Gui, Seesaw and Me

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Philosophical Questions: Who am I? Why am I here?

- I met **Gui** in February 2016 on the first edition of "Lisbon mini-school on Particle and Astroparticle Physics". I turned 21 some days later.
- One year later we started working in what would become a big part of my master thesis, along with G. Branco and J. Silva-Marcos. **Gui** becomes my master thesis supervisor.
- End of 2018: "Can one have significant deviations from leptonic 3×3 unitarity in the framework of type I seesaw mechanism?" is published and I present my master thesis.
- September 2019: I start my PhD with the supervision of **Gui**. (In the meantime we did some work on low-scale seesaw type one models that will be on arXiv very soon, make sure to check it out!)

Seesaw Type I

$$\frac{1}{2}\overline{\nu_R^c} M \nu_R \implies \nu = C\overline{\nu}^T = \nu^c \quad (1)$$

$$L^{SI\nu SM}_{M\nu Yukawa} = \frac{1}{2}\left[\begin{pmatrix}\nu_L^T & \nu_L'^T\end{pmatrix} C^{-1} \begin{pmatrix} 0 & m^* \\ m^\dagger & M^\dagger \end{pmatrix} \begin{pmatrix}\nu_L \\ \nu_L'\end{pmatrix}\right] - \overline{l_L} m_l l_R + h.c. \quad (2)$$

$$\mathcal{M}^\dagger = \mathcal{M}^* = \begin{pmatrix} 0 & m^* \\ m^\dagger & M^\dagger \end{pmatrix} \quad (3)$$

$$\mathcal{V}^T \mathcal{M}^* \mathcal{V} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix} = Diag(m_1, m_2, m_3, M_1, M_2, M_3) = D \quad (4)$$

Seesaw Type I

$$V = \begin{pmatrix} K & R \\ S & Z \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \nu_L \\ \nu'_L \end{pmatrix} = V \begin{pmatrix} n_L \\ N_L \end{pmatrix} \tag{6}$$

$$\nu_L^\delta = K^{\delta j} n_{Lj} + R^{\delta j} N_{Lj} \tag{7}$$

$$\nu_L'^\delta = S^{\delta j} n_{Lj} + Z^{\delta j} N_{Lj} \tag{8}$$

$$L_W^l = -\frac{g}{\sqrt{2}} \left[W_\mu^+ (\overline{n_L^i} K_{ik}^\dagger + \overline{N_L^i} R_{ik}^\dagger) \gamma^\mu l_L^k + W_\mu^- \overline{l_L^k} \gamma^\mu (K_{kj} n_L^j + R_{kj} N_L^j) \right] \tag{9}$$

$$\begin{aligned} L_{A,Z}^l = &-\frac{g}{2\cos\theta_w} \left[Z_\mu \left((\overline{n_L^i} K_{ik}^\dagger + \overline{N_L^i} R_{ik}^\dagger) \gamma^\mu (K_{kj} n_L^j + R_{kj} N_L^j) - \overline{l_L^i} \delta_{ij} \gamma^\mu l_L^j \right) \right] - \\ &\left[\left(\frac{g \sin^2\theta_w}{\cos\theta_w} Z_\mu + e A_\mu \right) (\overline{l_L^i} \delta_{ij} \gamma^\mu l_L^j + \overline{l_R^i} \delta_{ij} \gamma^\mu l_R^j) \right] \end{aligned} \tag{10}$$

Majorana Neutrinos

Pros

- Mass States $\neq$ Interaction States
- Neutrino Flavour Oscillations possible for small mass difference between light mass states n.
- Explains small light neutrino masses (Seesaw Approximation) : $M \sim D >> m \sim d \implies d = -K^{\dagger} m M^{-1} m^{T} (K^{T})^{-1}$, with $M_{eff} = -mM^{-1}m^{T}$.
- CP Violating phases
- Possibility of Leptogenesis due to CP Violation at High Energies (Heavy mass states involved).

Majorana Neutrinos

Cons

- 3 x 3 Leptonic Mixing Matrix (K) is not Unitary (FCNC).

BUT, FCNC are proportional to the deviations from unitarity which are also responsible for heavy-light neutrino mixing - through the matrix R.

Deviations from Unitarity are fundamental to test and characterize Seesaw Type I models

Fits to a Unitary Leptonic Mixing Matrix

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (11)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is a Dirac-type CP violating phase.

