Heavy Quarks on the Lattice

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UNIVERSITY OF Southampton

- Nomenclature and justification
- The RHQ Action
- Extracting form factors
- Illustrative plots

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$$S_{\text{Wilson}} = \sum_{n,m} \bar{\psi}_m \left((m+4)\delta_{n,m} - \frac{1}{2a} \sum_{\mu \pm 1}^{\pm 4} (1-\gamma_\mu) U_\mu \delta_{n+\hat{\mu},n} \right) \psi_n$$

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Must introduce anisotropy between spatial

and temporal components

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RHQ Action

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$$D^2_\mu \psi_n = (U_{n+\mu}\psi_{n+\mu} + U^\dagger_{n-\mu}\psi_{n-\mu}) - 2\psi_n$$

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- ma: Bare lattice mass/Cutoff Ratio
- ζ: Spatial/Temporal Anisotropy Ratio
- c_P: ma-dependent c_{sw} coefficient

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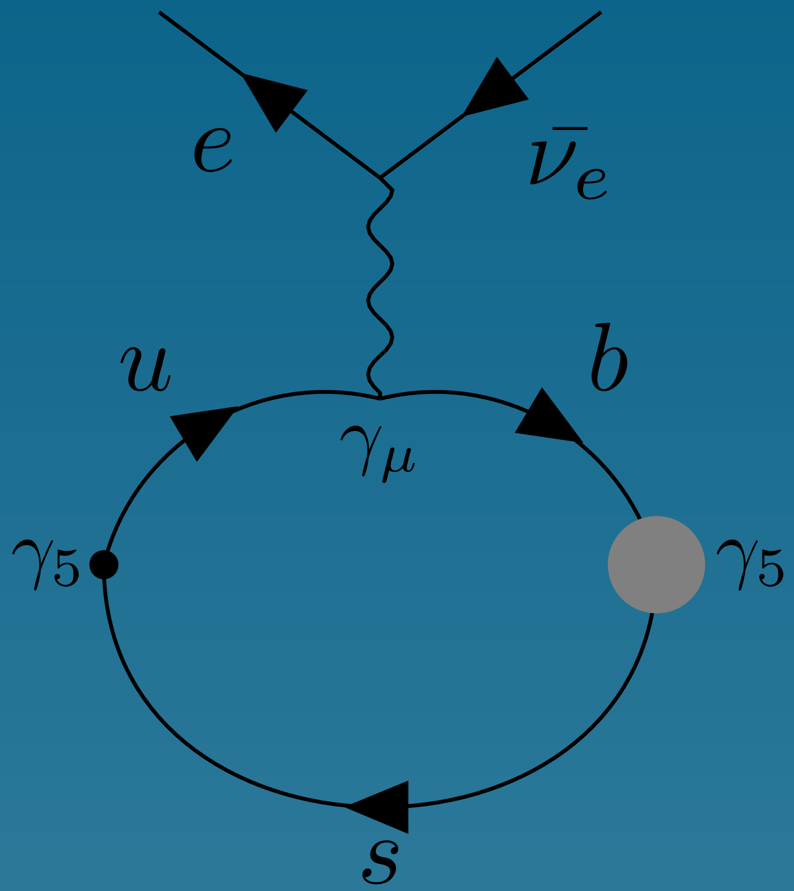
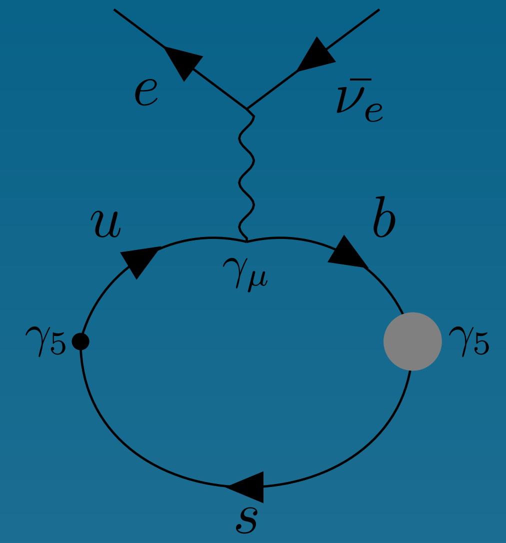


$$\langle P(k)|\mathcal{H}^{b\to q}|B_s(p)\rangle \propto V_{qb}\langle P(k)|\bar{q}\gamma_\mu b|B_s\rangle$$



