(Dirac) Neutrinos and Dark Matter

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The $\nu_L - \nu_R$ Dirac Connection

Under the SM symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, the fermion singlet ν_R transforms trivially. So it is not mandatory. But if it is added anyway, it would have the Yukawa interaction $\bar{\nu}_R(\nu_L \phi^0 - e_L \phi^+)$, and a Dirac neutrino appears to be born. However, since ν_R is not protected by any symmetry from having a Majorana mass, the 2×2 mass matrix spanning $(\nu_L, \bar{\nu}_R)$ is given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}.$$

If $m_D << m_R$, then the famous seesaw mechanism implies $m_{\nu_L} \simeq m_D^2/m_R$ and ν_L is Majorana. This is the usual thinking on neutrinos !

Suppose we back up one step and ask the question "Where does ν_R come from ?"

The usual answer is $SO(10)$ because its spinorial <u>16</u> representation contains all the SM fermions $+\ \nu_R$. Furthermore, it has the elegant left-right decomposition $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2}$ with $(\nu, e)_R \sim (1, 1, 2, -1/2)$.

To break $SU(2)_R \times U(1)_{(B-L)/2}$ to $U(1)_Y$, a scalar triplet $(\Delta_R^{++}, \Delta_R^+, \Delta_R^0) \sim (1,1,3,1)$ is routinely invoked. Since the term $\Delta_R^0 \nu_R \nu_R$ exists, ν_R again gets a large Majorana mass, and the canonical seesaw scenario is recovered. This is the usual justification of Majorana ν_L instead of Dirac $\nu_{L,R}$!!

In the SM without ν_R, global $U(1)_B$ and $U(1)_L$ are conserved automatically. In the LR model, the $B-L$ gauge symmetry broken by $\langle \Delta_R^0 \rangle$ results in the conservation of $U(1)_B$, but $U(1)_L$ is broken to $(-1)^L$, i.e. lepton parity.

To insist on Dirac $\nu_{L,R}$, a possible way is to break L by 3 units (from a scalar quartet of $\underline{672}$) instead of 2 units (from a scalar triplet of $\underline{126}$). Another option is to use the scalar doublet of $\underline{16}$, which breaks L by one unit, whereas the scalar bidoublet of $\underline{10}$ (which provides quarks and leptons with masses) does not break L at all. In fact, it has been well-known since superstring-inspired E_6 models, where the fundamental $\underline{27}$ representation of E_6 contains $\underline{16} + \underline{10} + \underline{1}$ of $SO(10)$, that neutrinos are Dirac in that context. The dimension-five operator $16^*_S 16^*_S 16_F 16_F$ is assumed to be negligible.

SO(10) $\to$ SU(5) $\times$ $U(1)_\chi$

To accommodate ν_R, keep $SO(10)$ but instead of the left-right decomposition, take $SU(5) \times U(1)_\chi$. Now the fermions belong to $\underline{16} = (5^*, 3) + (10, -1) + (1, -5)$, and the scalars belong to $\underline{10} = (5^*, -2) + (5, 2)$.

Under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$,

$(5^*, 3) = d^c[3^*, 1, 1/3, 3] + (\nu, e)[1, 2, -1/2, 3]$,

$(10, -1) = u^c[3^*, 1, -2/3, -1] + (u, d)[3, 2, 1/6, -1] + e^c[1, 1, 1, -1]$, $\quad (1, -5) = \nu^c[1, 1, 0, -5]$,

$\Phi_1 = (\phi_1^0, \phi_1^-)[1, 2, -1/2, -2]$,

$\Phi_2 = (\phi_2^+, \phi_2^0)[1, 2, 1/2, 2]$.

In previous theoretical studies, $U(1)_\chi$ is mostly ignored, although the Z_χ gauge boson is still routinely searched for, with $m_{Z_\chi} > 4.1$ TeV based on present LHC data. Since ν^c has $Q_\chi = -5$, it couples to ν through the allowed interaction $\nu^c(\nu\phi_2^0 - e\phi_2^+)$. Thus (ν, ν^c) form a Dirac neutrino pair. However, the breaking of $U(1)_\chi$ by the scalar singlet $\zeta \sim (1, -10)$ from the 126 of $SO(10)$ would also make ν^c massive. Hence the seesaw mechanism occurs as in the left-right case, only the context is changed. Since ζ and $\Phi_{1,2}$ all have even Q_χ, $U(1)_\chi$ breaks to $(-1)^{Q_\chi}$ just as L breaks to $(-1)^L$.