Parameter	Best fit	1σ range	3σ range
Δm^2_{21} $[10^{-5} eV^2]$	7.55	7.39 – 7.75	7.05 – 8.14
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3} eV^2](NO)$	2.50	2.47 – 2.53	2.41 – 2.60
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3} eV^2](IO)$	2.42	2.38 – 2.45	2.31 – 2.51
$\sin^2\theta_{12}$	0.320	0.304 – 0.340	0.273 – 0.379
$\sin^2\theta_{23}(NO)$	0.547	0.517 – 0.567	0.445 – 0.599
$\sin^2\theta_{23}(IO)$	0.551	0.521 – 0.569	0.453 – 0.598
$\sin^2\theta_{13}(NO)$	0.02160	0.02091 –0.02243	0.0196 –0.0241
$\sin^2\theta_{13}(IO)$	0.02220	0.02144 –0.02294	0.0199 –0.0244
$\delta/\pi(NO)$	1.32	1.17 –1.53	0.87 –1.94
$\delta/\pi(IO)$	1.56	1.41 –1.69	1.12 –1.94

Source: de Salas, P. F. and Forero, D. V. and Ternes, C. A. and Tortola, M. and Valle, J. W. F., Phys. Lett. B, (2018) 782: 633-640.

Bounds on Deviations from Unitarity of K

Taken from searches for Lepton Flavour Violating (LFV) decays, probes of the universality of weak interactions, CKM unitarity bounds and electroweak precision data.

$$I - |KK^\dagger| \sim |2\eta| \leq \begin{pmatrix} 2.5 \times 10^{-3} & 2.4 \times 10^{-5} & 2.7 \times 10^{-3} \\ 2.4 \times 10^{-5} & 4.0 \times 10^{-4} & 1.2 \times 10^{-3} \\ 2.7 \times 10^{-3} & 1.2 \times 10^{-3} & 5.6 \times 10^{-3} \end{pmatrix} \quad (12)$$

Valid when all heavy neutrinos have a mass above m_W.

Source: Fernandez-Martinez, E., Hernandez-Garcia, J. Lopez-Pavon, J. J. High Energ. Phys. (2016) 2016: 33. https://doi.org/10.1007/JHEP08(2016)033

Developing Appealing Models within the Seesaw Type I Framework

Detectable at the next round of experiments:

- Masses not bigger than $1 - 10\ TeV \rightarrow$ NO Seesaw approximation
- Sizable Deviations from Unitarity

Major Constraint: Finite loop corrections to light neutrino masses!

Necessary Equations for an Exact Treatment I

$$\mathcal{V} = \begin{pmatrix} K & KX^\dagger \\ -ZX & Z \end{pmatrix} \tag{13}$$

$$-X^\dagger Z^\dagger m^T = dK^T \tag{14}$$

$$K^\dagger m - X^\dagger Z^\dagger M = -dX^T Z^T \tag{15}$$

$$Z^\dagger m^T = DX^* K^T \tag{16}$$

$$XK^\dagger m + Z^\dagger M = DZ^T \tag{17}$$

$$d = -X^T \; D \; X \tag{18}$$

$$X = \pm i\sqrt{D^{-1}} O_c \sqrt{d} \tag{19}$$

Details: Nuno Rosa Agostinho, G. C. Branco, Pedro M. F. Pereira, M. N. Rebelo, J. I. Silva-Marcos, "Can One have Significant Deviations from Leptonic 3×3 Unitarity in the Framework of Type I Seesaw Mechanism?", Eur. Phys. J. C (2018) 78: 895

Necessary Equations for an Exact Treatment II

$$I + X^\dagger X = U \ \left(I + d_X^2\right) \ U^\dagger \tag{20}$$

$$\mathcal{V}\mathcal{V}^\dagger = I_{6\times 6} \rightarrow K(K^\dagger + X^\dagger X K^\dagger) = I \tag{21}$$

$$K \ U \ \left(I + d_X^2\right) \ U^\dagger \ K^\dagger = K \ U \ \sqrt{\left(I + d_X^2\right)} \ \cdot \sqrt{\left(I + d_X^2\right)} U^\dagger \ K^\dagger = I \tag{22}$$

