# Aligned neutron-proton pairs in $N=Z$ nuclei

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Motivation

Spin-aligned $T=0$ np pairs in $N=Z$ nuclei

A boson model with aligned $T=0$ np pairs

*Neutron-deficient nuclei, Valencia, February 2011*

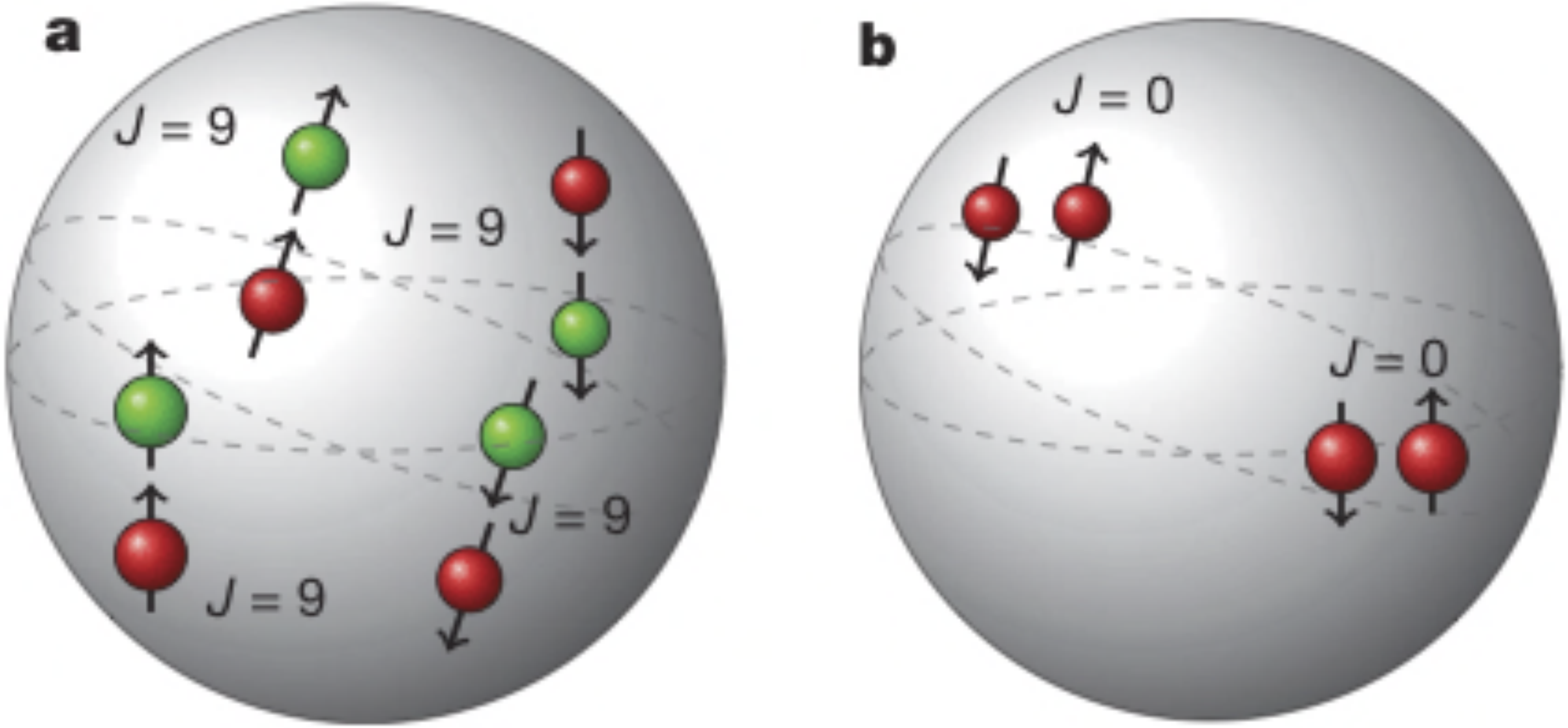
# Spin-aligned $T$=0 np pairs

Motivation: A simple description of $N=Z$ nuclei in the $1g_{9/2}$ shell ($^{98}$In, $^{96}$Cd, $^{94}$Ag, $^{92}$Pd, $^{90}$Rh).

Starting point: Shell-model interpretation in terms of spin-aligned $T$=0 np pairs (Blomqvist).

Experiments have been proposed and carried out at GANIL (Cederwall, de France,…).

# Nuclear belly dancer




B. Cederwall *et al.*, Nature **469** (2011) 68

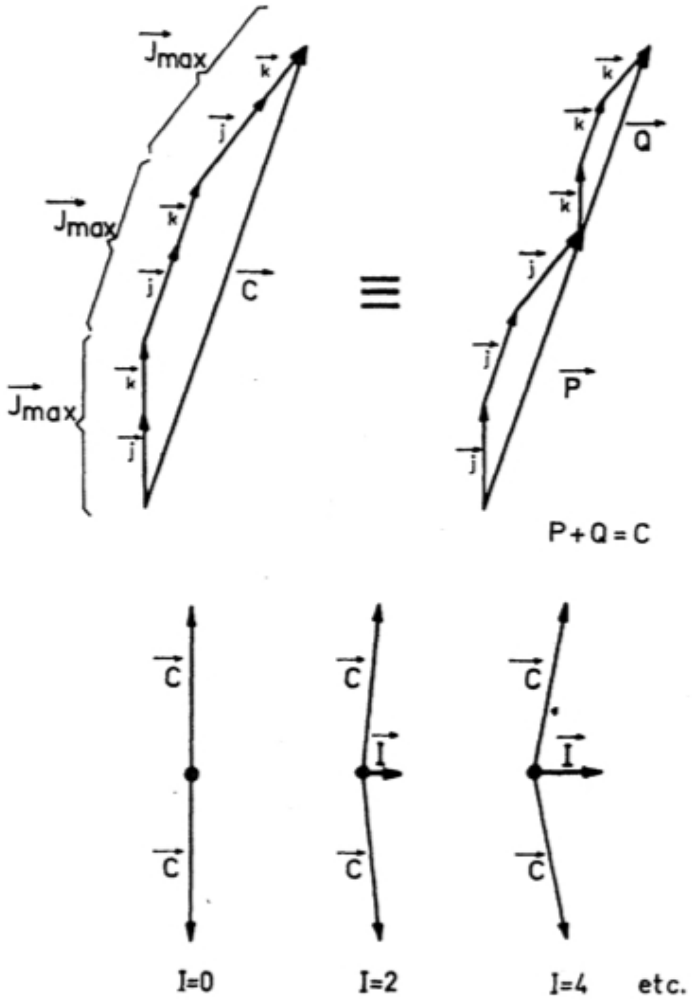
# The stretch scheme

A model for even-even nuclei.

For $^{96}$Cd: C is the $9^+$ np pair.

For other *N*=*Z* nuclei?

Approximation: “neglect of the anti-symmetrization between the two [C] chains”



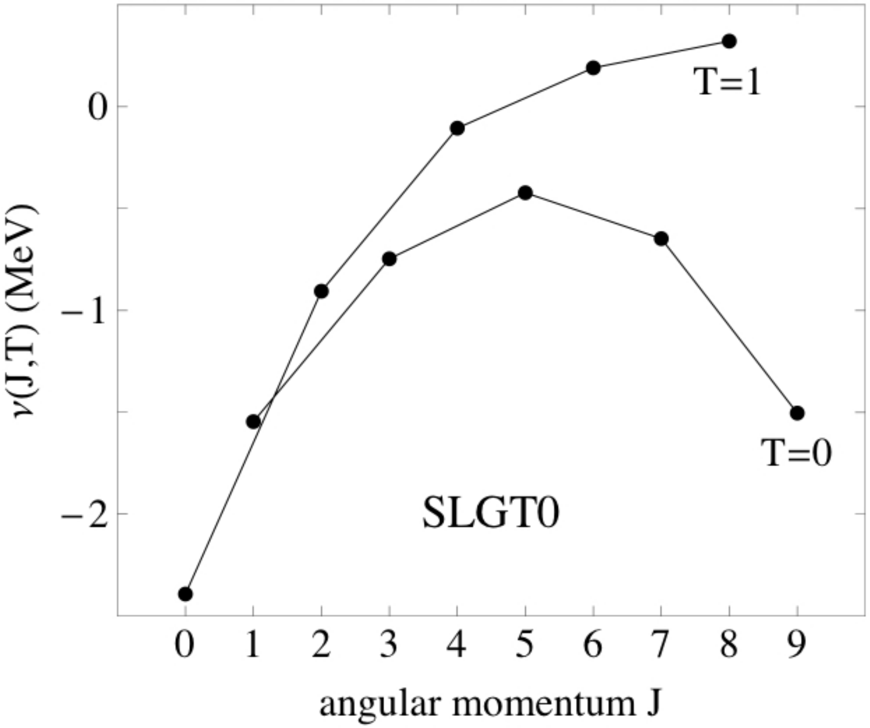

M. Danos and V. Gillet, Phys. Rev. **161** (1967) 1034

# Shell-model interaction

Assume nucleons in the $1g_{9/2}$ shell (i.e., don't get too close to $^{80}Zr$).

Take $1g_{9/2}+2p_{1/2}$ interaction SLGT0 and renormalize to $1g_{9/2}$.

Perform shell-model calculation and analyze wave functions.





