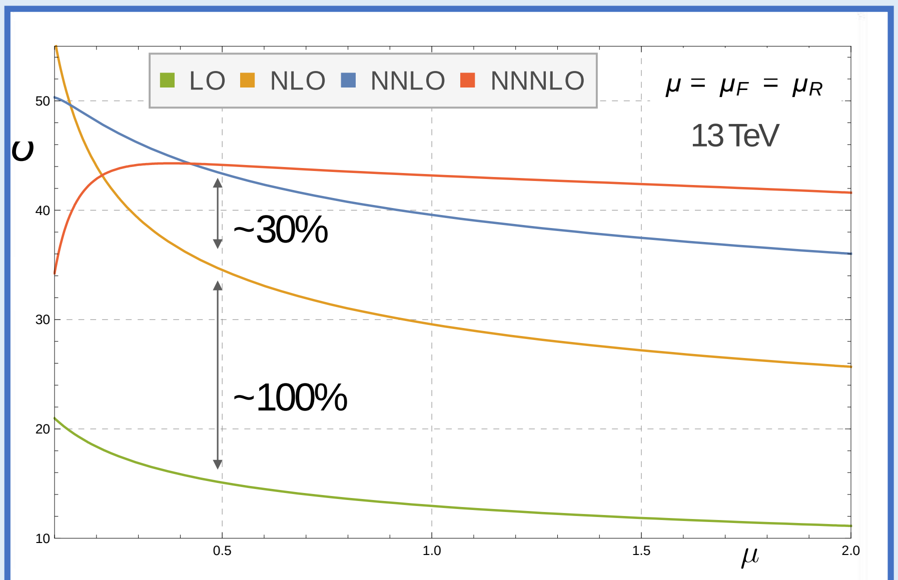
UV $\longrightarrow$ integrations in loop momenta $\longrightarrow$ ren. scale μ_R

IR

- External massive particle p radiates another particle with 4-momentum $p_r \rightarrow 0$ (soft singularities)
- External massless particle p radiates another particle with 4-momentum $p_r \rightarrow 0$ (soft singularities) or p // p_r (collinear singularities) $\rightarrow$ fact. scale μ_F
- Intermediate IR infinities in Feynman amplitudes, regulated e.g. with a fictitious mass which is set to zero in the end of calculation

Gluon Fusion Higgs Production at N^3 LO

Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2018) **JHEP 1905 (2019) 080**



- Observe stabilization of expansion
- Small correction (2% at $M_H/2$)
- Scale variation at N^3LO ~2%

Daniel de Florian
QCD @ LHC

N^2LO and N^3LO **automated** calculations in SM and extensions

REGULARIZATION

- Mathematical consistency
- Causality and unitarity
- Symmetries
- Quantum action principle
- **Computational eficiency**

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Application example: $e^+e^- \to \gamma^* \to q\bar{q}$ at NLO

To d, or not to d: recent developments and comparisons of regularization schemes

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COST is supported by the Framework Programme Horizon 2020

CA16201 - Unraveling new physics at the LHC through the precision frontier

- **Tradicional Schemes**: CDR (conventional Dimensional Regularization), t'Hooft Veltman Scheme (HV), Four dimensional Helicity Scheme (FDH), Dimensional Reduction (DRED)
- **Reformulations of dimensional schemes**: Four dimensional formulation of FDH (FDF), Six dimensional Formalism (SDF)
- **Non-dimensional schemes**: **Implicit Regularization (IREG)** , Four-dimensional Regularization/Renormalization (FDR), Four-Dimensional Un-subtraction (FDU), Differential Renormalization (DREN).
- **Mixed schemes**: High-Covariant-Derivatives (HCD), etc.

- **Algorithm for decomposing amplitudes into master integrals**
- **Subtract UV and IR divergences at the integrand level**
- **Facilitate resummation of Sudakov logs stemming from IR divergences**

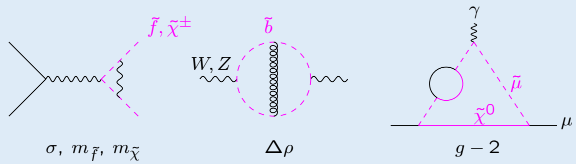
Why to explore non-dimensional methods ?