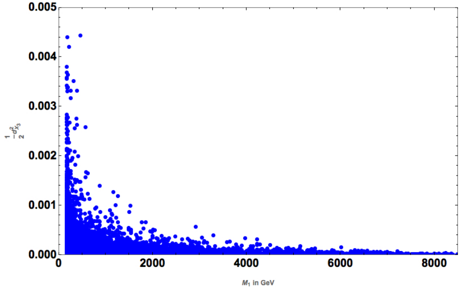
$$K \ U \ \sqrt{\left(I + d_X^2\right)} = U_K \tag{23}$$

Necessary Equations for an Exact Treatment III

$$\left(\sqrt{I + d_X^2}\right)^{-1} = \begin{pmatrix} \frac{1}{\sqrt{1+d_{X_1}^2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+d_{X_2}^2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1+d_{X_3}^2}} \end{pmatrix} \tag{24}$$

$$\begin{aligned} K &= \left(U_K \left(\sqrt{I + d_X^2}\right)^{-1} U_K^\dagger\right) \left(U_K . U^\dagger\right) \\ &= \left(U_K \; U^\dagger \left(\sqrt{I + X^\dagger X}\right)^{-1} U \; U_K^\dagger\right) \left(U_K . U^\dagger\right) \\ &= (I - \eta)\, \Omega \end{aligned} \tag{25}$$

Size of Deviations from Unitarity



Exact for all M:

$$m = KX^{\dagger} D \left(Z^{*}\right)^{-1} = -i\, K \sqrt{d}\, O_c^{\dagger} \sqrt{D} \left(Z^{-1}\right)^{*} \tag{26}$$

Casas-Ibarra approximate ($M >> m$) result:

$$m \simeq -i\, U_{\text{PMNS}} \sqrt{d}\, O_c^{\text{CI}} \sqrt{D} \tag{27}$$

On Finite Loop corrections

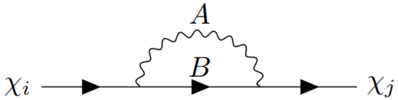


Figure: Diagrams which contribute to $\Sigma(p^2)$

Where if $A = Z, H, \phi_Z$ then $B = \chi_K$
or else if $A = \phi^{\pm}, W^{\pm}$, then $B = l^{\mp}$
and

$$\chi = \begin{pmatrix} n \\ N \end{pmatrix} \quad (28)$$

$$\nu_L = (K \;\; R) P_L \chi \quad (29)$$

as in [eq. 6].

$$M = M^{tree} + M^{loop} \quad (30)$$

$$M^{loop} = \begin{pmatrix} \delta M_L & \delta m \\ (\delta m)^T & \delta M \end{pmatrix} \quad (31)$$

$$\delta M_L = (K \;\; R) \Sigma^*(p^2) (K \;\; R)^T \quad (32)$$

Generating a Controlled δM_L

$$\delta M_L \approx \frac{g^2}{64\pi^2 m_W^2} R \left[3L^M(m_Z) + L^M(m_H)\right] R^T \quad (33)$$

$$L^M(m_B) = Diag\left(M_1^3 \frac{\log(M_1^2/m_B^2)}{M_1^2/m_B^2 - 1}, M_2^3 \frac{\log(M_2^2/m_B^2)}{M_2^2/m_B^2 - 1}, M_3^3 \frac{\log(M_3^2/m_B^2)}{M_3^2/m_B^2 - 1}\right) \quad (34)$$

Cases

1. Usual Seesaw: Having a very small R, such that $R\left(L^M(m_B)\right)R^T$ is suppressed.
2. Almost Conserved U(1)-like symmetry: Having a R with a given structure such that combined with at least two almost degenerate heavy neutrinos it yields a small $R\left(L^M(m_B)\right)R^T$ due to cancellations.

Almost Conserved U(1)-like symmetry

Table: Example for the case of Normal Ordering of light neutrino masses. This example gives the following phenomenological important quantities: $|m_{\beta\beta}| = 6.58 \times 10^{-3}\ eV$, $m_\beta = 1.01 \times 10^{-2}\ eV$, and $Tr\left[mm^\dagger\right] = 0.0488 m_t^2$.

