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$SU(5)$ theory with A_4 modular symmetry

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1. Orbifold $T^2/\mathbb{Z}_2$

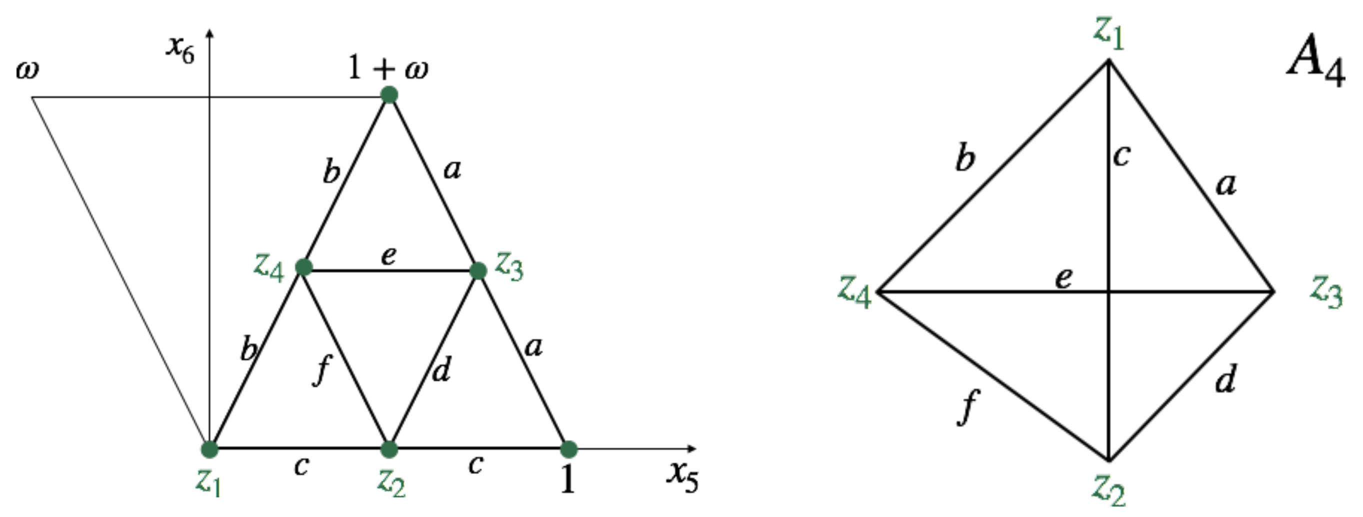
- Theory in **6 space-time dimensions**. The two extra dimensions, combined in the complex coordinate $z = x_5 + i x_6$ are finite in size and compactified on a $\boldsymbol{T^2/\mathbb{Z}_2}$ orbifold:

$$T^2: \begin{array}{l} z \to z+1 \\ z \to z+\omega \end{array} \quad \omega = e^{i2\pi/3} \qquad \mathbb{Z}_2: \ z \to -z$$

- The orbifold action leaves 4 invariant **fixed points** at

$$(z_1, z_2, z_3, z_4) = (0, \frac{1}{2}, \frac{\omega}{2}, \frac{1+\omega}{2})$$

- Sides a are identified, similarly for b and c, forming a tetrahedron



- The transformations $S: z \to z + \frac{1}{2}$ and $T: z \to \omega^2 z$ induce permutations of the four fixed points and generate the group

$$A_4: \quad S^2 = T^3 = (ST)^3 = 1$$

- The fixed points are also permuted under $U: z \to z^*$ generating the group: $S_4: \quad U^2 = (SU)^2 = (TU)^2 = (STU)^4 = 1$

Modular symmetry

- The torus T^2 is defined by a lattice with modulus ω.
- The modular transformations

$$\omega \to \omega' = \frac{a\omega + b}{c\omega + d}, \text{ generated by } \begin{array}{l} S: \ \omega \to -1/\omega \\ T: \ \omega \to \omega + 1 \end{array}$$

Modular group Γ: $S^2 = (ST)^3 = 1$

$a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$

define the same lattice, i.e. the same extra dimensional space-time.

- The effective 4D Lagrangian depends on the modulus ω:

$$\mathcal{L}_{eff} \supset Y(\omega)_{ij} \phi \, \bar{\psi}_i \psi_j$$

- The couplings and the fields on the bulk, depending on the extra dimensions, will transform under the modular symmetry as

$$Y(\omega) \to (c\omega + d)^k \rho Y(\omega)$$
$$\psi^I \to (c\omega + d)^{-k_I} \rho^I \psi^I$$

Yukawa couplings become **modular forms** of weight $\boldsymbol{k}$, with representation $\boldsymbol{\rho}$ under the modular group! They can give rise to mass matrix structure.

- To build an invariant, we have to satisfy two conditions:

1. The weight k compensates the overall weight of the fields: $k = \Sigma_I k_I$
2. The product of the representations $\rho \times \rho^1 \times \cdots \times \rho^n$ contains a singlet.

- We impose an extra condition $T^3 = 1$ such that we end up with the finite modular group $\Gamma_3 \simeq A_4$.

The orbifold has a remnant symmetry $A_4 \ltimes \mathbb{Z}_2$ on the branes. The action of the modular symmetry $\Gamma_3 \simeq A_4$ on the branes is the same as the remnant A_4 symmetry from orbifolding and therefore they are identified.

