Introductory remarks: Regularization and calculational schemes

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4th November 2019, Florence Thinkstart-Workshop on regularization/paving the way to alternative NNLO strategies

Motivation

Regularization:

Calculational methods: (not always related to new regularizations)

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Zurich Workshop, “To d, or not to d”

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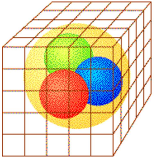

DREG

$\mu^{4-D} \int d^D p$

Motivation: fund. def. of QFT, study e.g. symmetries, puzzles renormalization theory

every regul. is also a calc. tool!

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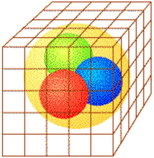

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New era: precision! LHC QCD/EW, BSM, low-E, dark matter
Community interest increasing

Florence Workshop: paving the way to alternative NNLO strategies

Outline

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Overview of schemes

- DREG and variants (CDR, HV, FDH, DRED)
  standard, well developed, $QS_{[d_s]} = QS_{[d]} \oplus QS_{[n_\epsilon]}$
- FDF, SFD (4-, 6-dimensional formulation)
  $=$DREG/FDH, exploit properties of evanescent quantities
- Implicit regularization and
  FDR (Four-dimensional renormalization)
  stay in 4-dim! regularize by "replacement rule"
  use constraints/make choices for divergent integrals
- FDU (4-dimensional unsubtraction)
  loop-tree duality, cutting rules, combine real+virtual

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Precise definitions

4S: ordinary 4-dimensional Minkowski/momentum space, metric $\bar{g}^{\mu\nu}$

QDS: "D-dimensional space" [Wilson'73],[Collins] :=
truly ∞-dimensional space with some D-dim characteristics:

- D-dimensional Integral = linear mapping $\int d^D k e^{-k^2} = \pi^{D/2}$
- $g_{(D)}^{\mu\nu}$: bilinear form $\mu = 0, 1, 2, \ldots \infty, \quad g_{(D)}{}^{\mu}{}_{\mu} = D$
- γ-matrices similar

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explicit construction $\Rightarrow$ no contradictions possible

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necessarily 4S $\subset$ QDS

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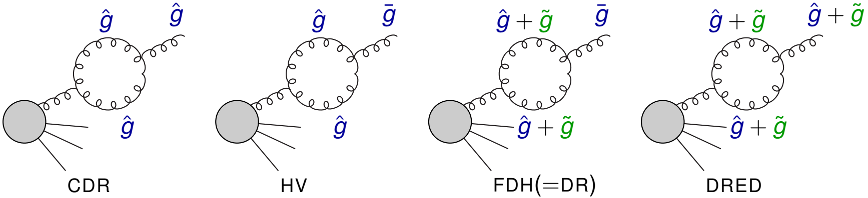
hierarchy of spaces 4S $\subset$ QDS $\subset$ Q4S [DS '05]

although $\mu = 0, 1, 2, \ldots \infty$, useful relations hold, e.g.

$$g^{\mu\nu} = g^{\mu\nu}_{(D)} + g^{\mu\nu}_{(2\epsilon)}, \qquad g^{\mu}{}_{\mu} = 4, \qquad g^{\mu\nu} k_{(D)\nu} = k^{\mu}_{(D)}$$



Summary regarding UV regularization:

- CDR and HV only need QDS, simpler but break SUSY
- FDH and DRED require Q4S to preserve gauge invariance

Common formulation of FDH (Bern, Dixon, Freitas; Kilgore, ...):

- "$4 < D < N_s$, internal gluons are N_s-dim.; at the end $N_s = 4$"

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Issue 1: Unitarity, Causality, Equivalence

unitarity determines imaginary terms, causality determines nonlocal terms [Bogoliubov et al, Epstein, Glaser]

Basic requirement (e.g. BPHZ or DREG+$\overline{\text{MS}}$ are correct):

any two correct regularizations may differ only by real, local terms

Dimensional regularization [Speer '74,Breitenlohner, Maison '77]

Need correct renormalization of $\tilde{\gamma}^\mu$-terms

Dimensional reduction [Jack, Jones, Roberts '93]

equivalence to DREG making use of indep. ϵ-scalar renormalization],

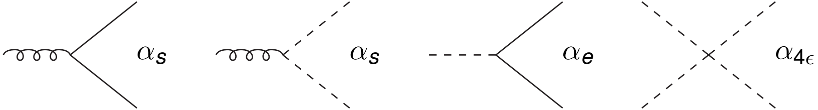
similar

other schemes? e.g. "take care" in FDF, "extra integrals" in FDR

Detail: 't Hooft, van Damme-problem: unitarity violation in DRED

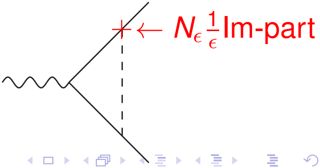
$D^\mu = \hat{\partial}^\mu + igA^\mu =$		$\hat{\partial}^\mu + ig\hat{A}^\mu$	$+$	$ig\tilde{A}^\mu$
4-component Gluon in DRED	$=$	D-component gauge field	$+$	ϵ-scalars
g		$\hat{g}$		$\tilde{g}$

