A solution to the strong CP problem in the two-Higgs-doublet model

Luís Lavoura

CFTP, Instituto Superior Técnico, Univ. Lisboa

Together with Pedro M. Ferreira (Inst. Sup. Eng. Lisboa)

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The inclusion of instantons in the path integral of QCD leads to the presence in the QCD Lagrangian of a term

$$\theta \, \frac{g_s^2}{64\pi^2} \sum_{\mu,\nu,\rho,\sigma=0}^{3} \epsilon_{\mu\nu\rho\sigma} \sum_{a=1}^{8} F_a^{\mu\nu} F_a^{\rho\sigma} .$$

This term breaks both the P and T symmetries. A (strong) observable that violates P and T is the electric dipole moment of the neutron; its experimental upper bound requires $\theta \lesssim 10^{-9}$ (with uncertainties). The 'strong CP problem' is the presence of this unnaturally small parameter in the QCD Lagrangian.

In reality,

$$\theta = \theta_{\mathrm{QCD}} + \theta_{\mathrm{QFD}}$$

is the sum of two terms: θ_{QCD}, that was originally present in the QCD Lagrangian, and

$$\theta_{\mathrm{QFD}} = \arg\det\left(M_p M_n\right)$$

that may arise when the quark mass matrices M_p (of the charge 2/3 quarks) and M_n (of the charge $-1/3$ quarks) are not real. Under a chiral transformation of the quark fields, both θ_{QFD} and θ_{QCD} change, but their sum remains invariant.

Solving the strong CP problem through electroweak model building requires three steps:

1. Imposing either P or CP symmetry on the Lagrangian, so that θ_{QCD} is set to zero.

2. Devising quark mass matrices of such a shape that their total determinant is real, *viz.* the phase of the determinant of M_p offsets the one of M_n.

3. Constraining the one-loop-level theory in such a way that, when the quark mass matrices are renormalized,

$$\arg\det\left(M_q + \Sigma_q\right) \approx \operatorname{Im}\operatorname{tr}\left(M_q^{-1}\Sigma_q\right)$$

   is not too large, for both $q = p$ and $q = n$.

We attempt to do this in the context of a two-Higgs-doublet model with a softly broken symmetry but without any extra quarks, *i.e.* this is not a Barr–Nelson-type model.

Both 2HDMs and softly broken symmetries have been leitmotivs in the work of Gui Rebelo, whom we honour today!

$$\mathcal{L}_{\mathrm{Yukawa}} = -\sum_{j,k=1}^{3} \sum_{a=1}^{2} \bar{Q}_{Lj} \left[\phi_a \left(\Gamma_a \right)_{jk} n_{Rk} + \tilde{\phi}_a \left(\Delta_a \right)_{jk} p_{Rk} \right] + \mathrm{H.c.}$$

$$\bar{Q}_{Lj} \equiv \begin{pmatrix} \bar{p}_{Lj} & \bar{n}_{Lj} \end{pmatrix},$$

$$\phi_a \equiv \begin{pmatrix} \varphi_a^{+} \\ \varphi_a^{0} \end{pmatrix}, \quad \tilde{\phi}_a \equiv \begin{pmatrix} {\varphi_a^{0}}^{*} \\ -\varphi_a^{-} \end{pmatrix}.$$

With VEVs $\left\langle 0 \left| \varphi_a^0 \right| 0 \right\rangle = v_a e^{i\aleph_a} / \sqrt{2}$ $(a = 1, 2)$, we have

$$M_n = \frac{v_1 e^{i\aleph_1} \Gamma_1 + v_2 e^{i\aleph_2} \Gamma_2}{\sqrt{2}},$$

$$M_p = \frac{v_1 e^{-i\aleph_1} \Delta_1 + v_2 e^{-i\aleph_2} \Delta_2}{\sqrt{2}},$$

and we furthermore define

$$N_n = \frac{v_2 e^{i\aleph_1} \Gamma_1 - v_1 e^{i\aleph_2} \Gamma_2}{\sqrt{2}},$$

$$N_p = \frac{v_2 e^{-i\aleph_1} \Delta_1 - v_1 e^{-i\aleph_2} \Delta_2}{\sqrt{2}}.$$

