# $SU(5)$ Grand Unified Theory with $A_4$ modular symmetry

Predicting Fermion Mass Matrices

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based on

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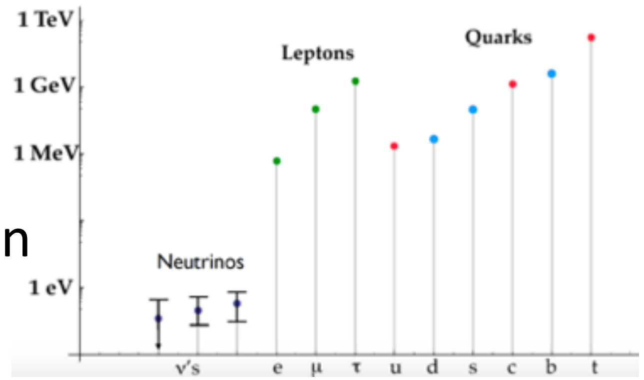
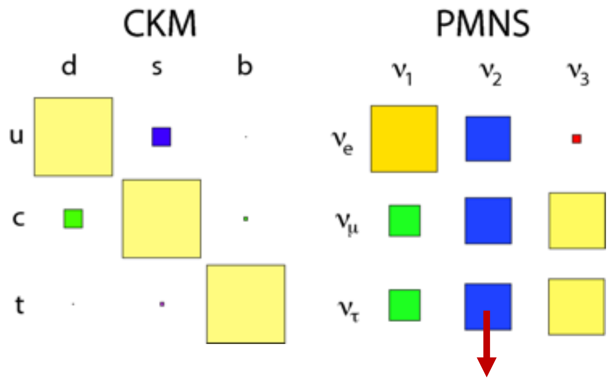
# Motivation

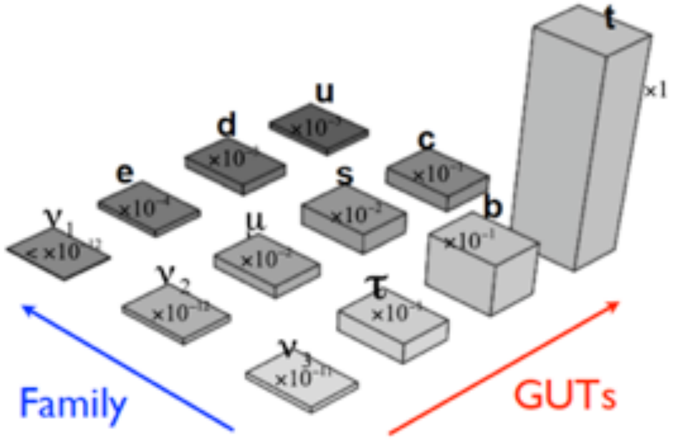
- The **flavour puzzle**:
  - ➢ Why 3 families?
  - ➢ Why such hierarchical masses?
  - ➢ Why small quark mixing and large lepton mixing?
  - ➢ What is the origin of CP violation?




**Flavour symmetries!**

- Origin of **neutrino masses**

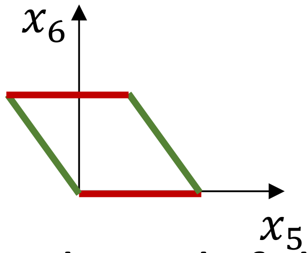



## Supersymmetric SU(5) model in 6 dimensions

- Extra dimensions:
  - ➢ **Origin of the family symmetry after orbifolding**.
  - ➢ GUT breaking through boundary conditions.
  - ➢ Higgs doublet-triplet splitting.
  - ➢ No need of extra fields and alignment superpotentials.

# Orbifold $\boldsymbol{T^2}/\mathbb{Z}_2$

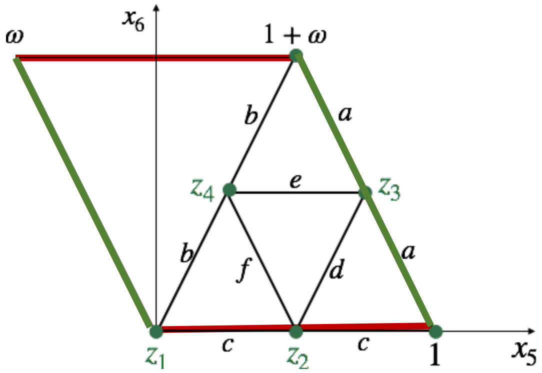
- Theory in **6 space-time dimensions**. The two extra dimensions, combined in the complex coordinate $z = x_5 + ix_6$ are finite in size and compactified on a $\boldsymbol{T^2}/\mathbb{Z}_2$ orbifold:

