Position-Space Approach to the Hadronic Light-by-Light Scattering Contribution to the Anomalous Magnetic Moment of the Muon on the Lattice

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The Future of Particle Physics in the Post-Higgs Landscape

4 April 2019

Gyromagnetic Moment (History)

gyromagnetic moment: $\boldsymbol{\mu} = g \frac{e}{2m} \mathbf{S}$

1924

Stern-Gerlach experiment
observed μ

1928

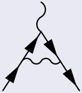
Dirac theory:
$g = 2$

1947

$g \approx 2 \times (1 + 0.00118(3))$
Foley and Kush

1948

$$g = 2 \times (1 + \frac{\alpha}{2\pi}) \approx 2 \times (1 + 0.001161)$$



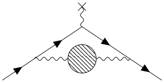
Schwinger

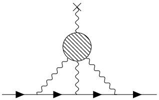
Where are we today?

Current Status of the Muon $g-2$

anomalous magnetic moment $a_\mu = \frac{g_\mu - 2}{2}$

contribution	$a_\mu[10^{-10}]$		reference
QED	11 658 471.895	$\pm$ 0.008	Aoyama *et al* '12
HVP LO	693.1	$\pm$ 3.4	Davier *et al* '17
HVP NLO	-9.84	$\pm$ 0.07	Hagiwara *et al* '11
HVP NNLO	1.24	$\pm$ 0.01	Kurz *et al* '14
HLBL LO	10.5	$\pm$ 2.6	Prades *et al* '09
HLBL NLO	0.3	$\pm$ 0.2	Colangelo *et al* '14
EW	15.36	$\pm$ 0.10	Gnendiger *et al* '13
total	11 659 182.3	$\pm$ 4.3	Davier *et al* '17
experimental	11 659 208.9	$\pm$ 6.3	Bennett *et al* '06

HVP LO: 

HLBL LO: 

Anomalous Magnetic Moment of the Muon

$\approx$ 3 to 4 standard deviations tension between a_μ^{exp} and a_μ^{theo}

$\rightarrow$ new physics?

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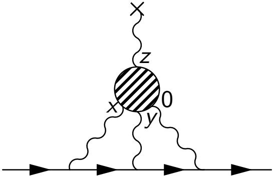
reduce uncertainties

experiment	theory for HLbL	
J-PARC Fermilab	**phenomenology** reduce model uncertainties for dominant contribution (π^0, η, η'; $\pi\pi$) using experimental input Colangelo *et al.* '14,...,'17 Pauk and Vanderhaeghen '14	**lattice QCD** model independent estimates Blum *et al.* ('05,...)'15,...,'17 Mainz lattice group

Euclidean position-space approach to a_μ^{HLbL}



master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int d^4y \Big[\int d^4x \, \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\mathrm{QED}} \; \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{\mathrm{QCD}} \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \; z_\rho \left\langle j_\mu(x) \, j_\nu(y) \, j_\sigma(z) \, j_\lambda(0) \right\rangle.$$

- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects from the photons
- manifest Lorentz covariance

Outline

1. Derivation of the Euclidean position-space approach to a_μ^{HLbL}
2. Tests of the QED Kernel
3. Tests of the Lattice Gauge Theory Code
4. Lattice QCD
5. Conclusion

Perturbation theory in Euclidean position-space

Scalar propagators:

$$G_0(x) = \frac{1}{4\pi^2 x^2}, \qquad G_m(x) = \frac{m}{4\pi^2|x|} K_1(m|x|).$$

Fermion propagator:

$$S(x) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{-ip_\mu\gamma_\mu + m}{p^2+m^2} e^{ipx} = \frac{m^2}{4\pi^2|x|}\Big[\gamma_\mu x_\mu \frac{K_2(m|x|)}{|x|} + K_1(m|x|)\Big],$$

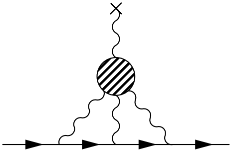
$U_n(z) =$ Chebyshev polynomials of the second kind:

$$U_0(z) = 1, \qquad U_1(z) = 2z, \qquad U_{n+1}(z) = 2zU_n(z) - U_{n-1}(z) \qquad (n \geq 1),$$

Key property: orthogonal basis on S_3; if $\hat{\epsilon}$ is a unit vector

$$\Big\langle U_n(\hat{\epsilon}\cdot\hat{x})\; U_m(\hat{\epsilon}\cdot\hat{y})\Big\rangle_{\hat{\epsilon}} = \frac{\delta_{nm}}{n+1}\, U_n(\hat{x}\cdot\hat{y}).$$

Sketch of the derivation



muon momentum $p = im\hat{\epsilon}$.

$$\hat{F}_2(0) = -\frac{i}{48m}\mathrm{Tr}\left\{[\gamma_\rho,\gamma_\tau]\,(-i\not{p}+m)\Gamma_{\rho\tau}(p,p)(-i\not{p}+m)\right\},$$

$$\begin{aligned}
\Gamma_{\rho\sigma}(p,p) &= -e^6\int_{x_1,x_2} K_{\mu\nu\lambda}(x_1,x_2,p)\;\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x_1,x_2),\\
K_{\mu\nu\lambda}(x_1,x_2,p) &= \gamma_\mu(i\not{p}+\not{\partial}^{(x_1)}-m)\gamma_\nu(i\not{p}+\not{\partial}^{(x_1)}+\not{\partial}^{(x_2)}-m)\gamma_\lambda\;\mathcal{I}(\hat{\epsilon},x_1,x_2),\\
\mathcal{I}(\hat{\epsilon},x,y) &= \int_{q,k}\frac{1}{q^2k^2(q+k)^2}\,\frac{1}{(p-q)^2+m^2}\,\frac{1}{(p-q-k)^2+m^2}\,e^{-i(qx+ky)}.
\end{aligned}$$

$$\hat{F}_2(0) = \frac{me^6}{3}\int_{x,y}\mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(\hat{\epsilon},x,y)\;i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)$$