To insist on Dirac neutrinos, ζ should be removed and replaced with the singlet scalar $\sigma \sim (1,-5)$. However, the allowed Yukawa Dirac coupling must then be very small for neutrino masses. To overcome this possible objection, a seesaw scenario may be implemented with the help of $U(1)_\chi$ and a softly broken Z_2 discrete symmetry.

particle	$SO(10)$	$SU(5)$	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_\chi$	Z_2
(ν, e)	16	5^*	1	2	$-1/2$	3	$+$
ν^c	16	1	1	1	0	-5	$-$
N	126^*	1	1	1	0	10	$-$
N^c	126	1	1	1	0	-10	$-$
(ϕ^+, ϕ^0)	10	5	1	2	1/2	2	$+$
(η^+, η^0)	144	5	1	2	1/2	7	$-$
σ	16	1	1	1	0	-5	$+$

The dimension-four terms must respect Z_2, so ν does not couple to ν^c through ϕ^0. However, it may couple to N^c through η^0. Furthermore, ν^c couples to N through σ, and (N, N^c) share an invariant Dirac mass. The Z_2 symmetry is broken explicitly but softly by the term $\mu\sigma\Phi^\dagger\eta$, and spontaneously by $\langle\eta^0\rangle = v_3$. The 4×4 neutrino mass matrix spanning (ν, ν^c, N, N^c) is then

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 & f_\eta v_3 \\ 0 & 0 & f_\sigma u & 0 \\ 0 & f_\sigma u & 0 & m_N \\ f_\eta v_3 & 0 & m_N & 0 \end{pmatrix}.$$

Dark Parity = $(-1)^{Q_\chi+2j}$

Going back to the canonical scenario with $\zeta \sim (1,-10)$ breaking $U(1)_\chi$ so that neutrinos are Majorana, we note that all SM fermions + ν_R have odd Q_χ, whereas the scalars $\Phi_{1,2}$ which couple to them have even Q_χ. Hence the VEVs of ζ and $\phi^0_{1,2}$ break $U(1)_\chi$ to $(-1)^{Q_\chi}$. The residual discrete symmetry $R_\chi = (-1)^{Q_\chi+2j}$ is thus even for all of the SM particles, including the gauge bosons because they have $Q_\chi = 0$ and $j = 1$. It is now an easy step to realize that any fermion/scalar with even/odd $U(1)_\chi$ has odd R_χ and belongs to the dark sector.

Previously, motivated by supersymmetry, $(-1)^{3(B-L)+2j}$ has been used as dark parity, assuming a left-right decomposition of $SO(10)$. Note that $15(B-L) = 12Y - 3Q_\chi$. Since $12Y$ is always even, odd/even Q_χ implies odd/even $3(B-L)$.

To justify the existence of $U(1)_\chi$, consider the breaking of $SO(10)$ to $SU(5) \times U(1)_\chi$, using the scalar $\underline{45}$. Since

$$\underline{45} = (24, 0) + (10, 4) + (10^*, -4) + (1, 0),$$

a large VEV from $(1, 0)$ works. To break $SU(5)$ to the

SM without breaking $U(1)_\chi$, consider

$$(24, 0) = (8, 1, 0, 0) + (1, 3, 0, 0) + (3, 2, 1/6, 0)$$

$$+(3^*, 2, -1/6, 0) + (1, 1, 0, 0).$$

Hence a large VEV from $(1, 1, 0, 0)$ works. Assuming both occur at M_U, then the renormalization-group evolution equations must be applied to find the observed values of the three gauge couplings of the SM at present energies.

