Dynamical gravitational corrections to the conformal anomaly

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Idea

Can we tell if **gravity is dynamical or not** by computing the conformal anomaly?

Is gravity a fundamental interaction?

Existence of gravitons

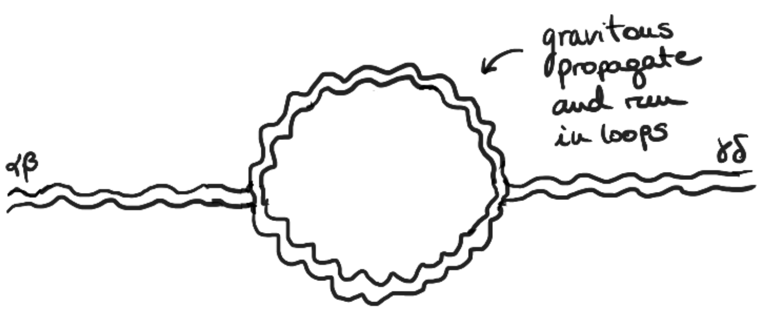
Entropic origin

Is gravity a fundamental interaction?

Existence of gravitons

Einstein's general relativity (GR) → Theory for the gravitational field

Fierz-Pauli lagrangian for free **spin 2 particles**



When interactions are included in a consistent way → GR

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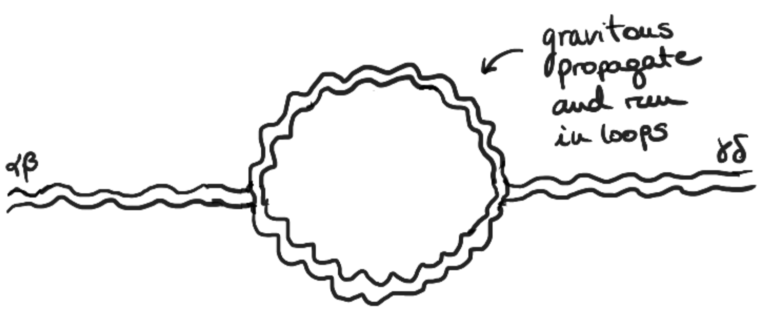


When interactions are included in a consistent way → GR

Entropic origin

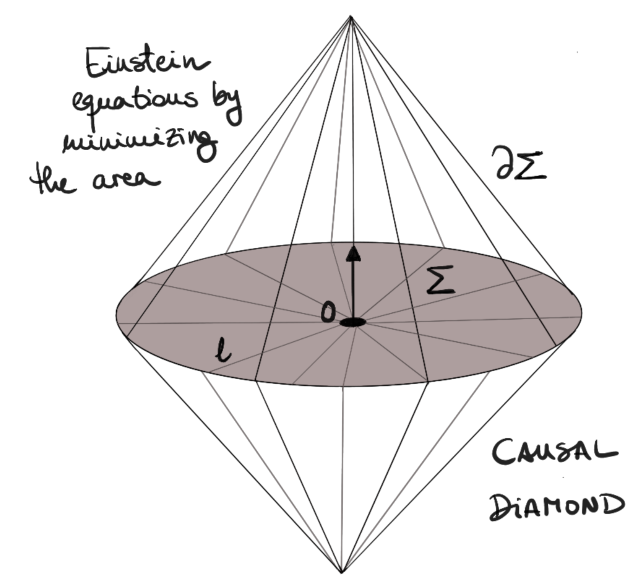
Dictionary between BH laws and thermodymacial laws

$$\delta S = \delta A$$

Instead of thinking in the fundamental dof → **Entropic force**

Jacobson, Padmanabhan, Verlinde,…

Einstein equations arise from minimizing the entropy (the area) with a fixed volume



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$x^\mu \to \lambda\, x^\mu$	Conformal group	$g_{\mu\nu} \to \Omega(x)^2\, g_{\mu\nu}$

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Anomaly in the trace of the energy momentum tensor

$$< T^\mu_\mu > = a_2$$

$\curvearrowright$ Proportional to the **one-loop counterterm**

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Proportional to the **one-loop counterterm**