The scalar function $\mathcal{I}(\hat{\epsilon}, x, y)$

position-space propagators

$$\mathcal{I}(\hat{\epsilon}, x, y) = \int_u G_0(u - y)\; J(\hat{\epsilon}, u)\; J(\hat{\epsilon}, x - u),$$
$$J(\hat{\epsilon}, y) = \int_u G_0(y - u)\, e^{m\hat{\epsilon}\cdot u} G_m(u)$$

Chebyshev expansion

$$J(\hat{\epsilon}, y) = \sum_{n \geq 0} z_n(y^2)\; U_n(\hat{\epsilon} \cdot \hat{y}),$$

averaging over $\hat{\epsilon}$

$$\hat{F}_2(0) = \frac{me^6}{3} \int_{x,y} \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y)\; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x, y)$$
$$\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y) = \langle \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(\hat{\epsilon}, x, y) \rangle_{\hat{\epsilon}}$$

evaluate the terms like $\langle \mathcal{I} \rangle_{\hat{\epsilon}}$ analytically

Explicit form of the QED kernel

$$\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) = \sum_{A=\mathrm{I,II,III}} \mathcal{G}^{A}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda}\, T^{(A)}_{\alpha\beta\delta}(x,y),$$

with e. g. $\mathcal{G}^{\mathrm{I}}_{\delta[\rho\sigma]\mu\alpha\nu\beta\lambda} \equiv \frac{1}{8}\mathrm{Tr}\Big\{\Big(\gamma_\delta[\gamma_\rho,\gamma_\sigma] + 2(\delta_{\delta\sigma}\gamma_\rho - \delta_{\delta\rho}\gamma_\sigma)\Big)\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_\lambda\Big\},$

$$T^{(\mathrm{I})}_{\alpha\beta\delta}(x,y) = \partial^{(x)}_\alpha(\partial^{(x)}_\beta + \partial^{(y)}_\beta)V_\delta(x,y),$$

$$T^{(\mathrm{II})}_{\alpha\beta\delta}(x,y) = m\partial^{(x)}_\alpha\Big(T_{\beta\delta}(x,y) + \frac{1}{4}\delta_{\beta\delta}S(x,y)\Big)$$

$$T^{(\mathrm{III})}_{\alpha\beta\delta}(x,y) = m(\partial^{(x)}_\beta + \partial^{(y)}_\beta)\Big(T_{\alpha\delta}(x,y) + \frac{1}{4}\delta_{\alpha\delta}S(x,y)\Big),$$

$$S(x,y) = \langle\mathcal{I}\rangle_{\hat\epsilon} = \bar{\mathfrak{g}}^{(0)}(|x|, \hat{x}\cdot\hat{y}, |y|),$$

$$V_\delta(x,y) = \langle\hat\epsilon_\delta\mathcal{I}\rangle_{\hat\epsilon} = x_\delta\bar{\mathfrak{g}}^{(1)}(|x|, \hat{x}\cdot\hat{y}, |y|) + y_\delta\bar{\mathfrak{g}}^{(2)}(|x|, \hat{x}\cdot\hat{y}, |y|),$$

$$T_{\alpha\beta}(x,y) = \langle(\hat\epsilon_\delta\hat\epsilon_\beta - \frac{1}{4}\delta_{\delta\beta})\mathcal{I}\rangle_{\hat\epsilon}$$

$$= (x_\alpha x_\beta - \frac{x^2}{4}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(1)} + (y_\alpha y_\beta - \frac{y^2}{4}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(2)} + (x_\alpha y_\beta + y_\alpha x_\beta - \frac{x\cdot y}{2}\delta_{\alpha\beta})\,\bar{\mathfrak{l}}^{(3)}.$$

The QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ is parametrized by six weight functions.

Example: Weight Function $g^{(2)}$

$$g^{(2)}(x^2, x\cdot y, y^2) = \frac{1}{8\pi y^2|x|\sin^3\beta}\int_0^\infty du\, u^2 \int_0^\pi d\phi_1$$

$$\left\{2\sin\beta + \left(\frac{y^2+u^2}{2|u||y|} - \cos\beta\cos\phi_1\right)\frac{\log\chi}{\sin\phi_1}\right\}\sum_{n=0}^{\infty}$$

$$\Big\{z_n(|u|)z_{n+1}(|x-u|)\left[|x-u|\cos\phi_1\frac{U_n}{n+1} + (|u|\cos\phi_1 - |x|)\frac{U_{n+1}}{n+2}\right]$$

$$+ z_{n+1}(|u|)z_n(|x-u|)\left[(|u|\cos\phi_1 - |x|)\frac{U_n}{n+1} + |x-u|\cos\phi_1\frac{U_{n+1}}{n+2}\right]\Big\}$$

