Beyond mean-field description of exotic structure and decay of proton-rich nuclei in A~70 region

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Characteristics of N~Z nuclei in A~70 mass region :

- *shape coexistence and mixing*
- *competition between T=0 and T=1 pairing correlations, isospin mixing*
- *rapid changes with particle number, angular momentum and excitation energy*

Open problems:

- *Precise data relevant for shape coexistence*
- *Isospin Symmetry Breaking effects on*
  - *Superallowed Fermi beta decay*
  - *Mirror Energy Differences*
- *Shape coexistence, shape isomers and Gamow-Teller β decay of rp-process (waiting-point) nuclei*

Self-consistent description of proton-rich A~70 nuclei based on:

- *beyond mean field VAMPIR approaches*
- *realistic effective interactions in large model spaces*

Complex EXCITED VAMPIR approach

- **the model space is defined by a finite dimensional set of spherical single particle states**
- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**
- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**
- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry**
- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

Beyond mean field variational procedure

complex VAMPIR

$$E^s[F_1^s] = \frac{\langle F_1^s | \hat{H} \hat{\Theta}_{00}^s | F_1^s \rangle}{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle} \qquad |\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s | F_1^s \rangle}{\sqrt{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}}$$

complex EXCITED VAMPIR

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^{i} \, |\phi(F_j^s)\rangle \, \alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}_{M0}^s | F_i^s \rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1} \, |\phi(F_j^s)\rangle \, \alpha_j^n + |\phi(F_n^s)\rangle \, \alpha_n^n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM> = \sum_{i=1}^{n} |\psi_i; sM> f_{i\alpha}^{(n)}, \qquad \alpha = 1, \ldots, n$$

A ~ 70 mass *region*

^{40}*Ca - core*

model space for both: protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$ $(2s_{1/2}$ $1d_{3/2}$ $0g_{7/2})$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

renormalized G-matrix (OBEP, Bonn A/Bonn CD)

- short range Gaussians in pp, np, nn channels
- monopole shifts:

  $<0g_{9/2}\ 0f;\ T=0|G|\ 0g_{9/2}\ 0f;T=0>$

  $<1p\ 1d_{5/2};\ T=0|G|\ 1p\ 1d_{5/2};T=0>$

Superallowed Fermi β decay between 0^+ T=1 analog states

test of the CVC hypothesis

test of the unitarity of CKM matrix

$$ft(1+\delta_R)(1-\delta_c) = \frac{K}{2G_v^2(1+\Delta_R^v)}$$

δc – isospin-symmetry-breaking correction

Strategy:

Charge-symmetric effective Hamiltonian:

- same single particle energies for π and υ
- Bonn A potential

Isospin-symmetry-breaking contributions:

∗ *electromagnetic interaction*

  - Coulomb contribution to the single particle energies resulting from the Ca core
  - Coulomb two-body matrix elements

∗ *charge-dependent strong interaction*

  - Bonn CD potential

Isospin-symmetry-breaking effective Hamiltonians:

∗ **Bonn A + Coulomb**

∗ **Bonn CD + Coulomb**

Isospin operator

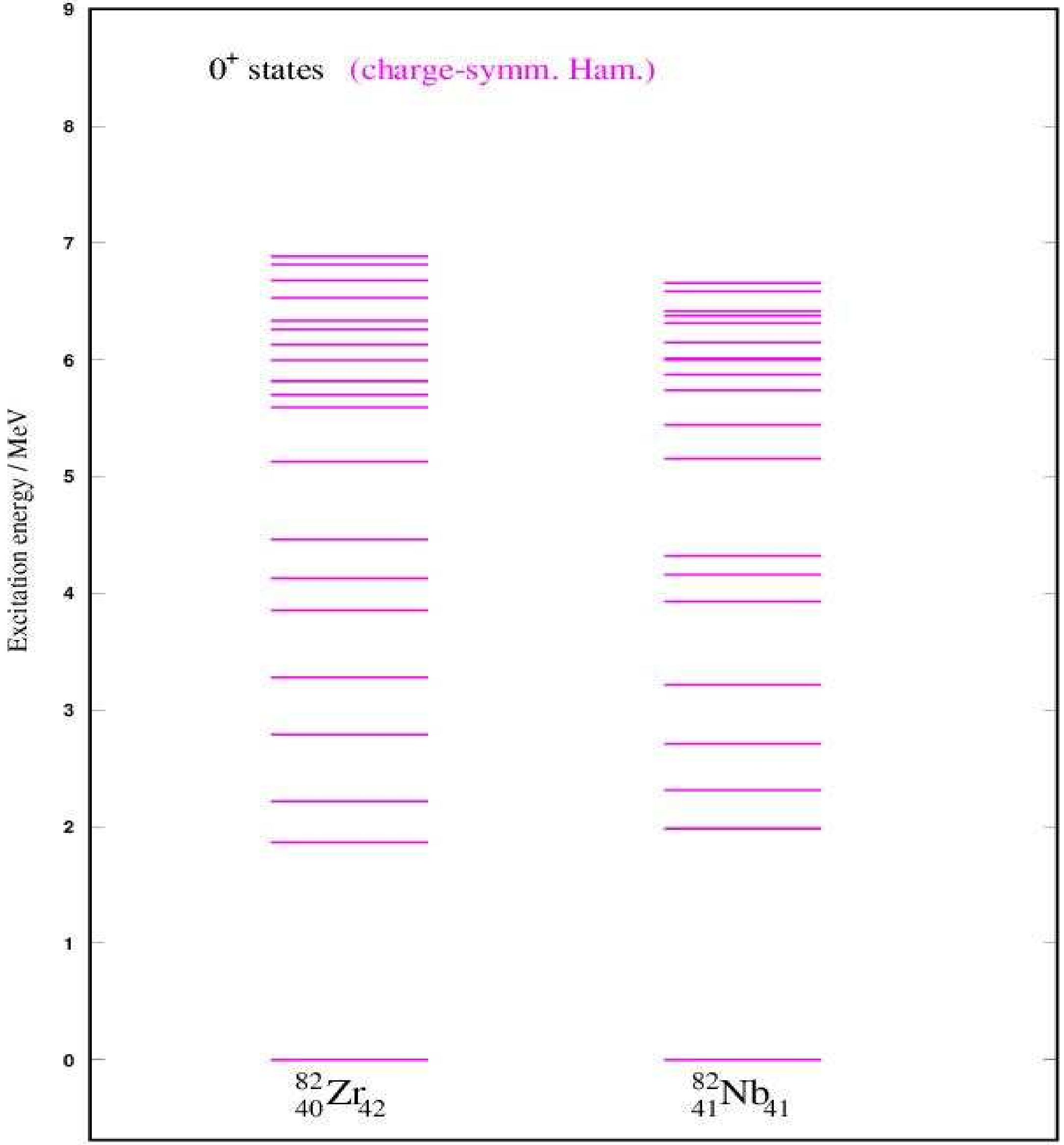
$\tau_+ = \Sigma_\alpha a_\alpha^+ b_\alpha$ *(α – single particle state of the model space)*

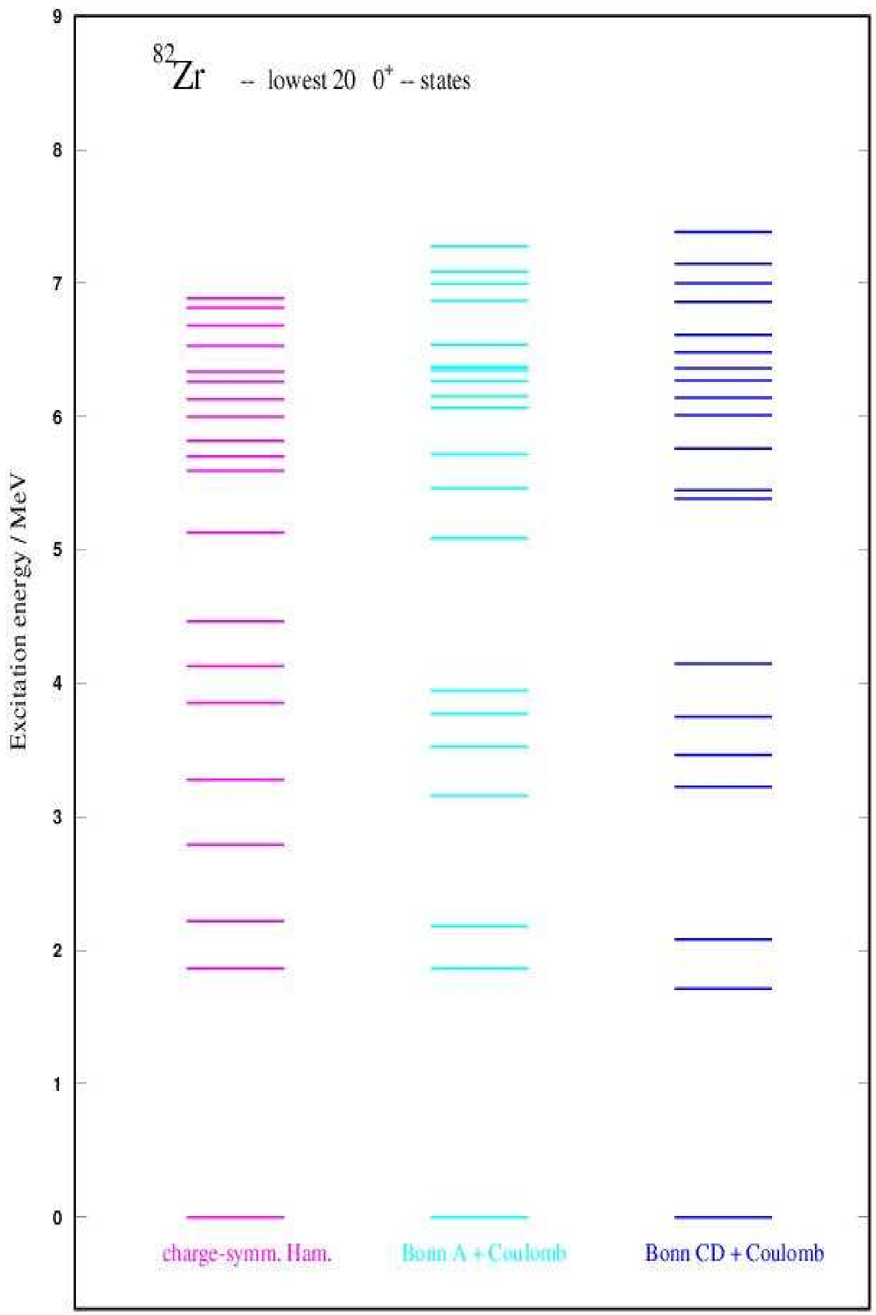
$M_F = \langle f \mid \tau_+ \mid i \rangle$

A=82 $_{41}\mathrm{Nb}_{41} \rightarrow {}_{40}\mathrm{Zr}_{42}$ $0^+ \rightarrow 0^+$

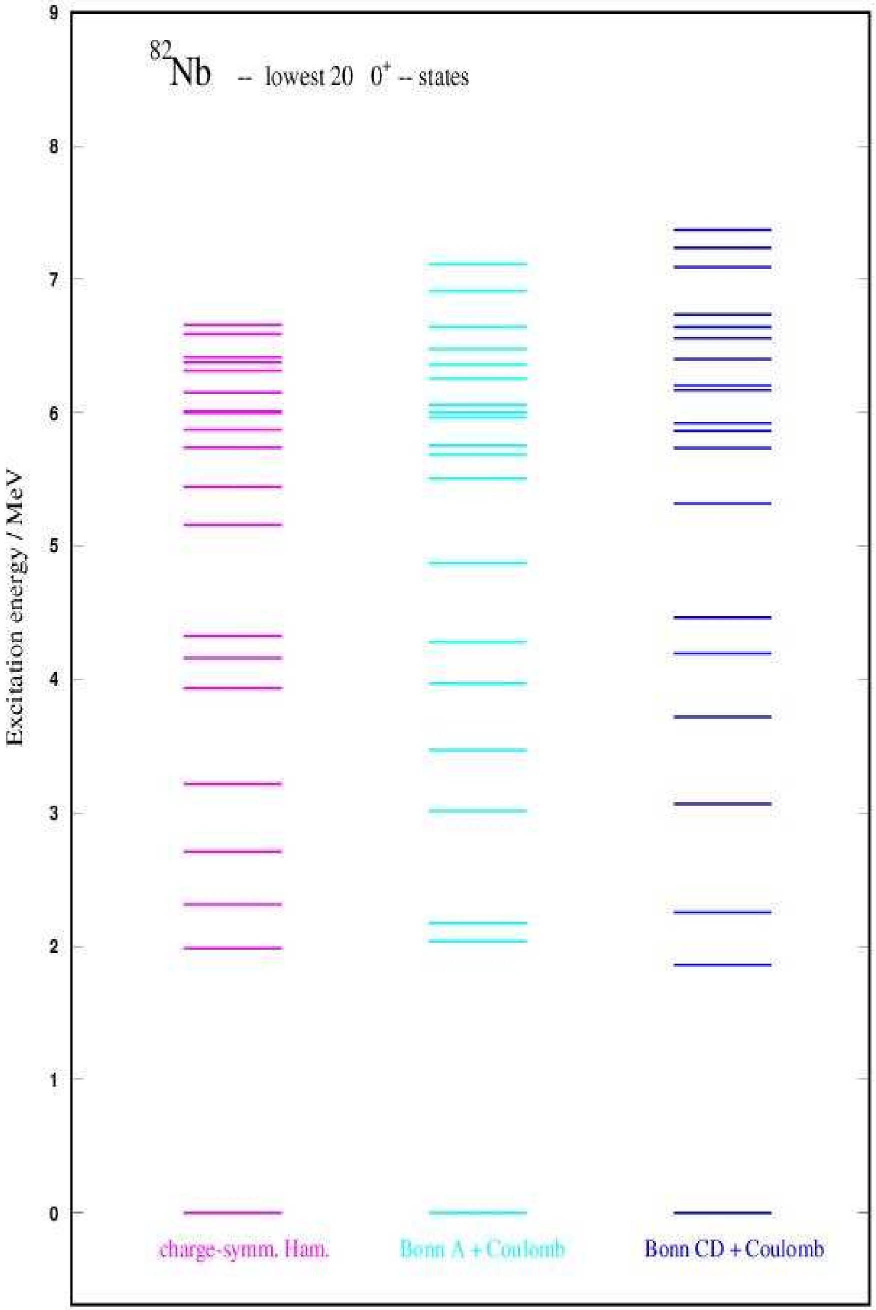
GANIL, J. Garces Narro et al, PRC63 (2001) 044307