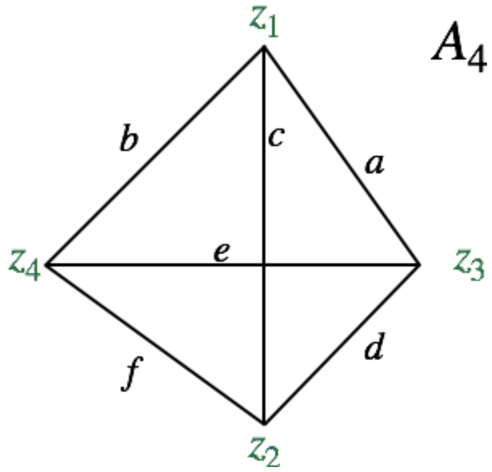



$$T^2: \quad \begin{array}{l} z \to z + 1 \\ z \to z + \omega \end{array} \qquad \omega = e^{i2\pi/3} \qquad \mathbb{Z}_2:\ z \to -z$$

- The orbifold action leaves 4 invariant **fixed points** at $(z_1, z_2, z_3, z_4) = (0, \frac{1}{2}, \frac{\omega}{2}, \frac{1+\omega}{2})$

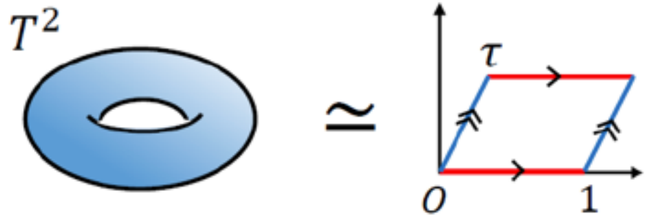
- Sides $a$ are identified, similarly for $b$ and $c$, forming a tetrahedron $A_4$

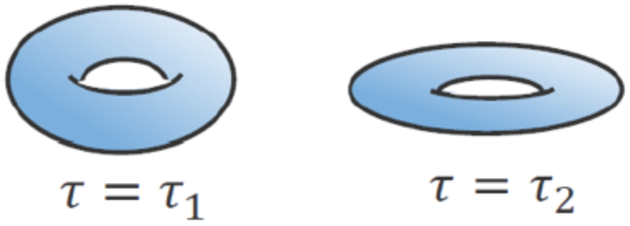


# Modular symmetry

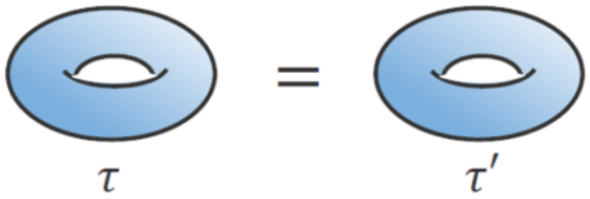
- The torus $T^2$ is defined by a lattice with **modulus $\boldsymbol{\tau}$**:




- Different values of $\tau$ corresponds to different shapes of the torus

- However, there are very specific transformations of $\tau$ which do not change the tours: **the modular transformations!**




$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}$$

$ad - bc = 1 \qquad a, b, c, d \in \mathbb{Z}$

generated by

$$S: \tau \to -1/\tau$$

$$T: \tau \to \tau + 1$$

Modular group $\Gamma$:

$$S^2 = (ST)^3 = 1$$

Finite modular group $\Gamma_N$:

Imposing an extra constraint $T^N = 1$ → $\Gamma_2 \simeq S_3 \;\; \Gamma_3 \simeq A_4 \;\; \Gamma_4 \simeq S_4 \;\; \cdots$