E.J.D. Serduke *et al.*, Nucl. Phys. A **256** (1976) 45

H. Herndl and B.A. Brown, Nucl Phys. A **627** (1997) 35

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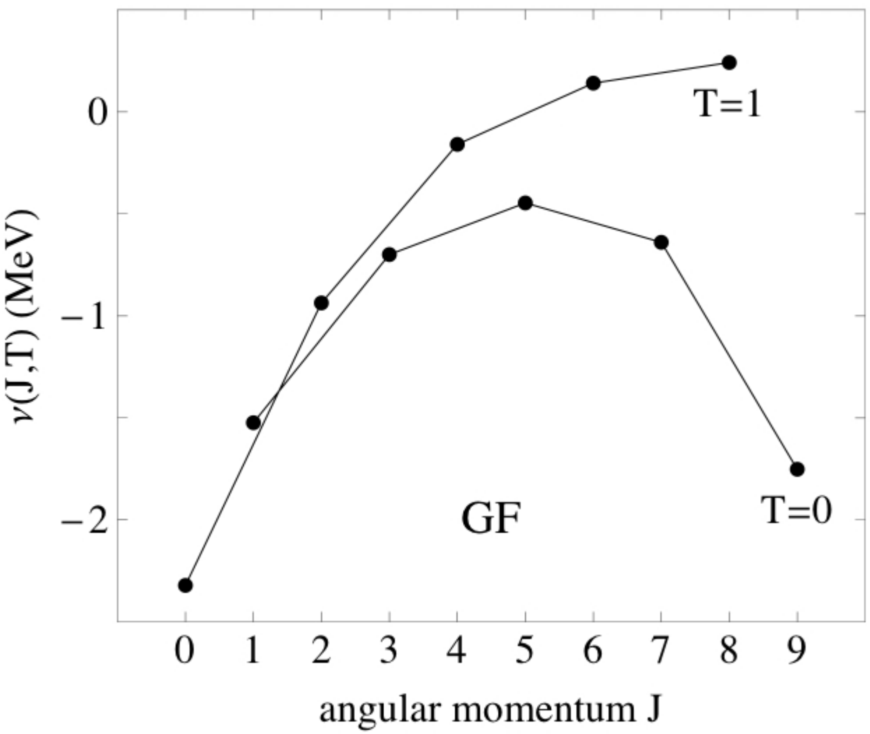


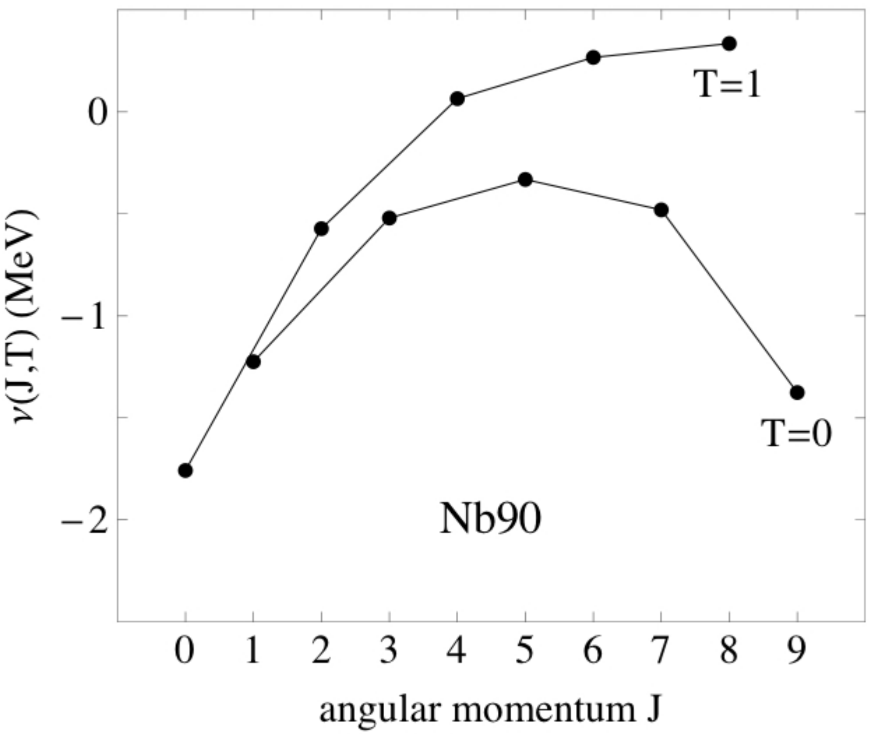

R. Gross and A. Frenkel, Nucl. Phys. A **267** (1976) 85

# Shell-model interaction

Assume nucleons in the $1g_{9/2}$ shell (i.e., don't get too close to $^{80}$Zr).

Derive hole-hole interaction from particle-hole spectrum of $^{90}$Nb.

Perform shell-model calculation and analyze wave functions.



# Structure of $^{96}$Cd

Define different types of nucleon pairs:

$$B^+_{JT} = \left(a^+_{9/2} \times a^+_{9/2}\right)^{(JT)}$$

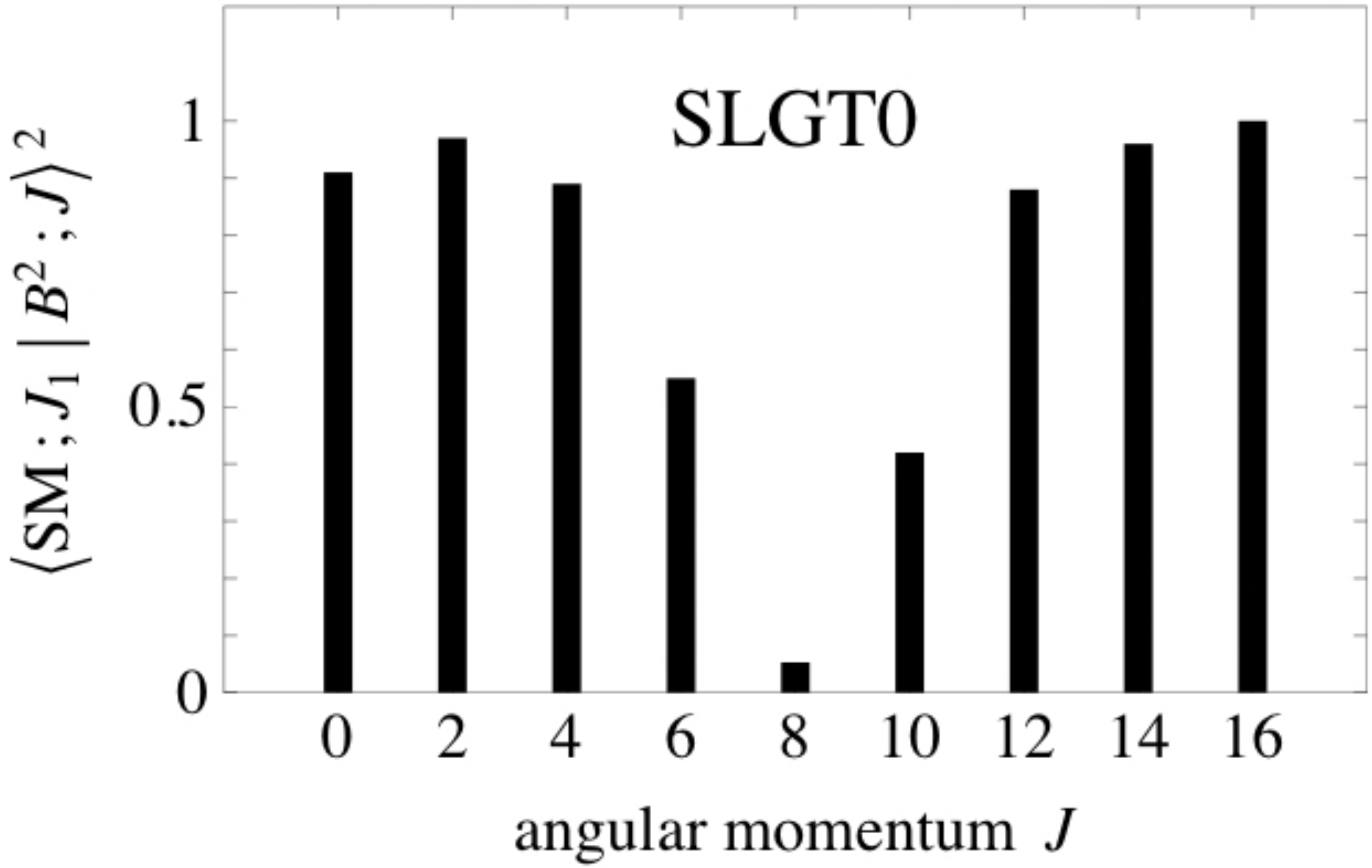
$$S^+ : J = 0, T = 1; \quad D^+ : J = 2, T = 1; \quad B^+ : J = 9, T = 0.$$

Calculate overlap with shell-model wave functions.
For example, for SLGT0

$$\left\langle 0^+_1 \middle| S^2;0 \right\rangle^2 = 0.80; \quad \left\langle 0^+_1 \middle| D^2;0 \right\rangle^2 = 0.35; \quad \left\langle 0^+_1 \middle| B^2;0 \right\rangle^2 = 0.91.$$

$$\left\langle 2^+_1 \middle| SD;2 \right\rangle^2 = 0.85; \quad \left\langle 2^+_1 \middle| D^2;2 \right\rangle^2 = 0.17; \quad \left\langle 2^+_1 \middle| B^2;2 \right\rangle^2 = 0.97.$$

# Structure of $^{96}$Cd



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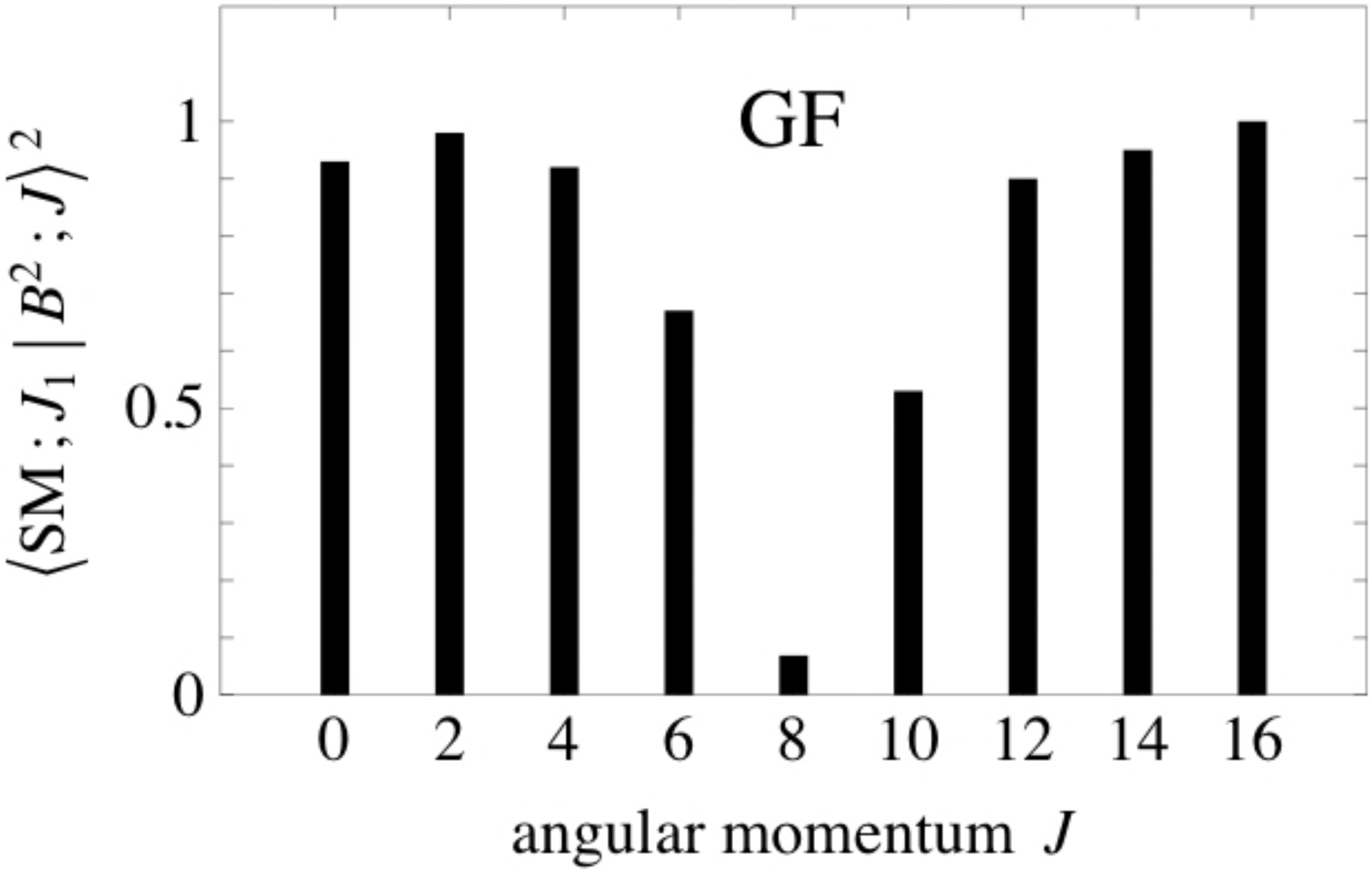