We bi-diagonalize the quark mass matrices:

$$\begin{aligned} U_L^{n\,\dagger} M_n U_R^n &= M_d \equiv \mathrm{diag}\left(m_d, m_s, m_b\right), \\ U_L^{p\,\dagger} M_p U_R^p &= M_u \equiv \mathrm{diag}\left(m_u, m_c, m_t\right), \end{aligned}$$

the CKM matrix being $V = U_L^{p\,\dagger} U_L^n$. We define

$$\begin{aligned} U_L^{n\,\dagger} N_n U_R^n &\equiv N_d, \\ U_L^{p\,\dagger} N_p U_R^p &\equiv N_u. \end{aligned}$$

Then, the Yukawa interactions in the physical basis are given (assuming there is no scalar–pseudoscalar mixing, *i.e.* no CP violation in the neutral-scalar mass matrix) by

$$
\begin{aligned}
\mathcal{L}_{\text{physical}} \quad = \quad & \frac{iA}{v}\,\bar{u}\left(N_u P_R - N_u^\dagger P_L\right) u \\
& + \frac{iA}{v}\,\bar{d}\left(N_d^\dagger P_L - N_d P_R\right) d \\
& + \frac{h}{v}\,\bar{u}\left[\left(sM_u - cN_u^\dagger\right) P_L\right. \\
& \left. + \left(sM_u - cN_u\right) P_R\right] u \\
& + \frac{h}{v}\,\bar{d}\left[\left(sM_d - cN_d^\dagger\right) P_L\right. \\
& \left. + \left(sM_d - cN_d\right) P_R\right] d \\
& + \frac{H}{v}\,\bar{u}\left[\left(cM_u + sN_u^\dagger\right) P_L\right. \\
& \left. + \left(cM_u + sN_u\right) P_R\right] u \\
& + \frac{H}{v}\,\bar{d}\left[\left(cM_d + sN_d^\dagger\right) P_L\right. \\
& \left. + \left(cM_d + sN_d\right) P_R\right] d \\
& + \frac{\sqrt{2}H^+}{v}\,\bar{u}\left(N_u^\dagger V P_L - V N_d P_R\right) d \\
& + \frac{\sqrt{2}H^-}{v}\,\bar{d}\left(V^\dagger N_u P_R - N_d^\dagger V^\dagger P_L\right) u,
\end{aligned}
$$

where P_L and P_R are the projectors of chirality, A is a pseudoscalar, h and H are neutral scalars, $H^\pm$ are charged scalars, and $c^2 + s^2 = 1$.

In our model, we impose a symmetry ($\omega \equiv e^{2i\pi/3}$)

$$\mathbb{Z}_3 : \quad \begin{cases} \bar{Q}_{L1} \to \omega \bar{Q}_{L1}, \ \bar{Q}_{L2} \to \omega^2 \bar{Q}_{L2}, \\ n_{R3} \to \omega n_{R3}, \ p_{R1} \to \omega p_{R1}, \ p_{R2} \to \omega p_{R2}, \\ \phi_2 \to \omega^2 \phi_2. \end{cases}$$

One then has

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_1 \\ d_1 & f_1 & 0 \end{pmatrix}, \ \Gamma_2 = \begin{pmatrix} d_2 & f_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_2 \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ p_1 & q_1 & 0 \\ 0 & 0 & r_1 \end{pmatrix}, \ \Delta_2 = \begin{pmatrix} p_2 & q_2 & 0 \\ 0 & 0 & r_2 \\ 0 & 0 & 0 \end{pmatrix}.$$

The quark mass matrices are

$$M_n = \frac{1}{\sqrt{2}} \begin{pmatrix} d_2 v_2 e^{i\aleph_2} & f_2 v_2 e^{i\aleph_2} & 0 \\ 0 & 0 & b_1 v_1 e^{i\aleph_1} \\ d_1 v_1 e^{i\aleph_1} & f_1 v_1 e^{i\aleph_1} & b_2 v_2 e^{i\aleph_2} \end{pmatrix},$$

$$M_p = \frac{1}{\sqrt{2}} \begin{pmatrix} p_2 v_2 e^{-i\aleph_2} & q_2 v_2 e^{-i\aleph_2} & 0 \\ p_1 v_1 e^{-i\aleph_1} & q_1 e^{-i\aleph_1} & r_2 v_2 e^{-i\aleph_2} \\ 0 & 0 & r_1 e^{-i\aleph_1} \end{pmatrix}.$$

If additionally one imposes CP symmetry, then $\theta_{\rm QCD} = 0$ and moreover the Yukawa couplings b_a, d_a, f_a, p_a, q_a, and r_a $(a = 1, 2)$ are real. Hence,

$$\arg\det M_n = 2\aleph_1 + \aleph_2 = -\arg\det M_p$$

as desired.