$T_{1/2}$= 52(6)ms





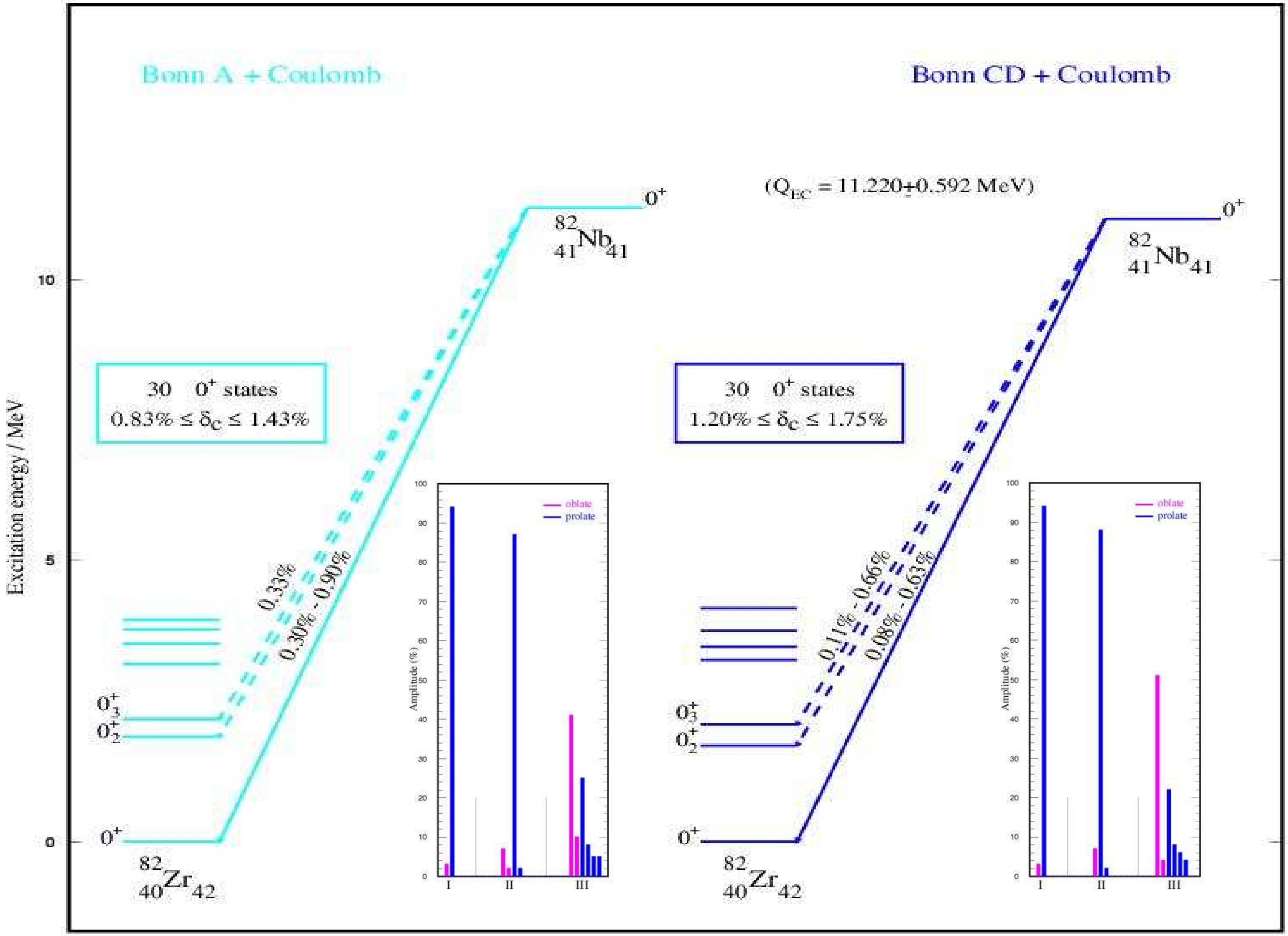
^{82}Zr -- lowest 20 0^+ -- states
Excitation energy / MeV
0
1
2
3
4
5
6
7
8
9
charge-symm. Ham.
Bonn A + Coulomb
Bonn CD + Coulomb



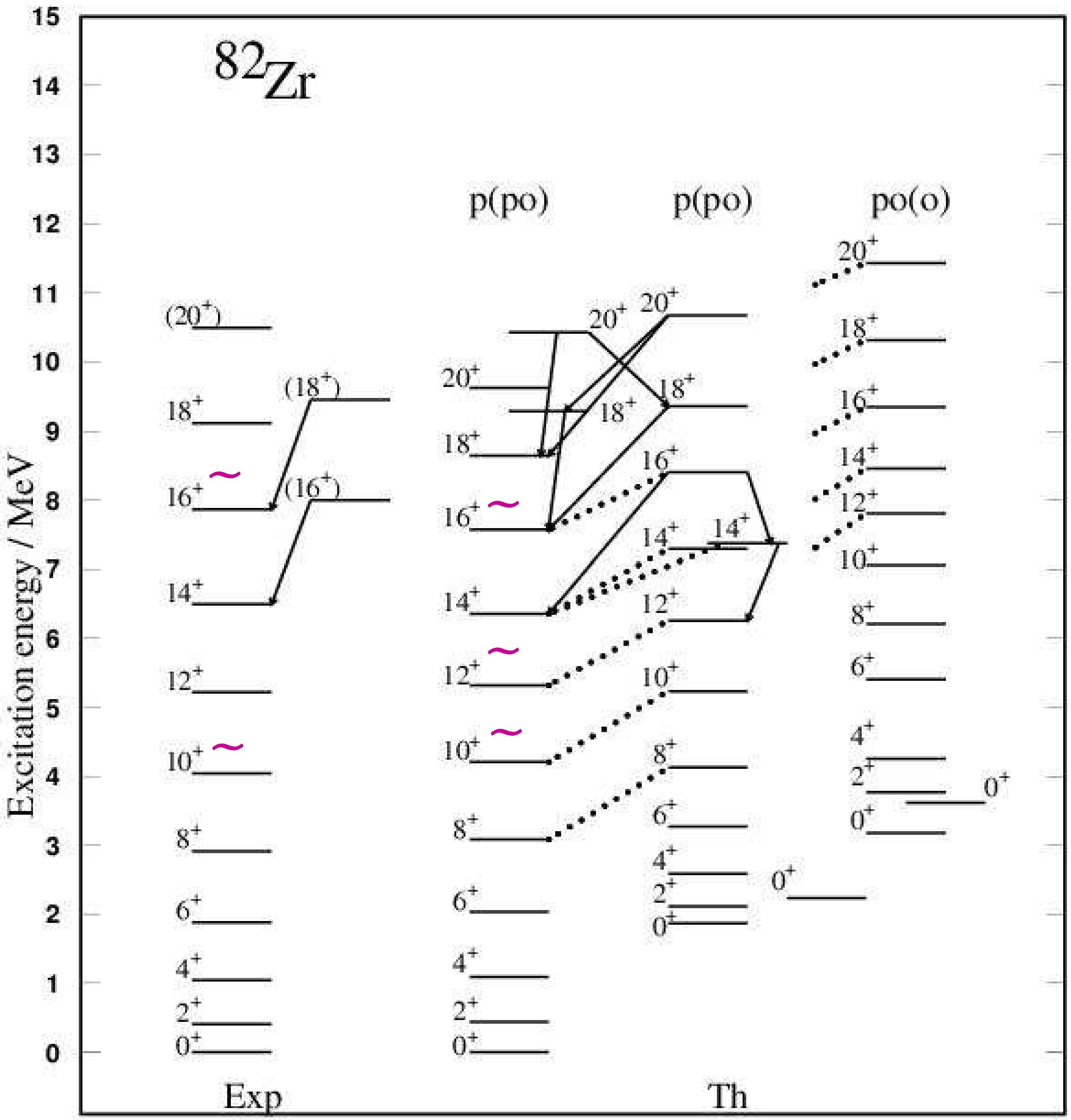
^{82}Nb -- lowest 20 0^+ -- states
Excitation energy / MeV
0
1
2
3
4
5
6
7
8
9
charge-symm. Ham.
Bonn A + Coulomb
Bonn CD + Coulomb

The total (S_T) and analog (S_{g-g}) Fermi β decay strengths for the charge-symmetric , Bonn A + Coulomb, and Bonn CD + Coulomb effective Hamiltonian

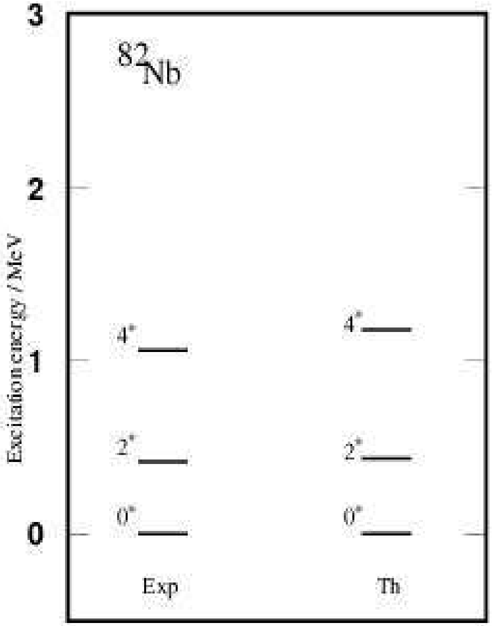
charge-symmetric Ham.		Bonn A + Coulomb		Bonn CD + Coulomb	
S_T	S_{g-g}	S_T	S_{g-g}	S_T	S_{g-g}
1.9715	1.9626	1.9761	1.9357	1.9752	1.9293

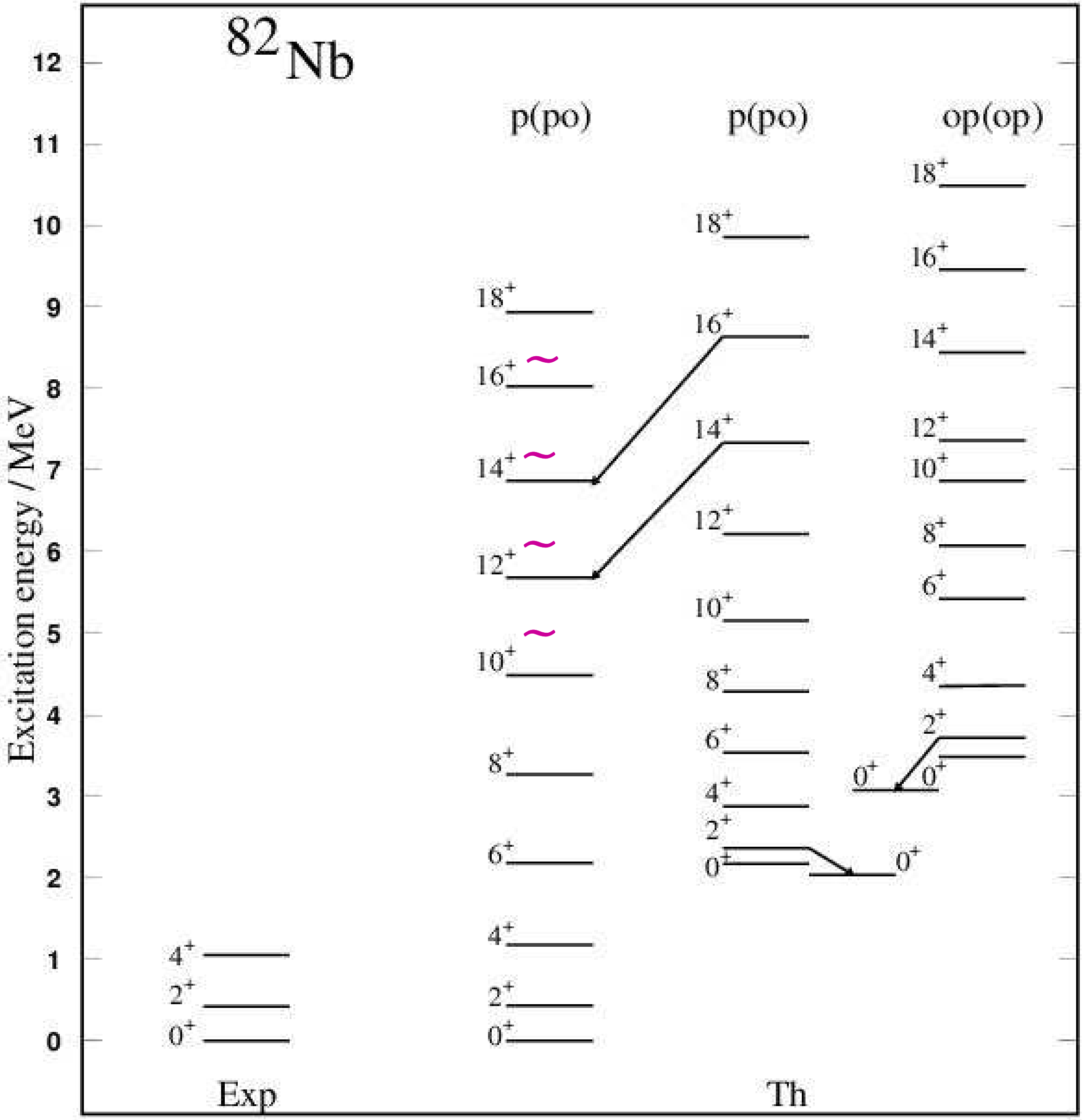


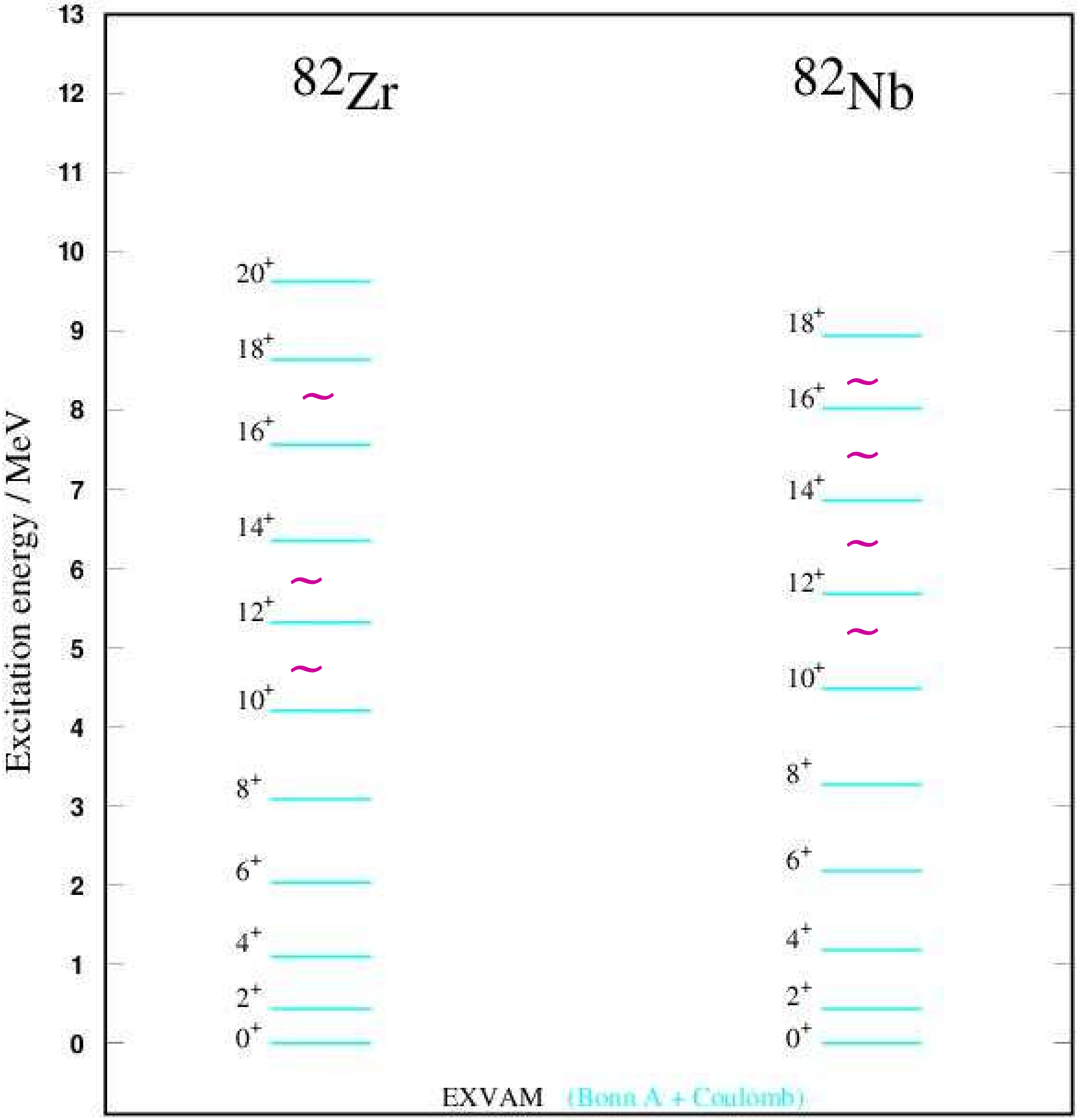
A. Petrovici et al, Phys. Rev. C78 (2008) 064311



High spin states obtained within the same approach (Bonn A + Coulomb)


82Nb
Excitation energy / MeV
3
2
1
0
4+
2+
0+
4+
2+
0+
Exp
Th


82Nb
Excitation energy / MeV
12
11
10
9
8
7
6
5
4
3
2
1
0
p(po)
p(po)
op(op)
18+
16+
14+
12+
10+
8+
6+
4+
2+
0+
0+
0+
0+
Exp
Th



Analog states in mirror nuclei

The amount of mixing for the lowest calculated states of ^{82}Zr (Bonn A + Coulomb)