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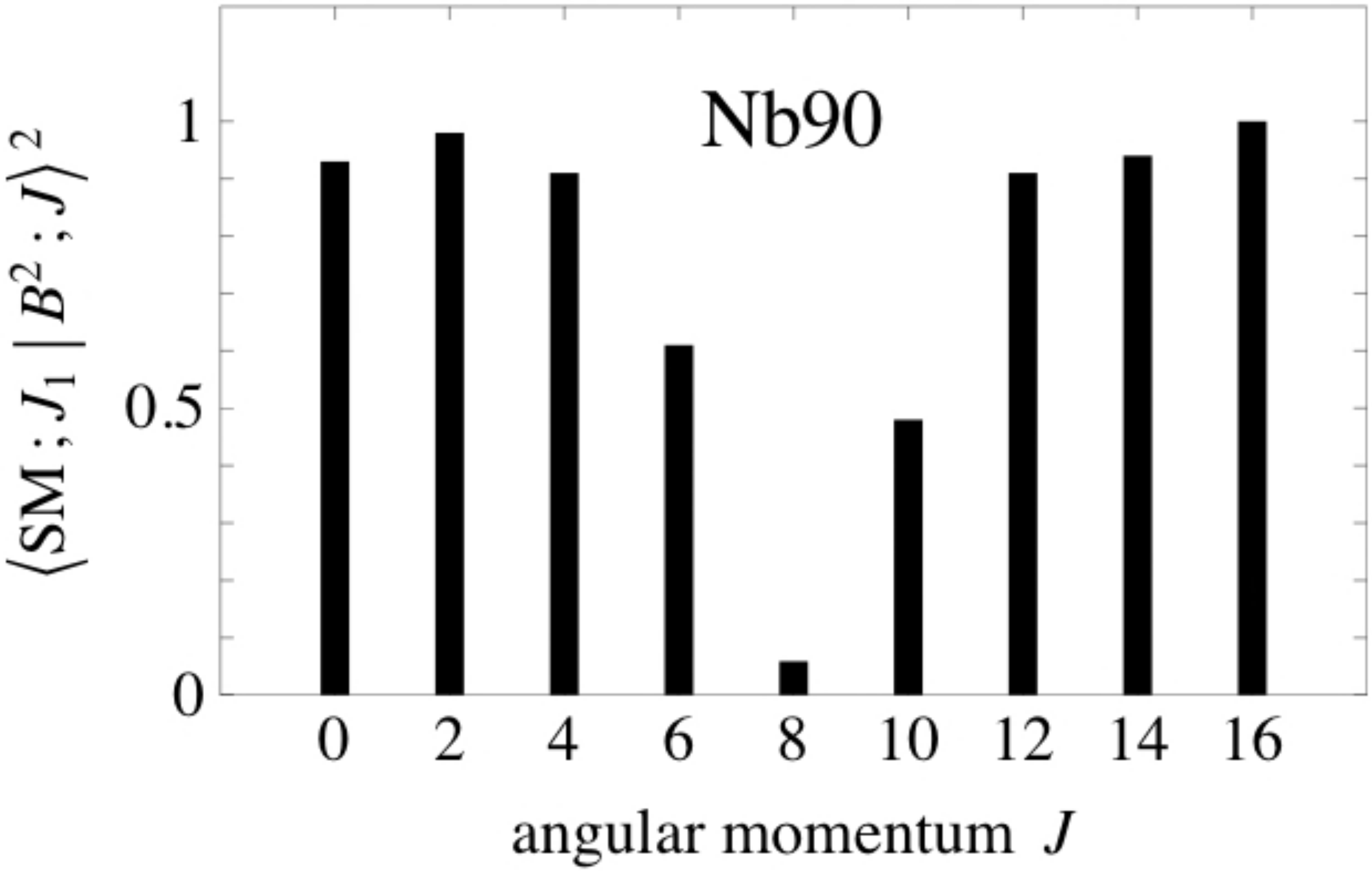


# Structure of $^{94}$Ag and $^{92}$Pd

Ideally, one needs overlaps of the type

$$^{94}\mathrm{Ag}: \left\langle 7_1^+ \middle| (B \times B)^{(L)} \times B; 7 \right\rangle = ?$$

$$^{92}\mathrm{Pd}: \left\langle 0_1^+ \middle| (B \times B)^{(L)} \times (B \times B)^{(L)}; 0 \right\rangle = ?$$

That's hard. One needs the nucleon-pair shell model in an isospin-invariant formulation.

# Mapping to bosons

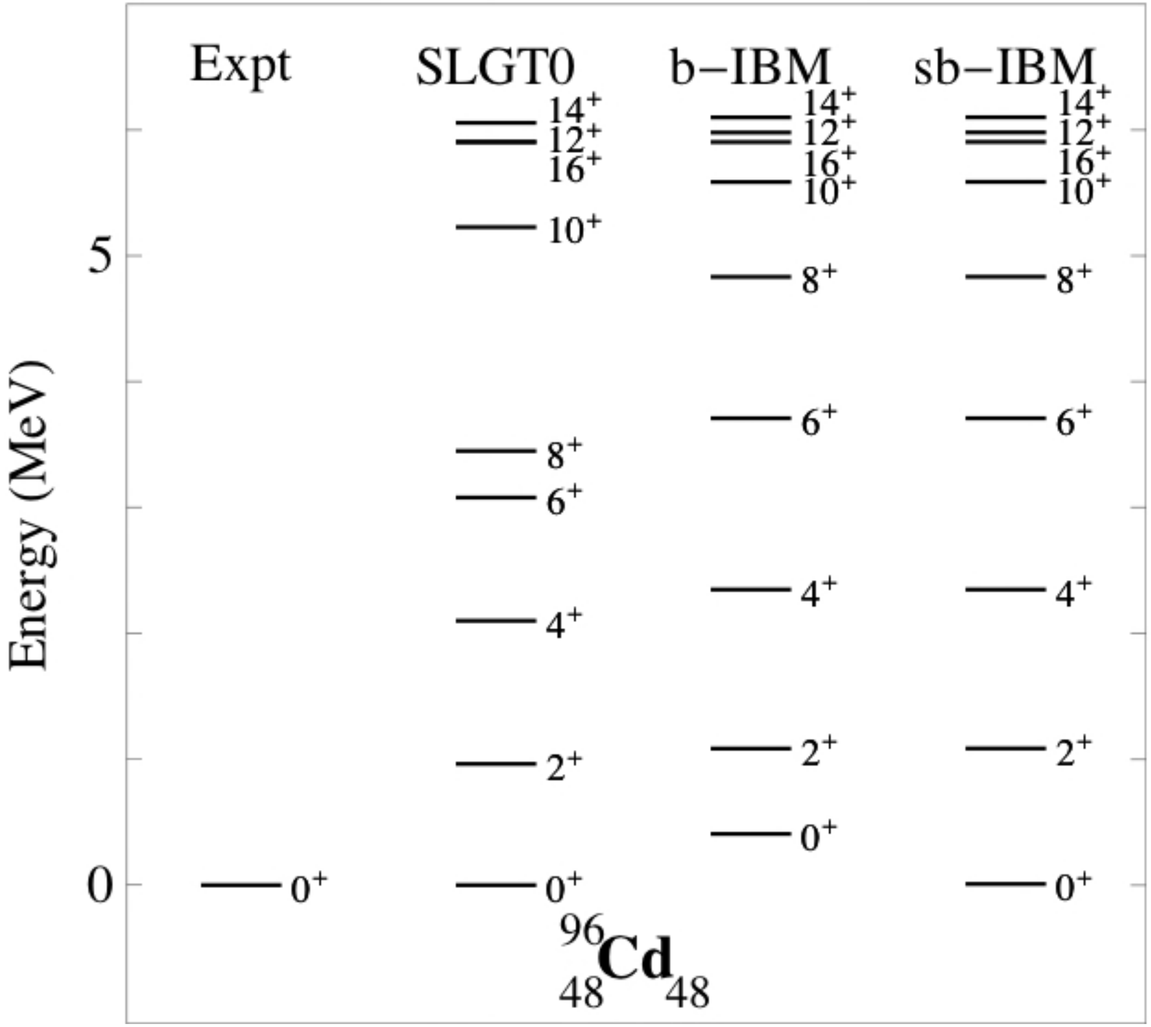
Transform to a much simpler problem in terms of interacting bosons: $B^{+}_{JT} \to b^{+}_{JT}$

We find that an adequate description of $^{94}$Ag and $^{92}$Pd is obtained with two types of bosons:

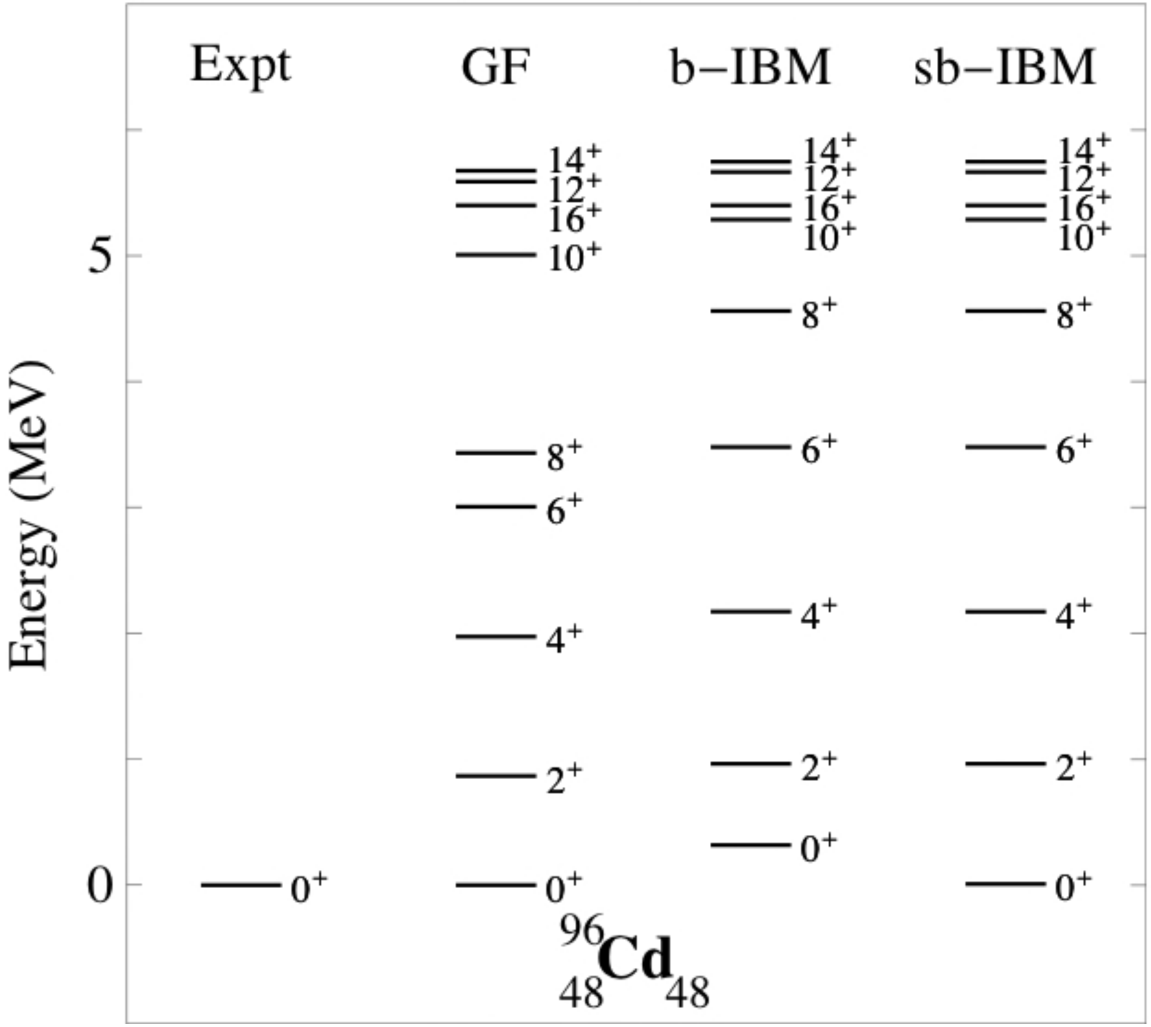
$B^{+} \to b^{+}; \quad S^{+} \to s^{+}$

Boson energies and boson-boson interactions are derived from the shell model (i.e., *not* fitted).

