Shapes of Self-Interacting Dark Matter

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Abstract

If dark matter has strong self-interactions, future astrophysical and cosmological observations, together with a clearer understanding of baryonic feedback effects, might be used to extract the velocity dependence of the dark matter scattering rate. To interpret such data, we should understand what predictions for this quantity are made by various models of the underlying particle nature of dark matter. We systematically compute this function for fermionic dark matter with light bosonic mediators of vector, scalar, axial vector, and pseudoscalar type. We do this by matching to the nonrelativistic effective theory of self-interacting dark matter and then computing the spin-averaged viscosity cross section nonperturbatively by solving the Schrodinger equation, thus accounting for any possible Sommerfeld enhancement of the low-velocity cross section. In the pseudoscalar case this requires a coupled-channel analysis of different angular momentum modes due to the structure of the effective potential experienced by the dark matter.

Collisional Dark Matter?

Dark matter constitutes the majority of the mass in our Universe but we don't know its underlying particle nature. Is dark matter collisional?

Many experiments are already looking for signals of dark matter interactions with the Standard Model, but if dark matter has significant self-interactions, it may also leave detectable astrophysical and cosmological signals. [1]

Self-interactions have been proposed to solve small scale discrepancies with ΛCDM

Figure 1: The Bullet Cluster showing gas in pink and dark matter in blue. Baryonic interactions slow the gas down and cause it to emit in X-rays while dark matter goes through largely unaffected and is displaced from the hot gas.

Sommerfeld Enhancement

Classical Analogy

w/o gravity $\qquad \sigma_0 = \pi R^2$

w/ gravity $\qquad \sigma = \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$

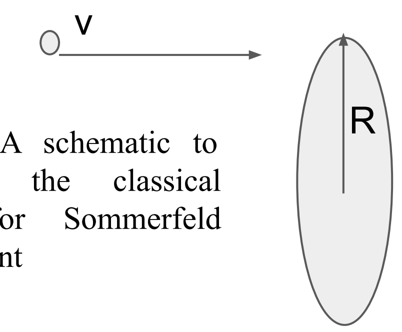

Figure 2: A schematic to understand the classical analogy for Sommerfeld Enhancement

Non-perturbative effect that can be treated quantum mechanically

- Match a field theory calculation onto a quantum mechanical potential
- Solve the Schrodinger Equation

Consider the Sommerfeld enhancement S for the Coulomb potential as an example [2]

$$V(r) = \frac{-\alpha}{2r} \qquad \epsilon_v \equiv \frac{v}{\alpha}$$

$$S = \left| \frac{\frac{\pi}{\epsilon_v}}{1 - exp[-\frac{\pi}{\epsilon_v}]} \right|$$

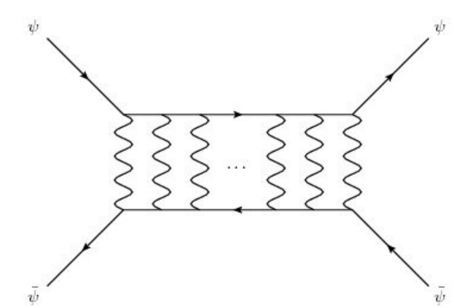

Figure 3: In terms of Feynman diagrams, Sommerfeld enhancement comes from summing up the infinite series of ladder diagrams. [3]

Analysis

Our analysis takes an EFT approach. We consider dark matter scattering and classify theories with a light mediator and fermionic dark matter.

The procedure is as follows. First, we write down the tree level $2 \rightarrow 2$ scattering amplitude. Then, we calculate the potential. Using the Lippmann-Schwinger Equation and the Born Approximation [4] we can solve for the wavefunction at small r (large q). This sets our initial conditions with which we can numerically solve the Schrodinger Equation.

We then match the asymptotics of our wavefunction solutions to those of a free particle and extract the phase shift, or more generally the S-matrix in the case of coupled channels. Using this, we then calculate the viscous cross section.

$$\sigma_V = \int d\Omega \sin^2\theta \frac{d\sigma}{d\Omega}$$

The viscous cross section removes enhancements from the forward and backward scattering regimes which don't affect the dark matter distribution in the galaxy.