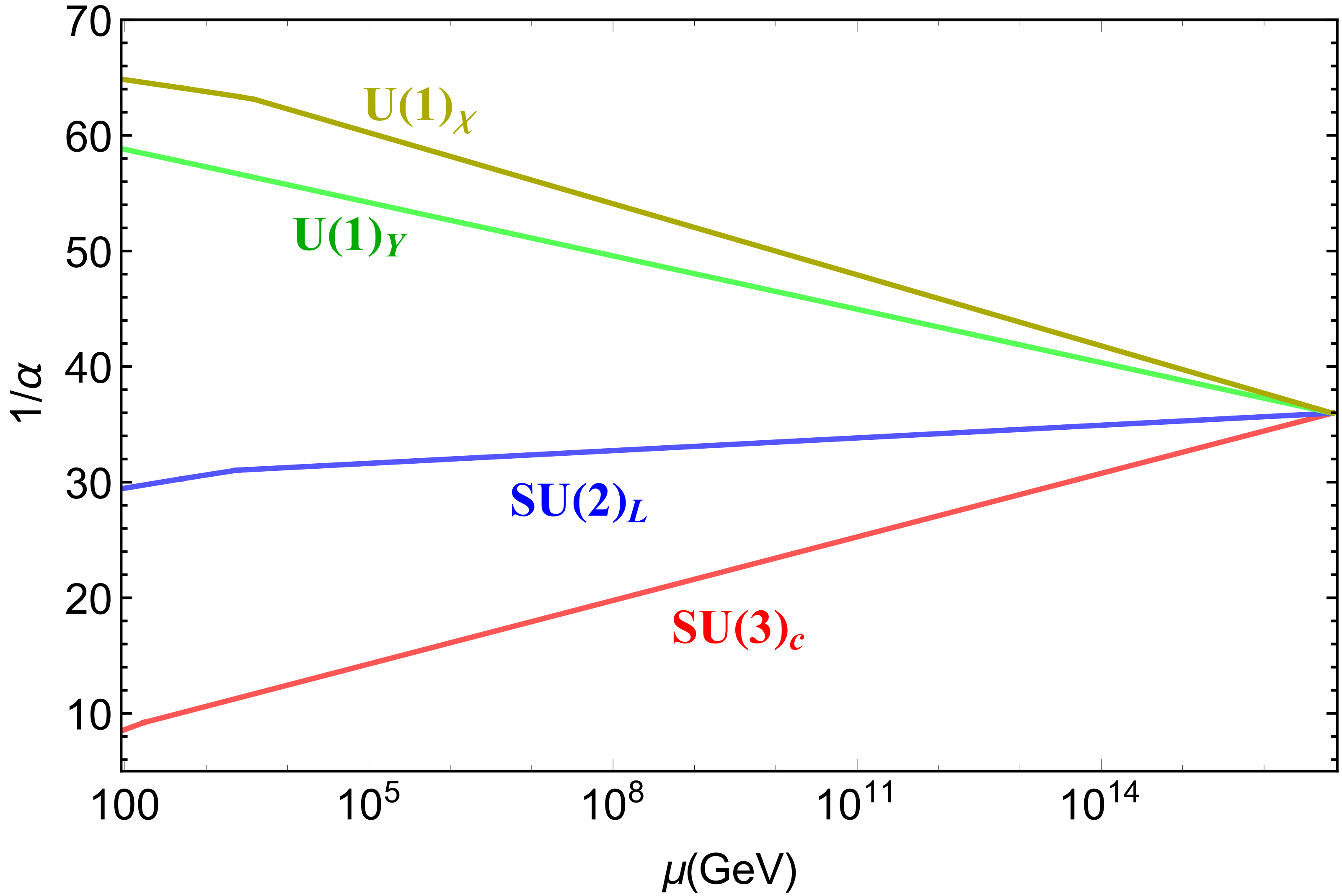
$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1}.$$

In the SM with three families ($N_F = 3$) and one Higgs doublet, $SU(3)_C : b_C = -11 + (4/3)N_F = -7$, $SU(2)_L : b_L = -22/3 + (4/3)N_F + 1/6 = -19/6$, $U(1)_Y : b_Y = (4/3)N_F + 1/10 = 41/10$, where b_Y has been normalized by 3/5. It is well-known that the three gauge couplings do not unify in this case. A second Higgs doublet at M_ϕ would contribute $\Delta b_L = 1/6$ and $\Delta b_Y = 1/10$.

Add now a colored fermion octet $\Omega \sim (8, 1, 0, 0)$ from $(24, 0)$ at M_Ω, contributing $\Delta b_C = (2/3)3 = 2$; an electroweak fermion triplet $\Sigma \sim (1, 3, 0, 0)$ from $(24, 0)$

at M_Σ, contributing $\Delta b_L = (2/3)2 = 4/3$; an electroweak scalar triplet $S \sim (1,3,0,-5)$ from $(24,-5)$ belonging to the $\underline{144}$ of $SO(10)$ at M_S, contributing $\Delta b_L = (1/3)2 = 2/3$. Note they all have odd R_χ, i.e. they belong to the dark sector.