- **Dimensional Specific QFT : SuSy, Chiral, Topological …** **SPURIOUS ANOMALIES**

E.g. SuSy :

EWPO

resolve %-level loop effects



- CDR : breaks SuSy   Loop momenta - D dim Photon – D dim , Photino 4d   **breaks SuSy**
- DRED is mathematically inconsistent and there is no full proof that SUSY is preserved   Loop momenta - D dim, photon and photino – 4d

However, in DRED the following relation is required: ${g^{(4)}}_{\mu\nu} {g^{(D)}}_{\rho}{}^{\nu} = {g^{(D)}}_{\mu}{}^{\rho}$ ➡
D-dimensional space is a subspace of 4-dimensional space

One can then calculate $\epsilon^{\mu\nu\rho\sigma}\,\epsilon_{\alpha\beta\gamma\delta}\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\alpha\beta\gamma\delta}$ in two different ways

$$\Rightarrow 0 = D(D-1)^2(D-2)^2(D-3)^2(D-4)$$

different calculational steps lead to different results,
mathematical inconsistency!!!   *[Siegel'80]*

Consistent formulation $\Rightarrow$ prove quantum action principle!   ➡

D. Stockinger and collaborators
Nucl.Phys. B935 (2018) 1-16
Nucl.Phys.Proc.Suppl. 160 (2006) 250-254
JHEP 0503 (2005) 076

Quantum action principle: $i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle \;=\; \langle T\phi_1\ldots\phi_n\Delta\rangle$

SUSY Ward/ST identities: $i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle \;\stackrel{?}{=}\; 0$

- $\Delta \equiv \delta_{\mathsf{SUSY}}\mathcal{L}$ in D dimensions
- if $\Delta = 0$, all SUSY Ward and Slavnov-Taylor identities are satisfied

⬇

SUSY: DREG breaks SUSY already in simplest cases, DRED preserves SUSY in many cases up to 2-Loop, but not at all orders

- **Facts of life with γ^5**

Jegerlehner - Eur.Phys.J. C18 (2001) 673

$\{\gamma_5, \gamma_\mu\} = 0$ **does not hold in divergent amplitudes**

γ^5 **Clifford algebra clashes with dim. cont. in space-time dimension – Breitenlohner-Maison extension of CDR to regularise/renormalise Chiral Gauge Theories – introduce gauge variants local counterms to restore WI**

In principle, we don't have to bother whether a regularization preserves symmetries

In practice, life is easier with a symmetry-preserving regularization!

Some recipes for maintaining gauge symmetry in Renormalised Chiral Gauge Theories

displace γ^5 to rightmost position in the amplitude

Tsai et. Al. Phys. Rev. D83 (2011) 065011

Symmetrise $Tr[\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma^5]$

Viglioni, Cherchiglia, Hiller, Vieira, Sampaio – Phys. Rev. D94(2016)065023

Similar problems in the physical dimension !!

γ_5 algebra ambiguities in Feynman amplitudes: Momentum routing invariance and anomalies in $D=4$ and $D=2$

Viglioni, Cherchiglia, Hiller, Vieira, Sampaio
Phys. Rev. D94(2016)065023

$$\int \frac{d^4k}{(2\pi)^4}\, k_\mu k_\nu\, f(k^2) = \frac{g_{\mu\nu}}{4} \int \frac{d^4k}{(2\pi)^4}\, k^2\, f(k^2)$$

Symmetric Integration is not valid in general in the integer dimension for *divergent integrals*.

Perez-Victoria (JHEP 0104 (2001) 032)

Consistent procedure

Dimensional regularization vs methods in fixed dimension with and without γ_5

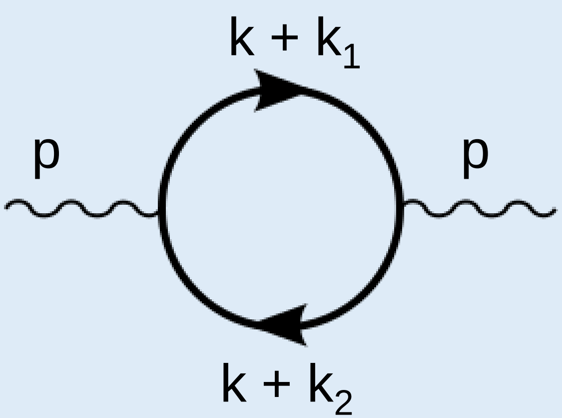
Bruque, Cherchiglia, Perez,
JHEP (2018) 2018: 109

Abstract: We study the Lorentz and Dirac algebra, including the antisymmetric ϵ tensor and the γ_5 matrix, in implicit gauge-invariant regularization/renormalization methods defined in fixed integer dimensions. They include constrained differential, implicit and four-dimensional renormalization. We find that these fixed-dimension methods face the same difficulties as the different versions of dimensional regularization. We propose a consistent procedure in these methods, similar to the consistent version of regularization by dimensional reduction.

