Probing New Physics with Atomic Precision

Elina Fuchs
Weizmann Institute of Science, Israel

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מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Minerva Stiftung

Unknown scale of New Physics

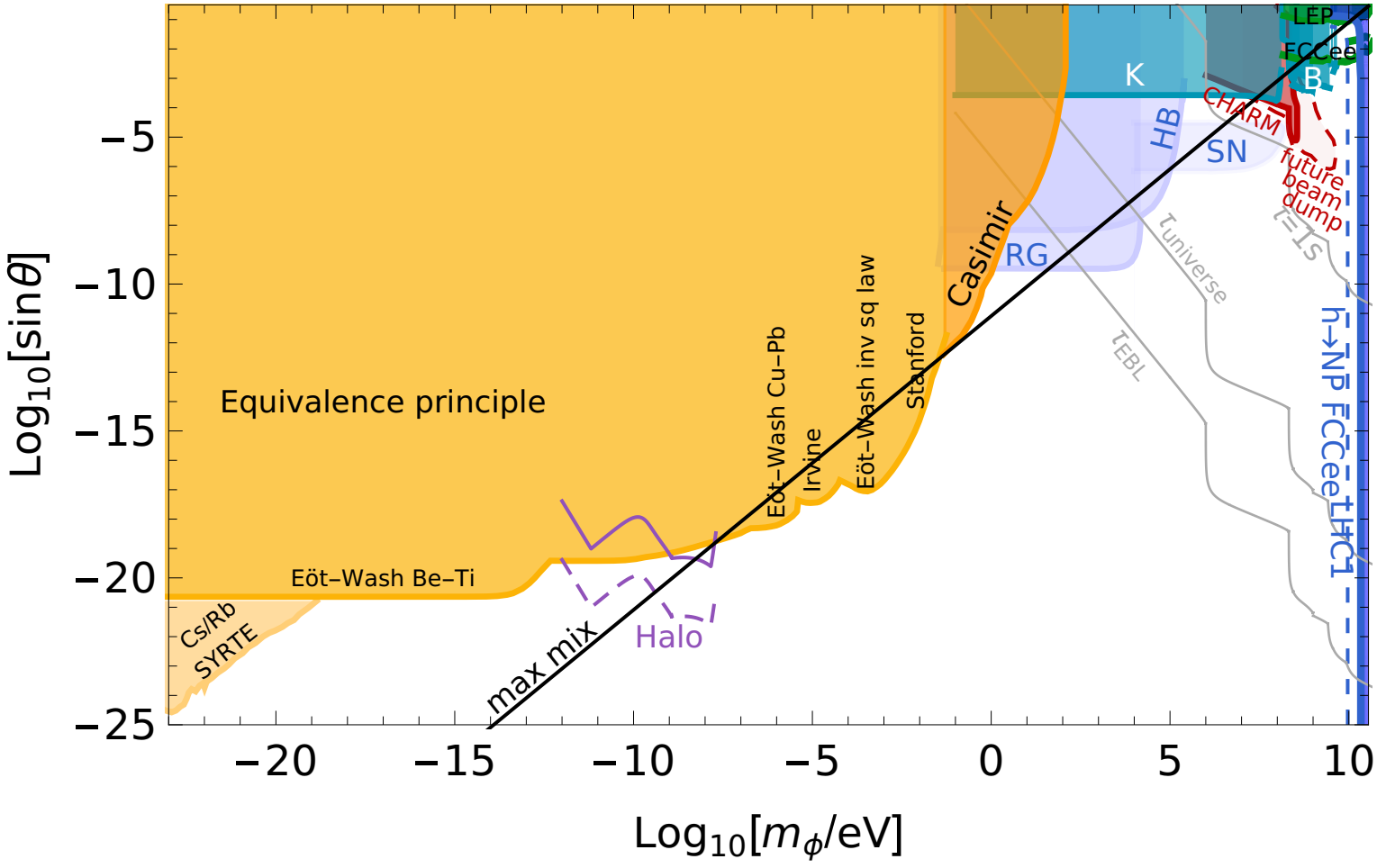
hierarchy problem, Dark Matter (DM), baryogenesis, ...