$I^\pi[\hbar]$	p(po)-band	p(po)-band	po(op)-band
	o-mixing /p-mixing	o-mixing /p-mixing	o-mixing /p-mixing
0^+	3(1)%94%	4(1)%91(1)(1)%	38(9)%40(10)(1)%
2^+	98%	97%	20(4)%30(21)(18)(4)(2)%
4^+	98(1)%	97%	64(2)(2)%16(11)(2)(1)(1)%
6^+	98(1)%	93(5)(2)%	36%58(4)%
8^+	98(1)%	75(22)(2)%	52(41)(1)%4(2)%
10^+	99%	64(34)%	76(19)(1)%2%
12^+	99%	1%83(15)%	71(25)%4%
14^+	96(2)%	43(2)%48(3)(2)%	64(13)(3)%12(6)(2)%
16^+	14%77(8)%	5(3)%79(12)%	93(2)%4%
18^+	11(11)(2)%43(31)(2)%	47(12)(5)(2)%24(8)(1)%	84%8(6)(1)%
20^+	24%55(14)(4)%	9(2)%59(26)(2)%	80(3)(2)%7(4)(4)%

The amount of mixing for the lowest calculated states of ^{82}Nb (Bonn A + Coulomb)

$I^\pi[\hbar]$	p(po)-band	p(po)-band	po(op)-band
	o-mixing /p-mixing	o-mixing /p-mixing	o-mixing /p-mixing
0^+	3(1)%95%	30(4)%44(12)(3)(1)(1)%	15(3)%43(23)(7)(2)(2)(1)%
2^+	98%	1(1)%94%	46(1)%27(13)(5)(4)(2)(1)%
4^+	98%	1%97%	58(3)%15(12)(6)(3)(2)%
6^+	98(2)%	95(3)(2)%	70%28%
8^+	95(4)(1)%	95(3)(2)%	63(25)(6)(2)%3%
10^+	98%	96(2)%	62(24)(8)%5%
12^+	99%	2%93(5)%	58(32)(3)%5(1)%
14^+	97(2)%	3%84(10)(2)%	80(8)(5)%4(2)(1)%
16^+	2(1)%89(5)(3)%	4%85(7)(2)%	84(3)%10(2)%
18^+	14(4)(2)(2)%72(4)(2)%	26(2)(1)(1)%57(7)(6)%	77%23%

$B(E2; I \rightarrow I-2)$ values (in e^2fm^4) for the yrast states of the nucleus ^{82}Zr and ^{82}Nb

$I^\pi[\hbar]$	^{82}Zr		^{82}Nb
	Experiment	Theory	Theory
2^+	2328(1058)	1322	1274
4^+	1672(360)	1970	1897
6^+	2539(1058)	2138	2064
8^+	2328(635)	2174	2042
10^+	1926(487)	2129	1912
12^+	1904(635)	1974	1824
14^+	> 610	1733	1696
16^+		1472	1479
18^+		809	774
20^+		808	

Spectroscopic quadrupole moments Q_2^{sp} (in efm^2) for selected states of the nucleus ^{82}Zr

I^π	p(po)	$p(po)$	po(op)
2^+	-74.93	-77.23	-38.73
4^+	-95.77	-97.98	22.52
6^+	-106.01	-106.85	-15.73
8^+	-109.72	-105.31	64.64
10^+	-109.54	-94.66	70.57
12^+	-105.08	-82.10	68.24
14^+	-98.58	-10.76	49.33
16^+	-71.42	-62.12	74.31
18^+	-37.36	1.008	55.17
20^+	-23.44	-52.55	57.43

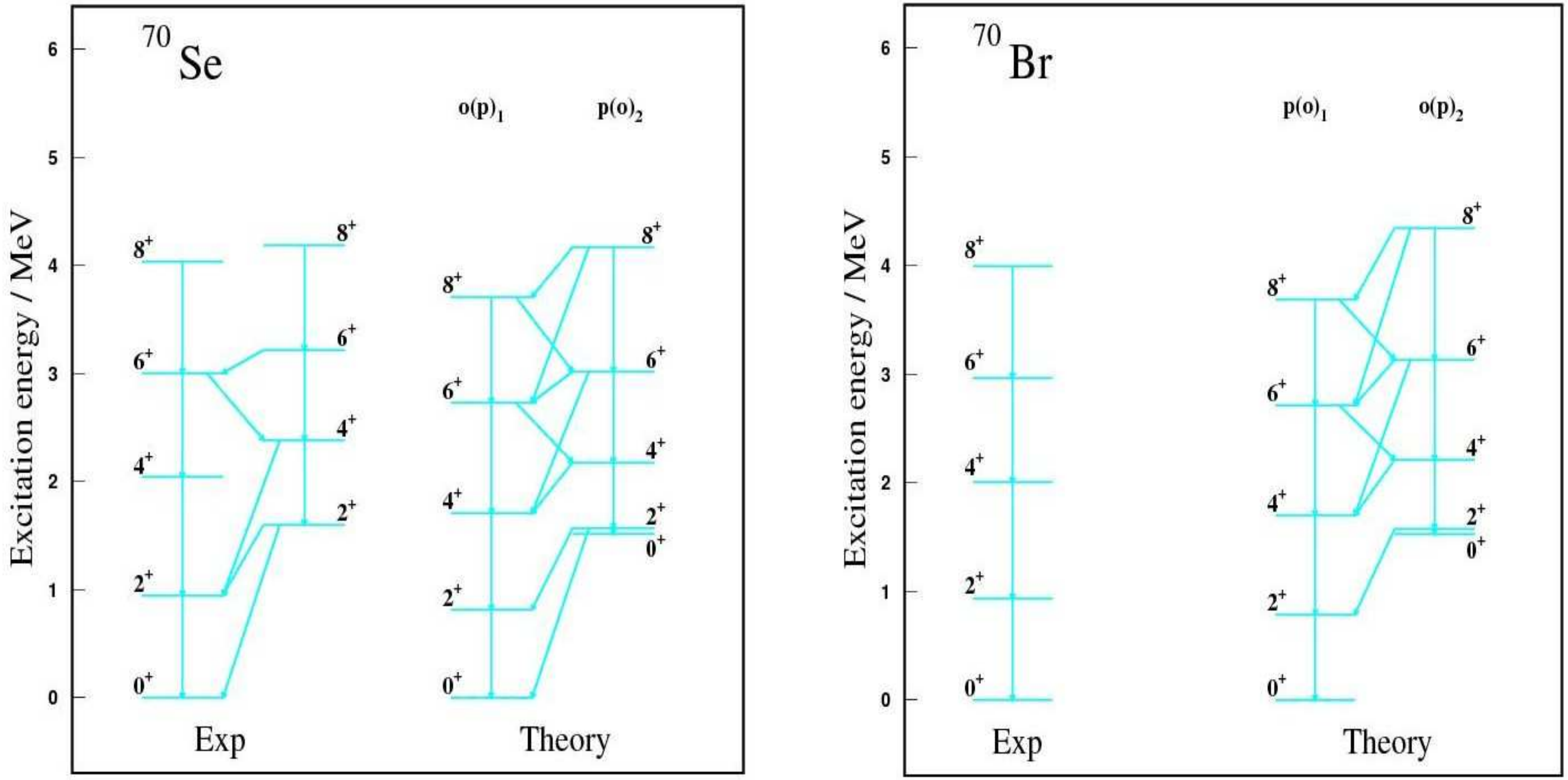
Mirror Energy Differences

A= 70, 82, 86

G. de Angelis et al, Eur. Phys. J. A12 (2001) 51 (^{70}Br)

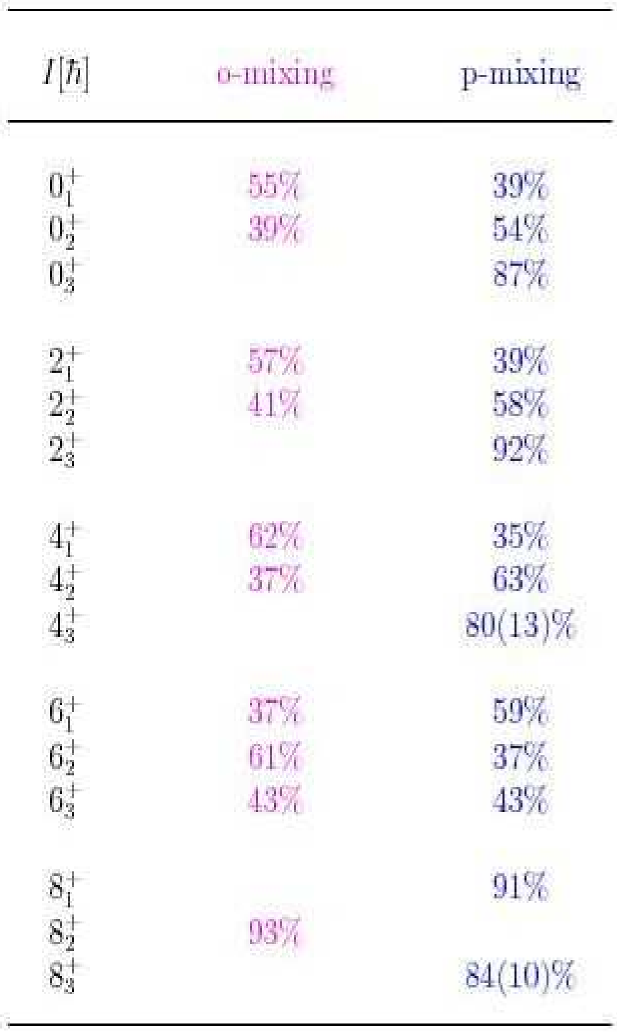
A. M. Hurst et al, Phys. Rev. Lett.98 (2007) 072501
(^{70}Se: No evidence for oblate shapes)

J. Ljungvall et al, Phys. Rev. Lett. 100 (2008) 102502
(^{70}Se: Evidence for oblate shapes)



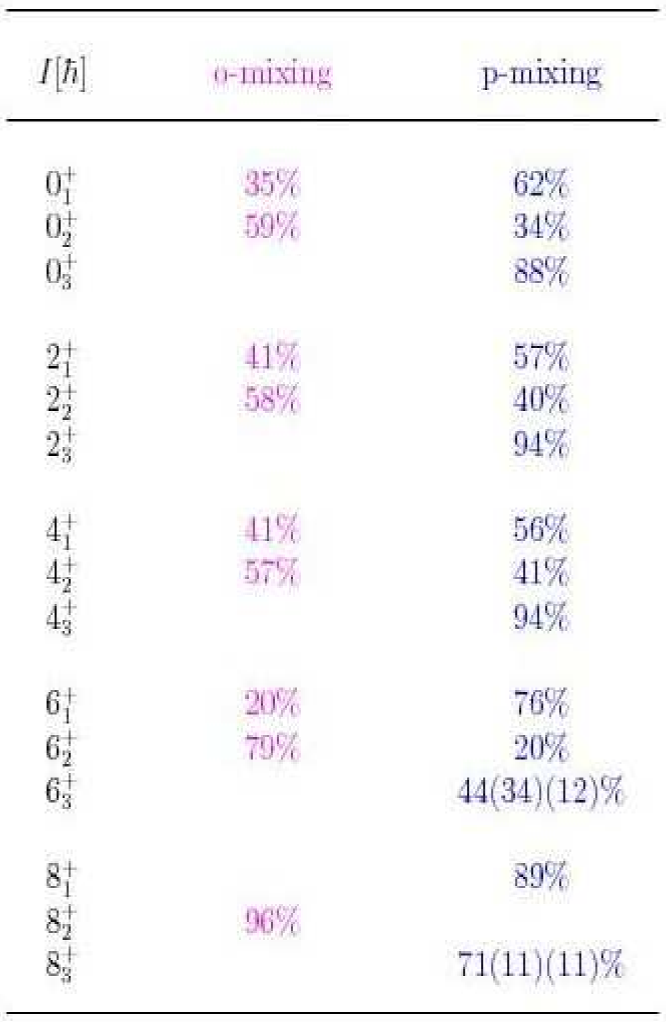
Complex Excited Vampir predictions: oblate-prolate mixing specific for each nucleus (varying with increasing spin).

The amount of mixing for the lowest states in ^{70}Se.



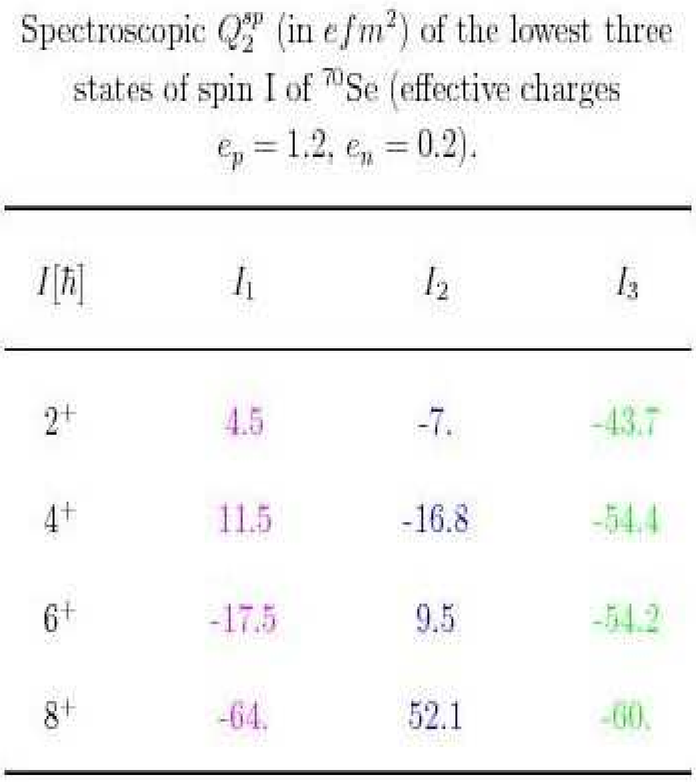
$I[\hbar]$	o-mixing	p-mixing
0^+_1	55%	39%
0^+_2	39%	54%
0^+_3		87%
2^+_1	57%	39%
2^+_2	41%	58%
2^+_3		92%
4^+_1	62%	35%
4^+_2	37%	63%
4^+_3		80(13)%
6^+_1	37%	59%
6^+_2	61%	37%
6^+_3	43%	43%
8^+_1		91%
8^+_2	93%	
8^+_3		84(10)%

The amount of mixing for the lowest states in ^{70}Br.



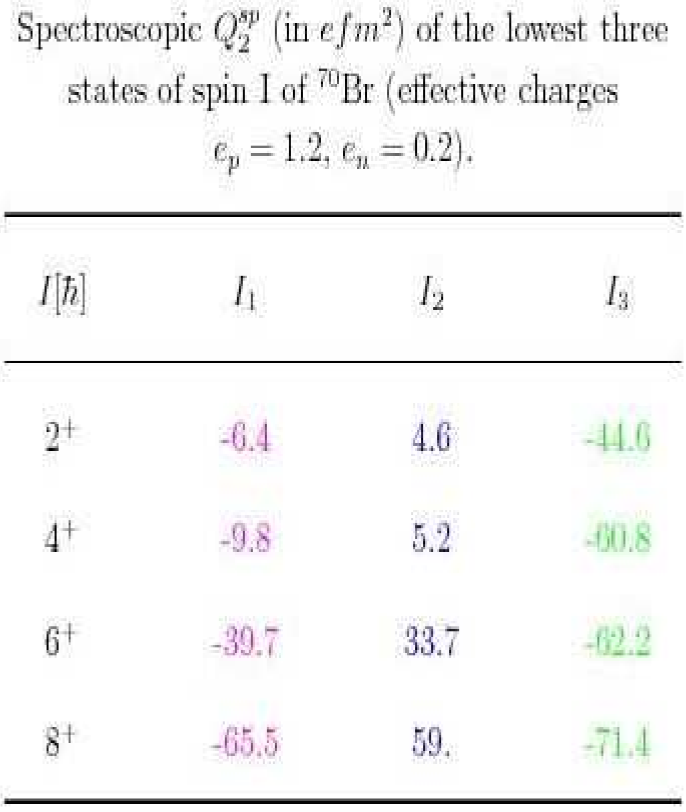
$I[\hbar]$	o-mixing	p-mixing
0^+_1	35%	62%
0^+_2	59%	34%
0^+_3		88%
2^+_1	41%	57%
2^+_2	58%	40%
2^+_3		94%
4^+_1	41%	56%
4^+_2	57%	41%
4^+_3		94%
6^+_1	20%	76%
6^+_2	79%	20%
6^+_3		44(34)(12)%
8^+_1		89%
8^+_2	96%	
8^+_3		71(11)(11)%

Strong oblate-prolate mixing up to spin 6^+ : oblate components dominate the yrast states of ^{70}Se, but the yrare states of ^{70}Br



Spectroscopic Q_2^{sp} (in efm^2) of the lowest three states of spin I of ^{70}Se (effective charges $e_p = 1.2$, $e_n = 0.2$).