Results

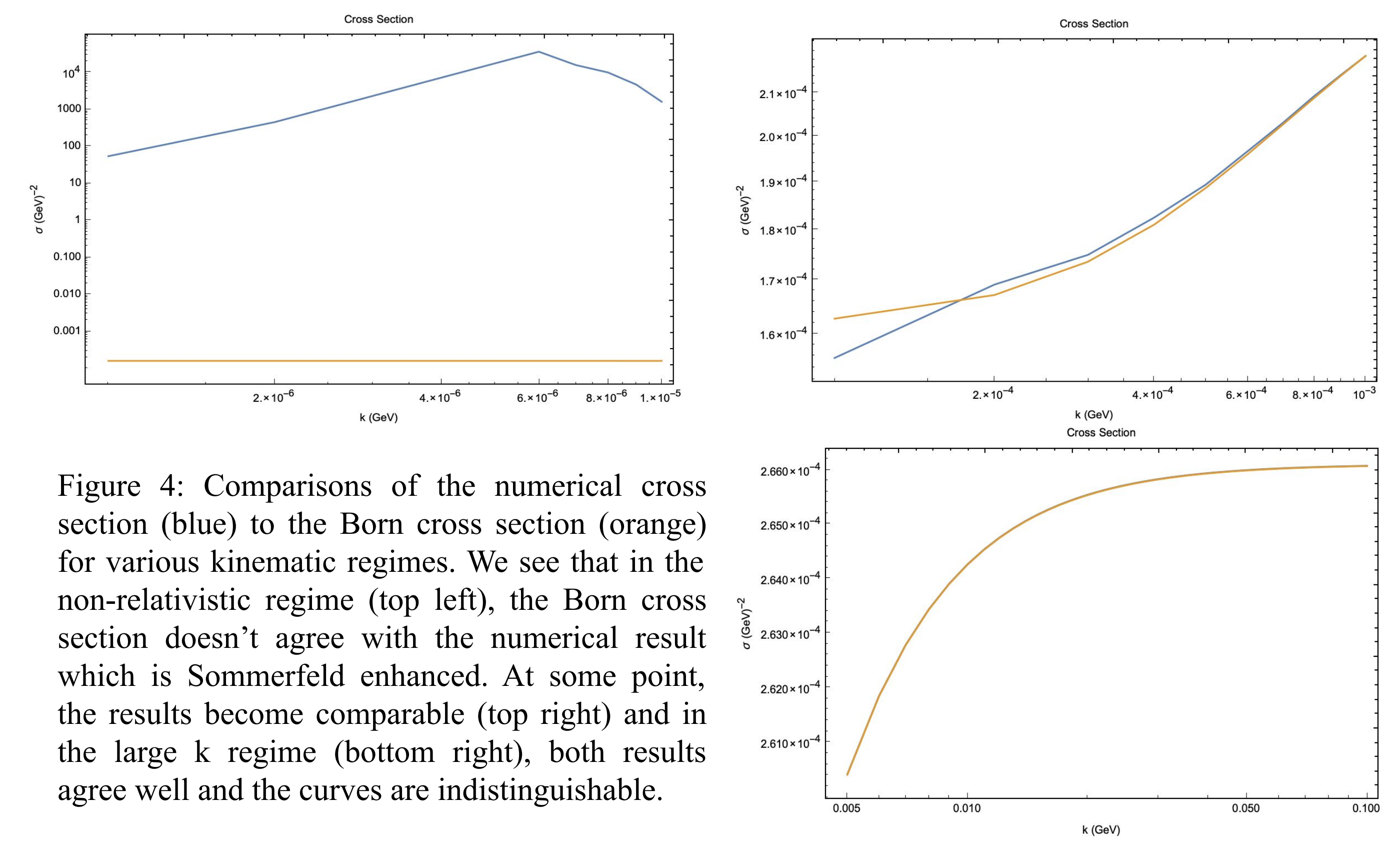

Figure 4: Comparisons of the numerical cross section (blue) to the Born cross section (orange) for various kinematic regimes. We see that in the non-relativistic regime (top left), the Born cross section doesn't agree with the numerical result which is Sommerfeld enhanced. At some point, the results become comparable (top right) and in the large k regime (bottom right), both results agree well and the curves are indistinguishable.

Conclusions

Understanding the space of theories of SIDM is an interesting theoretical problem to solve and we have a well defined procedure for computing the viscous cross section.

The viscous cross section is the relevant quantity of interest when inferring self-interaction cross sections from dark matter distributions in galaxies and galaxy clusters.

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References

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