$\rightsquigarrow$ where is the New Physics (NP)?

- heavy NP $\sim$ TeV
  - motivated by symmetry-based solutions to the hierarchy problem
- NP can also be light
  - can arise as pseudo Nambu-Goldstone bosons of a broken symmetry
  - examples of light NP: light DM, DM mediators, axion, dark photon, ...
  - even addressing hierarchy problem(s): relaxion

variety of methods necessary to probe different NP models

Relaxion searches across the scales and frontiers



5th force astro cosmo meson decays beam dump lepton collider LHC

want to probe natural region below the relaxion line

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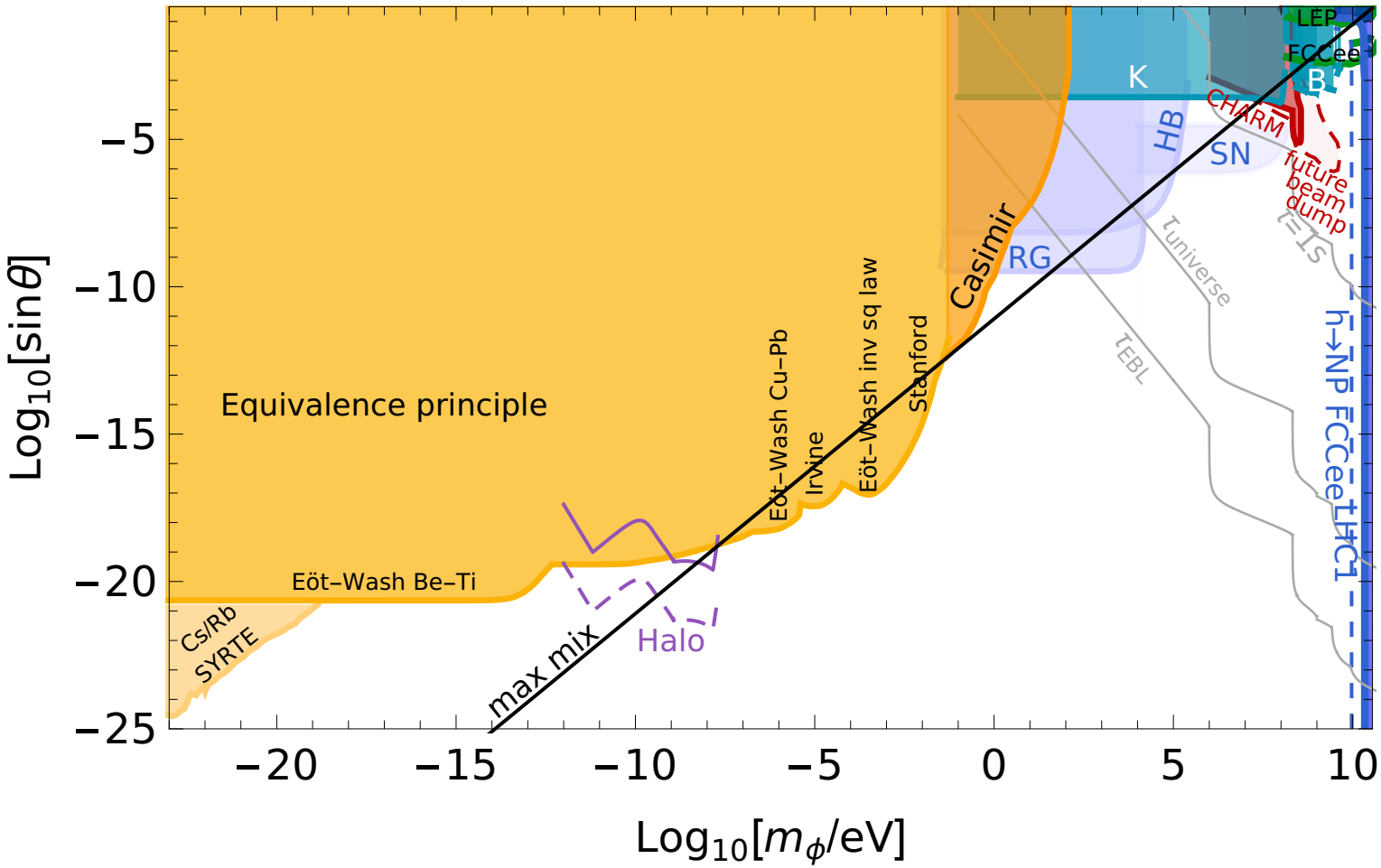