$I[\hbar]$	I_1	I_2	I_3
2^+	4.5	-7.	-43.7
4^+	11.5	-16.8	-54.4
6^+	-17.5	9.5	-54.2
8^+	-64.	52.1	-60.



Spectroscopic Q_2^{sp} (in efm^2) of the lowest three states of spin I of ^{70}Br (effective charges $e_p = 1.2$, $e_n = 0.2$).

$I[\hbar]$	I_1	I_2	I_3
2^+	-6.4	4.6	-44.6
4^+	-9.8	5.2	-60.8
6^+	-39.7	33.7	-62.2
8^+	-65.5	59.	-71.4

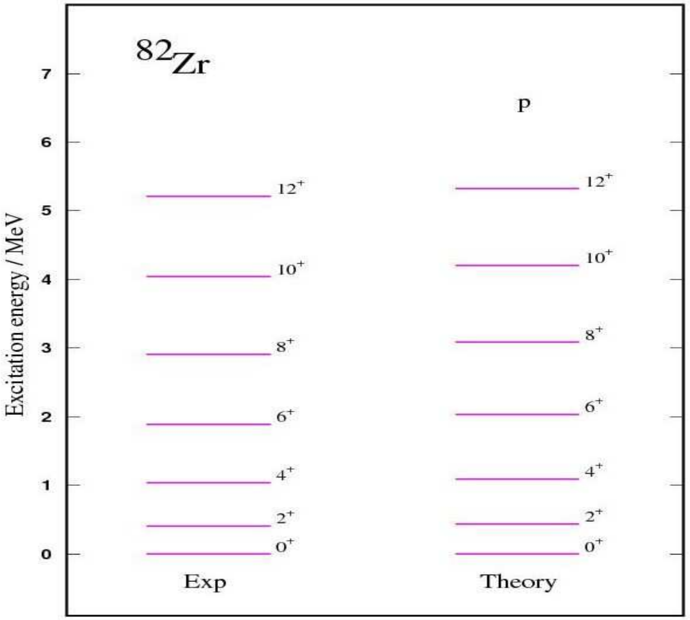
Precise quadrupole moments for low spin states could clarify the open problem.

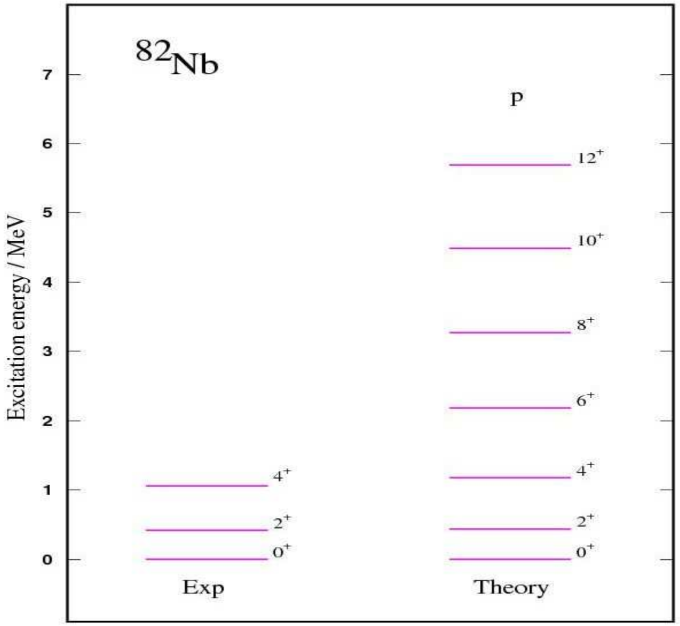
$B(E2; I \rightarrow I-2)$ values (in $e^2 fm^4$) for the lowest two bands of ^{70}Se (EXVAM). Strengths for secondary branches are given in parentheses (effective charges $e_p = 1.2$, $e_n = 0.2$) .

	EXVAM		Exp.	(HFB-based-config.mix.)
$I[\hbar]$	$o(p)_1$	$p(o)_2$		(Girod et al.)
2^+	492	501 (5)	342 ± 19	549
4^+	713	761	370 ± 24	955
6^+	779 (62)	792 (33)	530 ± 96	1404
8^+	717 (193)	666 (150)		

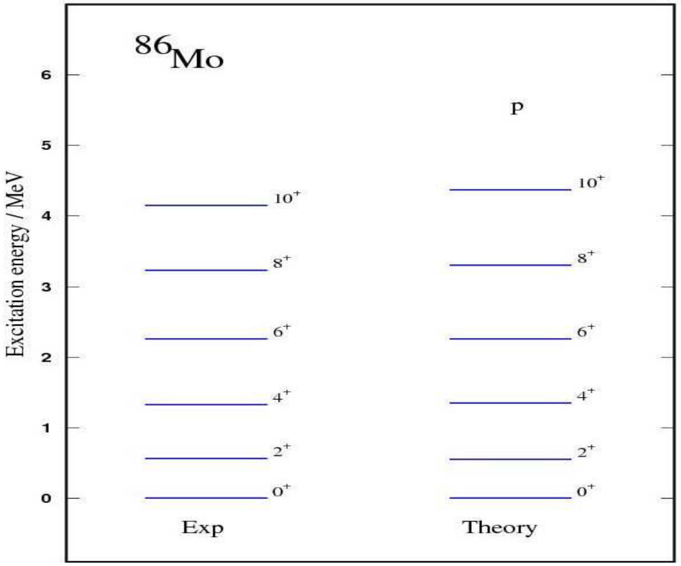
$B(E2; I \rightarrow I-2)$ values (in $e^2 fm^4$) for the lowest two bands of ^{70}Br (EXVAM). Strengths for secondary branches are given in parentheses (effective charges $e_p = 1.2$, $e_n = 0.2$).

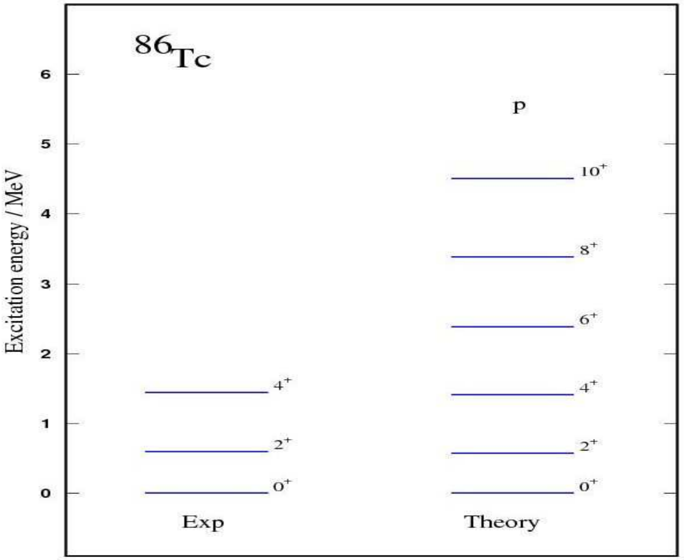
$I[\hbar]$	$p(o)_1$	$o(p)_2$
2^+	541	516
4^+	775	756
6^+	820 (60)	777 (44)
8^+	771 (81)	754 (84)

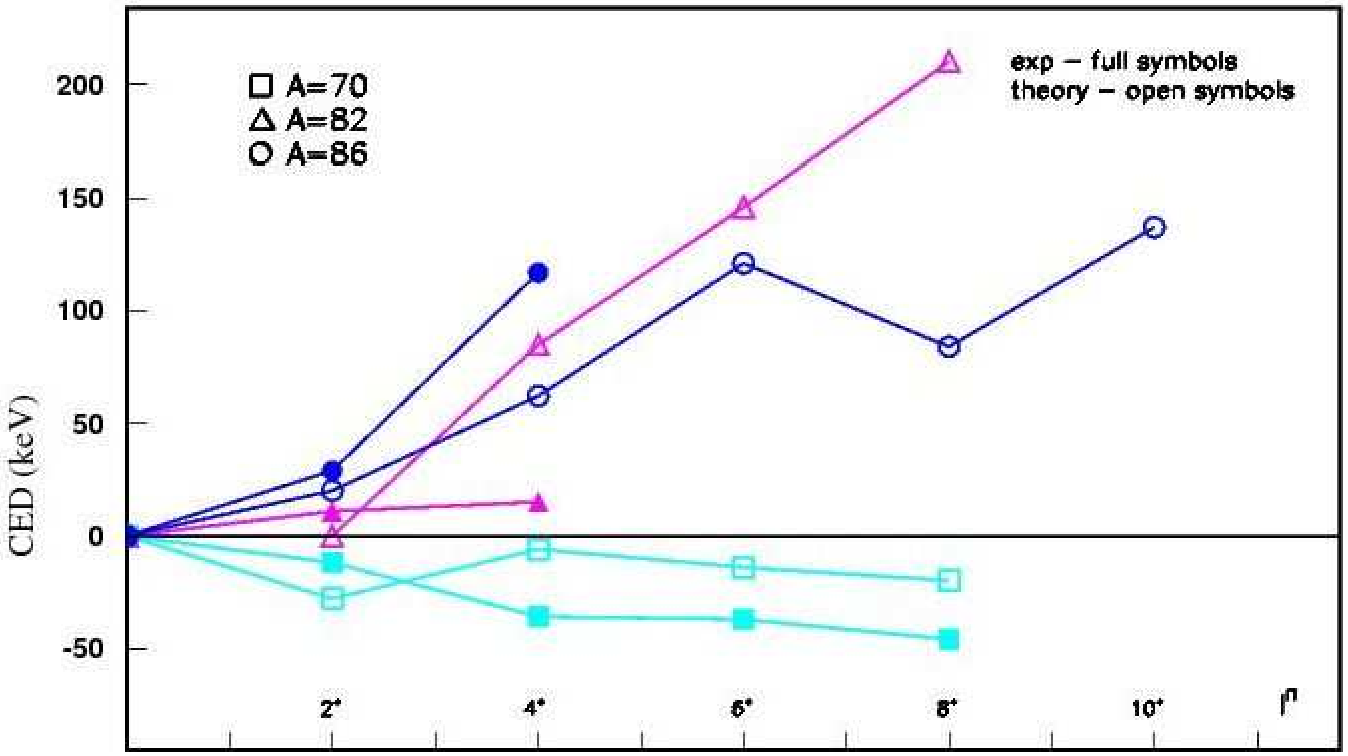




A. Petrovici et al, Phys. Rev. C78 (2008) 064311







A. Petrovici, J. Phys.G: Nucl. Part. Phys 37 (2010) 064036

Gamow-Teller β decay of the rp-process waiting point ^{72}Kr

CERN/ISOLDE I. Piqueras, Eur. Phys. J. A16(2003)313

$^{72}\mathrm{Kr} \rightarrow {}^{72}\mathrm{Br}$

$Q_{EC} = 5.040 \pm 0.375$ MeV

$0^{+}_{gs} \rightarrow 1^{+}$

open problem: *possible contribution of low-lying states to the effective half-life*

$0^{+}_{I} \rightarrow 1+$

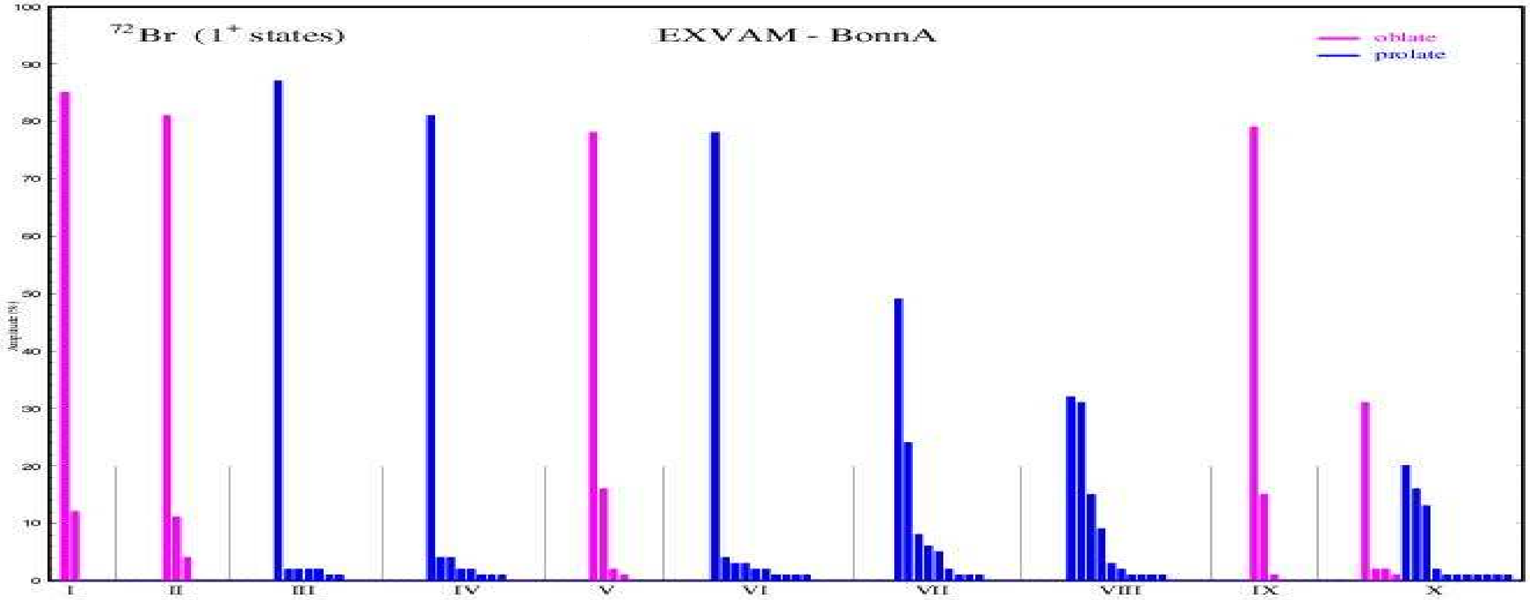
$E_{0^{+}_{I}} = 0.671$ MeV

$2^{+}_{yrast} \rightarrow 1^{+}$ / 2^{+} /3^{+}

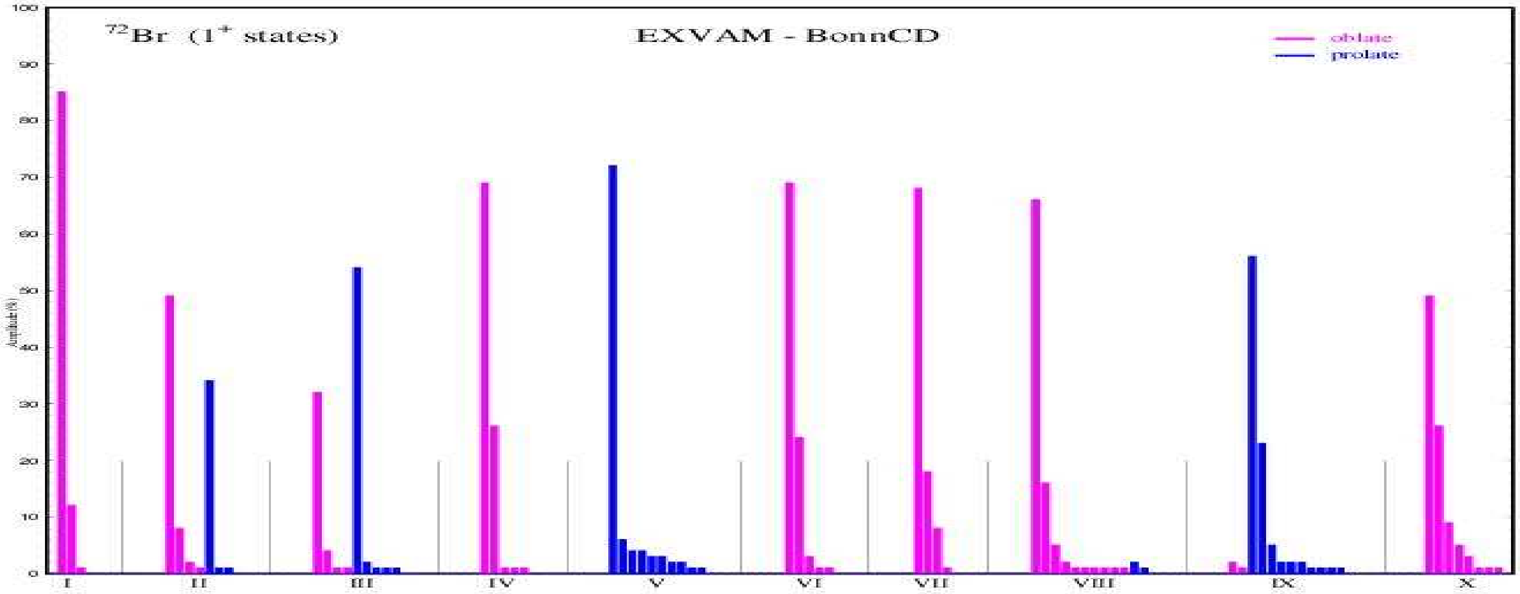
$E_{2^{+}_{yrast}} = 0.710$ MeV

The amount of mixing for the considered states of the ^{72}Kr nucleus (ms3).