One may withdraw the phases from the matrices M_n, N_n, M_p, and N_p through diagonal unitary transformations (rephasings)

$$X_{Ln} \begin{pmatrix} \aleph_2 & \aleph_2 & - \\ - & - & \aleph_1 \\ \aleph_1 & \aleph_1 & \aleph_2 \end{pmatrix} X_{Rn} = \begin{pmatrix} 0 & 0 & - \\ - & - & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$X_{Lp} \begin{pmatrix} -\aleph_2 & -\aleph_2 & - \\ -\aleph_1 & -\aleph_1 & -\aleph_2 \\ - & - & -\aleph_1 \end{pmatrix} X_{Rp} = \begin{pmatrix} 0 & 0 & - \\ 0 & 0 & 0 \\ - & - & 0 \end{pmatrix},$$

where

$$\begin{aligned} X_{Ln} &= \operatorname{diag}\left[1,\ e^{2i(\aleph_2-\aleph_1)},\ e^{i(\aleph_2-\aleph_1)}\right], \\ X_{Rn} &= \operatorname{diag}\left[e^{-i\aleph_2},\ e^{-i\aleph_2},\ e^{i(\aleph_1-2\aleph_2)}\right], \\ X_{Lp} &= \operatorname{diag}\left[1,\ e^{i(\aleph_1-\aleph_2)},\ e^{2i(\aleph_1-\aleph_2)}\right], \\ X_{Rp} &= \operatorname{diag}\left[e^{i\aleph_2},\ e^{i\aleph_2},\ e^{i(2\aleph_2-\aleph_1)}\right]. \end{aligned}$$

In this way,

$$X_{Lp}^{\dagger} X_{Ln} = \text{diag}\left(1,\ e^{3i(\aleph_2 - \aleph_1)},\ e^{3i(\aleph_2 - \aleph_1)}\right)$$

will render the CKM matrix

$$V = O_{Lp} X_{Lp}^{\dagger} X_{Ln} O_{Ln}$$

(O_{Lp} and O_{Ln} are real orthogonal matrices) complex. The phase $3\left(\aleph_2 - \aleph_1\right)$ is the only source of CP violation in this model.

The potential symmetric under $\phi_2 \to \omega^2 \phi_2$ is

$$\begin{aligned} V &= \mu_1 \phi_1^{\dagger} \phi_1 + \mu_2 \phi_2^{\dagger} \phi_2 + \lambda_1 \left(\phi_1^{\dagger} \phi_1\right)^2 + \lambda_2 \left(\phi_2^{\dagger} \phi_2\right)^2 \\ &\quad + \lambda_3 \left(\phi_1^{\dagger} \phi_1\right) \left(\phi_2^{\dagger} \phi_2\right) + \lambda_4 \left(\phi_1^{\dagger} \phi_2\right) \left(\phi_2^{\dagger} \phi_1\right) \\ &\quad - \left|\mu_3\right| \left[e^{i(\aleph_1 - \aleph_2)} \phi_1^{\dagger} \phi_2 + \text{H.c.}\right], \end{aligned}$$

where the last line breaks both the $\mathbb{Z}_3$ and CP symmetries softly. The phase $\aleph_1 - \aleph_2$ in the potential is the source of the phase $\aleph_1 - \aleph_2$ between the VEVs of φ_1^0 and φ_2^0. It is the ultimate source of all CP violation in this model.

So, in this model

- the matrices N_d and N_u are real;
- there is no CP violation in the interactions among the scalars; in particular, the scalars h and H do not mix with the pseudoscalar A;
- CP violation is located only in the CKM matrix, which is complex because of $3\left(\aleph_2 - \aleph_1\right) \neq 0$.
- The $\mathbb{Z}_3$ symmetry imposed is not anomalous (A. Trautner, private communication).

Making $\theta = 0$ at the tree level is not, as in the example above, much too difficult. It is much more tricky to get

$$\mathrm{Im}\,\mathrm{tr}\left(M_p^{-1}\Sigma_p + M_n^{-1}\Sigma_n\right) = 0$$

at the one-loop level. This is so, in particular, because of scalar–pseudoscalar mixing: if there is a neutral scalar S that couples to the fermion f as $S\,\bar{f}\left(a + ib\gamma_5\right) f$, with a and b simultaneously nonzero, *viz.* if S has simultaneously scalar and pseudoscalar couplings to f, then the self-energy of f will be complex.