Methodology: IREG

- Momentum space, nondimensional
- No modification at Lagrangian level
- Locality, Unitarity, Lorentz symmetry to N-loop
- Bogoliubov recursion formulae (BPHZ) → renorm. of subdivergencies
- UV and IR divergencies as BDI (internal momentum only)
- Parametrisation of reg. dep. terms as surface terms

Instructional Examples → **Vacuum polarization tensor massless QED in 3+1 d**



$k_1 - k_2 = p$

$e.g.\ \ k_1 = \alpha p\ ,\ \ k_2 = (\alpha - 1)p$

α arbitrary routing

BDI $I_{\log}(m^2) \equiv \int_k \frac{1}{(k^2 - m^2)^2}$ $\quad I_{\text{quad}}(m^2) \equiv \int_k \frac{1}{(k^2 - m^2)}$ $\quad I^{\mu\nu}_{\log}(m^2) = \int_k \frac{k^\mu k^\nu}{(k^2 - m^2)^3}$

etc.

ST

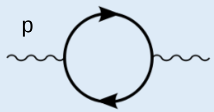
$$\Upsilon^{\mu\nu}_0 \equiv \int_k^d \frac{\partial}{\partial k_\mu} \frac{k^\nu}{(k^2 - m^2)^{\frac{d}{2}}} = d\left[\frac{g^{\mu\nu}}{d} I_{\log}(m^2) - I^{\mu\nu}_{\log}(m^2)\right]$$

Reg. Dep.

Scale Relations e.g. $I_{\log}(m^2) = I_{\log}(\lambda^2) + b \ln\left(\frac{\lambda^2}{m^2}\right)$

Reg. Indep.

$\lambda \neq 0$ plays the role of renormalization group scale



$k_1 - k_2 = p$

$$\Pi_{\mu\nu} = -\int_k tr\{\gamma_\mu S(k+k_1)\gamma_\nu S(k+k_2)\}$$

$$S(q) = \frac{i}{(\not{q}-\mu)} \quad \mu \;\; \text{infrared reg.}$$

Eliminate external momenta from BDI

$$\frac{1}{[(k+k_i)^2 - m^2]} = \sum_{j=0}^{N} \frac{(-1)^j (k_i^2 + 2k_i \cdot k)^j}{(k^2-m^2)^{j+1}} + \frac{(-1)^{N+1}(k_i^2 + 2k_i \cdot k)^{N+1}}{(k^2-m^2)^{N+1}[(k+k_i)^2 - m^2]}$$

$$\begin{aligned}\Pi_{\mu\nu} &= \tilde{\Pi}_{\mu\nu} + 4\Big(\Upsilon^2_{\mu\nu} - \frac{1}{2}(k_1^2+k_2^2)\Upsilon^0_{\mu\nu} + \frac{1}{3}(k_1^\alpha k_1^\beta + k_2^\alpha k_2^\beta + k_1^\alpha k_2^\beta)\Upsilon^0_{\mu\nu\alpha\beta} \\ &- (k_1+k_2)^\alpha (k_1+k_2)_\mu \Upsilon^0_{\nu\alpha} - \frac{1}{2}(k_1^\alpha k_1^\beta + k_2^\alpha k_2^\beta) g_{\mu\nu}\Upsilon^0_{\alpha\beta}\Big) \quad \text{where}\end{aligned}$$

$$\Upsilon = 0 \quad \text{MRI}$$

$$\tilde{\Pi}_{\mu\nu} = \frac{4}{3}\left((k_1-k_2)^2 g_{\mu\nu} - (k_1-k_2)_\mu (k_1-k_2)_\nu\right)\left(I_{log}(\mu^2) - \frac{i}{(4\pi)^2}\left(\frac{5}{3} + \ln\frac{-(k_1-k_2)^2}{\mu^2}\right)\right)$$

Scale relation

$$I_{\log}(m^2) = I_{\log}(\lambda^2) + b\ln\left(\frac{\lambda^2}{m^2}\right)$$

IR diverg. eliminated

$$\left(I_{log}(\lambda^2) - \frac{i}{(4\pi)^2}\left(\frac{5}{3} + \ln\frac{-(k_1-k_2)^2}{\lambda^2}\right)\right)$$

$$b_d \equiv \frac{i}{(4\pi)^{d/2}}\frac{(-1)^{d/2}}{\Gamma(d/2)}.$$

λ RG scale

IREG

- **Generalisable to n loop order in compliance with BPHZ forest formula**

$$I_{\log}^{(n)}(m^2) \equiv \int_k \frac{1}{(k^2-m^2)^2} \ln^{n-1}\left(-\frac{(k^2-m^2)}{\lambda^2}\right)$$ → Typical UV BDI n-loop order