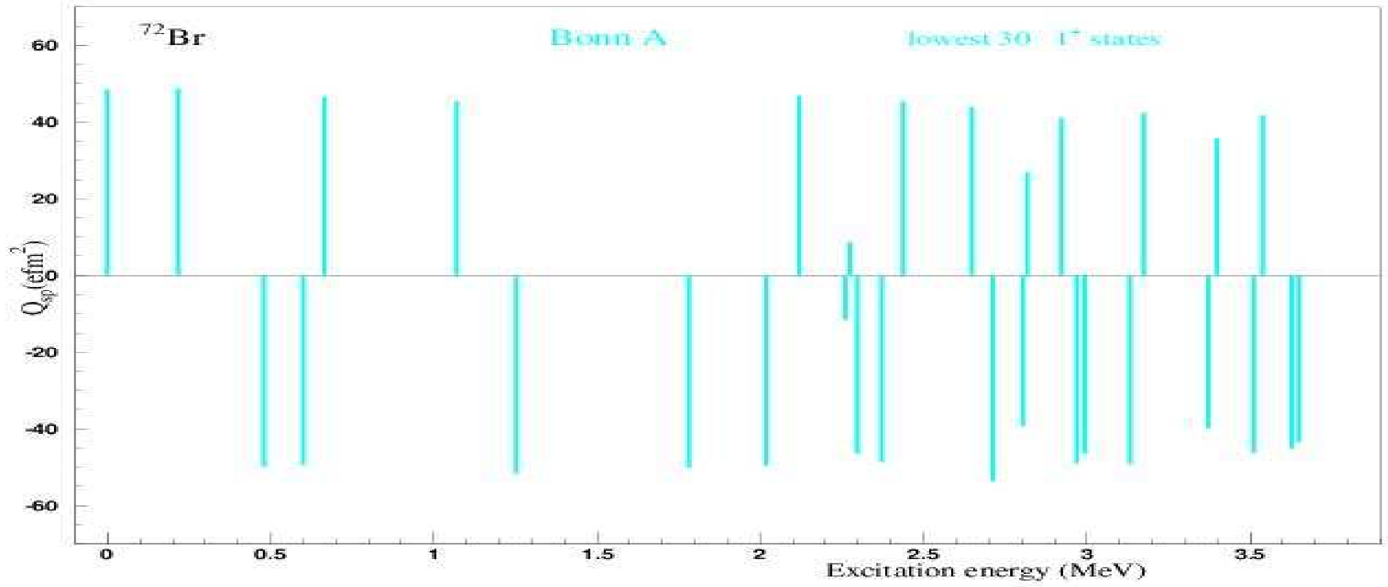
$I[\hbar]$	Bonn A o-mixing	p-mixing	Bonn CD o-mixing	p-mixing
0^{+}_{1}	64(2)%	29(2)(1)(1)%	50(3)%	38(5)(3)%
0^{+}_{2}	35(2)%	57(3)(1)(1)%	49(2)%	46(3)%
2^{+}_{1}	92(1)%	6%	76(1)%	20(3)%

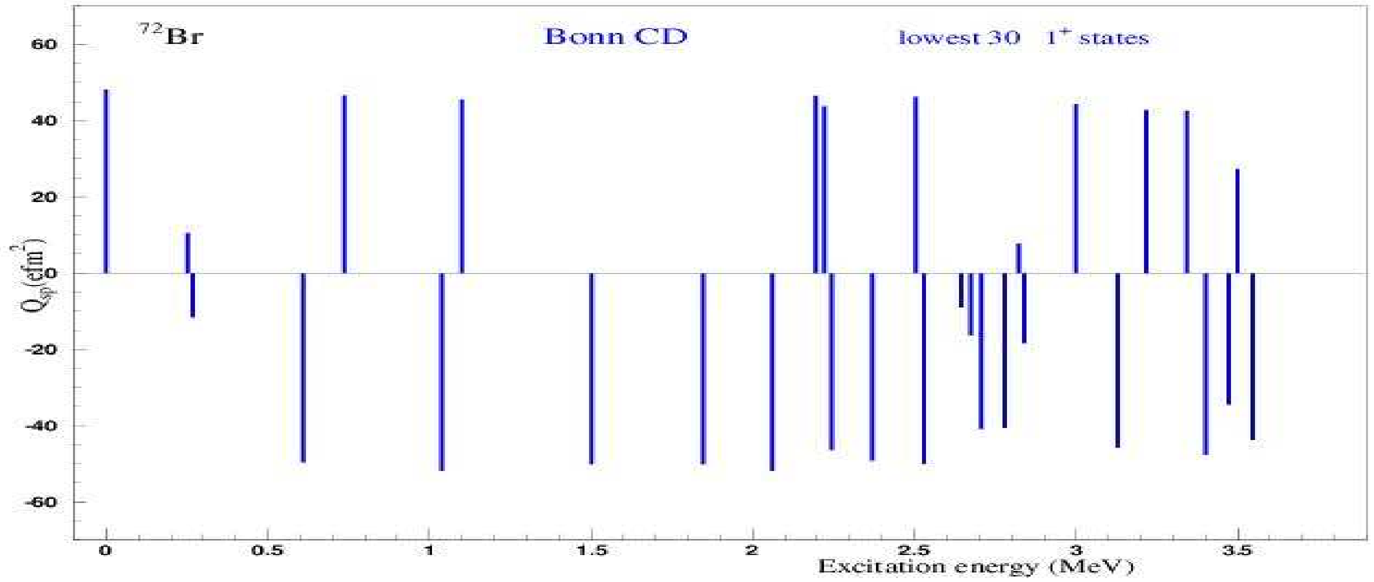


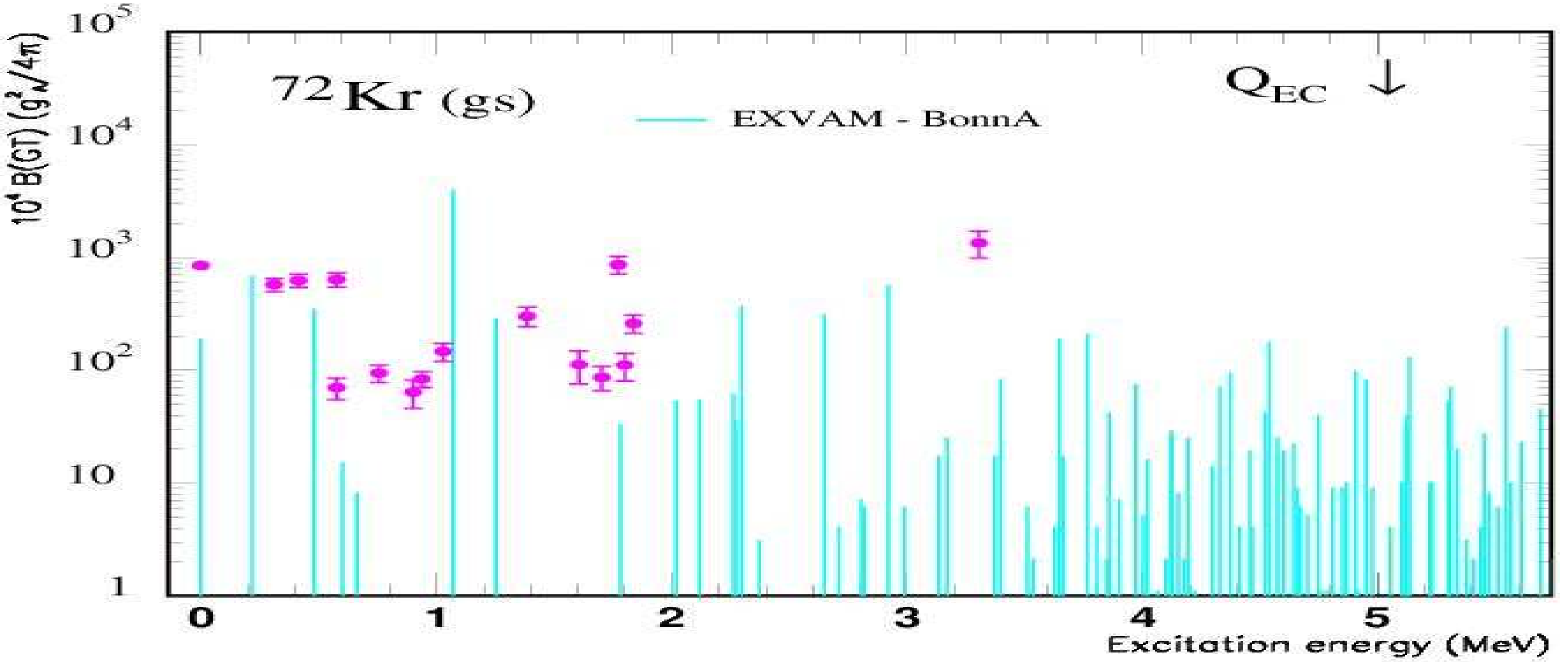
72Br (1+ states)
EXVAM - BonnA
oblate
prolate
Amplitude (%)
0
10
20
30
40
50
60
70
80
90
100
I
II
III
IV
V
VI
VII
VIII
IX
X

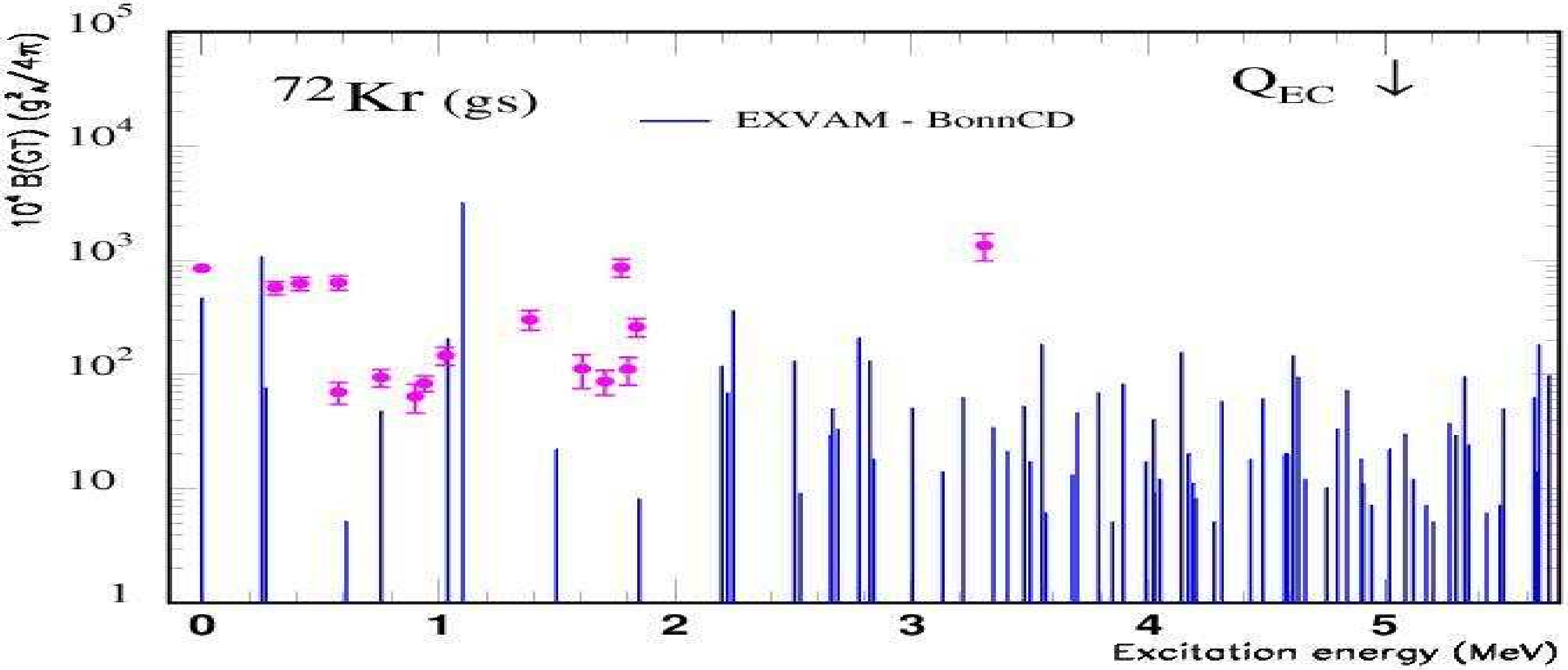


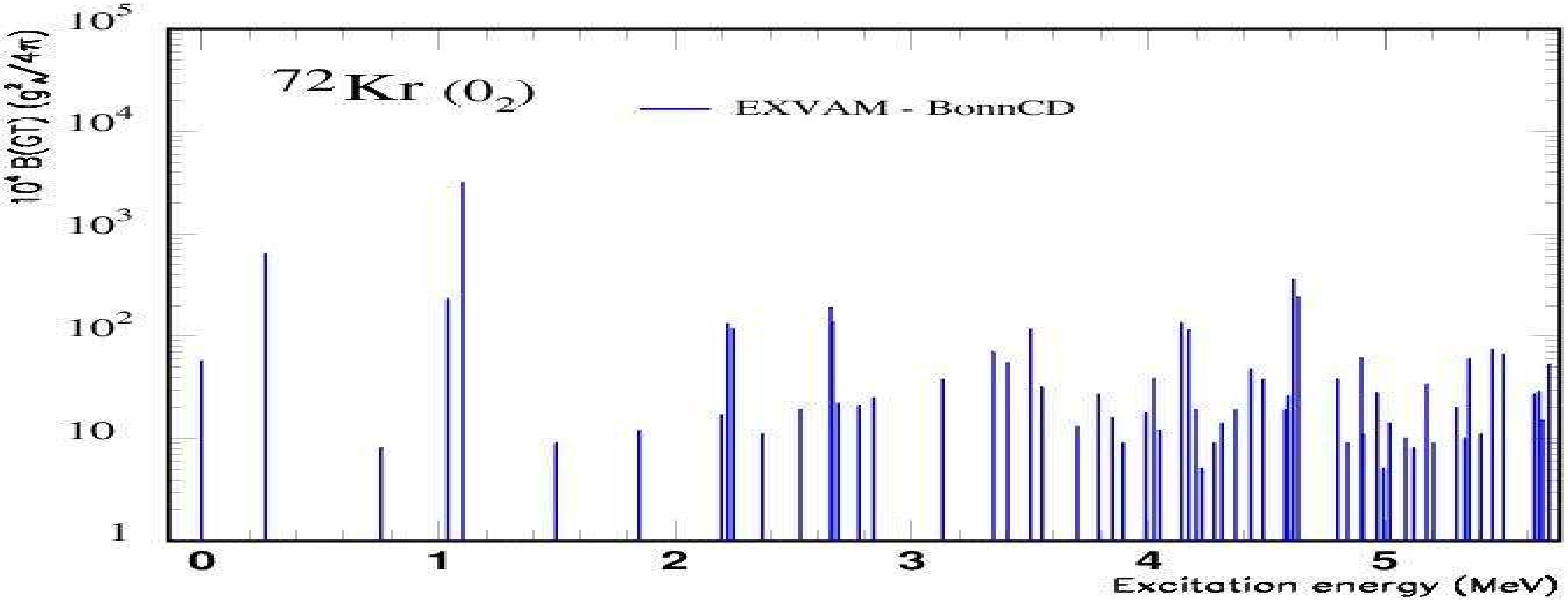
72Br (1+ states)
EXVAM - BonnCD
oblate
prolate
Amplitude (%)
0
10
20
30
40
50
60
70
80
90
100
I
II
III
IV
V
VI
VII
VIII
IX
X