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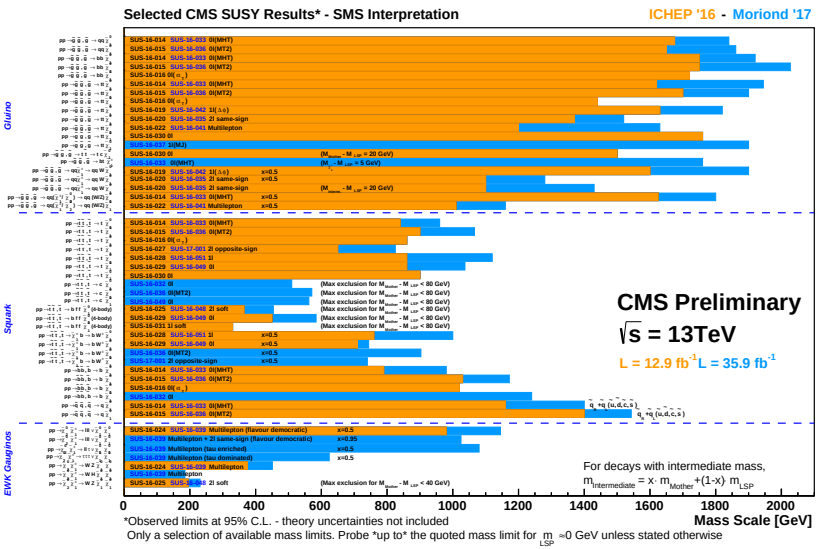
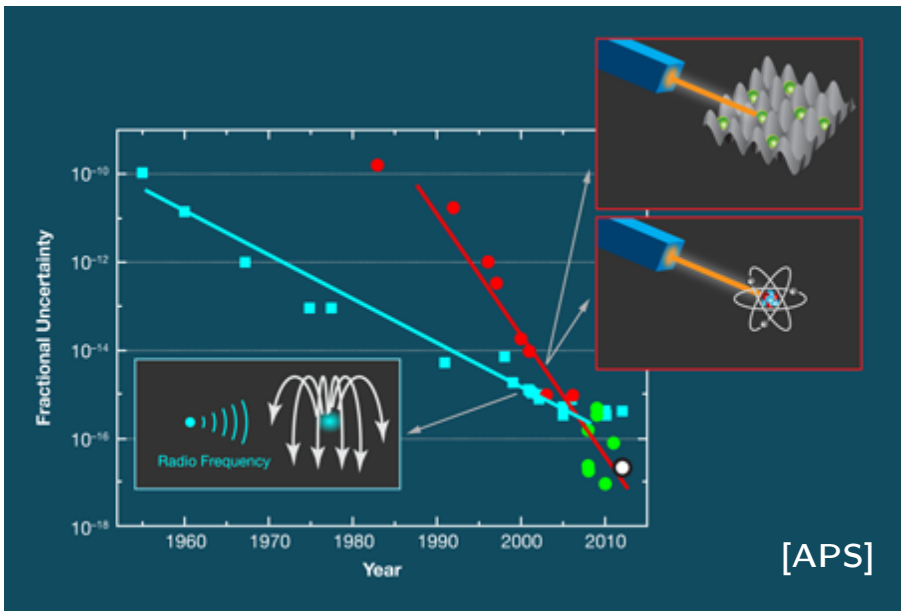
here: focus on high precision at low energy