In our model there is no scalar–pseudoscalar mixing (A does not mix with h and H), because there is no CP violation at all in the scalar sector. Moreover, the matrices N_d and N_u are real. This means that A has imaginary Yukawa couplings to the fermions, while h and H have real Yukawa couplings to the fermions. No loop of either A, h, or H can produce a complex contribution to any quark mass.

Loops of $H^{\pm}$ may do it, though. The matrices $X \equiv N_u^\dagger V$ and $Y \equiv V N_d$ are complex, therefore there is a complex one-loop contribution

$$\Sigma_\alpha = \sum_{j=d,s,b} m_j X_{\alpha j} Y_{j\alpha}^\dagger f\left(m_j^2\right)$$

to the mass of a charge 2/3 quark α, where $f\left(m_j^2\right)$ is a loop function. But, the relevant quantity is $\mathrm{Im}\,\mathrm{tr}\left(M_u^{-1}\Sigma_u\right)$. We compute

$$\begin{aligned}
\sum_{\alpha=u,c,t} \frac{X_{\alpha j} Y_{j\alpha}^\dagger}{m_\alpha} &= \sum_{\alpha=u,c,t} \frac{\left(N_u^\dagger V\right)_{\alpha j} \left(N_d^\dagger V^\dagger\right)_{j\alpha}}{m_\alpha} \\
&= \left(N_d^\dagger V^\dagger M_u^{-1\dagger} N_u^\dagger V\right)_{jj} \\
&= \left(U_R^{n\dagger} N_n^\dagger M_p^{-1\dagger} N_p^\dagger U_L^n\right)_{jj}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\sum_{j=d,s,b} m_j f\left(m_j^2\right) \sum_{\alpha=u,c,t} \frac{X_{\alpha j} Y_{j\alpha}^\dagger}{m_\alpha} \\
= \quad & \mathrm{tr}\left[f\left(M_d M_d^\dagger\right) M_d U_R^{n\dagger} N_n^\dagger M_p^{-1\dagger} N_p^\dagger U_L^n\right] \\
= \quad & \mathrm{tr}\left[f\left(M_n M_n^\dagger\right) M_n N_n^\dagger M_p^{-1\dagger} N_p^\dagger\right].
\end{aligned}$$

We now look at the structure of phases of the matrices $M_n M_n^\dagger$ and $M_n N_n^\dagger M_p^{-1\dagger} N_p^\dagger$ in our model; we find that both of them have matrix elements with phases

$$\begin{pmatrix} 0 & 2\aleph & \aleph \\ -2\aleph & 0 & -\aleph \\ -\aleph & \aleph & 0 \end{pmatrix},$$

where $\aleph \equiv \aleph_2 - \aleph_1$. The diagonal matrix elements of any product of such matrices are real, hence the trace is real too. Thus, $\mathrm{Im}\,\mathrm{tr}\left(M_u^{-1}\Sigma_u\right)$ is real in our model.

Analogously, one may demonstrate that $\mathrm{Im}\,\mathrm{tr}\left(M_d^{-1}\Sigma_d\right)$ is real, hence strong CP violation is not generated at the one-loop level.

At two-loop level strong CP violation will in general arise, and we cannot ascertain that it will be small enough; but, with a two-loop factor $\sim 10^{-5}$ and the Standard-Model CP-violation-suppressing factor $\sim 10^{-3}$, possibly it will.

In conclusion, in a—very special—2HDM strong CP violation is absent both at the tree and one-loop levels, but not at the two-loop level. That 2HDM features a soft origin for the breaking of CP in the scalar potential, and a non-anomalous $\mathbb{Z}_3$ symmetry that ensures that $\arg\det M_n = -\arg\det M_p$. There is no CP violation in scalar–pseudoscalar mixing; CP violation is only in the CKM matrix. No extra quarks are employed in this simple 2HDM.

Our model must of course be fitted to the data, by adjusting the ten Yukawa couplings and the eight parameters of the scalar potential. We have performed this job to demonstrate that, in spite of the model containing flavour-changing neutral Yukawa interactions, in some cases the masses of the neutral scalars need not be too large ($\gtrsim 500\,\mathrm{GeV}$).