Selective sensitivity of proton scattering to densities on the nuclear surface

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Abstract

Microscopic descriptions of nucleon scattering from nuclei depend on the matter distribution of their neutron and proton constituents. Additionally, the different behaviour of density-dependent effective interactions in the *pp* and *pn* channels offer a selective mechanism by which proton probes couple to proton and neutron distributions in the nucleus. Recent formal studies of the optical model potential have shown that intrinsic medium effects, i.e. nuclear mean fields and Pauli blocking, appear in the optical potential in the form of a the gradient of the density-dependent effective interaction. These properties set limits to the sensitivity of proton scattering to the matter distribution of proton-rich nuclei.

Microscopic optical model potentials (OMP) for nucleon-nucleus (*NA*) scattering usually take the form of a convolution of a two-body effective interaction with the target ground-state mixed density. Within the Brueckner-Bethe-Goldstone (BBG) g-matrix approach for the effective interaction, nuclear medium effects are made explicit by means volume integrals throughout the bulk of the nucleus. Even though these models account for a broad body of scattering data [1, 2], there remain puzzling limitations –specially at nucleon energies below 100 MeV– which require further investigation. This is particularly relevant considering the construction of the EURISOL and similar facilities, where radioactive isotope beams could be set to collide hydrogen targets. Thus, when the energy of these unstable beams reach 70A MeV, inverse kinematics tells us that the physics behind the collision is the same as that of *NA* scattering at 70 MeV, typical nucleon energies used in the seventies. Therefore, current trends involving radioactive beams provide stimulating grounds to revisit the challenges behind the interaction of nucleons with nuclei, to assess their structure as well as learn about the *in-medium* effective interaction itself.

The interaction between a nucleon with energy E and a composite nucleus can be described by means of an OMP, a one-body operator which in momentum space takes the general form

$$U(\boldsymbol{k}',\boldsymbol{k};E) = \int d\boldsymbol{p}'\, d\boldsymbol{p}\, \langle \boldsymbol{k}'\boldsymbol{p}' \mid \hat{T}(E) \mid \boldsymbol{k}\,\boldsymbol{p}\,\rangle_{\mathcal{A}}\, \hat{\rho}(\boldsymbol{p}',\boldsymbol{p}) \; . \tag{1}$$

Here $\hat{T}$ is a two-body effective interaction which, in general, contains information about the discrete spectrum of the many-body system. The one-body mixed density in momentum space, $\hat{\rho}(\boldsymbol{p}',\boldsymbol{p})$, describes the ground-state of the target. A complete evaluation of the optical potential considering all these elements is far from feasible even with nowadays computing capabilities. Part of the difficulties can be avoided by treating separately the ground-state of the target and the *NN* effective interaction. A remaining difficulty is the account for the Fermi motion of the target nucleons implied by the $d\boldsymbol{p}\,d\boldsymbol{p}'$ integration.

As demonstrated in Ref. [3], intrinsic medium contributions can be disentangled from their free-space counterpart in $\hat{T}$ following a general analysis of its momentum-space structure. The matrix elements of $\hat{T}$ in coordinate space are denoted with $\langle \boldsymbol{r}'\boldsymbol{s}' \mid \hat{T} \mid \boldsymbol{r}\ \boldsymbol{s}\rangle$, where the 'prior' coordinates of each particle are denoted by $\boldsymbol{r}$ and $\boldsymbol{s}$, respectively, while $\boldsymbol{r}'$ and $\boldsymbol{s}'$ refer to the 'post' coordinates of the same particles. These vectors define the *mean coordinate* $\boldsymbol{z}$, given by the simple average of the prior and post coordinates:

$$\boldsymbol{z} = \tfrac{1}{4}(\boldsymbol{r} + \boldsymbol{s} + \boldsymbol{r}' + \boldsymbol{s}')$$

In momentum space, the $\hat{T}$-matrix elements are given by $\langle \boldsymbol{k}'\boldsymbol{p}' \mid \hat{T} \mid \boldsymbol{k}\ \boldsymbol{p}\rangle$, where $\boldsymbol{k}$ ($\boldsymbol{k}'$) and $\boldsymbol{p}$ ($\boldsymbol{p}'$) represent the projectile and struck-nucleon momenta prior (post) interaction, respectively. The two representations of the $\hat{T}$-matrix are related by means of Fourier transforms, which according to Ref. [3] can be expressed as

$$\langle \boldsymbol{k}'\boldsymbol{p}' \mid \hat{T} \mid \boldsymbol{k}\ \boldsymbol{p}\rangle = \int \frac{d\boldsymbol{z}}{(2\pi)^3}\, e^{i\boldsymbol{z}\cdot\boldsymbol{K}_\perp} g_{\boldsymbol{z}}(\boldsymbol{K}_\parallel; (\boldsymbol{k}' - \boldsymbol{p}')/2, (\boldsymbol{k} - \boldsymbol{p})/2)\ . \quad (2)$$

Clearly the *reduced interaction*, $g_{\boldsymbol{z}}$, is evaluated at the mean coordinate $\boldsymbol{z}$ and pair momentum $\boldsymbol{K}_\parallel = (\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{k}'+\boldsymbol{p}')/2$. Here $\boldsymbol{K}_\perp = \boldsymbol{k}+\boldsymbol{p}-\boldsymbol{k}'-\boldsymbol{p}'$. Assuming that g_z depends only on the magnitude of the mean coordinate, $|\boldsymbol{z}| = z$, and that far away from the center of the nucleus g_z takes free-space t matrix form ($g_\infty = t$), then it can be shown [3] that

$$\langle \boldsymbol{k}'\boldsymbol{p}' \mid \hat{T} \mid \boldsymbol{k}\ \boldsymbol{p}\rangle = \delta(\boldsymbol{K}_\perp) g_\infty - \frac{1}{2\pi^2}\int_0^\infty z^3\, dz\, \frac{j_1(z\,K_\perp)}{z\,K_\perp}\, \frac{\partial g_z}{\partial z}\ . \quad (3)$$

What is curious about this result is that it disentangles very clearly the free-space contribution, the g_∞ term, from its medium-dependent counterpart. The medium dependence appears as the gradient of the reduced interaction, whereas the medium-independent contribution exhibits momentum conservation, as dictated by $\delta(\boldsymbol{K}_\perp)$.