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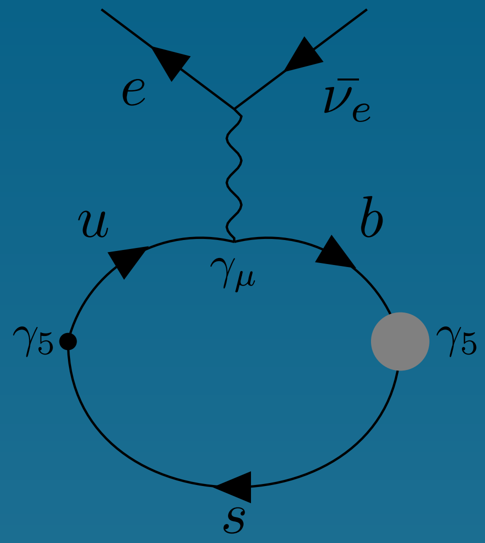
$$\begin{aligned} p^+ &= p + k \\ q &= p - k \end{aligned}$$



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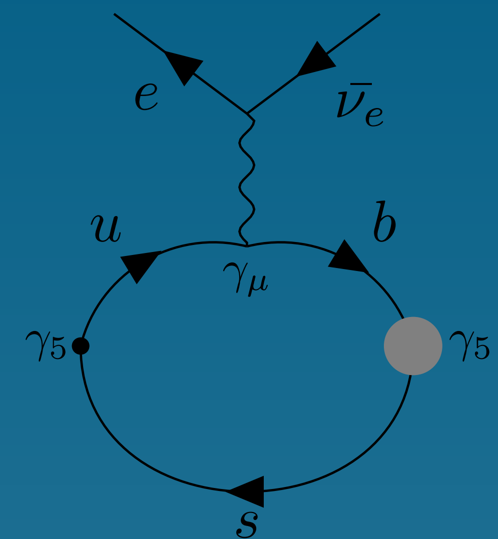

$$\langle P(k)|\bar{q}\gamma_\mu b|B_s(p)\rangle = \sqrt{M_{B_s}}[v^\mu f_{\|}(E_P) + p^\mu_\perp f_\perp(E_P)]$$

$$p^\mu_\perp = p^\mu_P - (p_P \cdot v)v^\mu$$

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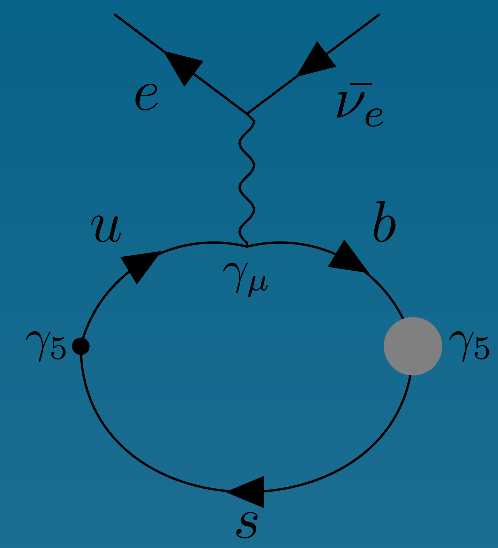

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B_s rest-frame

$$f_\parallel(E_P) = \frac{\langle P(k)|\bar{q}\gamma_0 b|B_s(p)\rangle}{\sqrt{2M_{B_s}}}$$

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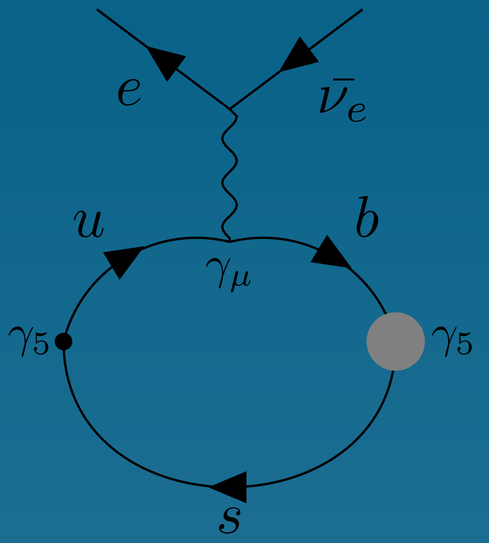