# Orbifold $T^2/\mathbb{Z}_2$ + modular symmetry

- The modulus is fixed to be $\tau = \omega = e^{i2\pi/3}$
- The orbifold has a remnant symmetry $A_4 \ltimes \mathbb{Z}_2$ on the branes.
- Fields on the bulk will transform under the finite modular group $\Gamma_3 \simeq A_4$

$$\omega \to \omega' = \frac{a\omega + b}{c\omega + d}$$

$$Y(\omega) \to (c\omega + d)^k \rho Y(\omega)$$

$$\psi^I \to (c\omega + d)^{-k_I} \rho^I \psi^I$$

Yukawa couplings become **modular forms** of weight ***k***, with representation ***ρ*** under the modular group! They can **give rise to mass matrix structure.**

# The model

$$F = \begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad T = \begin{pmatrix} 0 & u_g^c & -u_b^c & u_r & d_r \\ . & 0 & u_r^c & u_b & d_b \\ . & . & 0 & u_g & d_g \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}_L$$

- Supersymmetry $\boldsymbol{SU(5)}$ **GUT** model in 6d with an orbifold $\boldsymbol{T^2/\mathbb{Z}_2}$.

| Field | Representation | | |
|---|---|---|---|
| | $A_4 \ltimes \mathbb{Z}_2$ | $SU(5)$ | $U(1)$ |
| $F$ | $\mathbf{3}$ | $\bar{\mathbf{5}}$ | $a+2c$ |
| $N_s^c$ | $\mathbf{1}$ | $\mathbf{1}$ | $a$ |
| $N_a^c$ | $\mathbf{1}$ | $\mathbf{1}$ | $4a$ |
| $\xi$ | $\mathbf{1}$ | $\mathbf{1}$ | $-2a$ |

Brane fields

| Field | Representation | | | | Localization | | |
|---|---|---|---|---|---|---|---|
| | $A_4$ | $SU(5)$ | $U(1)$ | Weight | $P_0$ | $P_{1/2}$ | $P_{\omega/2}$ |
| $T_1^\pm$ | $\mathbf{1}''$ | $\mathbf{10}$ | $c+4a$ | $-\gamma$ | $+1$ | $\pm1$ | $\pm1$ |
| $T_2^\pm$ | $\mathbf{1}'$ | $\mathbf{10}$ | $c+2a$ | $-\gamma$ | $+1$ | $\pm1$ | $\pm1$ |
| $T_3^\pm$ | $\mathbf{1}$ | $\mathbf{10}$ | $c$ | $-\gamma$ | $+1$ | $\pm1$ | $\pm1$ |
| $H_5$ | $\mathbf{1}$ | $\mathbf{5}$ | $-2c$ | $-\alpha$ | $+1$ | $+1$ | $+1$ |
| $H_{\bar{5}}$ | $\mathbf{1}'$ | $\bar{\mathbf{5}}$ | $b$ | $\alpha+\gamma$ | $+1$ | $+1$ | $+1$ |
| $\phi_1$ | $\mathbf{3}$ | $\mathbf{1}$ | $-b-a-3c$ | $-\alpha$ | $+1$ | $+1$ | $+1$ |
| $\phi_2$ | $\mathbf{3}$ | $\mathbf{1}$ | $-3a$ | $\alpha-\beta$ | $+1$ | -1 | $+1$ |

Bulk fields

- The boundary conditions break $A_4$ completely and $SU(5) \rightarrow SM$.
- The only Yukawa couplings with weight different from zero are the two neutrino and the up Yukawa couplings: $y_s^\nu, y_a^\nu, y^u$.

# Mass Matrices

- Diagonal down quark and charged lepton mass matrices:

$$M^d = v_d \begin{pmatrix} y_1^d \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^d \tilde{\xi} & 0 \\ 0 & 0 & y_3^d \end{pmatrix} \qquad M^e = v_d \begin{pmatrix} y_1^e \tilde{\xi}^2 & 0 & 0 \\ 0 & y_2^e \tilde{\xi} & 0 \\ 0 & 0 & y_3^e \end{pmatrix}$$

where the hierarchy is given by the different powers of $\tilde{\xi} = \langle \xi \rangle / M_{GUT}$.

- All the contributions to quark mixing is coming from the up-sector:

$$M^u = v_u \begin{pmatrix} y_{11}^u \tilde{\xi}^4 & y_{12}^u \tilde{\xi}^3 & y_{13}^u \tilde{\xi}^2 \\ y_{21}^u \tilde{\xi}^3 & y_{22}^u \tilde{\xi}^2 & y_{23}^u \tilde{\xi} \\ y_{31}^u \tilde{\xi}^2 & y_{32}^u \tilde{\xi} & y_{33}^u \end{pmatrix} \tilde{v}_2$$

The up Yukawa coupling, $y^u$, has to have at least weight $\boldsymbol{\gamma = 7}$.