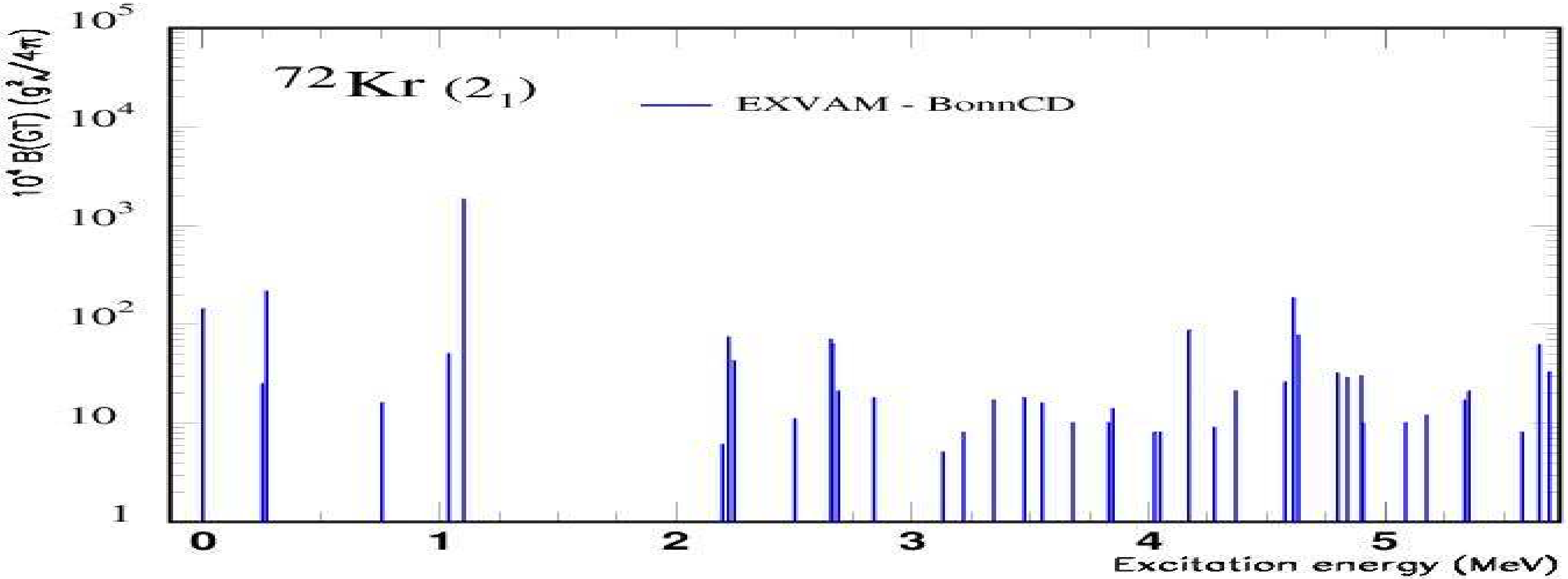

^{72}Br
Bonn A
lowest 30 1^+ states
$Q_{sp}(efm^2)$
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
3.5
Excitation energy (MeV)

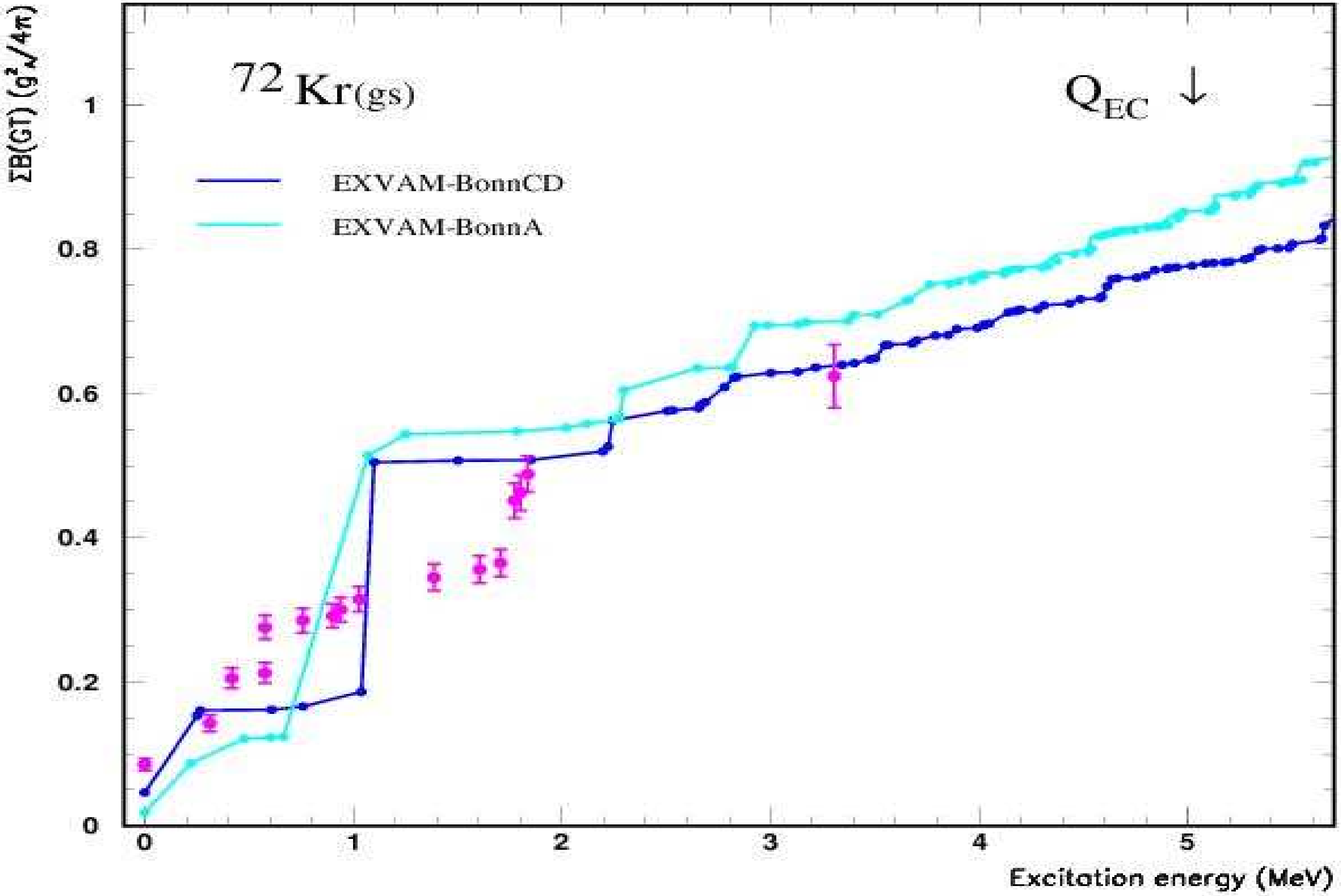

^{72}Br
Bonn CD
lowest 30 1^+ states
$Q_{sp}(efm^2)$
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
3.5
Excitation energy (MeV)

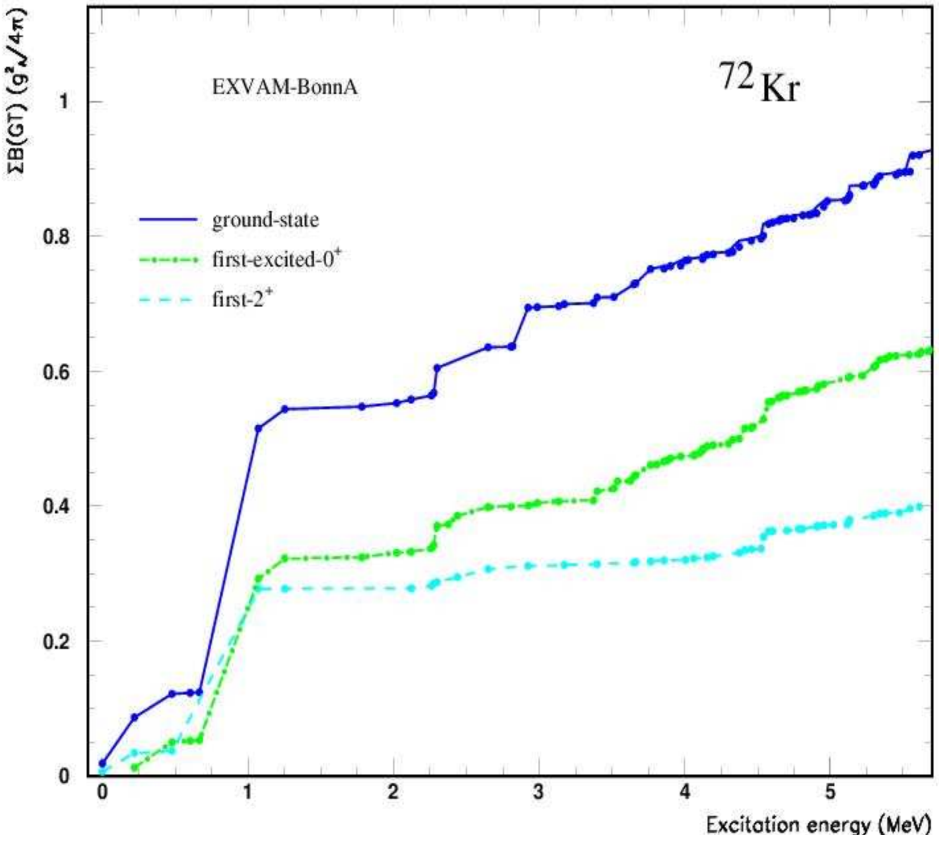

^{72}Kr (gs)
EXVAM - BonnA
Q_{EC} ↓
10^4 B(GT) $(g_A^2/4\pi)$
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5
Excitation energy (MeV)

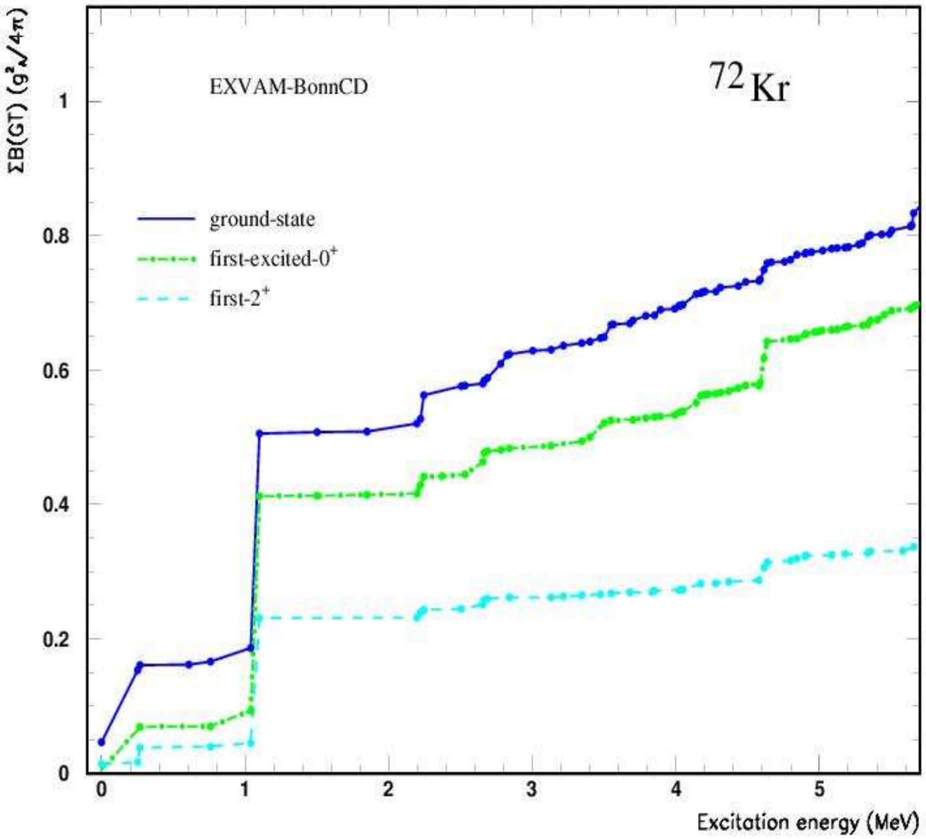

^{72}Kr (gs)
EXVAM - BonnCD
Q_{EC} ↓
10^4 B(GT) $(g_A^2/4\pi)$
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5
Excitation energy (MeV)

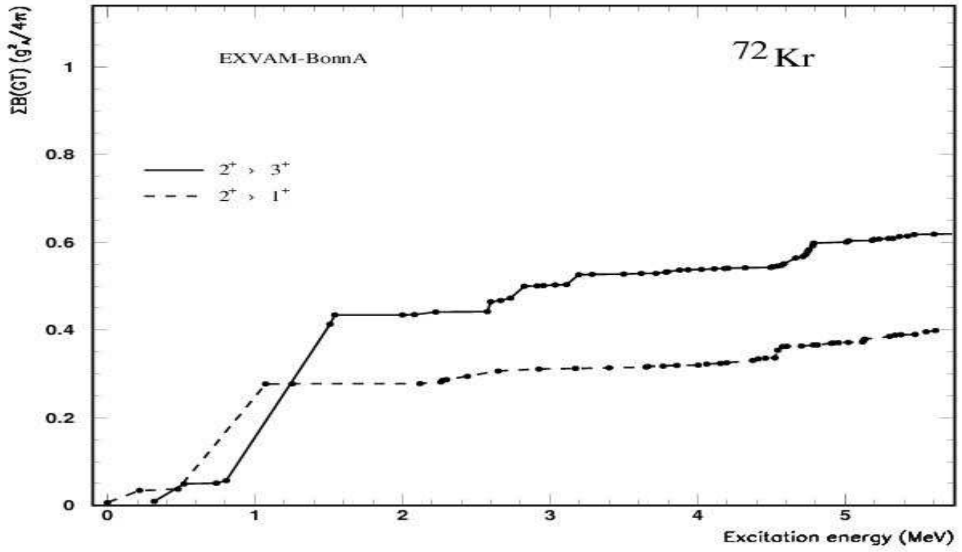

^{72}Kr (0_2)
EXVAM - BonnCD
10^4 B(GT) ($g_A^2/4\pi$)
1
10
10^2
10^3
10^4
10^5
0
1
2
3
4
5
Excitation energy (MeV)


^{72}Kr (2_1)
EXVAM - BonnCD
10^4 B(GT) ($g_A^2/4\pi$)
1
10
10^2
10^3
10^4
10^5
0
1
2
3
4
5
Excitation energy (MeV)


ΣB(GT) (g²A/4π)
72 Kr(gs)
QEC ↓
EXVAM-BonnCD
EXVAM-BonnA
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
Excitation energy (MeV)


EXVAM-BonnA
^{72}Kr
ground-state
first-excited-0^+
first-2^+
ΣB(GT) ($g_A^2/4\pi$)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
Excitation energy (MeV)


EXVAM-BonnCD
^{72}Kr
ground-state
first-excited-0^+
first-2^+
ΣB(GT) ($g_A^2/4\pi$)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
Excitation energy (MeV)


EXVAM-BonnA
^{72}Kr
$2^+ > 3^+$
$2^+ > 1^+$
ΣB(GT) ($g_A^2/4\pi$)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
Excitation energy (MeV)

$$\frac{1}{T_{1/2}} = \frac{g_A^2}{D} \sum_i f(Z, E_i) | \langle 1_i^+ || \beta^+ || 0^+ \rangle |^2$$

$D = 6146$ s $g_A = 1.26$

$T_{1/2}^{exp} = 17.1(2)$ s

$T_{1/2}$ (gs) = 20.8 s (Bonn A)   18.9 s (Bonn CD)

$T_{1/2}$ (first-excited 0^+) = 17.3 s (Bonn A)   12.9 s (Bonn CD)

$T_{1/2}$ (yrast $2^+ \to 1^+$) = 18.7 s (Bonn A)   21.6 s (Bonn CD)

$T_{1/2}$ (yrast $2^+ \to 3^+$) = 19.5 s (Bonn A)

$$\lambda = \ln 2/K\, \Sigma_i\, [(2J_i+1)\, e^{-E_i/(kT)}]/\, G(Z,A,T)\, \Sigma_j\, B_{ij} \Phi_{ij}$$

i – *parent states* $\quad$ j – *daughter states*

$G(Z,A,T) = \Sigma_i\, e^{-E_i/(\kappa T)}$ *(partition function of the parent nucleus)*