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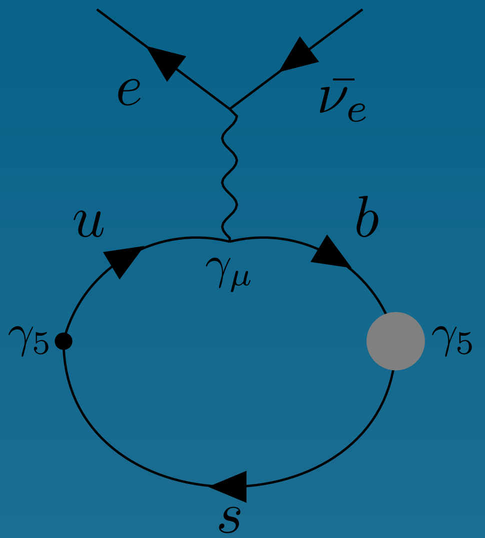

$$C_{3,\mu} = \sum_{\vec{x},\vec{y}} e^{i\vec{p}_P\cdot\vec{y}}\langle \mathcal{O}_{B_s}^\dagger V^\mu \mathcal{O}_P\rangle$$

$$C_P = \sum_{\vec{x}} e^{i\vec{p}_P\cdot\vec{x}}\langle \mathcal{O}_P^\dagger \mathcal{O}_P\rangle$$

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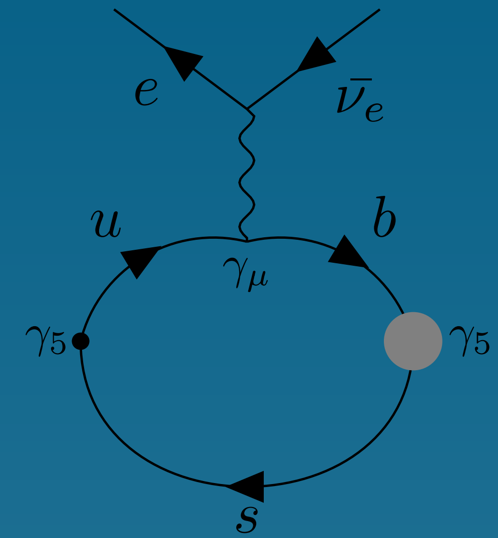

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$$R_{3,\mu} = \frac{C_{3\mu}}{C_P\widetilde{C}_{B_s}}\sqrt{\frac{2E_P}{e^{-E_P t}e^{-M_{B_s}(t-t_{snk})}}}$$