where

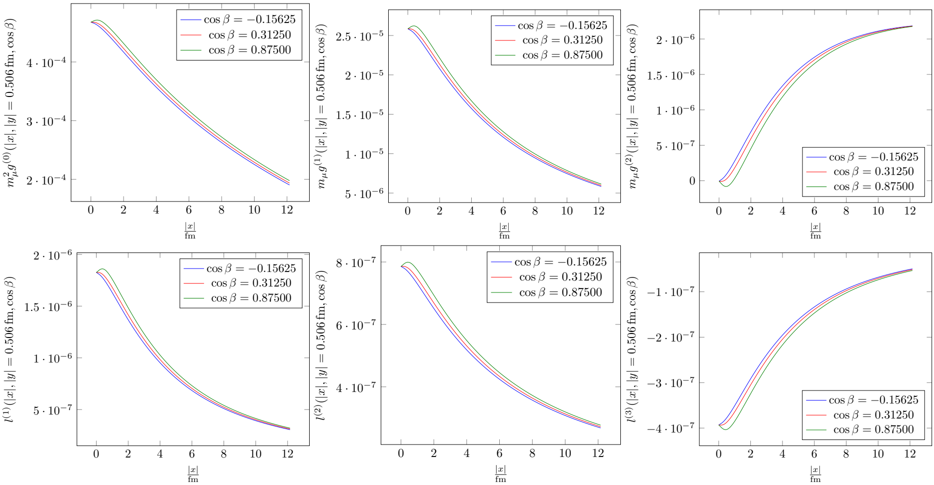
$$x\cdot y = |x||y|\cos\beta\,, \quad |x-u| = \sqrt{|x|^2 + |u|^2 - 2|x||u|\cos\phi_1}$$

$$\chi = \frac{y^2+u^2-2|u||y|\cos(\beta-\phi)}{y^2+u^2-2|u||y|\cos(\beta+\phi)}\,, \quad U_n = U_n\Big(\frac{|x|\cos\phi_1 - |u|}{|u-x|}\Big)$$

$z_n =$ linear combination of products of two modified Bessel functions.

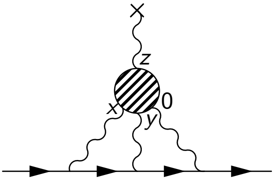
From [I]. Reminder: $V_\delta(x,y) = x_\delta \bar{g}^{(1)}(|x|, \hat{x}\cdot\hat{y}, |y|) + y_\delta \bar{g}^{(2)}(|x|, \hat{x}\cdot\hat{y}, |y|)$.

Complete set of weight functions: $|x|$ dependence



$\bar{g}^{(0)}(|x|, \hat{x} \cdot \hat{y}, |y|)$ contains an arbitrary additive constant (due to the IR divergence in $I(\hat{\epsilon}, x, y)$), which does not contribute to $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x, y)$.

Euclidean position-space approach to a_μ^{HLbL}



master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int d^4y \Big[\int d^4x \, \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\mathrm{QED}} \; \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{\mathrm{QCD}} \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \, z_\rho \left\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \right\rangle.$$

- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects from the photons
- manifest Lorentz covariance

Tests in Continuum, Infinite Volume

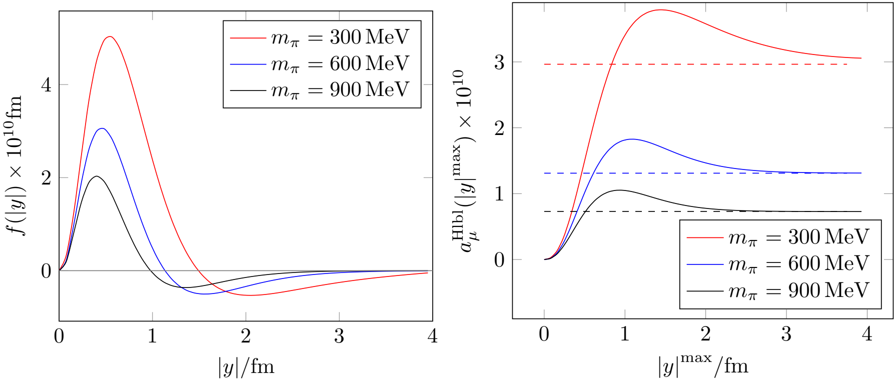
master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} 8\pi^3 \int_0^\infty d|y||y|^3 \Big[\int_0^\infty d|x||x|^3 \int_0^\pi d\beta \sin^2\beta \, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \; z_\rho \left\langle j_\mu(x) \, j_\nu(y) \, j_\sigma(z) \, j_\lambda(0) \right\rangle.$$

- $\int d|x|$, $\int d|y|$ and $\int d\beta$ evaluated numerically
- $\int d^4z$ evaluated (semi-)analytically

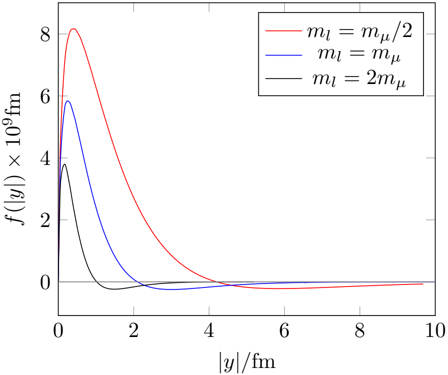
Contribution of the π^0 to a_μ^{HLbL} (Model!)



dashed line = result from momentum-space integration

- we reproduce the known result
- contribution is perhaps surprisingly long-range
- integrand peaked at short distances

Lepton loop integrand contribution to a_μ^{HLbL}



- we reproduce the known result
- contribution is long-range
- integrand sharply peaked at short distances

The QED kernel is correct

- we reproduce the π^0-pole in VMD model
- we reproduce the lepton loop

What next?

achievements

- method for a_μ^{HLbL} on the lattice
- verified the QED kernel
- learned about the integrand

challenges in the view of lattice computations

- contributions are quite long range
- integrand peaked at small distances

What next?