$$I_{\log}^{(n+1)}(m^2) = I_{\log}^{(n+1)}(\lambda^2) - b \sum_{i=1}^{n+1} \frac{n!}{i!} \ln^i\left(\frac{m^2}{\lambda^2}\right)$$ → Scale relation

$$\frac{dI_{\log}^{(n)}(\lambda^2)}{d\lambda^2} = -\frac{(n-1)}{\lambda^2} I_{\log}^{(n-1)}(\lambda^2) + \frac{b_d}{\lambda^2} A^{(n)},$$

$$\frac{dI_{\log}^{(n)\mu\nu}(\lambda^2)}{d\lambda^2} = -\frac{(n-1)}{\lambda^2} I_{\log}^{(n-1)\mu\nu}(\lambda^2) + \frac{g^{\mu\nu}}{2}\frac{b_d}{\lambda^2} B^{(n)}$$

Derivatives of BDI are BDI → evaluate RG functions

$$\frac{1}{2}\sum_{j=1}^{n}\left(\frac{2}{d}\right)^j \frac{(n-1)!}{(n-j)!} \Upsilon_0^{(n)\mu\nu} = -I_{\log}^{(n)\mu\nu}(\lambda^2) + \frac{g^{\mu\nu}}{2}\sum_{j=1}^{n}\left(\frac{2}{d}\right)^j \frac{(n-1)!}{(n-j)!} I_{\log}^{(l-j+1)}(\lambda^2)$$ → ST parametrises reg. dep. terms

- **MRI** ⟺ **ST = 0** ⟺ **Gauge Invariance** (proof for abelian case)

- **Clear separation UV and IR divergences**

  (IR can be expressed in dual (position) space as BDI)

- **When radiative corrections are finite but undetermined**

R. Jackiw Int.J.Mod.Phys. B14 (2000) 2011

Some Applications of IREG .

Y. R. Batista, Brigitte Hiller, Adriano Cherchiglia, and Marcos Sampaio. *Supercurrent anomaly and gauge invariance in the $N = 1$ supersymmetric Yang-Mills theory*, Phys. Rev. D98 (2018) 025018.

anomalies

A. L. Cherchiglia, Marcos Sampaio, Brigitte Hiller, and A. P. B. Scarpelli. *Subtleties in the beta-function calculation of $N = 1$ supersymmetric gauge theories*, Eur. Phys. J. C76 (2016) 47.

SuSy

A. R. Vieira, A. L. Cherchiglia, and Marcos Sampaio. *Momentum routing invariance in extended QED: Assuring gauge invariance beyond tree level*, Phys. Rev. D93 (2016) 025029.

Lorentz violation

A. C. D. Viglioni, A. L. Cherchiglia, A. R. Vieira, Brigitte Hiller, and Marcos Sampaio. *γ^5 algebra ambiguities in Feynman amplitudes: Momentum routing invariance and anomalies in $D = 4$ and $D = 2$*, Phys. Rev. D94 (2016) 065023.

Chiral Theories

A. L. Cherchiglia, L. A. Cabral, M. C. Nemes, and Marcos Sampaio. *(Un)determined finite regularization-dependent quantum corrections: The Higgs boson decay into two photons and the two-photon scattering examples*, Phys. Rev. D87 (2013) 065011.

Radiative undetermined parameters
NLO calculation

L. C. Ferreira, A. L. Cherchiglia, Brigitte Hiller, Marcos Sampaio, and M. C. Nemes. *Momentum routing invariance in Feynman diagrams and quantum symmetry breakings*, Phys. Rev. D 86 (2012) 025016.

MRI in Feynman diagrams
and gauge symmetry

H. G. Fargnoli, Brigitte Hiller, A. P. B. Scarpelli, Marcos Sampaio, and M.C. Nemes. *Regularization independent analysis of the origin of two loop contributions to $N = 1$ Super Yang–Mills beta function*, Eur. Phys. J. C71 (2011) 1633.