Heavy Neutrino Masses (m_t)	$\lvert m \rvert\ (m_t)$	$\lvert M \rvert\ (m_t)$
$\begin{bmatrix} M_1 = 3 \\ M_2 = 3 + 1 \times 10^{-10} \\ M_3 = 50 \end{bmatrix}$	$\begin{pmatrix} 0.140 & 4.12 \times 10^{-13} & 6.49 \times 10^{-7} \\ 0.000876 & 2.06 \times 10^{-12} & 2.32 \times 10^{-6} \\ 0.171 & 1.84 \times 10^{-12} & 3.17 \times 10^{-6} \end{pmatrix}$	$\begin{pmatrix} 7.15 \times 10^{-10} & 2.99 & 1.76 \times 10^{-4} \\ 2.99 & 2.14 \times 10^{-11} & 3.85 \times 10^{-5} \\ 1.76 \times 10^{-4} & 3.85 \times 10^{-5} & 5.00 \times 10^{1} \end{pmatrix}$
Tree Level Light Neutrino Masses (eV)	$\lvert \eta \rvert^{tree}$	R^{tree}
$\begin{bmatrix} m_1 = 0.00507 \\ m_2 = 0.0100 \\ m_3 = 0.0522 \end{bmatrix}$	$\begin{pmatrix} 1.09 \times 10^{-3} & 6.82 \times 10^{-6} & 1.33 \times 10^{-3} \\ 6.82 \times 10^{-6} & 4.27 \times 10^{-8} & 8.34 \times 10^{-6} \\ 1.33 \times 10^{-3} & 8.34 \times 10^{-6} & 1.63 \times 10^{-3} \end{pmatrix}$	$\begin{pmatrix} -0.0262 - 0.0201i & -0.0201 + 0.0262i & 4.78 \times 10^{-9} + \left(4.53 \times 10^{-10}\right) i \\ 0.000137 + 0.000154i & 0.000154 - 0.000137i & -4.46 \times 10^{-8} - \left(1.28 \times 10^{-8}\right) i \\ -0.0066 + 0.0398i & 0.0398 + 0.0066i & -5.12 \times 10^{-8} - \left(4.92 \times 10^{-9}\right) i \end{pmatrix}$
One Loop Light Neutrino Masses (eV)	$\lvert \eta \rvert^{loop}$	R^{loop}
$\begin{bmatrix} m_1 = 0.00491 \\ m_2 = 0.0100 \\ m_3 = 0.0504 \end{bmatrix}$	$\begin{pmatrix} 1.09 \times 10^{-3} & 6.82 \times 10^{-6} & 1.33 \times 10^{-3} \\ 6.82 \times 10^{-6} & 4.27 \times 10^{-8} & 8.33 \times 10^{-6} \\ 1.33 \times 10^{-3} & 8.33 \times 10^{-6} & 1.63 \times 10^{-3} \end{pmatrix}$	$\begin{pmatrix} -0.0262 - 0.0201i & -0.0201 + 0.0262i & 4.78 \times 10^{-9} + \left(4.53 \times 10^{-10}\right) i \\ 0.000137 + 0.000154i & 0.000154 - 0.000137i & -4.46 \times 10^{-8} - \left(1.28 \times 10^{-8}\right) i \\ -0.0066 + 0.0398i & 0.0398 + 0.0066i & -5.12 \times 10^{-8} - \left(4.92 \times 10^{-9}\right) i \end{pmatrix}$

Conclusions

Usual Seesaw

Not appealing because:

- Small Deviations from unitarity.
- No constraints on heavy masses $\Longrightarrow$ Very difficult to test at colliders.

Conclusions

Symmetry-Protected Seesaw with *GeV*/*TeV* Heavy Neutrinos

Appealing because:

- No unnaturally small Yukawa couplings.
- Can be observed in the next round of experiments at the LHC.
- Possibility of resonant Leptogenesis?

Experimental input from KATRIN (& similar experiments), GERDA (& similar experiments), the LHC and neutrino oscillation experiments will be fundamental to discern if the proposed solution matches with Nature.

A Midsummer Night's Dream

Trieste, June 2019

(Someone is talking about their harsh relationship with their supervisor...)
A playful German PhD Student enters the scene:
But your supervisor is like your mother in Physics!! Don't you love your mother?!
Next February I will turn 25, which means **Gui** has been my mother in Physics for the last four years. **Thank you and Happy Birthday.**