achievements

- method for a_μ^{HLbL} on the lattice
- verified the QED kernel
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challenges in the view of lattice computations

- contributions are quite long range
- integrand peaked at small distances

a way to improve

- do subtractions on the kernel (first proposed by Blum *et al.* '17)
- exploit $\int_x i\hat{\Pi}(x,y) = \int_y i\hat{\Pi}(x,y) = 0$
- example:
  - $\mathcal{L}^{(0)} = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$
  - $\mathcal{L}^{(1)} = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) - \frac{1}{2}\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,x) - \frac{1}{2}\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(y,y)$
  - $a_\mu^{\text{HLbL}} = \frac{me^6}{3}\int_{x,y}\mathcal{L}^{(0)}(x,y)i\hat{\Pi}(x,y) = \frac{me^6}{3}\int_{x,y}\mathcal{L}^{(1)}(x,y)i\hat{\Pi}(x,y)$

Continuum, Infinite Volume

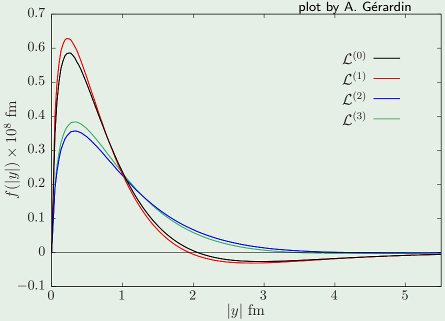
master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} 8\pi^3 \int_0^\infty d|y||y|^3 \Big[\int_0^\infty d|x||x|^3 \int_0^\pi d\beta \sin^2\beta \, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \Big].$$

subtractions on the kernel

- we try (short notation):
  $\mathcal{L}^{(0)} = \bar{\mathcal{L}}(x,y)$ (standard kernel)
  $\mathcal{L}^{(1)} = \bar{\mathcal{L}}(x,y) - \frac{1}{2}\bar{\mathcal{L}}(x,x) - \frac{1}{2}\bar{\mathcal{L}}(y,y)$
  $\mathcal{L}^{(2)} = \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,0)$
  $\mathcal{L}^{(3)} = \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,x) + \bar{\mathcal{L}}(0,x)$
- $\mathcal{L}^{(0)}(0,0) = 0$
  $\mathcal{L}^{(1)}(x,x) = 0$
  $\mathcal{L}^{(2)}(0,y) = \mathcal{L}^{(2)}(x,0) = 0$
  $\mathcal{L}^{(3)}(x,x) = \mathcal{L}^{(3)}(0,y) = 0$

y integrand lepton loop $m_l = m_\mu$



- with all kernels $\mathcal{L}^{(0,1,2,3)}$ we can reproduce the known result

Lattice QED Computation

master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} 2\pi^2 \sum_{|y|} a_{|y|} |y|^3 \left[a^4 \sum_{x\in\Lambda} \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \right].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -a^4 \sum_{z\in\Lambda} z_\rho \left\langle j_\mu(x)\, j_\nu(y)\, j_\sigma(z)\, j_\lambda(0) \right\rangle.$$

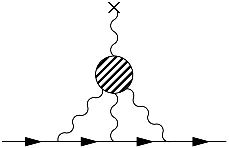
- $i\hat{\Pi}$ in Lattice QED

goal

- reproduce known lepton loop result
- validate Lattice QCD code

focus on standard kernel $\mathcal{L}^{(0)}$ and subtracted kernel $\mathcal{L}^{(2)}$

Lattice QED Computation with Wilson Fermions



lattice gauge theory

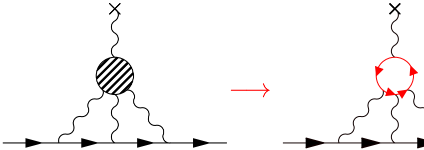
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] e^{-S_G[U]} \mathcal{D}[\psi, \bar{\psi}] e^{-S_F[\psi,\bar{\psi},U]} O[\psi, \bar{\psi}, U]$$

$$S_G[U] = \beta \sum_{n \in \Lambda} \sum_{\mu < \nu} \mathrm{Re}\,\mathrm{Tr}[1 - U_{\mu\nu}(n)]$$

$$S_F[\psi, \bar{\psi}, U] = \text{Wilson fermions}$$

- local vector currents: $j^l_\lambda(x) = \bar{q}_x \gamma_\lambda q_x$
- conserved vector currents: $j^c_\lambda(x) = \frac{1}{2}\left(\bar{q}_{x+\hat{\lambda}}(\gamma_\lambda + 1) U^\dagger_{\lambda,x} q_x + \bar{q}_x(\gamma_\lambda - 1) U_{\lambda,x} q_{x+\hat{\lambda}}\right)$

Lattice QED Computation with Wilson Fermions



lattice gauge theory

$$\langle O \rangle = \frac{1}{Z} \int \cancel{\mathcal{D}[U] e^{-S_G[U]}} \mathcal{D}[\psi, \bar{\psi}] e^{-S_F[\psi, \bar{\psi}, U]} O[\psi, \bar{\psi}, U]$$