Holomorphy and
exact beta functions
In N=1 SYM

NNLO (and Beyond) Implementation of IReg

- At multiloop level, the IReg prescription does not give a natural order to perform integrations

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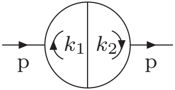
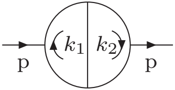
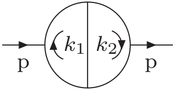
A. Cherchiglia, M. Sampaio, M. Nemes, Int. J. Mod. Phys. A26 (2011) 2591


k_1
k_2
p
p



$$\frac{ig^4}{2}\int_{k_1 k_2} \Delta(k_1)\Delta(k_1-p)\Delta(k_1-k_2)\Delta(k_2)\Delta(k_2-p), \quad \Delta(k_i)\equiv\frac{1}{k_i^2-\mu^2}$$



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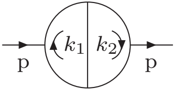
$$\frac{1}{(k_i-p)^2-\mu^2} = \sum_{l=0}^{2(n^{(k_i)}-1)} f_l^{\,(k_i,\,p)} + \bar{f}^{\,(k_i,\,p)}, \quad f_l^{\,(k_i,\,p)} \propto k_i^{-(l+2)} \quad \text{if} \quad k_i \to \infty$$



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$$\frac{1}{(k_i-p)^2-\mu^2} = \sum_{l=0}^{2(n^{(k_i)}-1)} f_l^{\,(k_i,\,p)} + \bar{f}^{\,(k_i,\,p)}, \quad f_l^{\,(k_i,\,p)} \propto k_i^{-(l+2)} \quad \text{if} \quad k_i\to\infty$$

Finiteness of the terms that contain $\bar{f}^{\,(k_i,\,p)}$ when $k_i\to\infty$ in all possible ways $\Rightarrow$ $n^{(k_1)} = n^{(k_2)} = 3$

- Divergences when $k_1 \to \infty$ and k_2 is fixed

$$
\begin{aligned}
A_1 &= \int\limits_{k_1 k_2} \Delta(k_1) f_0^{(k_1,\, p)} \Delta(k_1 - k_2)\Delta(k_2) \left[\sum_{m=0}^{4} f_m^{(k_2,\, p)} + \bar{f}^{\,(k_2,\, p)} \right] \\
&= \int\limits_{k_1 k_2} \Delta^2(k_1)\Delta(k_1 - k_2)\Delta(k_2)\Delta(k_2 - p)
\end{aligned}
$$

- Divergences when $k_2 \to \infty$ and k_1 is fixed

$$A_2 = \int\limits_{k_1 k_2} \Delta(k_1) \left[\sum_{l=0}^{4} f_l^{\,(k_1,\, p)} + \bar{f}^{\,(k_1,\, p)} \right] \Delta(k_1 - k_2) \Delta(k_2) f_0^{\,(k_2,\, p)}$$

$$= \int\limits_{k_1 k_2} \Delta(k_1)\Delta(k_1 - p)\Delta(k_1 - k_2)\Delta^2(k_2)$$

- Divergences when $k_1 \to \infty$ and $k_2 \to \infty$ simultaneously

$$A_3 = \int_{k_1 k_2} \Delta(k_1) f_1^{(k_1,\, p)} \Delta(k_1 - k_2) \Delta(k_2) f_1^{(k_2,\, p)}$$

$$= \int_{k_1 k_2} \Delta^3(k_1)(2p \cdot k_1) \Delta(k_1 - k_2) \Delta^3(k_2)(2p \cdot k_2)$$

- Term double counted

$$A_4 = \int_{k_1 k_2} \Delta(k_1) f_0^{(k_1,\, p)} \Delta(k_1 - k_2) \Delta(k_2) f_0^{(k_2,\, p)}$$

$$= \int_{k_1 k_2} \Delta^2(k_1) \Delta(k_1 - k_2) \Delta^2(k_2)$$

- Apply IReg rules according to their classification

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  1. A_1: Perform the integral in k_1 and consider k_2 as external momenta

- Apply IReg rules according to their classification
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  4. A_4: Perform the integrals in k_1 and k_2 (intersection of the cases $k_1 \to \infty$ and k_2 fixed, $k_2 \to \infty$ and k_1 fixed)

- Results

$$A_1 = \bar{A}_1 + \alpha_1$$

$$\bar{A}_1 = \int_{k_2} \Delta(k_2)\Delta(k_2 - p)\left[I_{log}(\lambda^2)\right]$$

$$\alpha_1 = \int_{k_2} \Delta(k_2)\Delta(k_2 - p)\left[2b_6 - b_6 \ln\left(-\frac{k_2^2 - \mu^2}{\lambda^2}\right)\right]$$

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$$A_3 = \bar{\alpha}_3 = b_6 p^2 \left[\frac{I_{log}(\lambda^2)}{3} + 2\Upsilon_1 \right]$$

$$A_4 = 0$$

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- Terms which contain $I_{log}(\lambda^2)$ multiplying an integral $\longrightarrow$ Subtracted by Bogoliubov's recursion formula