After replacing the above expression for $\hat{T}$ into Eq. (1) for U we obtain the *unabridged* OMP, $U(\boldsymbol{k}', \boldsymbol{k}; E) \equiv U_0(\boldsymbol{k}', \boldsymbol{k}; E) + U_1(\boldsymbol{k}', \boldsymbol{k}; E)$, with U_0 the full-folding optical potential based on the free t-matrix. The medium-dependent contribution U_1, in turn, is given by

$$U_1(\boldsymbol{k}', \boldsymbol{k}; E) = \frac{1}{2\pi^2}\int d\boldsymbol{p}\, d\boldsymbol{p}'\hat{\rho}(\boldsymbol{p}', \boldsymbol{p})\int_0^\infty z^3 dz \frac{j_1(zK_\perp)}{zK_\perp}\left(-\frac{\partial g_z}{\partial z}\right)\ . \quad (4)$$

A realization of the optical potential is made if we model the reduced interaction g_z by means of the nuclear matter g matrix in the BBG theory. Here at each coordinate z with nuclear density $\rho(z)$, g_z is represented by the antisymmetized g matrix. As shown in Ref. [3], this model reproduces the *in-medium* folding model introduced by Arellano, Brieva and Love [4] if one assumes a Slater approximation for the mixed density. However, the above expression is general enough to allow the use of the full (off-shell) mixed density.

The medium-dependent term U_1 can be conveniently expressed as $U_1 = \int_0^\infty u(z)\, dz$, with u representing a potential density. Thus, by examining the behavior of $u(z)$ for selected matrix elements of U we can assess the importance of the various contributions to the OMP. To illustrate this feature we consider protons of mass m and energy $E = 40$ MeV colliding a given nucleus. We examine the forward on-shell ($\boldsymbol{k} = \boldsymbol{k}'$; $k = \sqrt{2mE}$) matrix elements of U. Here the targets are ^{40}Ca, ^{48}Ca, ^{48}Ni and ^{56}Ni. In the left panel of Fig. (1) we plot the real (solid curves) and imaginary (dashed curves) components of $u(z)$ as function of the radial distance z. This figure evidences quite neatly that intrinsic medium effects are confined to the region $4 - 6$ fm, with clear dominance of the coupling of the

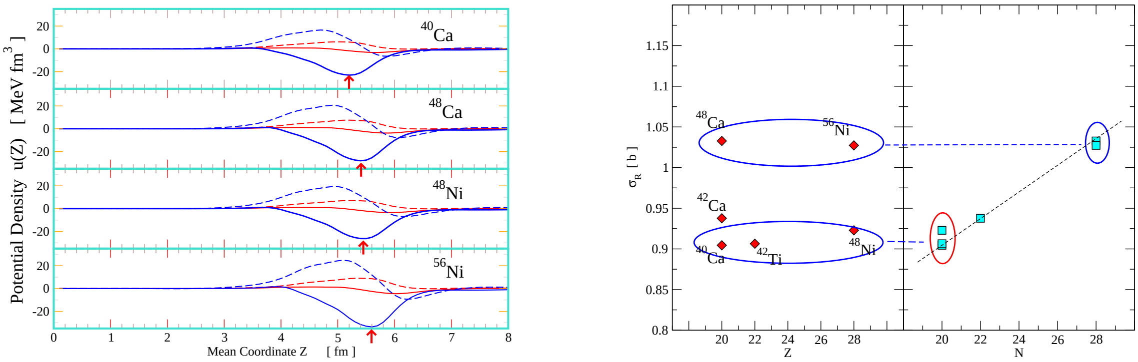

Figure 1: Left: Potential density $u(z)$ for selected targets as function of the radius z ($E = 40$ MeV). Right: Reaction cross sections as function of the proton (Z) and neutron (N) number of the target.

incoming proton to the target neutrons (blue curves) over that to protons (red curves). The strength of these contributions is dictated by the area under the curves. Hence, judging from these figures, the main sensitivity to intrinsic medium effects should come mainly from neutron densities. The sensitivity to proton densities come mainly from U_0, whose strength scales with the proton number Z.

To illustrate the above remarks, we have calculated OMP considering Hartree-Fock-Bogoliubov densities based on the D1S Gogny interaction [6]. The bare two-nucleon interaction is the Argonne v_{18} potential, from which off-shell g matrices are calculated at various densities. These matrices are then folded, without localization whatsoever, to obtain non-local optical potentials which are then used to evaluate scattering observables. In the right frame of Fig. 1 we show the reaction cross section σ_R as functions of the proton and neutron numbers. Clearly σ_R exhibits a uniform sensitivity with respect to the neutron number N, in contrast with a weak sensitivity to variations in Z.

In summary, based on formal properties implicit in the structure of the OMP, we are able to gauge –to some extent– the degree of sensitivity of NA scattering observables to proton and neutron distributions of the target. These features are energy- as well as channel-dependent. An analysis of reaction cross sections for Ca and Ni isotopes shows stronger sensitivity to the neutron number. Although not discussed here, this feature is generally true for medium energy proton scattering observables, being more sensitive to the details of the neutron than the proton density.

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