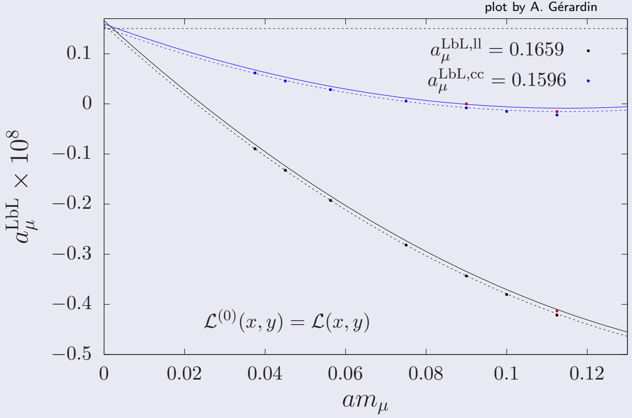
$$S_G[U] = \beta \sum_{n \in \Lambda} \sum_{\mu < \nu} \mathrm{Re}\, \mathrm{Tr}[1 - \cancelto{1}{U_{\mu\nu}(n)}] = 0$$

$$S_F[\psi, \bar{\psi}, U] = \text{Wilson fermions}$$

- QED leading order
- local vector currents: $j^l_\lambda(x) = \bar{q}_x \gamma_\lambda q_x$
- conserved vector currents: $j^c_\lambda(x) = \frac{1}{2}\left(\bar{q}_{x+\hat{\lambda}}(\gamma_\lambda + 1) U^\dagger_{\lambda,x} q_x + \bar{q}_x(\gamma_\lambda - 1) U_{\lambda,x} q_{x+\hat{\lambda}}\right)$

Lattice QED Computation with Wilson Fermions

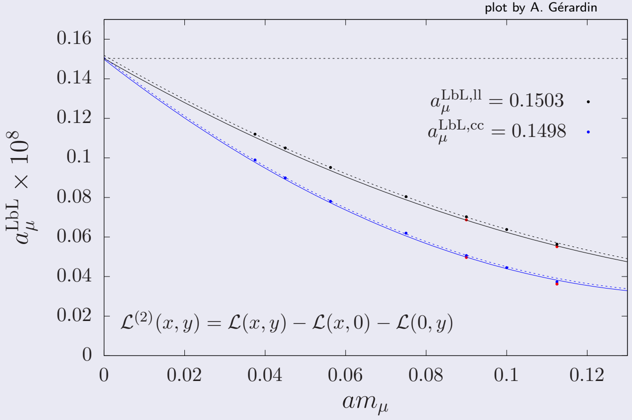
continuum extrapolation lepton loop ($m_l = 2m_\mu$) $\mathcal{L}^{(0)}$



- dashed line: continuum extrapolation for $m_\mu = 7.2$ using a quadratic fit
- solid line: volume extrapolation: curve shifted by the difference between the results for lattice extents $m_\mu L = 7.2$ and 14.4 at fixed a

Lattice QED Computation with Wilson Fermions

continuum extrapolation lepton loop ($m_l = 2m_\mu$) $\mathcal{L}^{(2)}$



- less discretisation effects
- it is advantageous to use the subtracted kernel $\mathcal{L}^{(2)}$

Lattice Setup

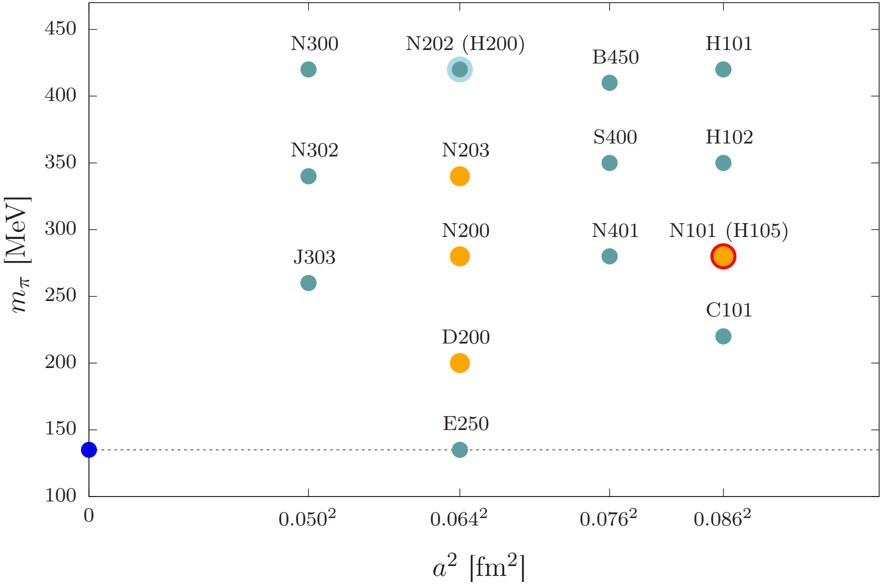
CLS $N_f = 2+1$ ensembles

CLS	$L^3 \times T$	a [fm]	m_π [MeV]	$m_\pi L$	L [fm]	#confs
H105	$32^3 \times 96$	0.086	285	3.9	2.7	1000
N101	$48^3 \times 128$		285	5.9	4.1	400
N203	$48^3 \times 128$	0.064	340	5.4	3.1	750
N200	$48^3 \times 128$		285	4.4	3.1	800
D200	$64^3 \times 128$		200	4.2	4.2	1100

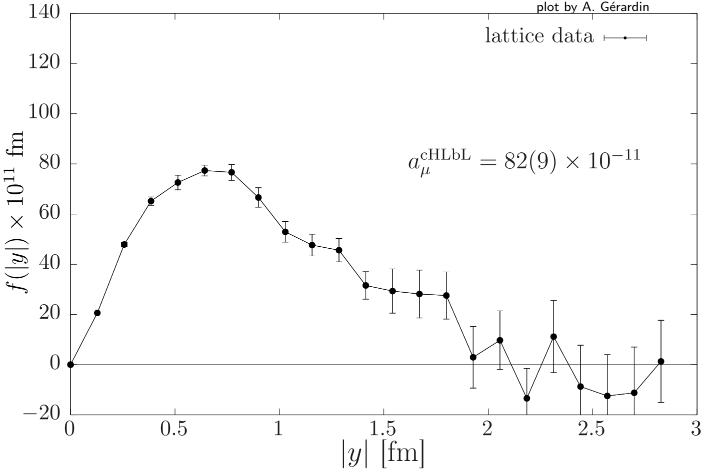
- $\mathcal{O}(a)$ improved Wilson fermions