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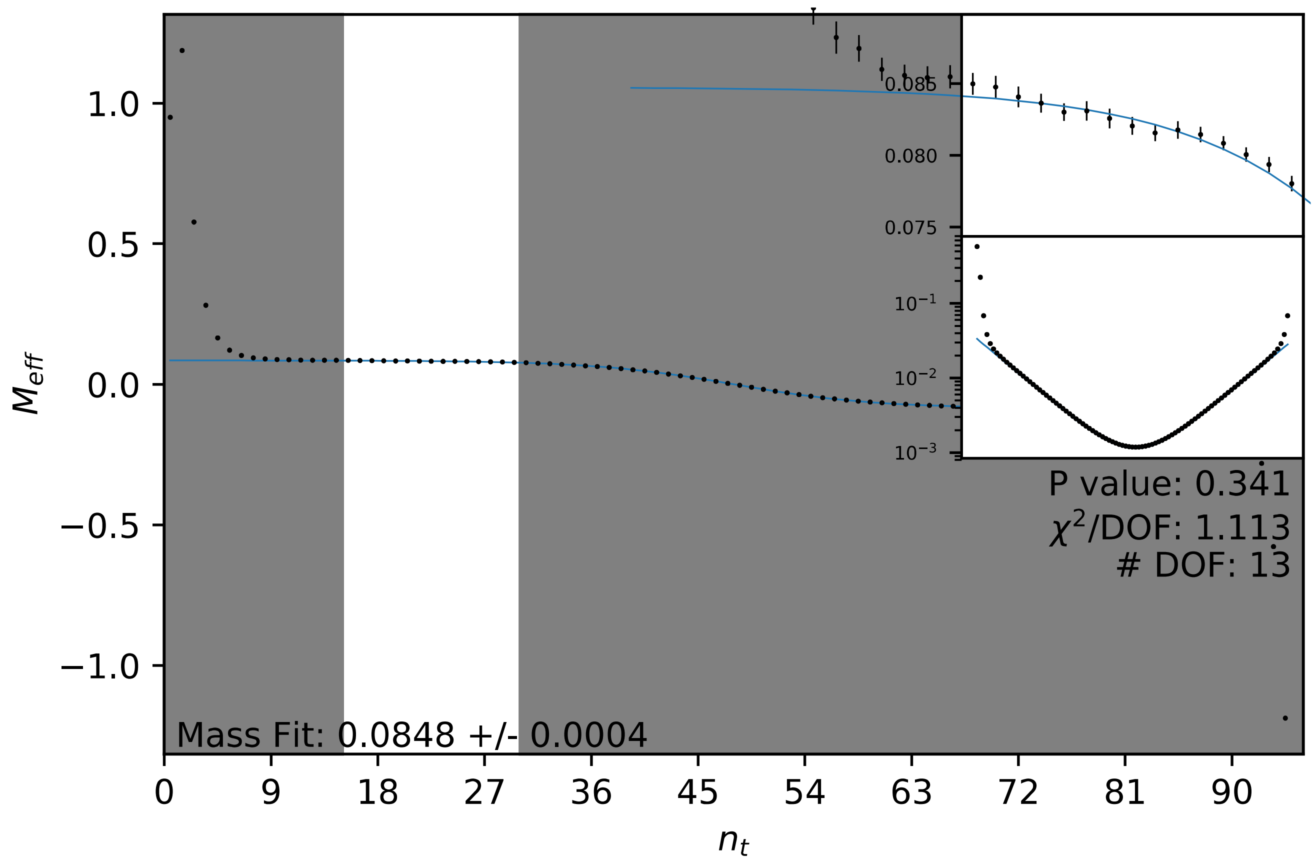

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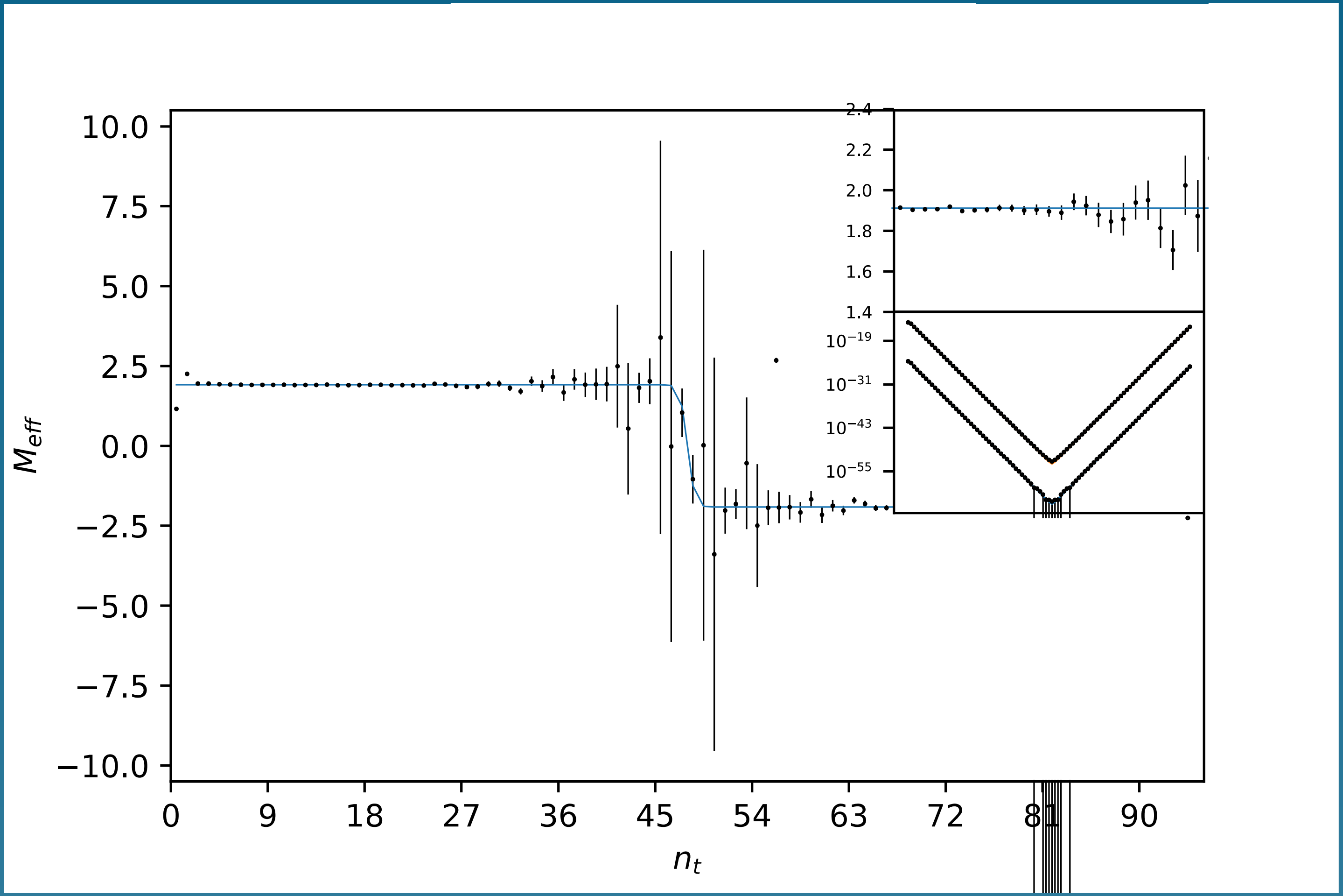