It has been shown that Σ, which mimics the wino of supersymmetry, is a realistic dark-matter candidate at $M_\Sigma \simeq 2.3$ TeV. Using this and assuming that $M_\phi \simeq 500$ GeV, the constraint from unification is $0.654 = 0.2949 \ln(M_S/M_Z) - 0.5 \ln(M_\Omega/M_Z)$. A solution is for example $M_S = 2.5$ TeV, $M_\Omega = 174$ GeV, $M_U = 6.93 \times 10^{16}$ GeV, $\alpha_U = 0.0278$, and $\alpha_\chi(M_Z) = 0.0154$.


70
60
50
40
30
20
10
1/α
U(1)χ
U(1)Y
SU(2)L
SU(3)c
100
10^5
10^8
10^11
10^14
μ(GeV)

In this scenario, Σ^0 is dark matter. It does not couple to Z or Z_χ or any scalar at tree level. Its interaction to quarks may occur in one and two loops, but the effect is negligible. Hence it is not detectable in direct-search experiments. The scalar triplet S decays to $\nu\Sigma$ through the allowed $f_S S^* \nu^c \Sigma$ Yukawa coupling and the neutrino $\nu - \nu^c$ mixing $\sim \sqrt{m_\nu/M_R}$, with a lifetime $\sim 10^{-7}$ s. As for Ω, which mimics the gluino of supersymmetry, it is stable by itself but its bound states, i.e. gluinonia, would decay into quark pairs. Since M_Ω is predicted to be relatively light, this may be detectable at the LHC.

Light Scalar Dilepton Mediator

In the case of Dirac neutrinos in the context of $U(1)_\chi$, both baryon number B and lepton number L are conserved even after the symmetry breaking of $U(1)_\chi$ and $SU(2)_L \times U(1)_Y$. This means that if a new particle is added, it may be assigned B and L numbers appropriately, according to its assumed interactions with the known quarks and leptons. These assignments lie outside $U(1)_\chi$, hence Q_χ is now not a marker of dark matter. Take $\zeta \sim (1, -10)$ as before. It has the allowed coupling $\zeta^* \nu^c \nu^c$, so it should have $L = -2$. If L is to be conserved, ζ must not have a VEV. Recall the $U(1)_\chi$ is broken by $\sigma \sim (1, -5)$ instead.

scalar	$SO(10)$	$SU(5)$	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_\chi$	L
ζ	126	1	1	1	0	-10	-2
ρ	16	1	1	1	0	-5	-1

Note that ρ is assigned $L = -1$, to distinguish it from σ which has $L = 0$. Being a scalar, ρ is absolutely stable because of L conservation. It also has the allowed $\zeta^* \rho\rho$ interaction, so it is self-interacting dark matter. For a light ζ, it is able to explain the central flatness of the density of dwarf galaxies (the cusp-core problem) by its large elastic scattering cross section, i.e. $\rho\rho^* \to \rho^*\rho$ through ζ exchange.

One nagging problem with self-interacting dark matter is that the light mediator should decay. If it is a scalar, then it does so by mixing with the Higgs boson. If it is an Abelian vector gauge boson, then it does so by mixing kinetically with the $U(1)_Y$ gauge boson. In either case, the final states would consist of electrons and photons. Since the production cross section $\rho\rho^* \to \zeta\zeta^*$ is enhanced by the Sommerfeld effect at late times, such decay of ζ would disrupt the cosmic microwave background (CMB) and be inconsistent with PLANCK data. The elegant solution here is that ζ decays only into two neutrinos.

As a scalar ρ must have the interaction $\lambda_{01}\Phi^\dagger\Phi\rho^*\rho$, the direct detection of ρ through Higgs exchange is always possible. Present data imply a bound of about $\lambda_{01} < 4.4 \times 10^{-4}$ if $m_\rho = 150$ GeV. To avoid this interaction, a Dirac fermion triplet $(1, 3, 0, \pm 5)$ from the $\underline{144}$ of $SO(10)$ with odd Z_2 may be used.