$N_f = 2 + 1$ CLS Ensembles

graphic by A. Gérardin

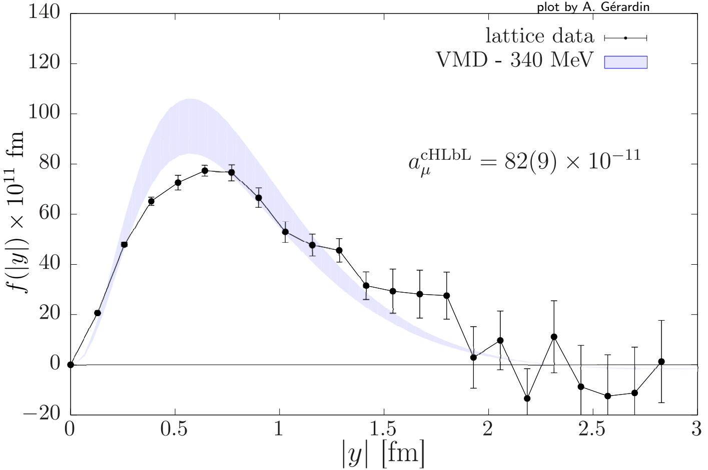


Integrand of a_μ^{cHLbL} with $\mathcal{L}^{(2)}$, $m_\pi = 340\,\text{MeV}$, $a = 0.064\,\text{fm}$



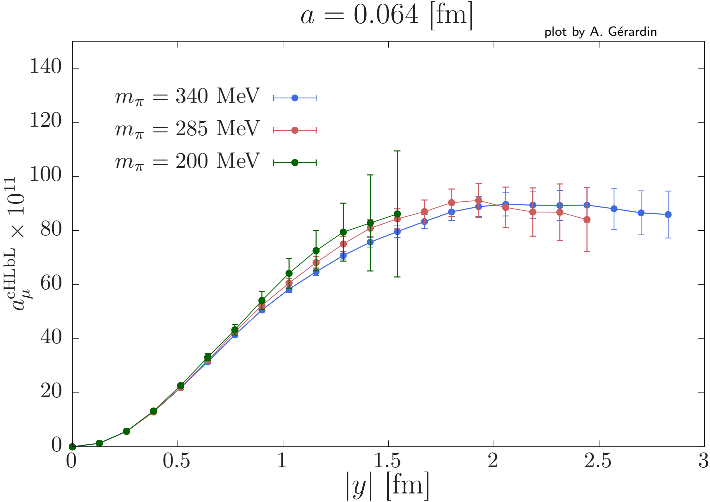
- fully connected contribution only
- we already observe a good signal
- integrand non-zero up to 2 fm

Integrand of a_μ^{cHLbL} with $\mathcal{L}^{(2)}$, $m_\pi = 340\,\text{MeV}$, $a = 0.064\,\text{fm}$



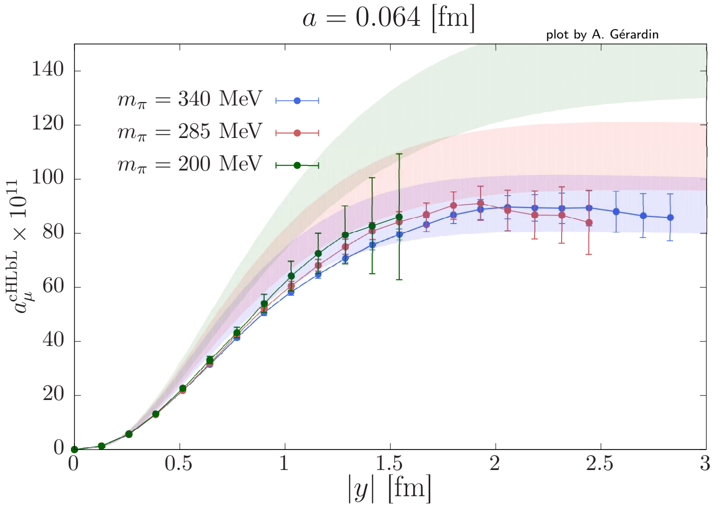
- for long distances the simple VMD Model seems to provide a good approximation to the full QCD computation
- the size of the box $L = 3.1\,\text{fm}$ is large enough to capture the HLbL contribution for this pion mass

Pion Mass Dependence of a_μ^{cHLbL}



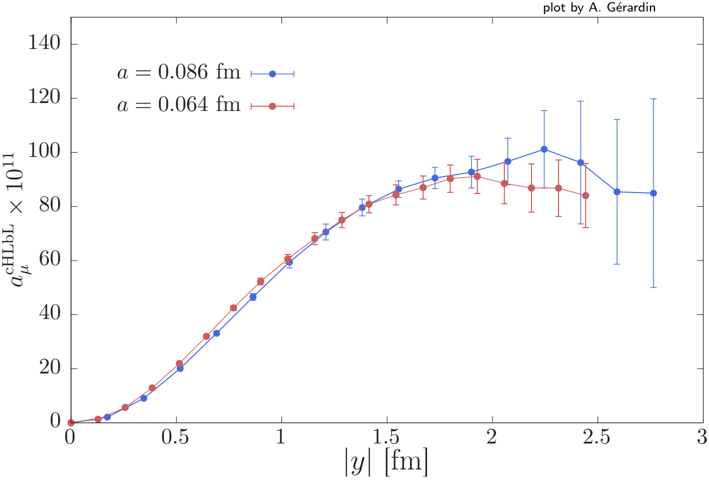
- the results show an upward trend for decreasing pion mass
- currently collecting more statistics in long distance regime