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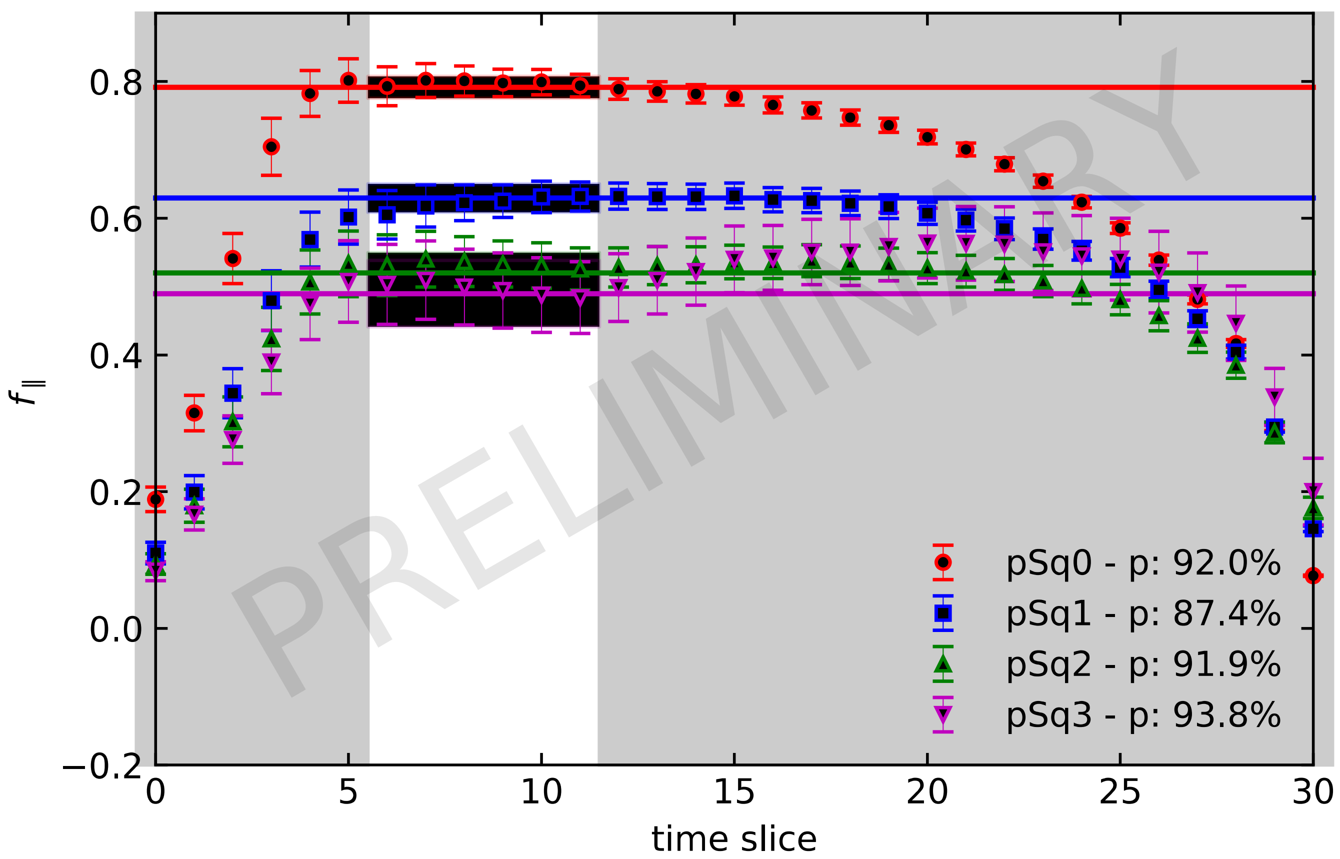