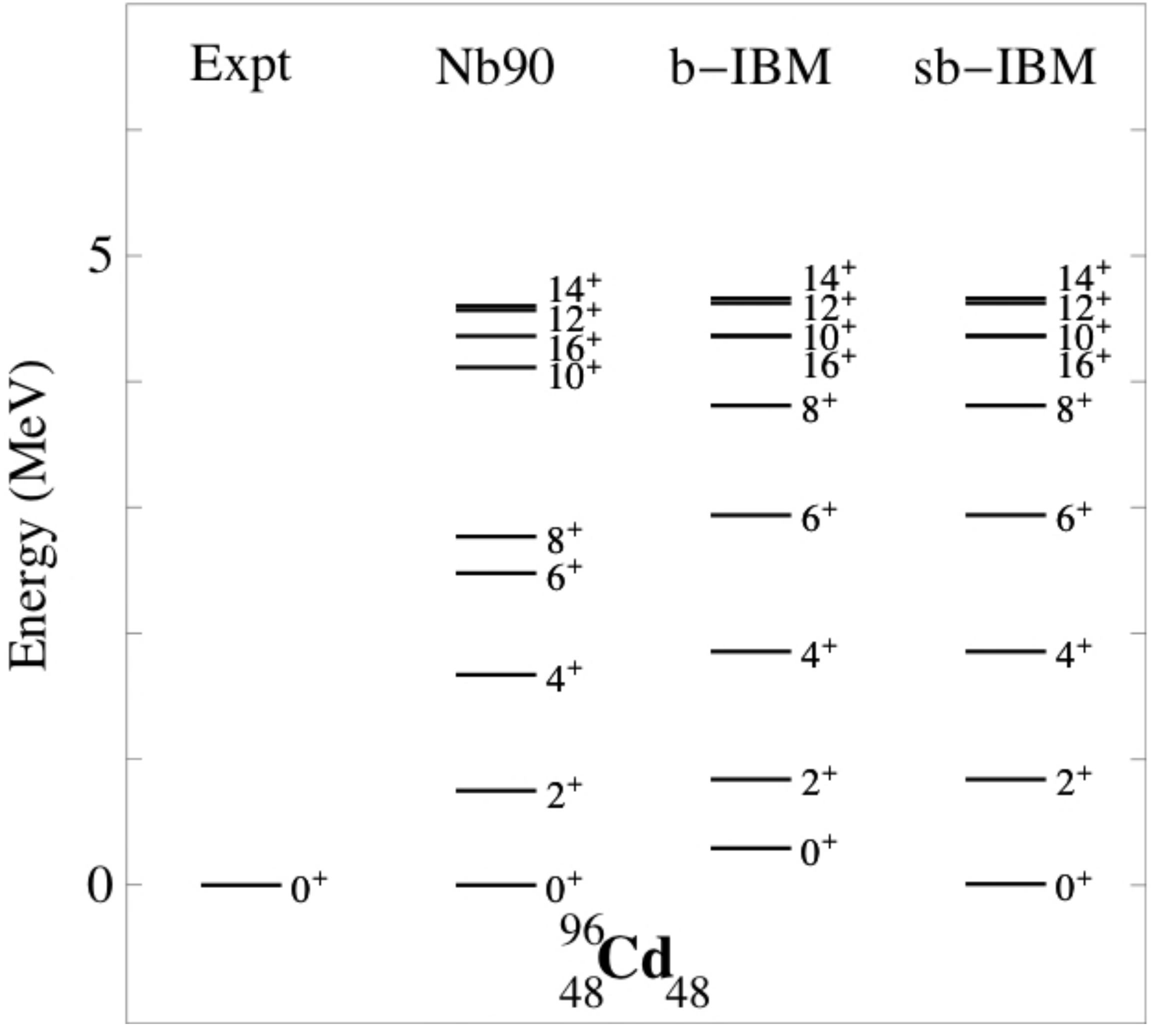
# Spectrum of $^{96}Cd$



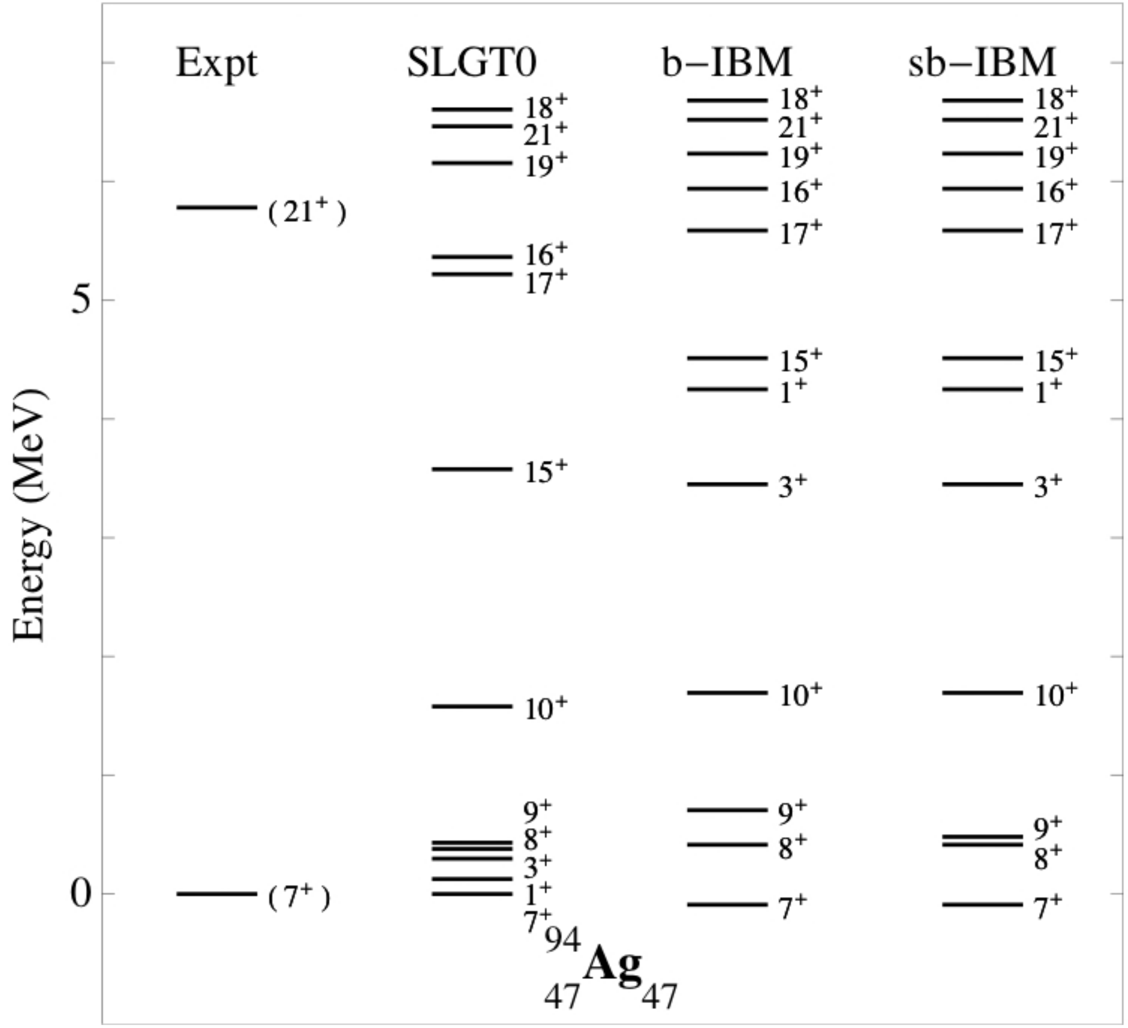
# Spectrum of $^{96}$Cd



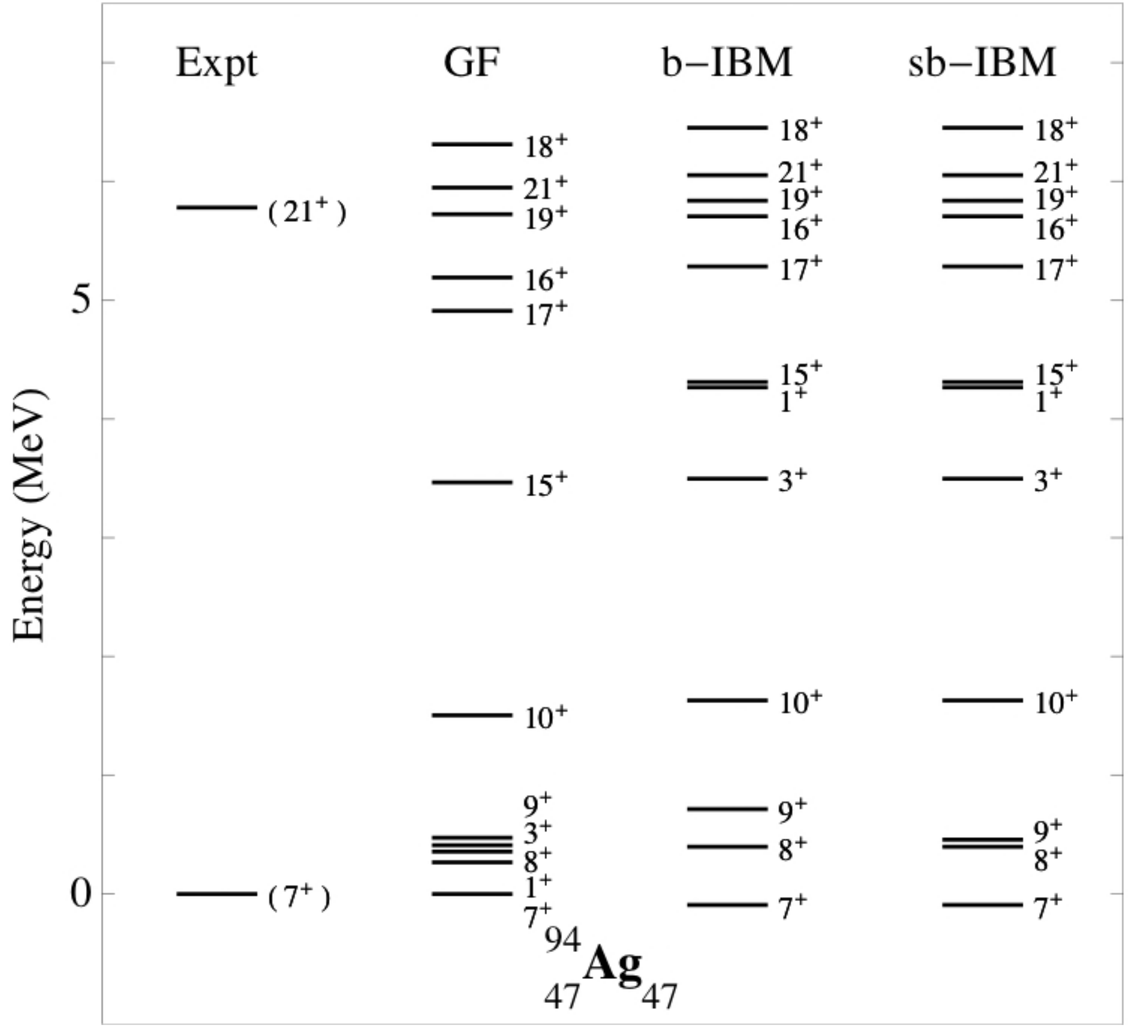
# Spectrum of $^{96}$Cd



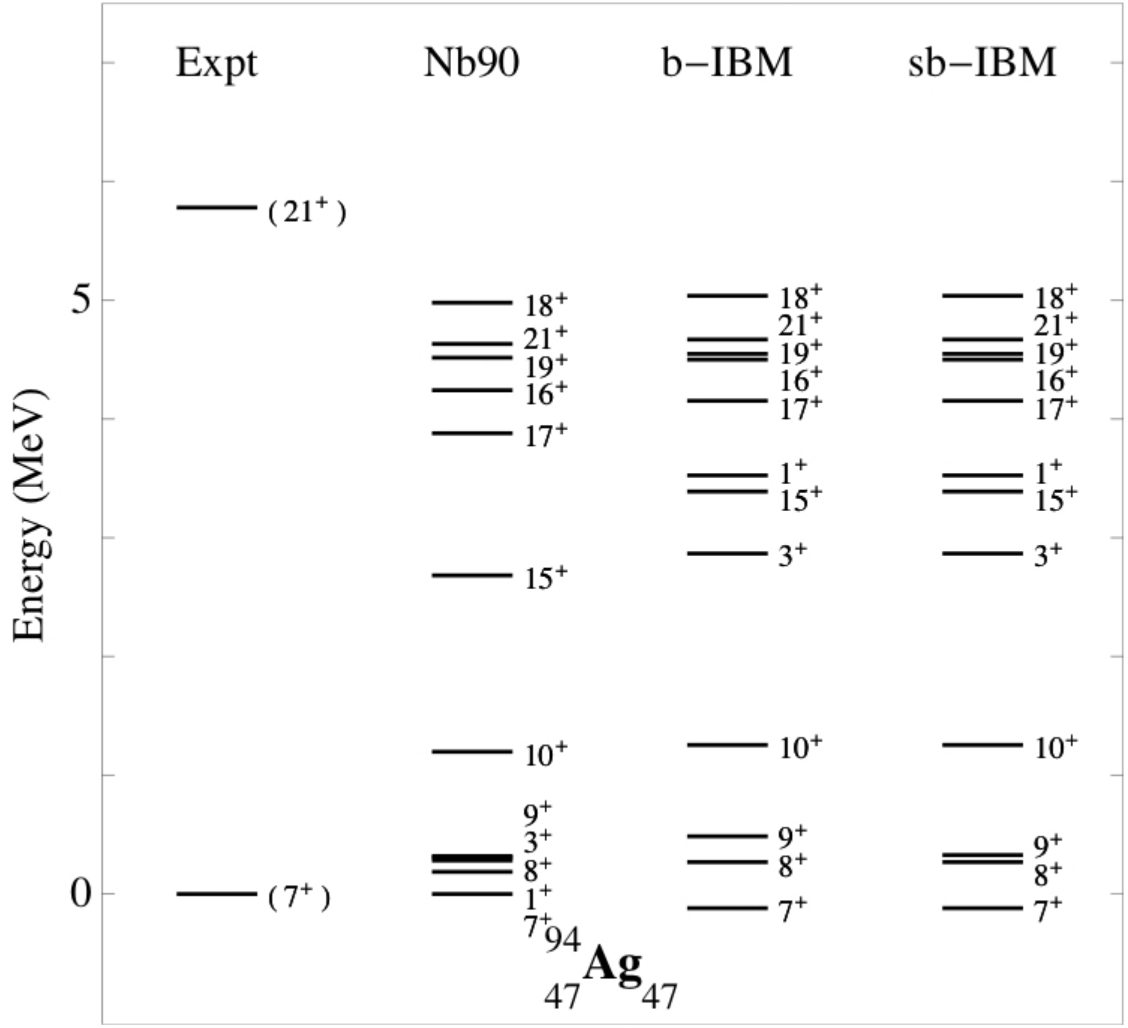
# Spectrum of $^{94}Ag$



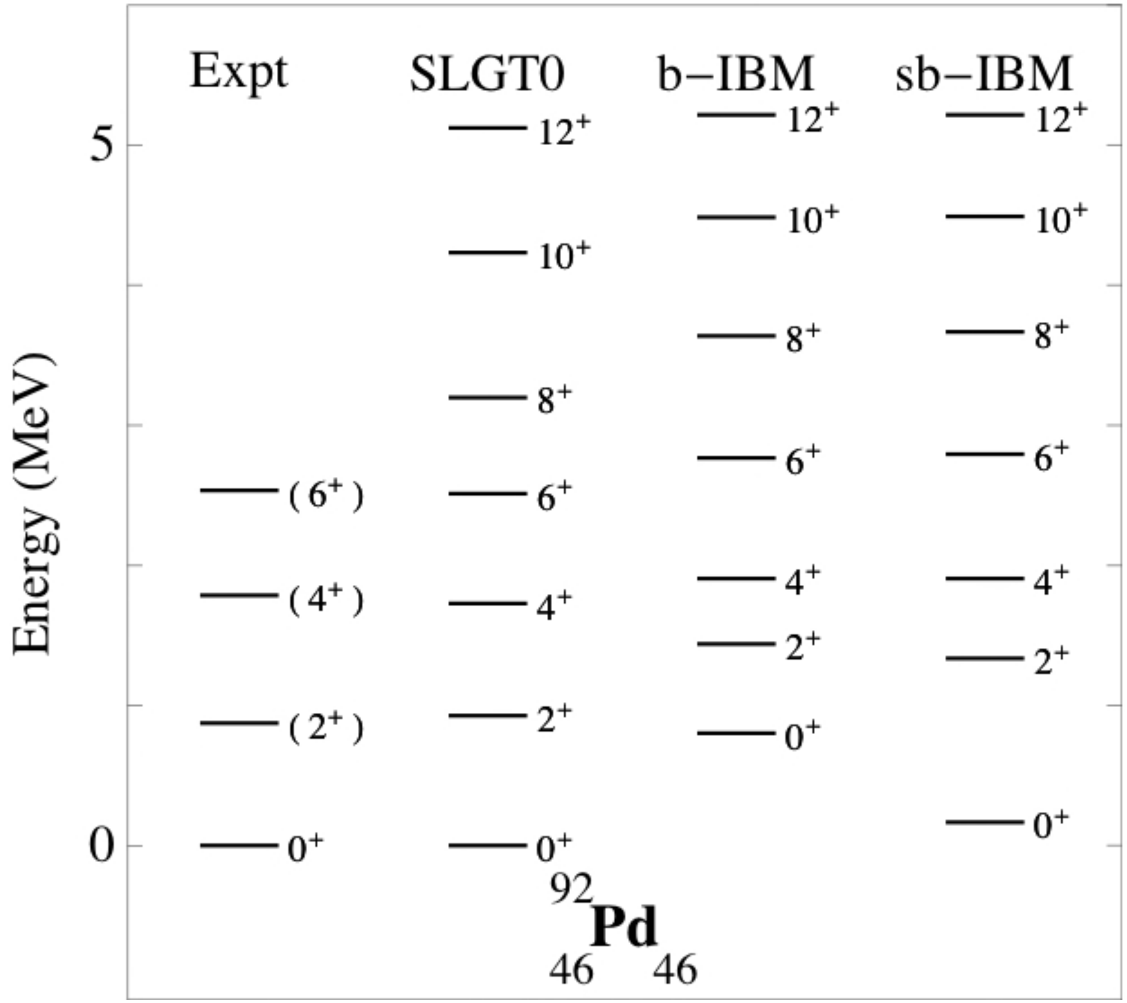
# Spectrum of $^{94}Ag$



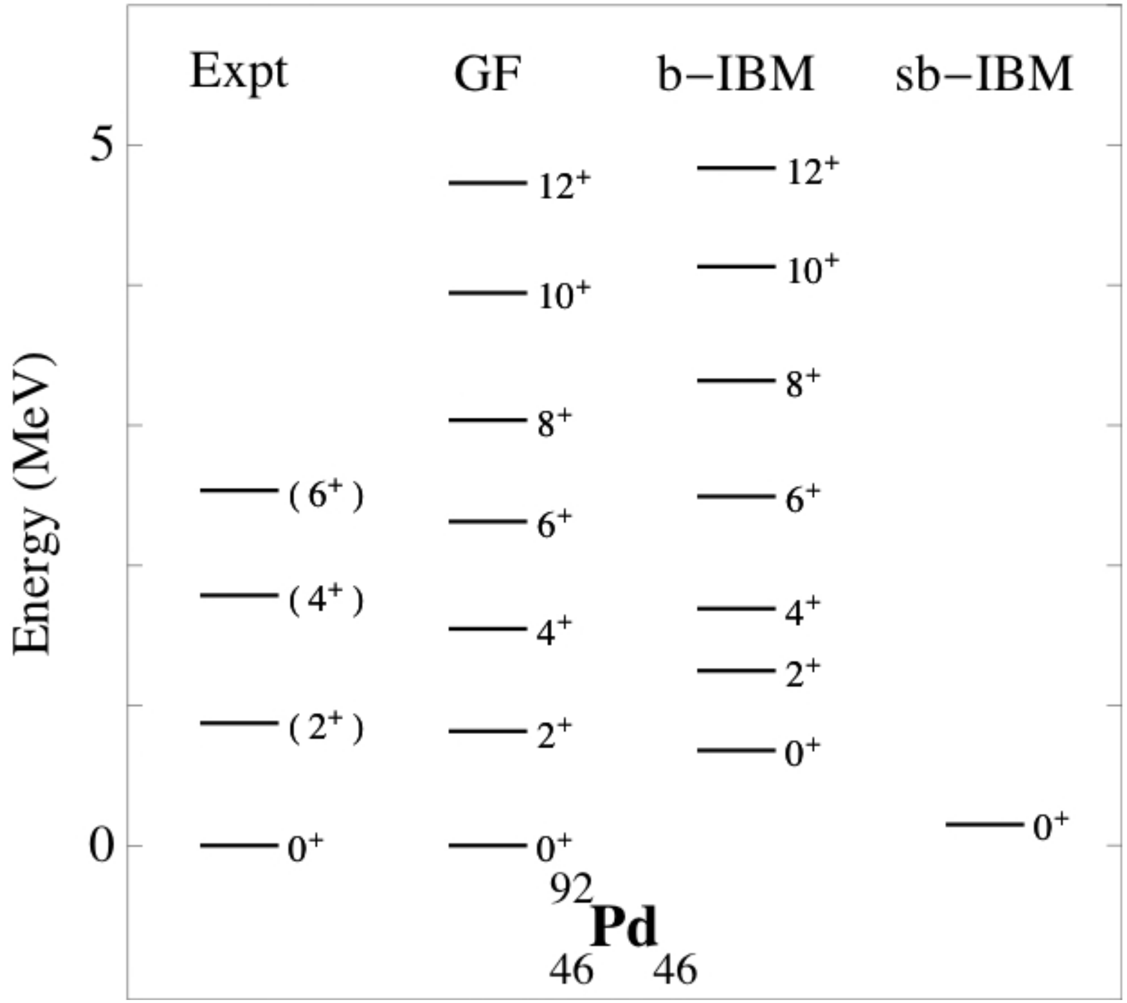