- ϵ-scalars not "protected" by gauge invariance



- Different couplings $\alpha_s, \alpha_e, \alpha_{4\epsilon}$, especially $\delta\alpha_s \neq \delta\alpha_e, \beta^s \neq \beta^e, \ldots$
- Distinction required, otherwise divergent/non-unitary results

  [Jack, Jones, Roberts '94][Harlander, Kant, Mihaila, Steinhauser '06][Kilgore '11]



Issue 2: Mathematical consistency

Basic requirement: (e.g. consistent version of DREG, DRED ok)

Mathematical consistency is required for any scheme

- "inconsistency" means: one initial expression leads to different results, depending on the order of calculational steps

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Problems:

- Siegel’s inconsistency of DRED ($\rightsquigarrow$ must not use Fierz ident.!)
- γ_5-problem in DREG, anticommuting γ_5 is dangerous

New schemes — new mathematics (?):

- FDR, IReg: **replacement rules for integrands!**
- different kinds of (in)consistency checks
- **Crucial criterion/property:** “representation independence” or “subintegration consistency”

Detail: Representation independence: an optional property

- "representation independence"

$$\int \frac{k^2}{k^2 - m^2} \stackrel{?}{=} \int 1 + \int \frac{m^2}{k^2 - m^2}$$

- "Momentum routing invariance"

$$\int_k \frac{k^2}{k^2 - m^2} \stackrel{?}{=} \int_k \frac{(k+p)^2}{(k+p)^2 - m^2}$$

- diagram must be unambiguous wherever it appears as subdiagram!

Issues 3 and 4: Regularized quantum action principle and symmetries

Basic statement:

Naive result from symmetry variation in path integral:

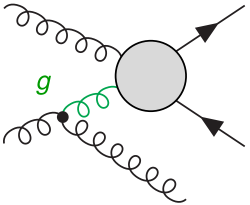
$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

- does this hold for regularized diagrams? What is $\delta\mathcal{L}$?
- E.g. holds in consistent DREG, DRED with $\delta\mathcal{L}^{(D)}$ — very useful!

Applications:

- DREG and DRED preserve gauge invariance in QED and QCD
- DRED preserves SUSY to some extent (but not exactly)
- Can one prove a QAP for IReg/FDR/FDU?

Issue 5: behaviour w.r.t. IR-divs and NNLO calculations — see Christoph Gnendiger's talk



$\sigma^{\text{DRED}}(gg \to t\bar{t}g) \xrightarrow{2\|3} ?$

- Used to be a "problem" in DRED
- Reconcile DRED with factorization by decomposing gluon [Signer, DS '05,'08, Gnendiger et al '14–'18]

$$\sim P_{g\to\hat{g}g}\,\sigma_{g\hat{g}} + P_{g\to\tilde{g}g}\,\sigma_{g\tilde{g}}$$

- Hence: "Which scheme is most efficient?"

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1. Overview of schemes
2. Definition of DREG/DRED
3. Five Issues with regularization schemes
4. Further remarks
5. Conclusions

News on γ_5-problem

- Gnendiger, Signer: γ_5 in FDH:
  1-/2-loop computation of axial form factors, compare γ_5^{AC}, γ_5^{BM}:
  "at one-loop, FDF simplifies calculations since the two γ_5's effectively become equal!"
  "However, (for γ_5-odd traces) it is not clear at all, if the use of γ_5^{AC} leads to a perceptible simplification"
- Bruque, Cherchiglia, Perez-Victoria: Dimensional regularization vs methods in fixed dimension with and without γ_5:
  Very powerful simple observation:
  "Renormalization does not commute with index contraction if it commutes with shifts of integration momenta and respects numerator-denominator consistency."
  As a result, similar γ_5-problem! *"Methods are not better or worse than DREG."*

News on SUSY

One of the questions for DRED is: “to what extent does DRED preserve SUSY?”
“is SUSY sufficiently preserved for the required precision computations of M_h^{MSSM}?”

YES, for 3-loop calculations of $\mathcal{O}(\alpha_{s,t}^3)$ [DS, Unger ’18]

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Summary

- Issue 1: Unitarity, Causality, Equivalence of schemes
  - Problems: evanescent renorm. in DREG/DRED, "extra terms" in FDR, construction of two-loop FDF, obvious in FDU?
- Issue 2: Consistent definition, representation independence
  - Problems: γ_5 in DRED **and** IReg/FDR, subdiagram consistency required in IReg/FDR, repres.-indep.
- Issues 3 and 4: Quantum action principle and symmetries

  $\langle(\delta\phi_1)\phi_2\ldots\rangle+\ldots=-i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$
  - Open Q: SUSY of DRED, improvement of γ_5? QAP-like statement/SUSY in IReg/FDR?
- Issue 5: IR-divs and NNLO efficiency
  - DREG/DRED: consistent! Sometimes FDH/DRED advantageous! Other schemes promising
  - Looking forward to a fruitful and inspiring Workshop!