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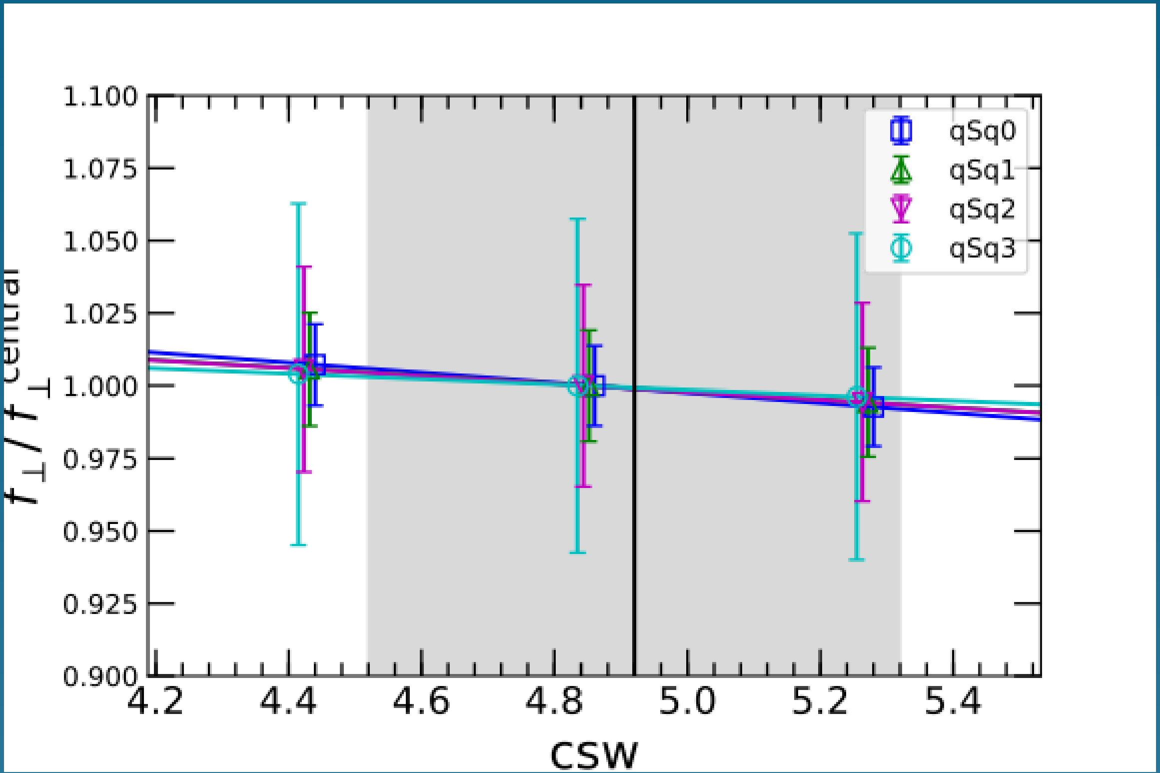

In conclusion:

- We can can parameterise the Clover action to control discretisation errors
- Requires a tuning of 3 parameters depending on *(ma)*
- Gives a general way to compute CKM matrix elements