2. The Model

$$F = \begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad T = \begin{pmatrix} 0 & u_g^c & -u_b^c & u_r & d_r \\ . & 0 & u_r^c & u_b & d_b \\ . & . & 0 & u_g & d_g \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}_L$$

- Supersymmetry $\boldsymbol{SU(5)}$ **GUT** model in 6d with an orbifold $\boldsymbol{T^2/\mathbb{Z}_2}$.

Field	Representation		
	$A_4 \ltimes \mathbb{Z}_2$	$SU(5)$	$U(1)$
F	3	$\bar{5}$	$a + 2c$
N_s^c	1	1	a
N_a^c	1	1	$4a$
ξ	1	1	$-2a$

Field	Representation			Localization			
	A_4	$SU(5)$	$U(1)$	Weight	P_0	$P_{1/2}$	$P_{\omega/2}$
$T_1^{\pm}$	$\mathbf{1''}$	**10**	$c + 4a$	$-\gamma$	+1	±1	±1
$T_2^{\pm}$	$\mathbf{1'}$	**10**	$c + 2a$	$-\gamma$	+1	±1	±1
$T_3^{\pm}$	**1**	**10**	c	$-\gamma$	+1	±1	±1
H_5	**1**	**5**	$-2c$	$-\alpha$	+1	+1	+1
$H_{\bar{5}}$	$\mathbf{1'}$	$\bar{\mathbf{5}}$	b	$\alpha + \gamma$	+1	+1	+1
ϕ_1	**3**	**1**	$-b - a - 3c$	$-\alpha$	+1	+1	+1
ϕ_2	**3**	**1**	$-3a$	$\alpha - \beta$	+1	-1	+1

- Boundary conditions break A_4 completely and $SU(5) \to SM$.
- The only Yukawa couplings with weight different from zero are the two neutrino and the up Yukawa couplings: y_s^ν, y_a^ν, y^u.
- Diagonal down quark and charged lepton mass matrices:

$$M^d = v_d \begin{pmatrix} y_1^d \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^d \tilde{\xi} & 0 \\ 0 & 0 & y_3^d \end{pmatrix} \qquad M^e = v_d \begin{pmatrix} y_1^e \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^e \tilde{\xi} & 0 \\ 0 & 0 & y_3^e \end{pmatrix}$$

where the hierarchy is given by the different powers of $\tilde{\xi} = \langle \xi \rangle / M_{GUT}$.

- All the contributions to quark mixing is coming from the up-sector:

$$M^u = v_u \begin{pmatrix} y_{11}^u \tilde{\xi}^4 & y_{12}^u \tilde{\xi}^3 & y_{13}^u \tilde{\xi}^2 \\ y_{21}^u \tilde{\xi}^3 & y_{22}^u \tilde{\xi}^2 & y_{23}^u \tilde{\xi} \\ y_{31}^u \tilde{\xi}^2 & y_{32}^u \tilde{\xi} & y_{33}^u \end{pmatrix} \tilde{v}_2$$

- The up Yukawa coupling, y^u, has to have at least weight $\boldsymbol{\gamma = 7}$.

3. Predictions in the neutrino sector

- The neutrino Yukawa couplings are given by

$$y_s^\nu \frac{\xi}{\Lambda} F H_5 N_s^c + y_a^\nu \frac{\phi_2 \xi}{\Lambda^2} F H_5 N_a^c$$

y_s^ν and y_a^ν are modular forms with weight α and β.

Triplet of A_4 — Singlet or triplet of A_4

- The only modular forms satisfying the extended symmetry $\boldsymbol{A_4 \times \mathbb{Z}_2}$ are the ones with weight $\boldsymbol{\alpha = 6}$ and $\boldsymbol{\beta = 6}$:

$$y_s^\nu = y \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}, \quad y_a^\nu = y_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + y_2 \begin{pmatrix} 2 \\ 4\omega^2 + 1 \\ 4\omega + 1 \end{pmatrix} + y_3 \begin{pmatrix} 2\omega^2 - 2\omega \\ 1 \\ -1 \end{pmatrix}$$

The $\mathbb{Z}_2$ symmetry fixes the parameters $\{y, y_2, y_3\}$ to be real while y_3 is purely imaginary.

- We apply the **type-I Seesaw Mechanism**:

$$m^\nu = v_u^2 \left(\frac{\tilde{\xi}^2}{M_s} y_s^\nu (y_s^\nu)^T + \frac{\tilde{v}_2^2 \tilde{\xi}^2}{M_a} y_a^\nu (y_a^\nu)^T \right)$$

- The neutrino mass matrix is $\boldsymbol{\mu - \tau}$ **reflection symmetric:**

$$\nu_e \to \nu_e^*, \ \nu_\mu \to \nu_\tau^*, \ \nu_\tau \to \nu_\mu^* \quad \Rightarrow \quad \theta_{13} \neq 0, \theta_{23} = 45^\circ, \delta^l = \pm 90^\circ$$

- The parameters $\{y, y_1, y_2, y_3\}$ fit the rest of the *PMNS* observables.
- The model predictions lie inside the 3σ region given by NuFit4.0.

References

F. De Anda, S. F. King and E. Perdomo. *SU(5) Grand Unified Theory with A_4* modular symmetry. [arXiv: 1812.05620].

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