NNLO (and Beyond) Implementation of IReg

Example: ϕ^3 in 6D

● $= (-1)\times$ Divergence of 

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$$\frac{ig^4}{2}\int_{k_2}\Delta(k_2)\Delta(k_2-p)\left[-I_{log}(\lambda^2)\right]=\frac{ig^4}{2}\left(-\bar{A}_1\right),$$

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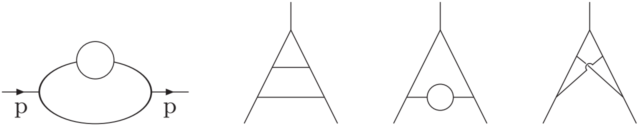

- 2-loop results:

$$\bar{\Xi}^{(2)} = \frac{ig^4 p^2}{6}\left\{ \frac{5b_6}{3} I_{log}^{(2)}(\lambda^2) - \frac{29b_6}{9} I_{log}(\lambda^2) - 20\Upsilon_2 - 4\Upsilon_1 \left[I_{log}(\lambda^2) - \frac{77b_6}{6} + 3\Upsilon_1 \right] + \text{finite} \right\}$$

$$\bar{\Lambda}^{(2)} = ig^5 \left[\frac{5b_6}{2} I_{log}^{(2)}(\lambda^2) - \left(\frac{17b_6}{3} + 6\Upsilon_1 \right) I_{log}(\lambda^2) + \text{finite} \right]$$

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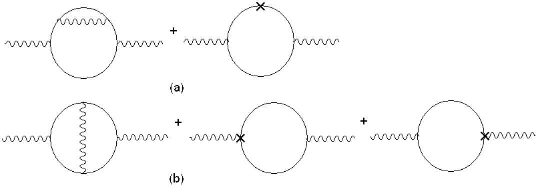
- Surface terms multiply divergence $\longrightarrow$ Affect the renormalization group functions

- Described method of implementing BPHZ in IReg used to obtain NNLO corrections and beyond in QED

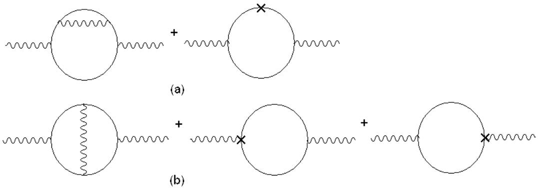
Dias et al., Eur. Phys. J. C **55**, 667 (2008)

Example: polarization tensor at NNLO



$$T_{\mu\nu} = \frac{8}{3}\imath e^4 b(p_\mu p_\nu - g_{\mu\nu}p^2)\left\{\frac{3}{2}I^2_{log}(\lambda^2) - 3bI^{(2)}_{log}(\lambda^2) + \frac{31}{6}bI_{log}(\lambda^2) - \right.$$

$$\left. -\frac{3}{2}b\ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{3}{2}b + 6b^2\zeta(3)\right\}.$$



$$T_{\mu\nu} = \frac{8}{3}\imath e^4 b \overbrace{(p_\mu p_\nu - g_{\mu\nu}p^2)}^{\text{Gauge invariant}} \left\{ \frac{3}{2} I_{log}^2(\lambda^2) - 3bI_{log}^{(2)}(\lambda^2) + \frac{31}{6} bI_{log}(\lambda^2) - \right.$$
$$\left. - \frac{3}{2} b \ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{3}{2}b + 6b^2\zeta(3) \right\}.$$

All surface terms were set to zero on gauge invariance grounds

A - Anomaly Puzzle

$U(1)_R$ current is in the same multiplet as T^{μ}_{μ}, **however**

chiral and trace anomaly subjected to the AB theorem

(no corrections beyond one-loop)

As $\beta \propto T^{\mu}_{\mu} \longrightarrow$ no corrections beyond one-loop

$$\beta(g)_{NSVZ} = \frac{-3C_A}{16\pi^2} \frac{g^3}{1 - \frac{C_A g^2}{8\pi^2}}$$

NSVZ Exact beta function

A nice solution

Arkani-Hamed and **Murayama**

JHEP 0006 (2000) 030

Different definitions of gauge couplings depending on renormalization schemes → different β functions

N=1 SYM
- holomorphic gauge coupling whose beta function is one loop exact
- canonical gauge coupling whose beta function is given by NSVZ β function.

Additional controversies: IR modes contributions to the beta-function

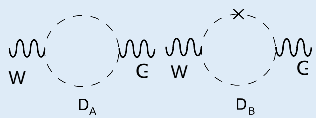
- **Instanton analysis** (NSVZ ´83 , ´85) → **it is clear that corrections to the one-loop result have an IR origin in an imbalance in the number of fermionic and bosonic zero modes**

- **A. Hamed and Murayama** → canonical **Wilsonian** coupling constant obeys a **NSVZ flow** → depends on **UV properties** of the theory, and thus questioned the IR origin of the corrections.