Pion Mass Dependence of a_μ^{cHLbL}



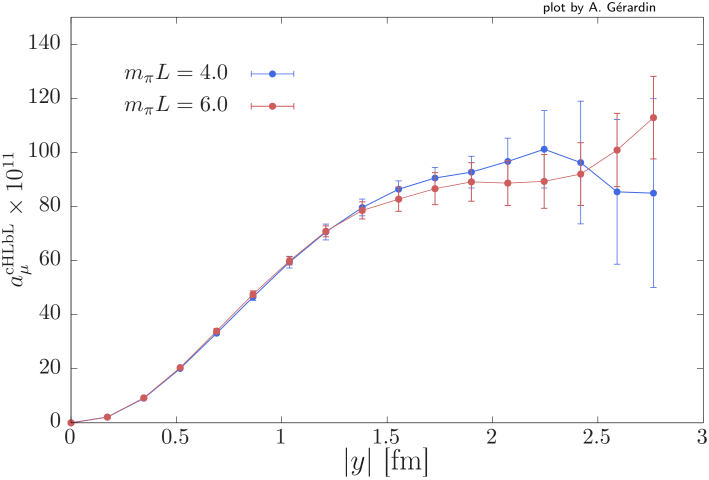
- the results show an upward trend for decreasing pion mass
- currently collecting more statistics in long distance regime

Discretisation Effects, $m_\pi = 285\,\text{MeV}$



- discretisation effects seem to be small (we are increasing statistics)

Finite Size Effects, $a = 0.086\,\text{fm}$



- finite size effects seem to be small (we are increasing statistics)

Conclusions

- Explicit formula for a_μ^{HLbL}
  - QED kernel function multiplying the position-space QCD correlation function
- Tests
  - QED kernel: reproduce known results for π^0 pole and lepton loop in the continuum for the standard kernel $\mathcal{L}^{(0)}$ and subtracted kernels $\mathcal{L}^{(1,2,3)}$
  - Lattice implementation: Reproduce lepton loop result in Lattice QED
- Lattice QCD
  - First Mainz results for the fully connected contribution (in QED_∞)
  - Subtractions are needed to obtain a signal at long distances
  - The discretisation and finite-size effects seem to be small
- Future
  - Collect more statistics
  - Perform chiral and continuum extrapolations
  - Implement disconnected contribution

Publications

H. B. Meyer, NA, A. Gérardin, A. Nyffeler. *Hadronic light-by-light scattering in the anomalous magnetic moment of the muon*. TAU2018, arXiv:1811.08320, 2018

A. Nyffeler, NA, A. Gérardin, J. Green, O. Gryniuk, G. von Hippel, H. B. Meyer, V. Pascalutsa, and H. Wittig. *Hadronic light-by-light scattering contribution to the muon g-2 on the lattice*. EPJ Web Conf., 179:01017, 2018

NA, A. Gérardin, H. B. Meyer, and A. Nyffeler. *Exploratory studies for the position-space approach to hadronic light-by-light scattering in the muon g-2*. EPJ Web Conf., 175:06023, 2018

NA, J. Green, H. B. Meyer, and A. Nyffeler. *Position-space approach to hadronic light-by-light scattering in the muon g-2 on the lattice*. PoS, LATTICE2016:164, 2016

J. Green, NA, O. Gryniuk, G. von Hippel, H. B. Meyer, A. Nyffeler, and V. Pascalutsa. *Direct calculation of hadronic light-by-light scattering*. PoS, LATTICE2015:109, 2016

The π^0 pole contribution

Assume a vector-meson-dominance transition form factor (parameters: m_V, m_π and overall normalization)

$$\mathcal{F}(-q_1^2, -q_2^2) = \frac{c}{(q_1^2 + m_V^2)(q_2^2 + m_V^2)}, \qquad c = -\frac{N_c m_V^4}{12\pi^2 F_\pi}.$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = \frac{c^2}{m_V^2(m_V^2 - m_\pi^2)} \frac{\partial}{\partial x_\alpha} \frac{\partial}{\partial y_\beta} \Big\{ \epsilon_{\mu\nu\alpha\beta}\epsilon_{\sigma\lambda\rho\gamma} \Big(\frac{\partial}{\partial x_\gamma} + \frac{\partial}{\partial y_\gamma} \Big) K_\pi(x,y)$$
$$+ \epsilon_{\mu\lambda\alpha\beta}\epsilon_{\nu\sigma\gamma\rho} \frac{\partial}{\partial y_\gamma} K_\pi(y-x, y) + \epsilon_{\mu\sigma\alpha\rho}\epsilon_{\nu\lambda\beta\gamma} \frac{\partial}{\partial x_\gamma} K_\pi(x, x-y) \Big\}.$$

where

$$K_\pi(x,y) \equiv \int d^4u \Big(G_{m_\pi}(u) - G_{m_V}(u) \Big) G_{m_V}(x-u) G_{m_V}(y-u) = K_\pi(y,x).$$