# Neutrino sector

- The neutrino Yukawa couplings are given by: $y_s^\nu \frac{\xi}{\Lambda} F H_5 N_s^c + y_a^\nu \frac{\phi_2 \xi}{\Lambda^2} F H_5 N_a^c$

Triplet of $A_4$ with **weight $\boldsymbol{\alpha}$** Singlet or triplet of $A_4$ with **weight $\boldsymbol{\beta}$**

- The only modular forms satisfying the extended symmetry $\boldsymbol{A_4 \ltimes \mathbb{Z}_2}$ are the ones with weight $\boldsymbol{\alpha} = \mathbf{6}$ and $\boldsymbol{\beta} = \mathbf{6}$:

$$y_s^\nu = y \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}, \quad y_a^\nu = y_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + y_2 \begin{pmatrix} 2 \\ 4\omega^2 + 1 \\ 4\omega + 1 \end{pmatrix} + y_3 \begin{pmatrix} 2\omega^2 - 2\omega \\ 1 \\ -1 \end{pmatrix}$$

- We apply the **type-I Seesaw Mechanism**:

$$m^\nu = v_u^2 \left( \frac{\tilde{\xi}^2}{M_s} y_s^\nu (y_s^\nu)^T + \frac{\tilde{v}_2^2 \tilde{\xi}^2}{M_a} y_a^\nu (y_a^\nu)^T \right)$$

# Neutrino sector

- The $\mathbb{Z}_2$ symmetry fixes the parameters $\{y, y_2, y_3\}$ to be real while $y_3$ is purely imaginary.

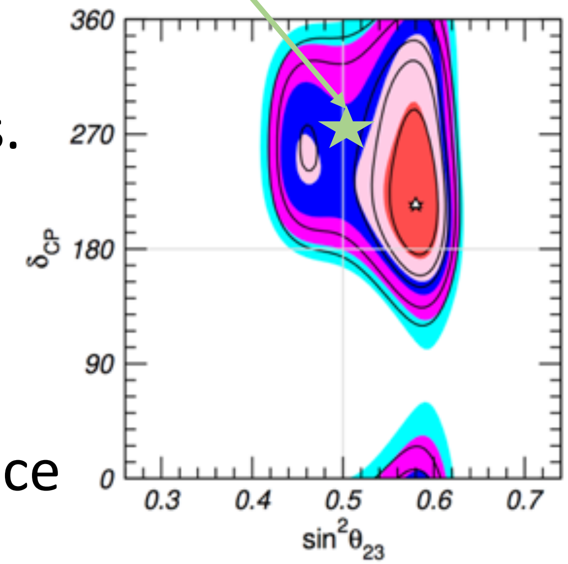
- The neutrino mass matrix is **$\boldsymbol{\mu - \tau}$ reflection symmetric:**

$$\nu_e \to \nu_e^*, \;\; \nu_\mu \to \nu_\tau^*, \quad \nu_\tau \to \nu_\mu^* \quad \Rightarrow \quad \theta_{13} \neq 0, \theta_{23} = 45°, \delta^l = \pm 90°$$

- The parameters $\{y, y_1, y_2, y_3\}$ fit the rest of the *PMNS* observables.

- The model predictions lie inside the $3\sigma$ region given by NuFit4.0.

- It also predicts normal mass ordering and a massless neutrino since we are only adding two right-handed neutrinos.




[NuFit4.0]

# Conclusions

- **Orbifold** compactification of extra dimensions can give rise to **flavour symmetries**.
- The **Yukawa couplings** become **modular forms** giving rise to mass matrix structure, which implies reducing the number of flavon fields in model building.
- Supersymmetric $SU(5)$ model in 6d, where the two extra dimensions are compactified on a $T^2/\mathbb{Z}_2$ orbifold, with modulus fixed $\tau = \omega = e^{i2\pi/3}$.
- The fields on the branes respect an **enhanced symmetry** $A_4 \ltimes \mathbb{Z}_2$ which leads to an **effective $\mu - \tau$ reflection symmetry** at low energies, predicting **maximal atmospheric mixing angle and maximal CP phase.**

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