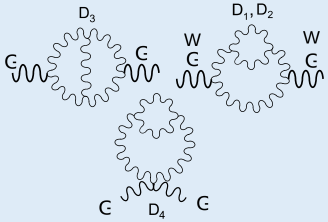
Idea: Feynman diagram calculation to 2 loop order

- **Framework that clearly separate UV divergences from the off-shell IR divergences that afflict these calculations**
- **Use IReg (physical dimension) since in dimensional methods UV and IR divergences are mixed**.
- **Use SuSy - BFM**

1-loop



2-loops



$$D_A = \frac{3}{4} C_A i \int d^4p \, d^2\theta \; \tilde{\mathbf{W}}^{\alpha}(p) \tilde{\mathbf{W}}_{\alpha}(-p) I(p),$$

$$D_B = i \frac{\xi}{4} C_A \int d^4p \, d^2\theta \; \tilde{\mathbf{W}}^{\alpha}(p) \tilde{\mathbf{W}}_{\alpha}(-p) p^2 U(p)$$

$$U(p) \equiv \int_k \frac{1}{k^4 (k-p)^2}, \qquad I(p) \equiv \int_k \frac{1}{k^2 (p-k)^2}$$

IR div **UV div**

Momentum space

$$I(p) = I_{\log}(\lambda^2) - b \ln\left(-\frac{p^2}{\lambda^2}\right) + 2b$$

where $b \equiv \frac{i}{(4\pi)^2}$, λ is a nonvanishing arbitrary parameter which plays the role of renormalization group scale

Configuration space

$$U = \frac{1}{p^2}\left(\tilde{I}_{\log}(\tilde{\lambda}^{-2}) + b \ln\left(-\frac{p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b \right)$$

where $\tilde{\lambda}$ is an infrared scale independent of the ultraviolet

$\bar{\tilde{\lambda}} \equiv (4/e^{2\gamma})\tilde{\lambda}$ $\gamma \rightarrow$ Euler-Mascheroni constant

Results:

$$G^{(2)}_{\text{ren}}(p^2)$$
$$= \frac{1}{2g^2} + \frac{C_A}{4} i\left[-3b\ln\left(-\frac{p^2}{\lambda^2}\right) + 6b + \xi p^2 U(p)\right]$$
$$- \frac{3g^2 C_A^2}{8}\left[-2bp^2 U(p) + bp^2 U^{(2)}(p)\right.$$
$$\left. + 2b^2 \ln\left(-\frac{p^2}{\lambda^2}\right) - 8b^2 + 3b^2\zeta(3) + \frac{b^2\pi^2}{6}\right],$$

Separately off-shell IR divergent

$$U = \frac{1}{p^2}\left(\tilde{I}_{\log}(\tilde{\lambda}^{-2}) + b\ln\left(-\frac{p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b\right),$$

$$U^{(2)} = \frac{1}{p^2}\ln\left(\frac{\bar{\tilde{\lambda}}^2}{\lambda^2}\right)\left(\tilde{I}_{\log}(\tilde{\lambda}^{-2}) + b\ln\left(-\frac{p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b\right)$$
$$+ \frac{b}{p^2}\left[\frac{1}{2}\ln^2\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right)\right]$$
$$- \frac{1}{p^2}\tilde{I}^{(2)}_{\log}(\tilde{\lambda}^{-2}),$$

$$\beta(g) = b_1 g^3 + b_2 g^5 + \ \ldots$$

Infrared modes contributes to beta function

$$b_1 = -\frac{3}{4}\frac{C_A}{(4\pi)^2},$$

$$b_2 = -\frac{3C_A^2}{8}\left[bp^2\lambda^2\frac{\partial}{\partial\lambda^2}U^{(2)}(p) - 2b^2 + bp^2 U(p)\right]$$
$$= -\frac{3}{4}\frac{C_A^2}{(4\pi)^4}$$

$$\lambda^2 \frac{\partial}{\partial\lambda^2}U^{(2)}(p) = -U(p)$$

SQED and conventional BFM

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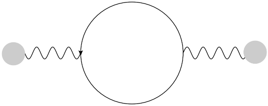
A. Cherchiglia et al, Eur. Phys. J. C76 (2016) no.2, 47