$Bij = Bij\ (GT)$

Φ_{ij} – *phase space integral*

$T < 2$ GK *X-ray bursts*

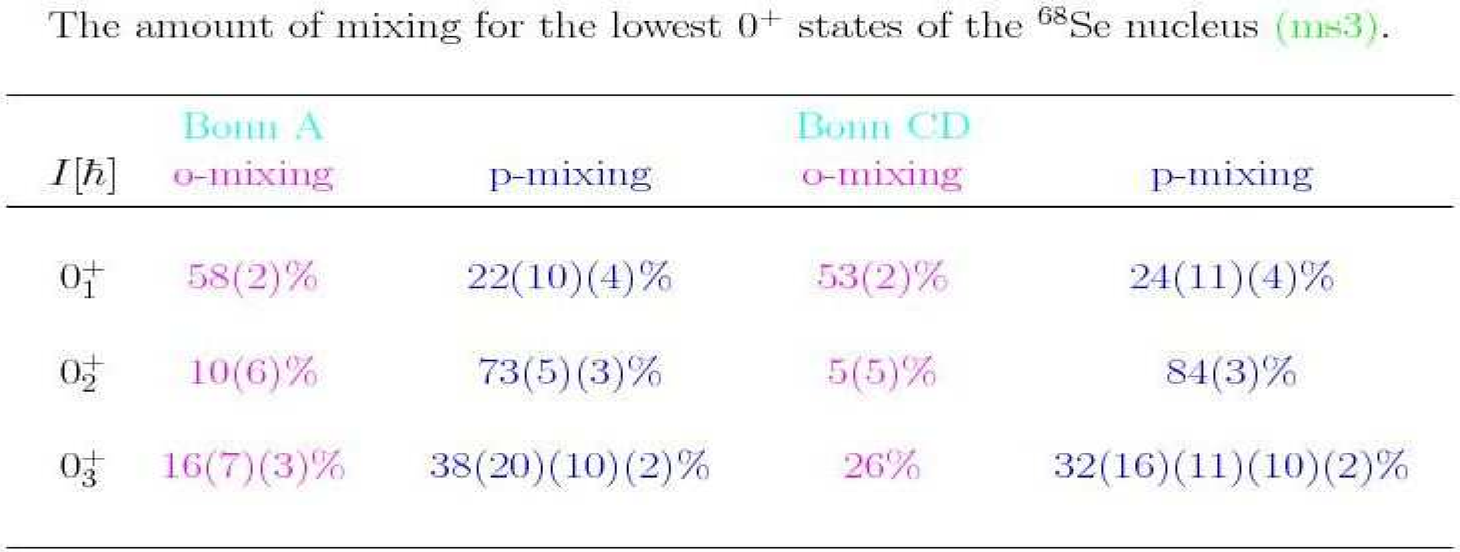
In the astrophysical environment of the X-ray bursts the effect of the decay of the first excited 0^+ state of ^{72}Kr is within the uncertainty of the ground-state half-life

A. Petrovici et al, Phys. Rev. C78 (2008) 044315

Gamow-Teller β decay of the rp-process waiting point ^{68}Se

CERN/ISOLDE P. Baumann et al, Phys. Rev. C50 (1994) 1180

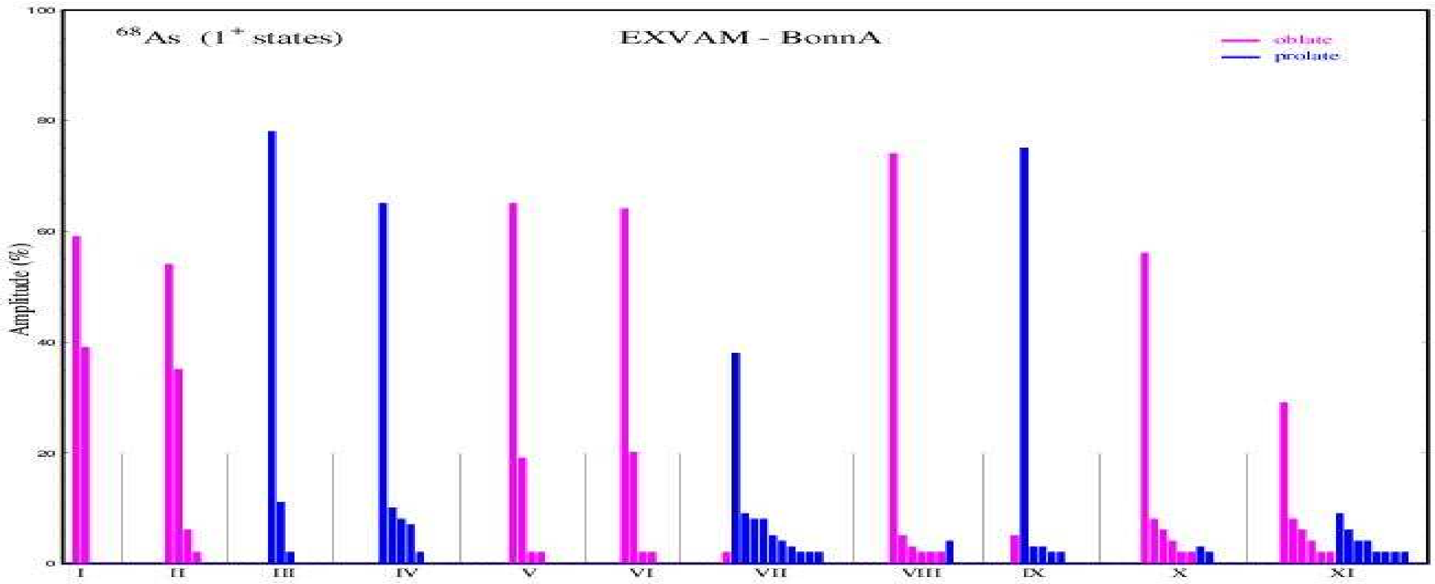
$^{68}Se \rightarrow {}^{68}As$ $0^{+} \rightarrow 1^{+}$ $Q_{EC} = 4.730 \pm 0.310$ MeV



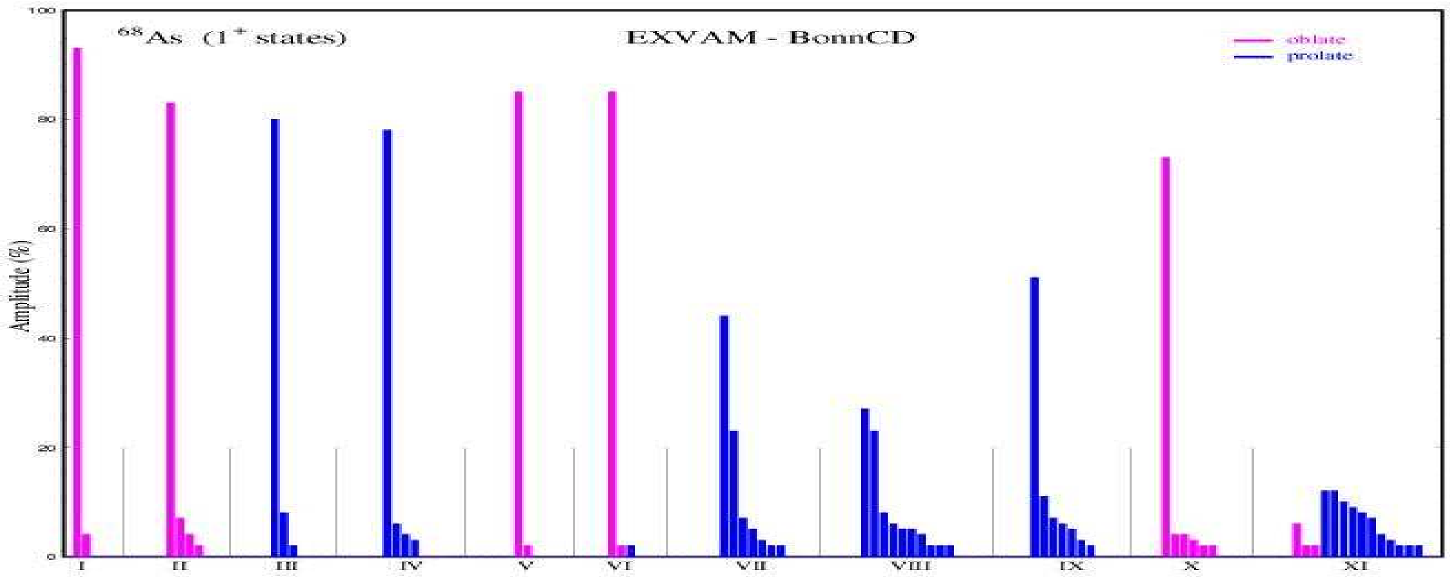
The amount of mixing for the lowest 0^+ states of the ^{68}Se nucleus (ms3).

$I[\hbar]$	Bonn A o-mixing	p-mixing	Bonn CD o-mixing	p-mixing
0_1^+	58(2)%	22(10)(4)%	53(2)%	24(11)(4)%
0_2^+	10(6)%	73(5)(3)%	5(5)%	84(3)%
0_3^+	16(7)(3)%	38(20)(10)(2)%	26%	32(16)(11)(10)(2)%

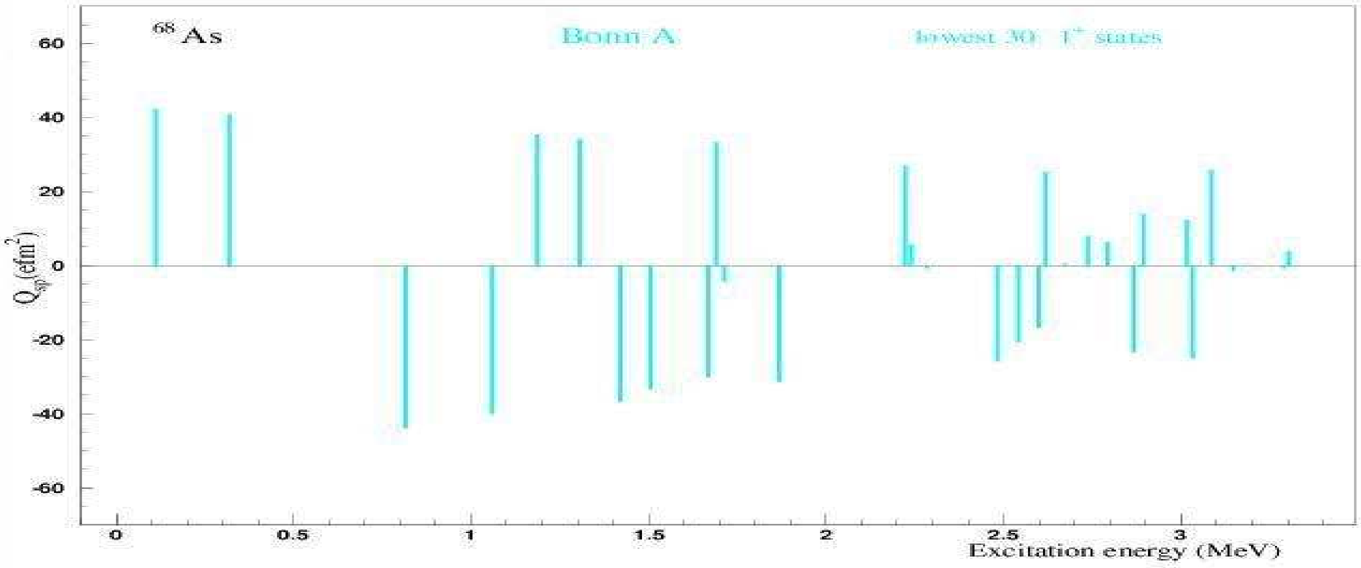
Excited 0^+ states still not identified experimentally.



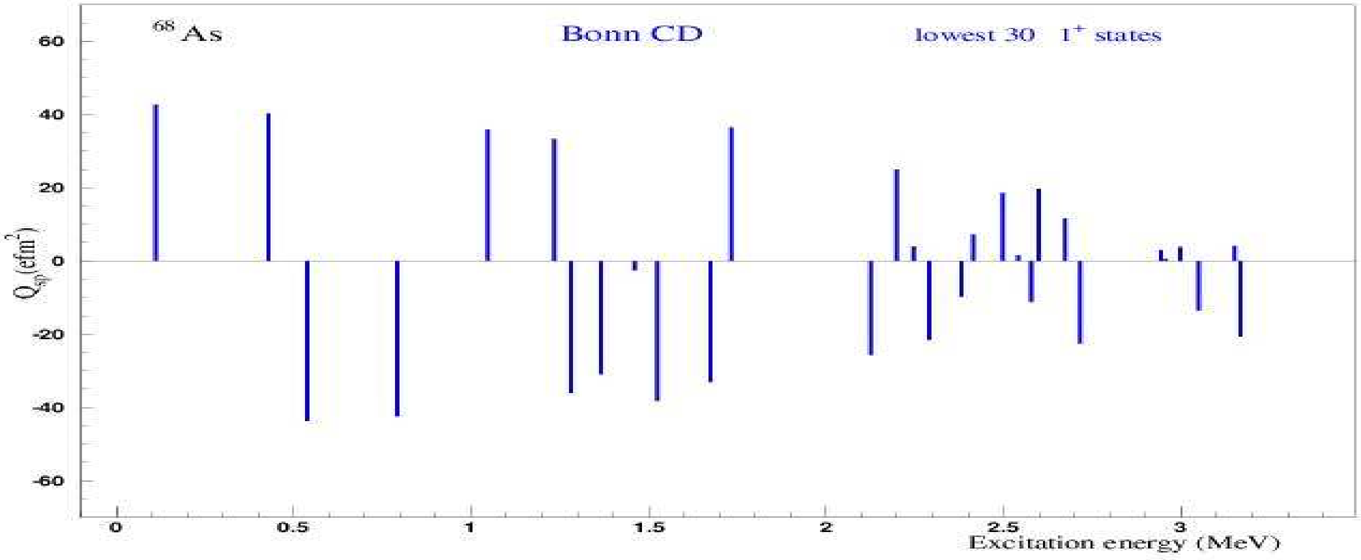
^{68}As (1^+ states)
EXVAM - BonnA
oblate
prolate
Amplitude (%)
100
80
60
40
20
0
I
II
III
IV
V
VI
VII
VIII
IX
X
XI



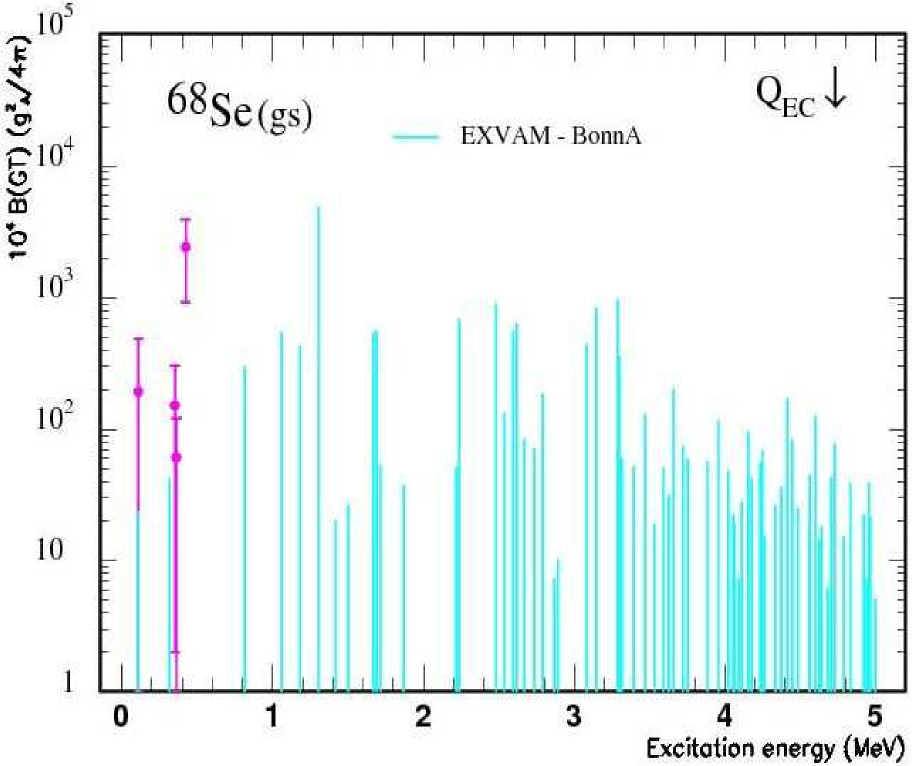
^{68}As (1^+ states)
EXVAM - BonnCD
oblate
prolate
Amplitude (%)
100
80
60
40
20
0
I
II
III
IV
V
VI
VII
VIII
IX
X
XI