# Spectrum of $^{94}Ag$



# Spectrum of $^{92}$Pd



# Spectrum of $^{92}$Pd



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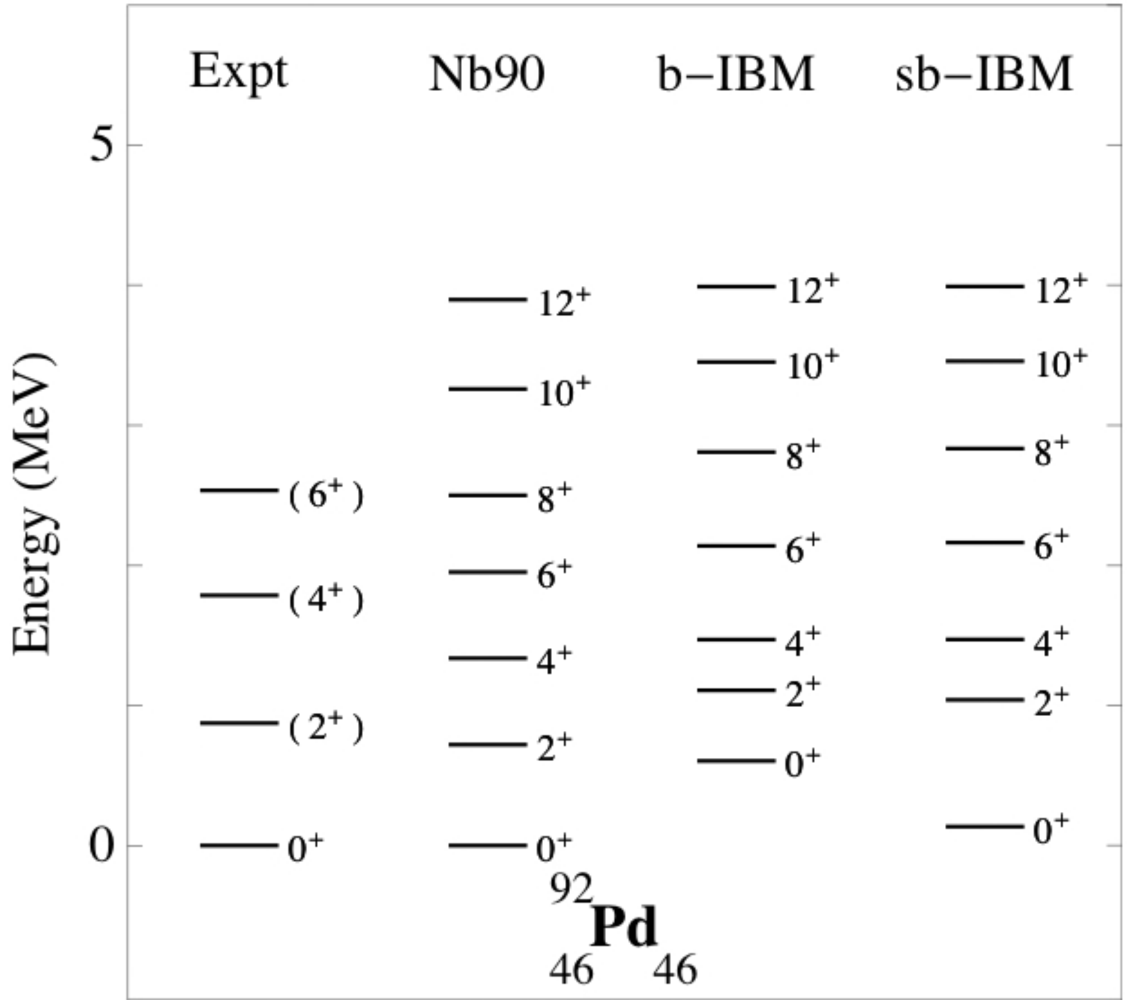


# E2 properties

The E2 operator in the shell model:

$$\hat{T}^{\mathrm{F}}_{\mu}(\mathrm{E2}) = e_{\nu}\sum_{i\in\nu} r_i^2 Y_{2\mu}(\theta_i,\phi_i) + e_{\pi}\sum_{i\in\pi} r_i^2 Y_{2\mu}(\theta_i,\phi_i)$$

The E2 operator in terms of *b* bosons:

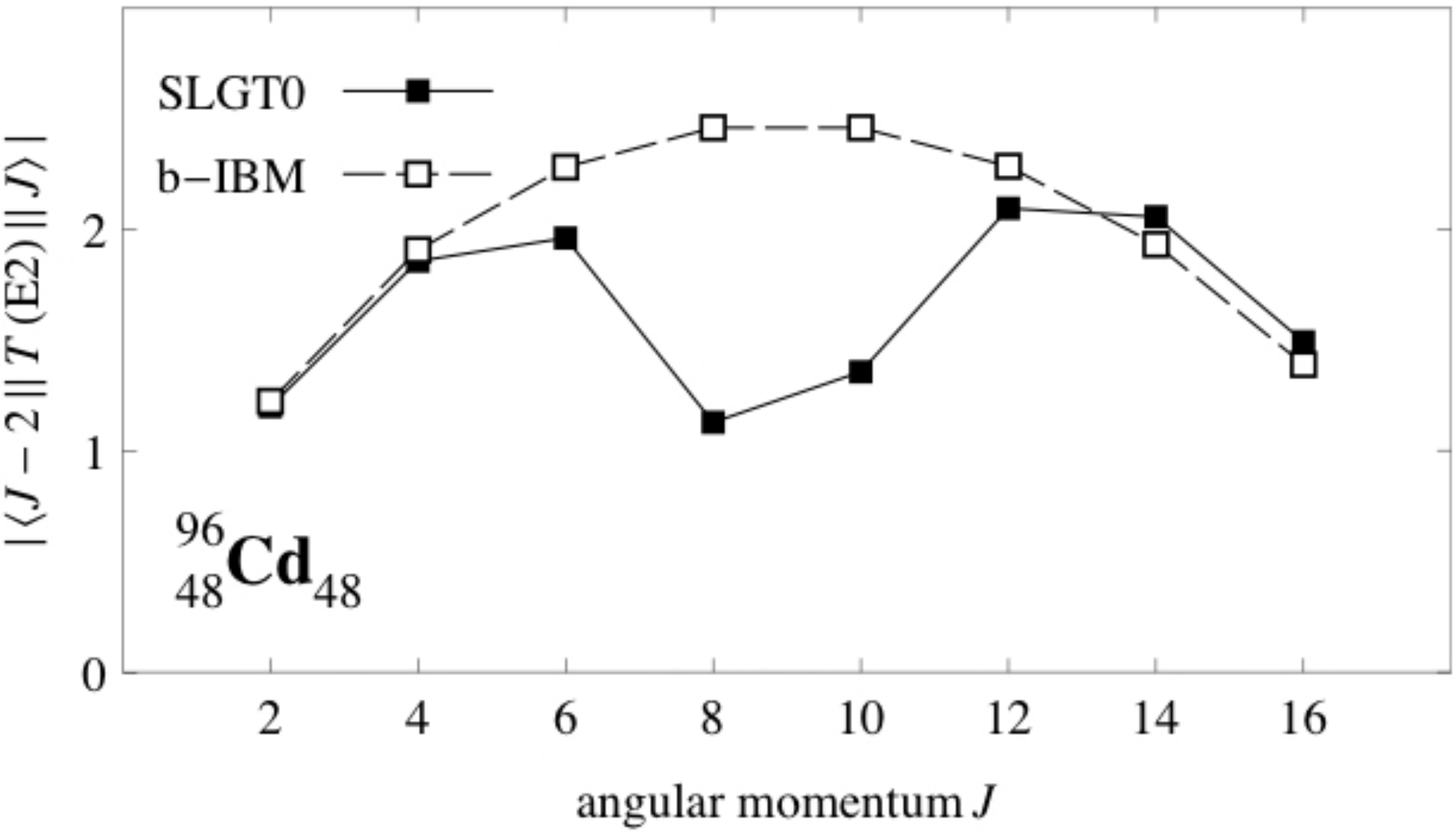
$$\hat{T}^{\mathrm{B}}_{\mu}(\mathrm{E2}) = e_{\mathrm{b}}\left(b^{+}\times\tilde{b}\right)^{(2)}_{\mu}$$

The mapping implies the relation:

$$\left\langle (g_{9/2})^2;9^+ \left\| \hat{T}^{\mathrm{F}}(\mathrm{E2}) \right\| (g_{9/2})^2;9^+ \right\rangle = \left\langle b \left\| \hat{T}^{\mathrm{B}}(\mathrm{E2}) \right\| b \right\rangle$$

$$\Rightarrow e_{\mathrm{b}} = -\sqrt{\frac{55}{3\pi}}\left(\ell_{\mathrm{ho}}\right)^2 \times \sqrt{\frac{266}{187}}\left(e_{\nu}+e_{\pi}\right)$$

# B(E2) values in $^{96}$Cd

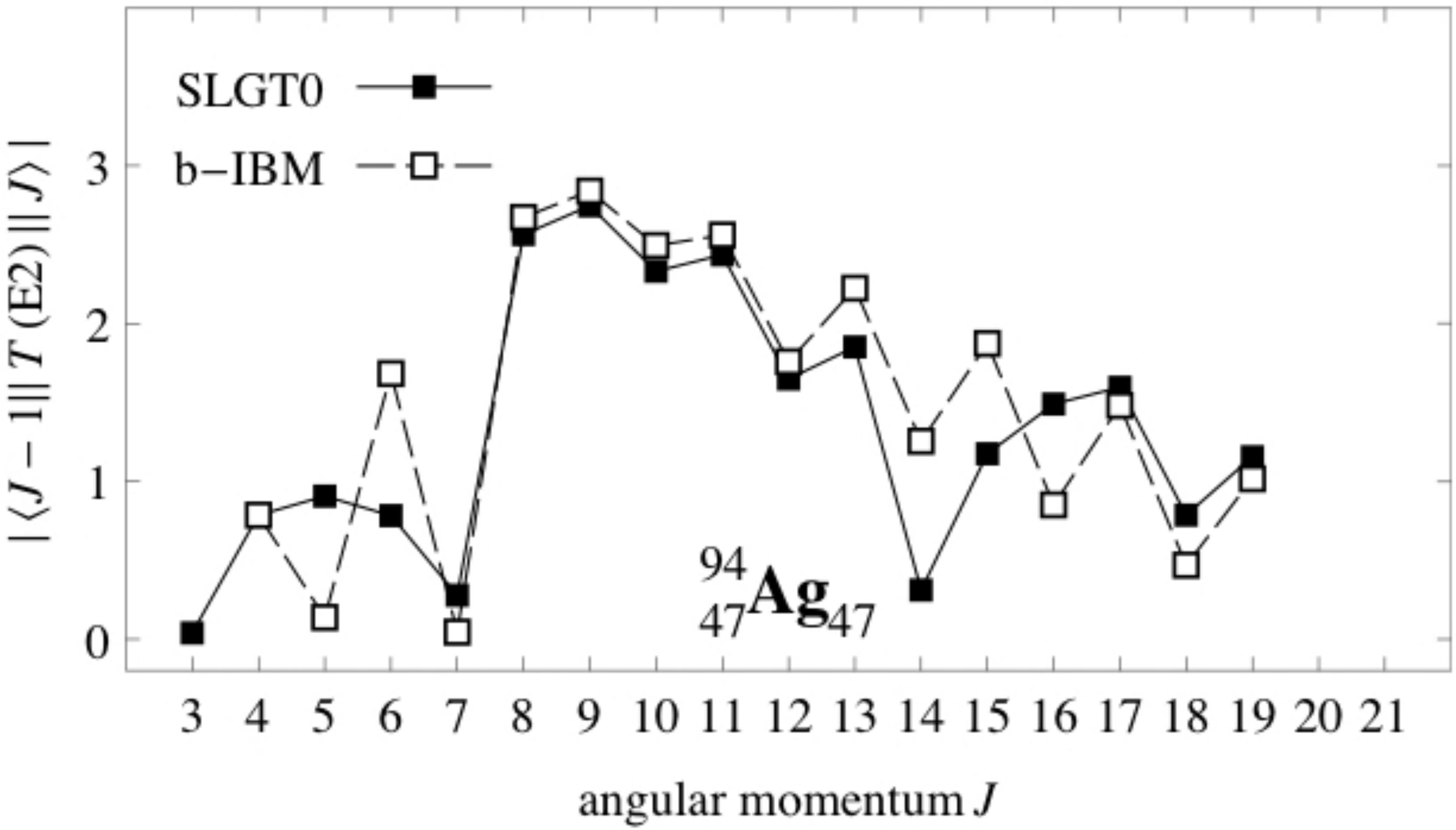


# B(E2) values in $^{96}$Cd

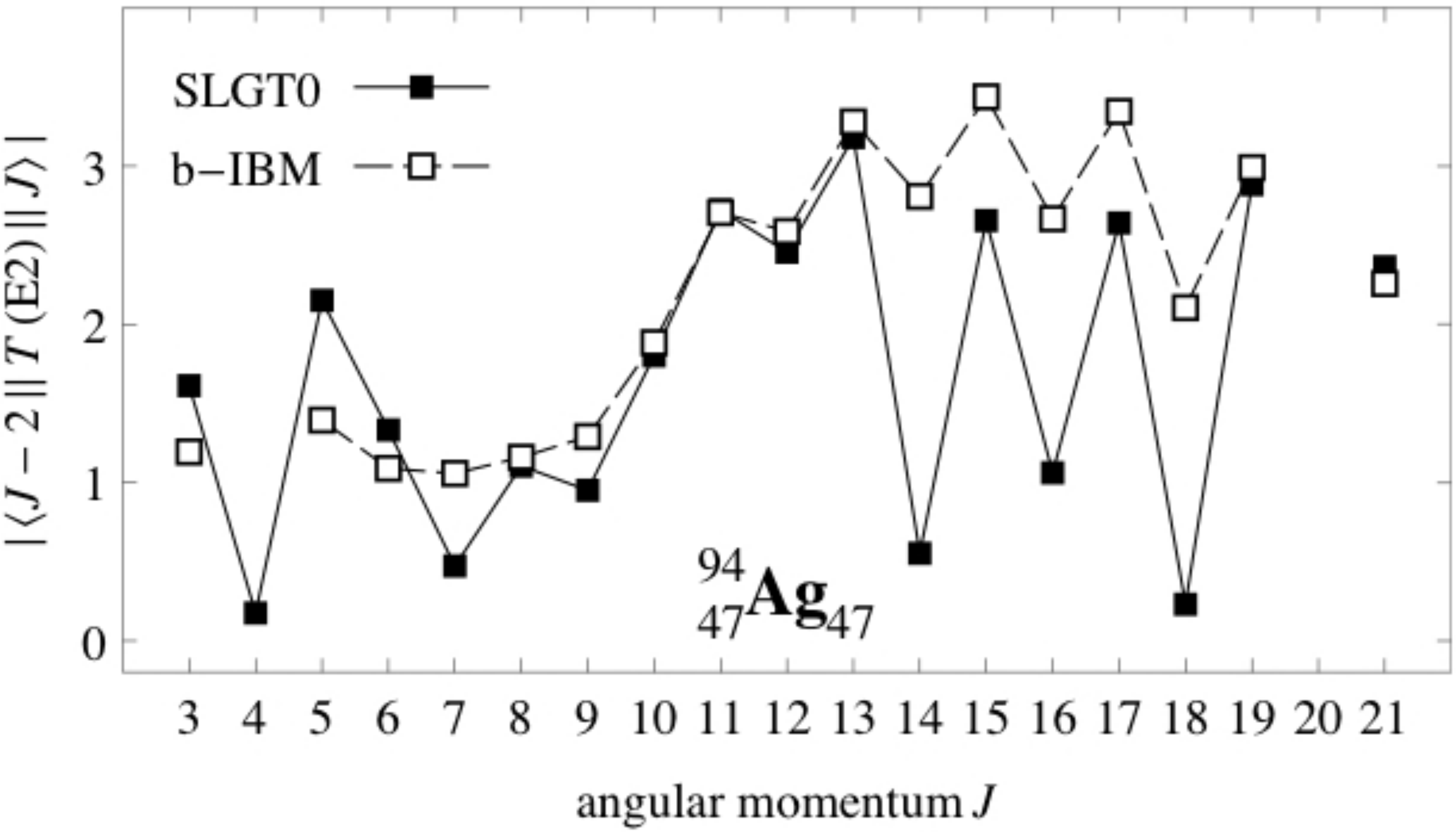
A simple consequence of the aligned-pair assumption:

$$B(\mathrm{E2}; J \to J') = e_{\mathrm{b}}^2 20(2J'+1) \begin{Bmatrix} 9 & 9 & 2 \\ J & J' & 9 \end{Bmatrix}^2$$

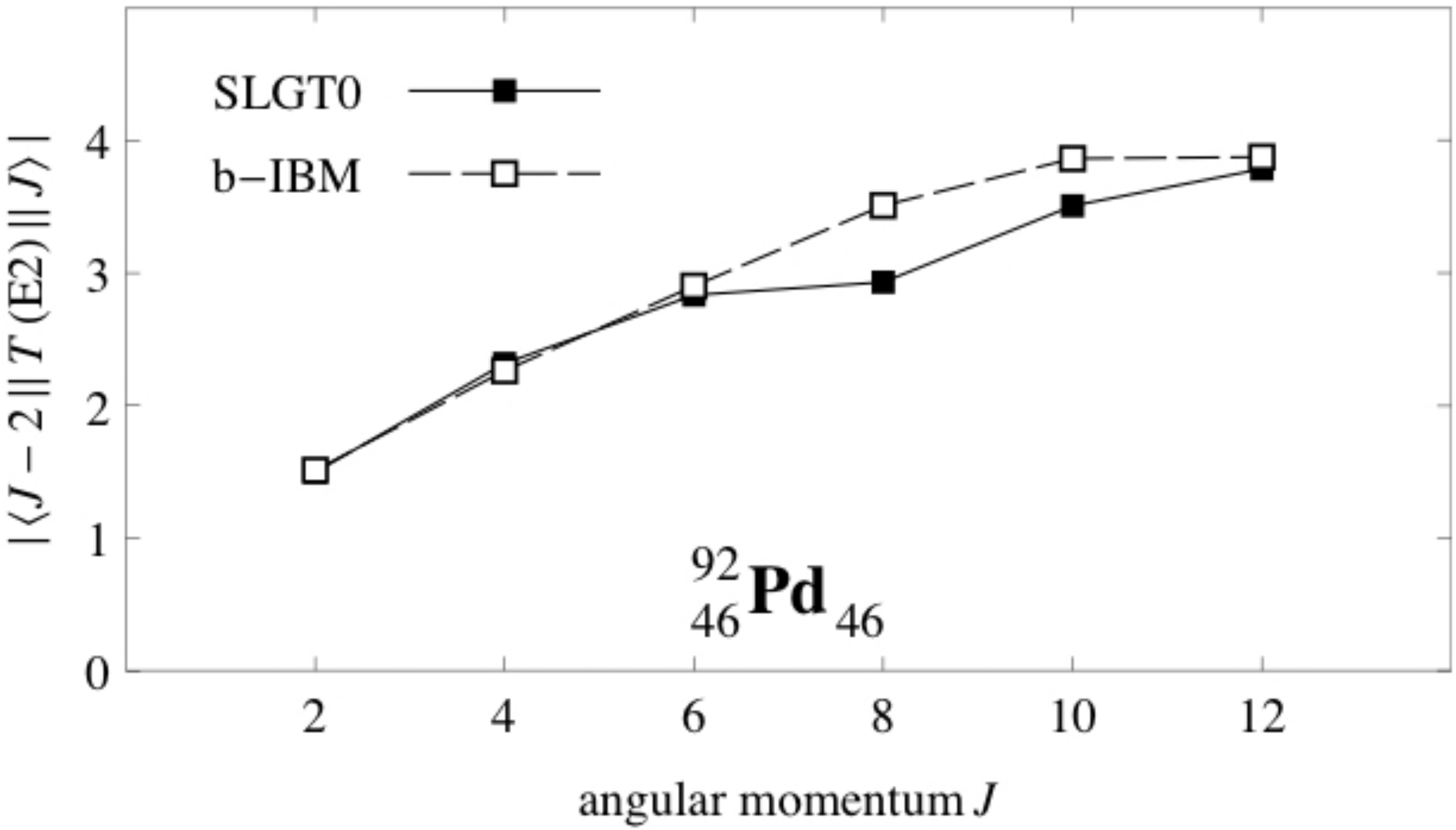
# B(E2) values in $^{94}Ag$



# B(E2) values in $^{94}$Ag



# B(E2) values in $^{92}$Pd



# Structure of $N$=$Z$, $T$=0 states

A reasonable approximation is therefore

$$^{96}\mathrm{Cd}\left(J^{+}\right) \approx \frac{1}{2}\left(b^{+} \times b^{+}\right)^{(J)}|\mathrm{o}\rangle$$

$$^{94}\,\mathrm{Ag}\left(J^{+}\right) \approx \sum_{L} c_{JL}\left(\left(b^{+} \times b^{+}\right)^{(L)} \times b^{+}\right)^{(J)}|\mathrm{o}\rangle$$

$$^{92}\mathrm{Pd}\left(J^{+}\right) \approx \sum_{LL'} d_{JLL'}\left(\left(b^{+} \times b^{+}\right)^{(L)} \times \left(b^{+} \times b^{+}\right)^{(L')}\right)^{(J)}|\mathrm{o}\rangle$$

The coefficients $c_{JL}$ and $d_{JLL'}$ are *sometimes* fixed by the ‘geometry’ of the $g_{9/2}$ shell.

# Example: $21^+$ isomer in $^{94}$Ag

How many independent states of three *b*-bosons can couple to angular momentum 21?

$$d(\upsilon,\ell,J)=\frac{i}{2\pi}\oint_{|z|=1}\frac{\left(z^{2J+1}-1\right)\left(z^{2\upsilon+2\ell-1}-1\right)\prod_{k=1}^{2\ell-2}\left(z^{\upsilon+k}-1\right)}{z^{\ell\upsilon+J+2}\prod_{k=1}^{2\ell-2}\left(z^{k+1}-1\right)}$$

Answer: *d* (3,9,21)=2 of which one is spurious.

After elimination of the spurious state, the energy of the remaining $21^+$ state is:

$$E\left(21^+\right)=3\varepsilon_b+\frac{6851}{20155}\nu_{12}^b+\frac{15488}{21545}\nu_{14}^b+\frac{1212882}{624805}\nu_{16}^b$$

# Example: $21^+$ isomer in $^{94}$Ag

In turn, we know the boson matrix elements in terms of the shell-model matrix elements:

$$\nu_{12}^b = \frac{1218}{69355}\nu_3 + \frac{63423}{138710}\nu_4 + \frac{29957}{63050}\nu_5 + \frac{109881}{53350}\nu_6 + \frac{1148337}{2358070}\nu_7 + \frac{15231}{31525}\nu_8 + \frac{10893}{535925}\nu_9$$

$$\nu_{14}^b = \frac{868}{8515}\nu_5 + \frac{1953}{1310}\nu_6 + \frac{46251}{57902}\nu_7 + \frac{1977}{1310}\nu_8 + \frac{2211}{22270}\nu_9$$

$$\nu_{16}^b = \frac{8}{17}\nu_7 + 3\nu_8 + \frac{9}{17}\nu_9$$

# Example: $21^+$ isomer in $^{94}$Ag

Therefore, we know the energy of the $21^+$ isomer in $^{94}$Ag in terms of the shell-model interaction:

$$E(21^+) = \frac{22134}{3707825}\,\nu_3 + \frac{1152549}{7415650}\,\nu_4 + \frac{1347751953}{5740387250}\,\nu_5 +$$

$$\frac{8606149749}{4857250750}\,\nu_6 + \frac{354940047213}{214690483150}\,\nu_7 +$$

$$\frac{1561553973}{220784125}\,\nu_8 + \frac{15411107094}{3753330125}\,\nu_9$$

$$\approx 0.006\,\nu_3 + 0.155\,\nu_4 + 0.235\,\nu_5 + 1.772\,\nu_6 +$$

$$1.653\,\nu_7 + 7.073\,\nu_8 + 4.106\,\nu_9$$

# Structure of the $21^+$ isomer in $^{94}$Ag

The structure of the $21^+$ isomer is fixed, independent of the interaction:

$$\left|21^+\right\rangle \propto -0.826\left|b^3[12]J=21\right\rangle + 0.862\left|b^3[14]J=21\right\rangle$$

The quadrupole moment of the $21^+$ state in $^{94}$Ag is related to that of the $9^+$ (isomer) in $^{98}$In:

$$Q\left(21^+;\ ^{94}\mathrm{Ag}\right) \approx 0.028\ Q\left(9^+;\ ^{98}\mathrm{In}\right)$$

# Conclusions

- Spectroscopy of neutron-proton systems in a high-*j* shell can be described in terms of aligned np pairs (Blomqvist).
- These np pairs can be represented as bosons.
- The *b*-IBM gives rise to interaction-independent relations between nuclear properties of $N=Z$ nuclei.
- Questions: Other relations? Can we transfer an aligned np pair between states in these nuclei?