If gravity is dynamical or just a background is important in the path integral

$$a_2 \sim \int \mathscr{D}\phi_i\, \mathscr{D} g_{\mu\nu}$$

Sensitive to the **graviton dynamics**

Background vs. dynamical gravity

We expand the gravitational field in a background and a perturbation

$$g_{\mu\nu} = \underbrace{\bar{g}_{\mu\nu}}_{\text{Background field}} + \kappa \underbrace{h_{\mu\nu}}_{\text{Quantum field}}$$

(constant curvature spaces for simplicity)

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We take for example an scalar field coupled to gravity

Lagrangian	Anomaly
$\sqrt{\bar{g}}\,\bar{\nabla}_\mu\phi\,\bar{\nabla}^\mu\phi$	$\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{29}{135}\lambda^2$
$\sqrt{\bar{g}}\left(\bar{\nabla}_\mu\phi\,\bar{\nabla}^\mu\phi + \frac{1}{6}\phi^2\bar{R}\right)$	$-\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{\lambda^2}{135}$
$\sqrt{g}\left(\nabla_\mu\phi\,\nabla^\mu\phi + \frac{1}{6}\phi^2 R\right)$	0
$\sqrt{g}\left(\nabla_\mu\phi\,\nabla^\mu\phi + R_{\mu\nu}^2 - \frac{1}{2}R^2\right)$	$\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{1078}{135}\lambda^2$

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Non-dynamical	$\sqrt{\bar{g}}\,\bar{\nabla}_\mu\phi\,\bar{\nabla}^\mu\phi$	$\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{29}{135}\lambda^2$ (λ: curvature of CCS)
Non-dynamical	$\sqrt{\bar{g}}\left(\bar{\nabla}_\mu\phi\,\bar{\nabla}^\mu\phi + \frac{1}{6}\phi^2\bar{R}\right)$	$-\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{\lambda^2}{135}$
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Background field — Quantum field

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Dynamical	$\sqrt{g}\left(\nabla_\mu\phi\,\nabla^\mu\phi + \overbrace{\frac{1}{6}\phi^2 R}^{\text{Non-minimal coupling}}\right)$	0
Dynamical	$\sqrt{g}\left(\nabla_\mu\phi\,\nabla^\mu\phi + \underbrace{R^2_{\mu\nu} - \frac{1}{2}R^2}_{\text{Quadratic gravity}}\right)$	$\frac{\sqrt{\bar{g}}}{(4\pi)^2}\frac{1078}{135}\lambda^2$

Outlook

Can we measure the **conformal anomaly** and be sensitive to the character of the gravitational field?

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Starobinsky's inflation driven by the trace anomaly of conformally coupled matter fields

Precisely models with the coupling of scalar fields to gravity

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Transport effects due to conformal anomaly (similar to the ones due to chiral anomalies)

Measurements of scale transport, SEE and SME, in Weyl semimetals

Work ongoing regarding same type of computations for fermions

Thank you!

Backup

Let us go back to see the origin of the anomaly

$$\underline{\Gamma_0[\bar{g}_{\mu\nu}, \bar{\phi}]} = \lim_{n\to 4} \left\{ \underline{\Gamma[\bar{g}_{\mu\nu}, \bar{\phi}; n]} - \Gamma_\infty[\bar{g}_{\mu\nu}, \bar{\phi}; n] \right\}$$

Renormalized effective action One-loop effective action Divergent piece

The full on-shell effective action has the same symmetries as the original theory in any dimension

Breitenlohner, Maison

$$\mathfrak{D}\Gamma[\bar{g}_{\mu\nu}, \bar{\phi}; n] = \left[2\bar{g}^{\mu\nu}\frac{\delta}{\delta \bar{g}^{\mu\nu}} + \frac{n-2}{2}\bar{\phi}\frac{\delta}{\delta\bar{\phi}} \right] \Gamma[\bar{g}_{\mu\nu}, \bar{\phi}; n] = 0$$

But we have evanescent operators coming from the counterterm

$$\mathfrak{D}\Gamma_0[\bar{g}_{\mu\nu}, \bar{\phi}] = \mathfrak{D}\left[\frac{1}{(n-4)}\mathscr{B}[\bar{g}_{\mu\nu}, \bar{\phi}] \right] = \mathscr{A}[\bar{g}_{\mu\nu}, \bar{\phi}]$$

Anomaly unless we find a proper counterterm