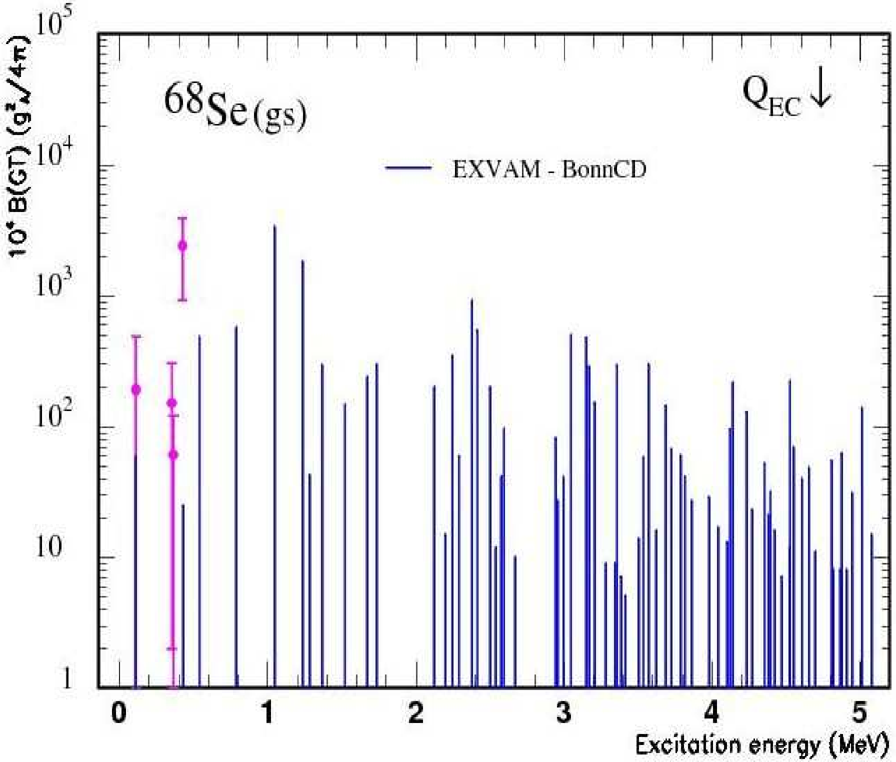


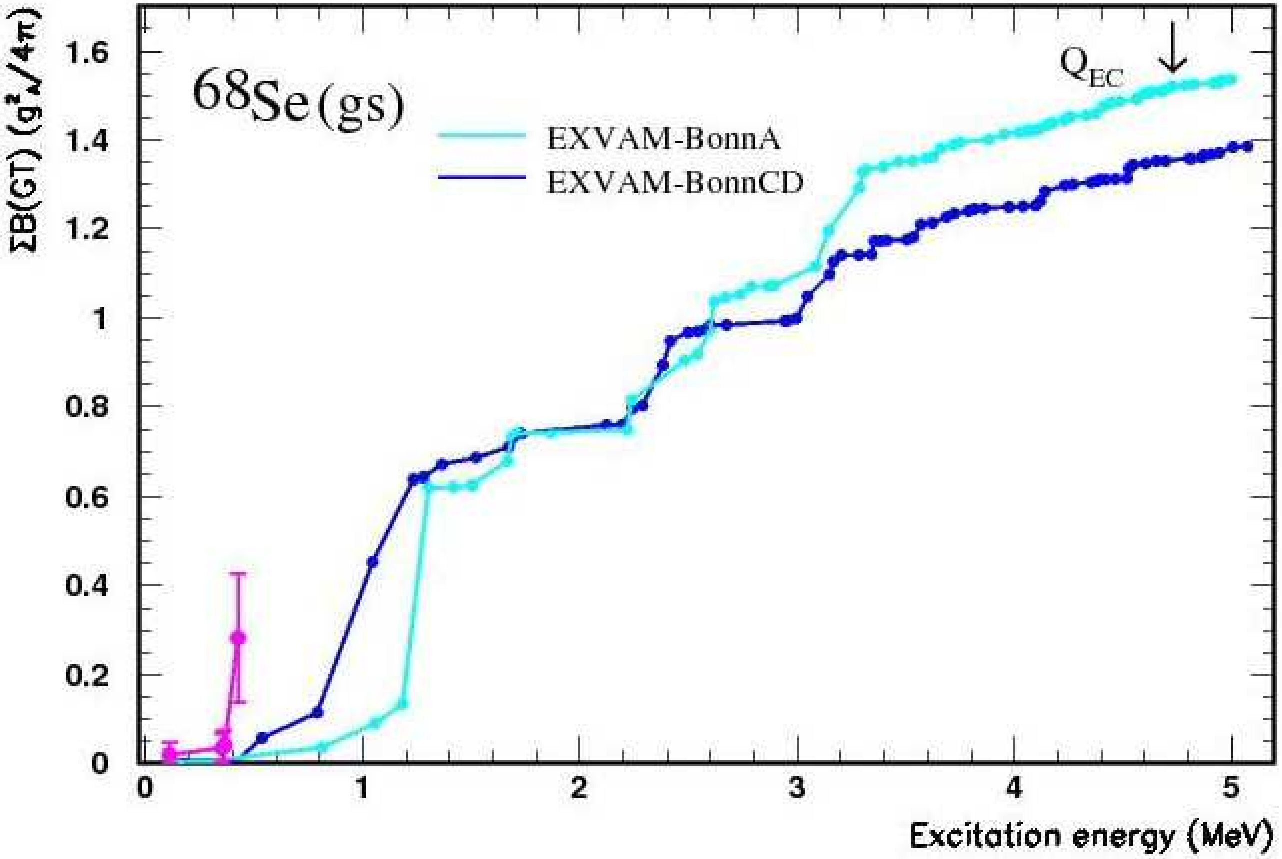
^{68}As
Bonn A
lowest 30 1^+ states
$Q_{sp}(efm^2)$
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
Excitation energy (MeV)



^{68}As
Bonn CD
lowest 30 1^+ states
$Q_{sp}(efm^2)$
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
Excitation energy (MeV)


^{68}Se(gs)
EXVAM - BonnA
$Q_{EC}\downarrow$
10^4 B(GT) ($g_A^2/4\pi$)
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5
Excitation energy (MeV)

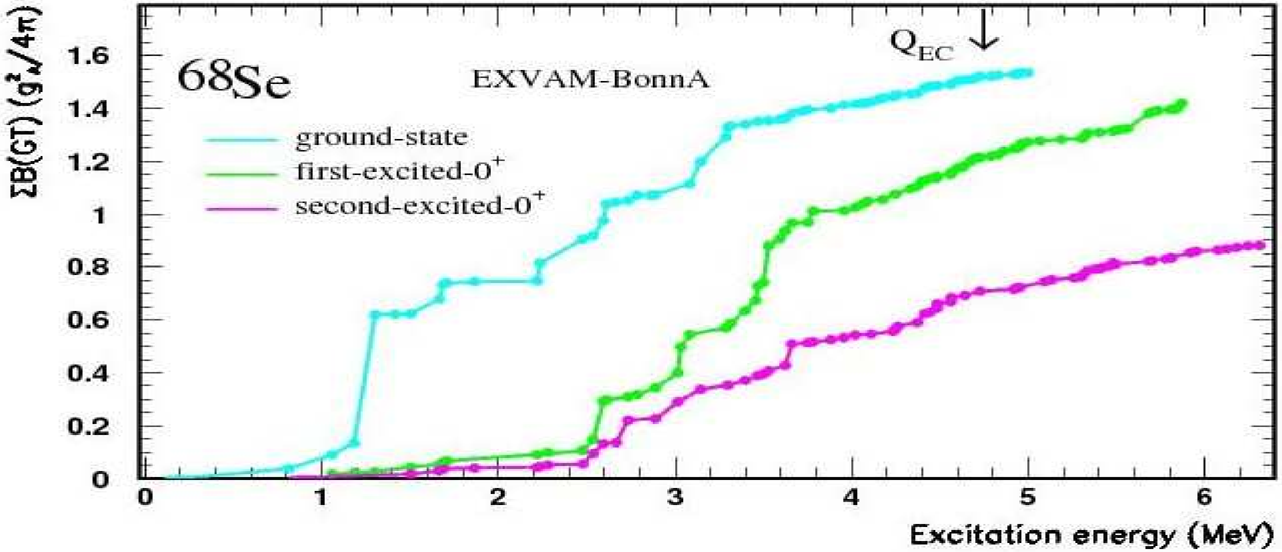

^{68}Se(gs)
EXVAM - BonnCD
$Q_{EC}\downarrow$
10^4 B(GT) ($g_A^2/4\pi$)
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5
Excitation energy (MeV)

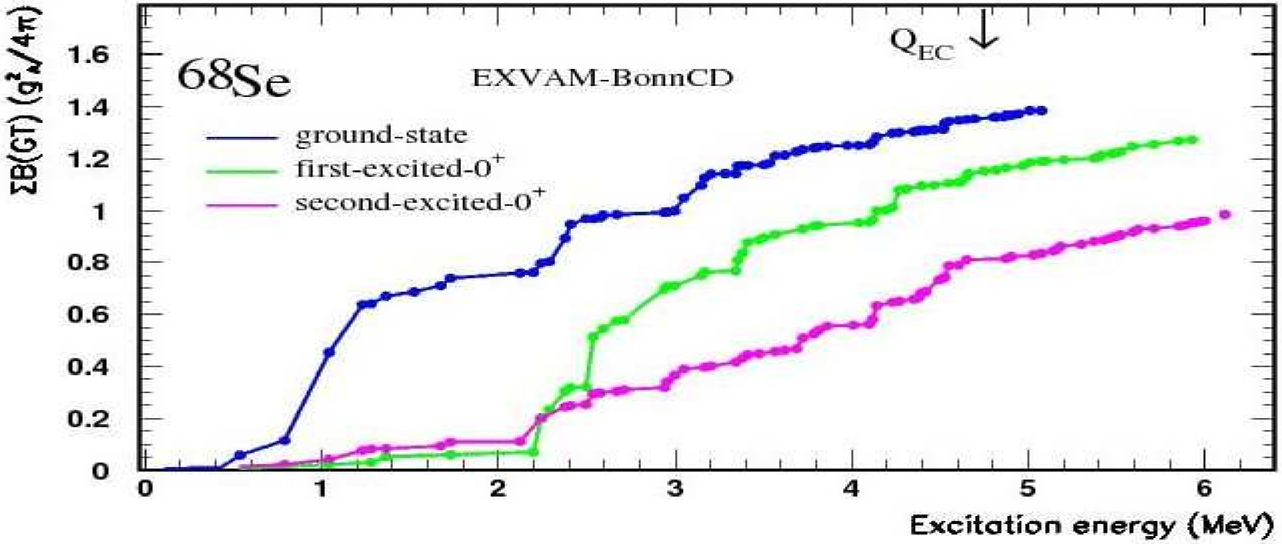


$T_{1/2}^{exp} = 35.5(7)$ s

$T_{1/2}^{BonnCD} = 33.9$ s

$T_{1/2}^{BonnA} = 48.5$ s

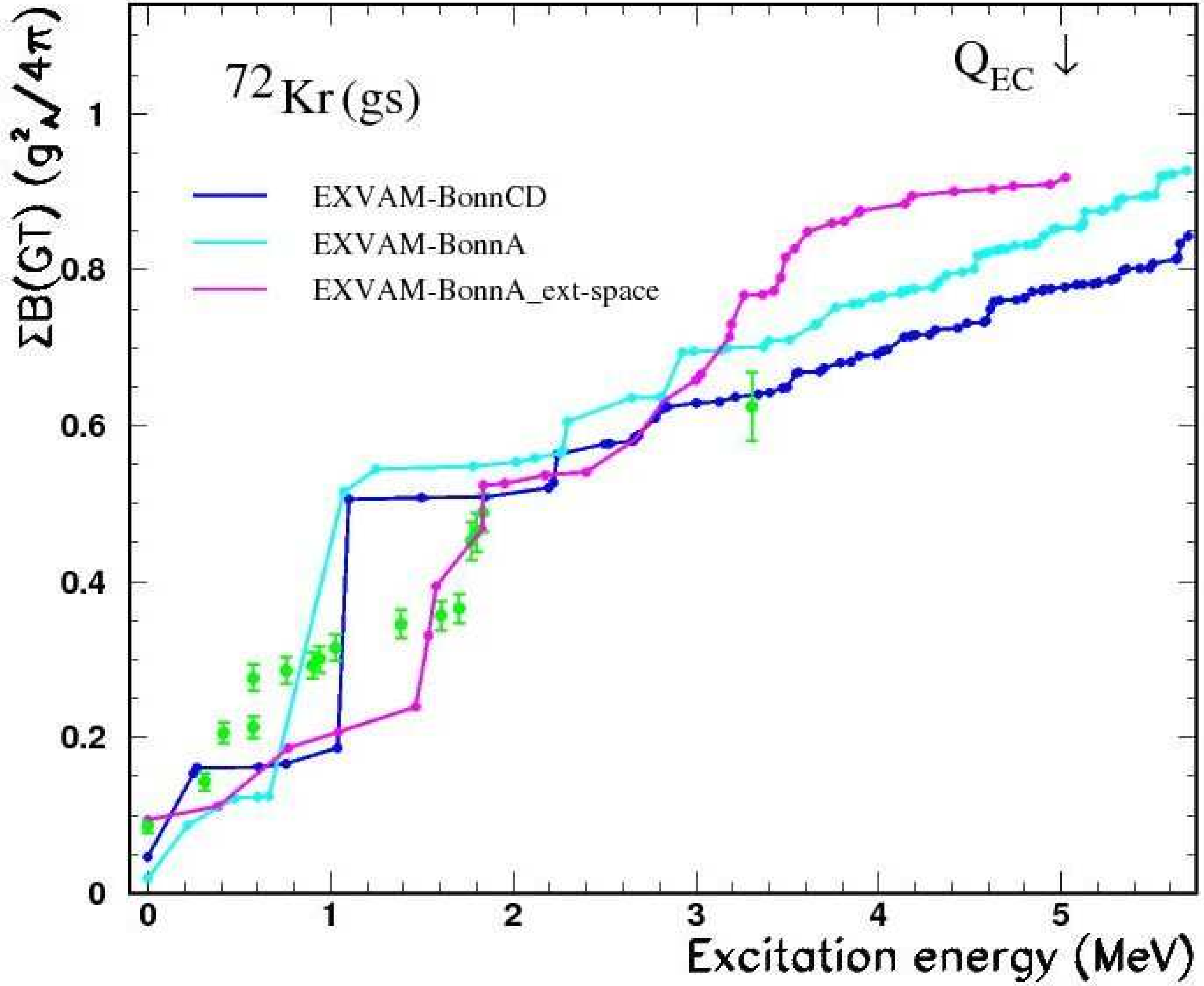




In the astrophysical environment of the X-ray bursts the decay of the isomeric 0^+ states of ^{68}Se will not influence the effective half-life

A. Petrovici et al., Phys. Rev. C80 (2009) 044319

no quenching needed



A. Petrovici, J. Phys.G: Nucl. Part. Phys 37 (2010) 064036

Summary and outlook

complex Excited Vampir *–* ***robust model*** *for describing* ***coexistence phenomena***

- ***strategy*** for calculating the ***isospin-symmetry-breaking effects*** on the superallowed Fermi β decay based on a self-consistent description of the analog and non-analog branches (*experimental* ***branching ratios*** *are needed*)

- trends in ***mirror energy differences*** based on ***different shape mixing of the analog states*** (***precise quadrupole moment*** *measurements are required)*

- ***Gamow-Teller strength distributions in the whole β window*** *are needed* to ***test*** and ***support*** the beyond mean field ***predictions*** required by the ***astrophysical rp-process***

- ***quenching issue*** discussed within the *complex* Excited Vampir model using realistic effective interactions ***in large model spaces*** (all spin-isospin partners included in the basis) requires ***strength distributions over the whole β window***

- in progress: ***systematic investigations*** to improve ***the effective interaction*** in larger model spaces

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