Methods to search for NP: energy vs. precision

- Energy frontier



- Precision frontier



LHC: $13\,\mathrm{TeV} \simeq 3 \cdot 10^9\, a_0^{-1}$
precision of m_Z: 10^{-5}

Atomic frequency measurements:
Cs clock 10^{-18} relative precision
Ca+ S-D states: 10^{-7}

Atomic tests of symmetries (and their breaking)

- **charge $\mathcal{C}$**: anti-hydrogen spectroscopy
- **parity $\mathcal{P}$**: atomic parity violation (APV) experiments
- **$\mathcal{CP}$**: electric dipole moments, particularly of the electron
- **time variation of fundamental constants**: e.g. $\dot{\alpha}\,/\alpha \lesssim 10^{-17}$
- **Local Lorentz invariance (LLI)**: e.g. with highly charged ions

Outline: different atomic methods

1. King plot nonlinearity
2. Rydberg states
3. Dynamical decoupling constrains Dark Matter
4. Direct data-theory comparison

NP as nonlinearity of King plot

1704.05068, PRL 120 (2018) 091801
Berengut, Budker, Delaunay, Flambaum, Frugiuele, EF, Grojean, Harnik, Ozeri, Perez, Soreq

1602.04822, PRD 96 (2017) no.1, 015011
Frugiuele, EF, Perez, Schlaffer

Possible new interactions

- extend the SM by adding a new boson ϕ
- interacts with e and n
- mass $m_\phi \;\leftrightarrow\;$ interaction range $1/m_\phi$

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- Yukawa potential $V_{\rm NP} = \frac{y_e y_n}{4\pi r} e^{-m_\phi r}$
- change of transition $(i : a \to b)$ frequency caused by new particle ϕ:

$$\boxed{\Delta\nu_i^{\rm NP} = \int {\rm d}r\, r^2\, V_{\rm NP}(r)\left(|\psi_a(r)|^2 - |\psi_b(r)|^2\right)}$$

Sensitive to this frequency shift?

Factorisation of isotope shifts

frequency shift of isotopes at LO factorised into nuclear and electronic:
[King '63]

$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} = K_i \, \mu_{AA'} + F_i \, \delta\langle r^2 \rangle_{AA'}$$

K_i, F_i: electronic mass shift (MS), field shift (FS) coefficients
reduced mass $\mu_{AA'} = 1/m_A - 1/m_{A'}$
nuclear charge radius variance $\delta\langle r^2 \rangle_{AA'}$ **poorly known**

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↷ data-driven observable free of $\delta\langle r^2 \rangle_{AA'}$? ↷ replace by $\nu_i^{AA'}$!
↷ King relation: linear at leading order

$$m\nu_2 = K_{21} + F_{21} \, m\nu_1$$

reduced frequencies: $m\nu_i^{AA'} \equiv \nu_i^{AA'}/\mu^{AA'}$

Vector space of isotopes

arrange measured IS in vectors of isotope pairs:

$$\overrightarrow{m\nu}_i \equiv \left(m\nu_i^{AA'_1}, m\nu_i^{AA'_2}, m\nu_i^{AA'_3} \right) ,$$
$$\overrightarrow{m\mu} \equiv (1, 1, 1)$$