With neutrinos as Dirac fermions and the interaction $\zeta^*\nu^c\nu^c$, the two other astrophysical anomalies regarding dark matter, i.e. the missing-satellite and too-big-to-fail problems, may also be solved, using the drag on dark matter by the cosmic neutrino background.

Other Applications

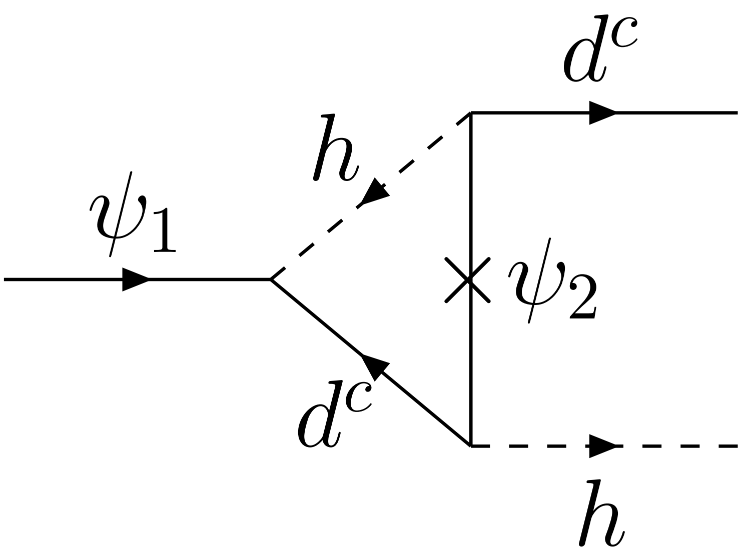
Baryogenesis :

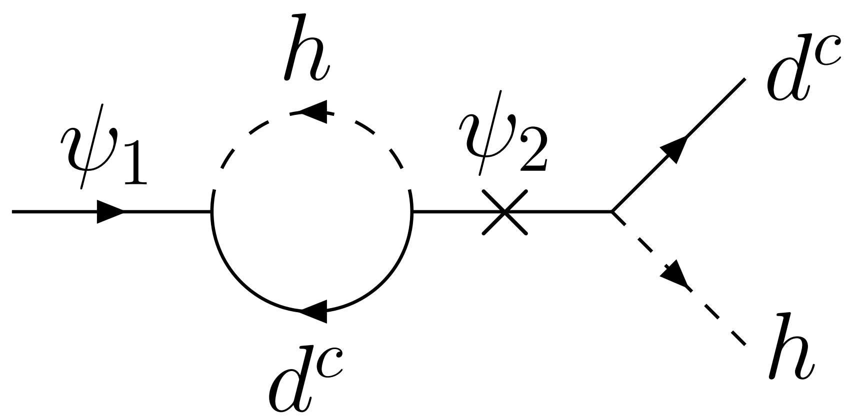
Since lepton number is strictly conserved for Dirac neutrinos and the scalar dilepton interaction, leptogenesis is not available for the explanation of the baryon asymmetry of the Universe. However, the analog process of having a heavy Majorana fermion ψ decay to $B = \pm 1$ final states may be implemented.

particle	$SO(10)$	$SU(5)$	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_\chi$	B
h_1	16^*	5	3	1	$-1/3$	-3	$-2/3$
h_2	10	5	3	1	$-1/3$	2	$-2/3$
ψ	45	24	1	1	0	0	1

Using the allowed couplings $h_2 ud$, $h_2^* u^c d^c$, $h_1 d^c \psi$, $h_1^* h_2 \sigma$, and assuming that ψ has a large Majorana mass, thus breaking B to $(-1)^{3B}$, it is clear that ψ may decay to $h_1^* \bar{d}^c (B = 1)$ and $h_1 d^c (B = -1)$. The subsequent decay of h_1 to $u^c d^c$ through $h_1 - h_2$ mixing from $\langle \sigma \rangle$ establishes a baryon asymmetry, through the usual one-loop diagrams involving a second ψ and a CP phase.