The lepton loop: fully analytic result for $i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)$

$$
\begin{aligned}
i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \;=\;& \widehat{\Pi}^{(1)}_{\rho;\mu\nu\lambda\sigma}(x,y) \\
&+\widehat{\Pi}^{(1)}_{\rho;\nu\lambda\mu\sigma}(y-x,-x) + x_\rho\,\Pi^{(r,1)}_{\nu\lambda\mu\sigma}(y-x,-x) \\
&+\widehat{\Pi}^{(1)}_{\rho;\lambda\nu\mu\sigma}(-x,y-x) + x_\rho\,\Pi^{(r,1)}_{\lambda\nu\mu\sigma}(-x,y-x).
\end{aligned}
$$

$$
\begin{aligned}
&\Pi^{(r,1)}_{\mu\nu\lambda\sigma}(x,y) \\
&= 2\Big(\frac{m}{2\pi}\Big)^8\Big[\frac{(-x_\alpha)(x-y)_\beta\;K_2(m|x|)K_2(m|x-y|)}{|x|^2|x-y|^2}\cdot I_{\gamma\delta}(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+\frac{K_1(m|x|)K_1(m|x-y|)}{|x||x-y|}\cdot p(|y|)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\lambda\} \\
&+\frac{(-x_\alpha)(x-y)_\beta\;K_2(m|x|)K_2(m|x-y|)}{|x|^2|x-y|^2}\cdot p(|y|)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\sigma\gamma_\lambda\} \\
&+\frac{(-x_\alpha)\;K_2(m|x|)K_1(m|x-y|)}{|x|^2|x-y|}\cdot q_\gamma(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\lambda\} \\
&+\frac{(x-y)_\beta\;K_1(m|x|)K_2(m|x-y|)}{|x||x-y|^2}\cdot q_\gamma(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\lambda\} \\
&+\frac{(-x_\alpha)\;K_2(m|x|)K_1(m|x-y|)}{|x|^2|x-y|}\cdot q_\delta(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+\frac{(x-y)_\beta\;K_1(m|x|)K_2(m|x-y|)}{|x||x-y|^2}\cdot q_\delta(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+\frac{K_1(m|x|)K_1(m|x-y|)}{|x||x-y|}\cdot I_{\gamma\delta}(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\delta\gamma_\lambda\}\Big]
\end{aligned}
$$

The lepton loop (continued)

$$\begin{aligned}
\widehat{\Pi}^{(1)}_{\rho;\mu\nu\lambda\sigma}(x,y) \quad &= 2\Big(\frac{m}{2\pi}\Big)^8 \Big[\frac{(-x_\alpha)(x-y)_\beta K_2(m|x|)K_2(m|x-y|)}{|x|^2|x-y|^2} \cdot f_{\rho\delta\gamma}(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+ \frac{K_1(m|x|)K_1(m|x-y|)}{|x||x-y|} \cdot f_{\rho\delta\gamma}(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+ \frac{K_1(m|x|)K_1(m|x-y|)}{|x||x-y|} g_\rho(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\lambda\} \\
&+ \frac{(-x_\alpha)(x-y)_\beta\ K_2(m|x|)K_2(m|x-y|)}{|x|^2|x-y|^2} g_\rho(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\sigma\gamma_\lambda\} \\
&+ \frac{(-x_\alpha)\ K_2(m|x|)K_1(m|x-y|)}{|x|^2|x-y|} h_{\rho\gamma}(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\lambda\} \\
&+ \frac{(x-y)_\beta\ K_1(m|x|)K_2(m|x-y|)}{|x||x-y|^2} h_{\rho\gamma}(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\gamma\gamma_\sigma\gamma_\lambda\} \\
&+ \frac{(-x_\alpha)\ K_2(m|x|)K_1(m|x-y|)}{|x|^2|x-y|} \hat{f}_{\rho\delta}(y)\cdot \mathrm{Tr}\{\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\lambda\} \\
&+ \frac{(x-y)_\beta\ K_1(m|x|)K_2(m|x-y|)}{|x||x-y|^2} \hat{f}_{\rho\delta}(y)\cdot \mathrm{Tr}\{\gamma_\mu\gamma_\beta\gamma_\nu\gamma_\sigma\gamma_\delta\gamma_\lambda\}\Big]
\end{aligned}$$

$$I_{\gamma\delta}(y) = \frac{2\pi^2}{m^2}\Big(\hat{y}_\gamma\hat{y}_\delta\ K_2(m|y|) - \delta_{\gamma\delta}\ \frac{K_1(m|y|)}{m|y|}\Big), \quad h_{\rho\gamma}(y) = \frac{\pi^2}{m^3}\Big(\hat{y}_\gamma\hat{y}_\rho\ m|y|\ K_1(m|y|) - \delta_{\gamma\rho}K_0(m|y|)\Big),$$

$$\hat{f}_{\rho\delta}(y) = \frac{\pi^2}{m^3}\Big\{\hat{y}_\rho\hat{y}_\delta\ \ m|y|K_1(m|y|) + \delta_{\rho\delta}K_0(m|y|)\Big\} \qquad q_\gamma(y) = \frac{2\pi^2}{m^2}\ \hat{y}_\gamma\ K_1(m|y|),$$

$$f_{\rho\delta\gamma}(y) = \frac{\pi^2}{m^3}\Big\{\hat{y}_\gamma\hat{y}_\delta\hat{y}_\rho\ m|y|K_2(m|y|) + (\delta_{\rho\delta}\hat{y}_\gamma - \delta_{\gamma\rho}\hat{y}_\delta - \delta_{\gamma\delta}\hat{y}_\rho)\,K_1(m|y|)\Big\}, \quad p(|y|) = \frac{2\pi^2}{m^2}\ K_0(m|y|).$$