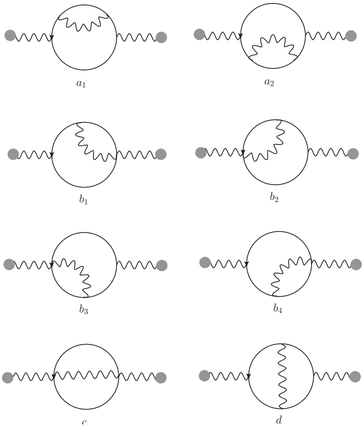


After after omitting surface terms on gauge invariant grounds

$$\Lambda^{(1)} = (-i)\frac{g^2}{2}\int_{p,\theta} B(-p,\theta) D^{\beta}\bar{D}^2 D_{\beta} B(p,\theta)\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right]$$

SQED in the conventional BFM

Results: 2 loops



$$\Lambda^{(2)} = -2\frac{g^4}{2}\int_{p,\theta} B(-p,\theta)D^{\beta}\bar{D}^2 D_{\beta}B(p,\theta)b\left[I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right)\right]$$

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Conclusions

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$$\beta = \frac{1}{(4\pi)^2} g^3 + \frac{1}{8(4\pi^2)^2} g^5 + \mathcal{O}(g^7)$$

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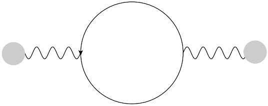

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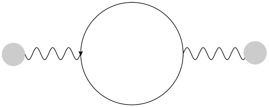

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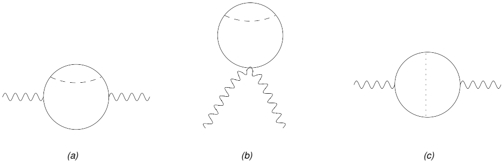
- No ambiguous term occurs, the result is automatically invariant;
- This invariant result is identical to the one obtained with the conventional BFM.



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SQED and BFM

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- Regarding divergencies, there exists a controversy related with the use of BFM
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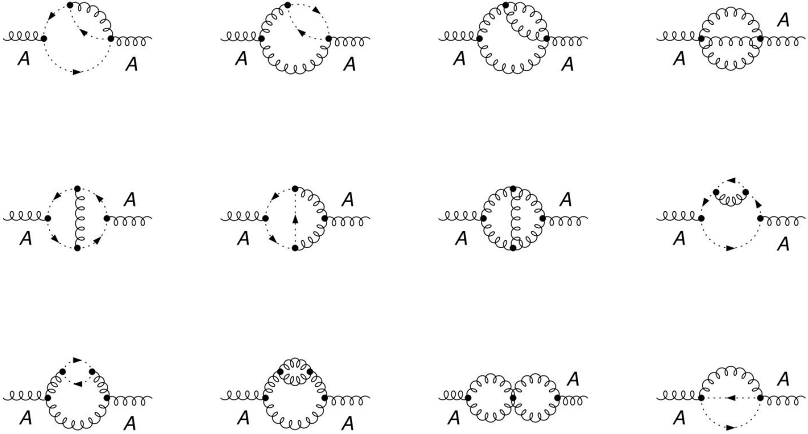
- Our aim is to compute the NNLO correction to the β function.

The diagrams



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- Local terms under investigation.

Research Proposals

1. **Regularization Scheme Dependence of QCD Amplitudes in IREG**
   (massless and massive cases – Gnendiger, Signer, Visconti JHEP - 2016)

- **cancellation of IR and UV singularities → compare with CDR, HV, FDH, DRED at NNLO**
- **study factorisation properties in IREG: calculation of anomalous dimensions** γ_{cusp} γ_q γ_g
- **advantages of IREG: no** ϵ **- scalars**
- **processes:** $\gamma^* \to q\bar{q}$ $H \to gg$ $q\bar{q} \to g\gamma$
- **heavy quark and heavy-to-light form factors (massive case)**
- **study compatibility of IREG with algorithms that isolate IR and UV divergencies: FKS subtraction** NPB 467 (1996) 399 **, Sector Decomposition** NPB 585 (2000) 741**.**

2. $q\bar{q} \to gg$

- **Issues with FDH: incorrect finite terms which violate unitarity → inconsistencies with IR structure→ solution: correct UV renormalization of subdivergencies**
  W. B. Kilgore, The Four Dimensional Helicity Scheme Beyond One Loop, Phys.Rev. D86 (2012) 014019, and
  Bern et al. , JHEP 0306 (2003) 028

3. $gg \to H$

- **NLO and NNLO in SM**
- **simple process (2→1)**
- **pedagogical avail: easy to see where divergencies come from and cancel each other**

4. $H \to b\bar{b}$

- **useful to test factorisation of overlapping singularities in phase space and loop integrations**
  JHEP03(2012)035