Whereas leptogenesis is constrained by lepton masses in the Yukawa coupling $\phi^+ l \nu^c_{1,2}$, the analog baryogenesis here is unrestricted.


d^c
h
ψ_1
ψ_2
d^c
h


h
ψ_1
ψ_2
d^c
d^c
h

Axionic dark matter :

Instead of the usual colored fermion triplet with an anomalous Peccei-Quinn charge, a colored fermion octet Ω, which mimics the gluino of supersymmetry, may be used. It obtains a large Majorana mass from the interaction $S^*\Omega\Omega$. The dynamical phase of S is the invisible axion and becomes a component of dark matter.

particle	$SO(10)$	$SU(5)$	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_\chi$	PQ
Ω	45	24	8	1	0	0	1
S	54	24	1	1	0	0	2

Seesaw dark matter and Higgs decay :

In the scenario where Q_χ is the marker of dark matter with conserved dark parity $R_\chi = (-1)^{Q_\chi}$, with seesaw Majorana neutrino masses using $\zeta \sim (1, -10)$ for $\zeta^* \nu^c \nu^c$, light dark matter may have the same seesaw origin. Consider the fermions $N \sim (1, 0)$ from the 45 and $D_\chi \sim (1, -10)$ from the 126. They have odd R_χ, thus belonging to the dark sector. They have the analog Dirac mass coming from $f_D \zeta^* D_\chi N$. Since N has an allowed invariant Majorana mass m_N, the seesaw mechanism works here as well, giving D_χ a mass $f_D^2 u^2 / m_N$.

Now D_χ interacts mainly with Z_χ. Just as ν decouples in the early Universe at a temperature of order 1 MeV, D_χ decouples at a temperature of order $T \sim 1\ \mathrm{MeV}(m_{Z_\chi}/m_Z)^{4/3}$. There remains however a suppressed Yukawa coupling to $\zeta_R = \sqrt{2}(Re(\zeta) - u)$, i.e.

$$\frac{f_D}{\sqrt{2}} \frac{f_D u}{m_N} \zeta_R D_\chi D_\chi = \frac{m_{D_\chi}}{\sqrt{2}u} \zeta_R D_\chi D_\chi.$$

Since ζ_R is heavy, the above interaction is only realized through $f_H H D_\chi D_\chi$, coming from the mixing of the SM Higgs boson H with ζ which is itself suppressed.

With this double suppression, the Higgs decay to D_χ allows the latter to be FIMP (Feebly Interacting Massive Particle) dark matter. Let λ_3 be the $(\Phi^\dagger\Phi)(\zeta^*\zeta)$ coupling, then $f_H = \sqrt{2}\lambda_3 v m_{D_\chi}/m^2_{\zeta_R}$. The decay rate is

$$\Gamma_H = \frac{f_H^2 m_H}{8\pi}\sqrt{1-4x^2}(1-2x^2),$$

where $x = m_{D_\chi}/m_H$. If the reheating temperature of the Universe after inflation is below the decoupling temperature of D_χ for thermal equilibrium and above m_H, its only production mechanism is freeze-in through

H decay before the latter decouples from the thermal bath. If $x << 1$, the correct relic abundance is obtained for $f_H \sim 10^{-12} x^{-1/2}$. As a numerical example, let $m_{D_\chi} = 5$ GeV, then $x = 0.04$. Assuming $\lambda_3 = 0.4$, then $f_H = 5 \times 10^{-12}$ is obtained with $m_{\zeta_R} = 10^7$ GeV. Assuming that this is also the mass of Z_χ, then the decoupling temperature of D_χ is about 5.2 TeV.

In this scenario, Z_χ is too heavy to be discovered at the LHC. Furthermore, D_χ interacts negligibly with quarks and is not detectable in direct-search experiments.

Concluding Remarks

In the framework of $SO(10) \to SU(5) \times U(1)_\chi$, dark matter and neutrino masses appear in a new light. There are several possible new insights. One such is the emergence of naturally light seesaw Dirac neutrinos. Another is self-interacting dark matter with a light scalar dilepton mediator which couples only to neutrinos. Other compatible phenomena include baryogenesis using $B \to (-1)^{3B}$ and axionic dark matter using a colored fermion octet.

In the canonical case of seesaw Majorana neutrinos, $R_\chi = (-1)^{Q_\chi + 2j}$ becomes dark parity, and allows for the explicit $SU(5)$ unification of gauge couplings with the addition of dark matter. There is also the exotic possibility of seesaw FIMP dark matter, produced through Higgs decay.

Previous simple (or simplified) models of dark matter may now be reinterpreted in terms of $U(1)_\chi$, with the hope that Z_χ itself would be discovered at the LHC in the future.