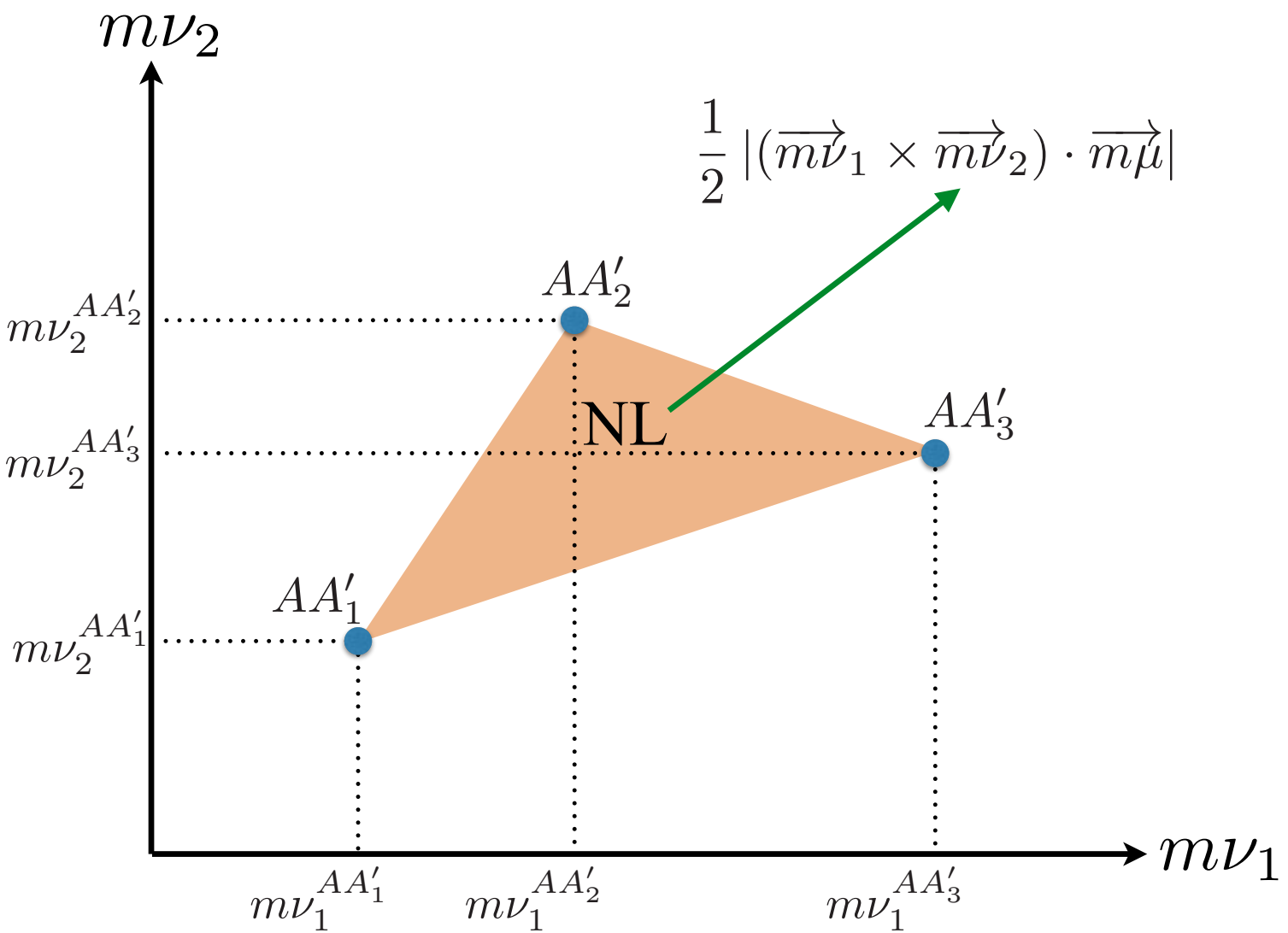
King relation in vector form

$$\overrightarrow{m\nu}_i = K_i \, \overrightarrow{m\mu} + F_i \, \overrightarrow{m\delta \langle r^2 \rangle} ,$$
$$\overrightarrow{m\nu}_2 = K_{21} \, \overrightarrow{m\mu} + F_{21} \, \overrightarrow{m\nu}_1$$

nonlinearity measure

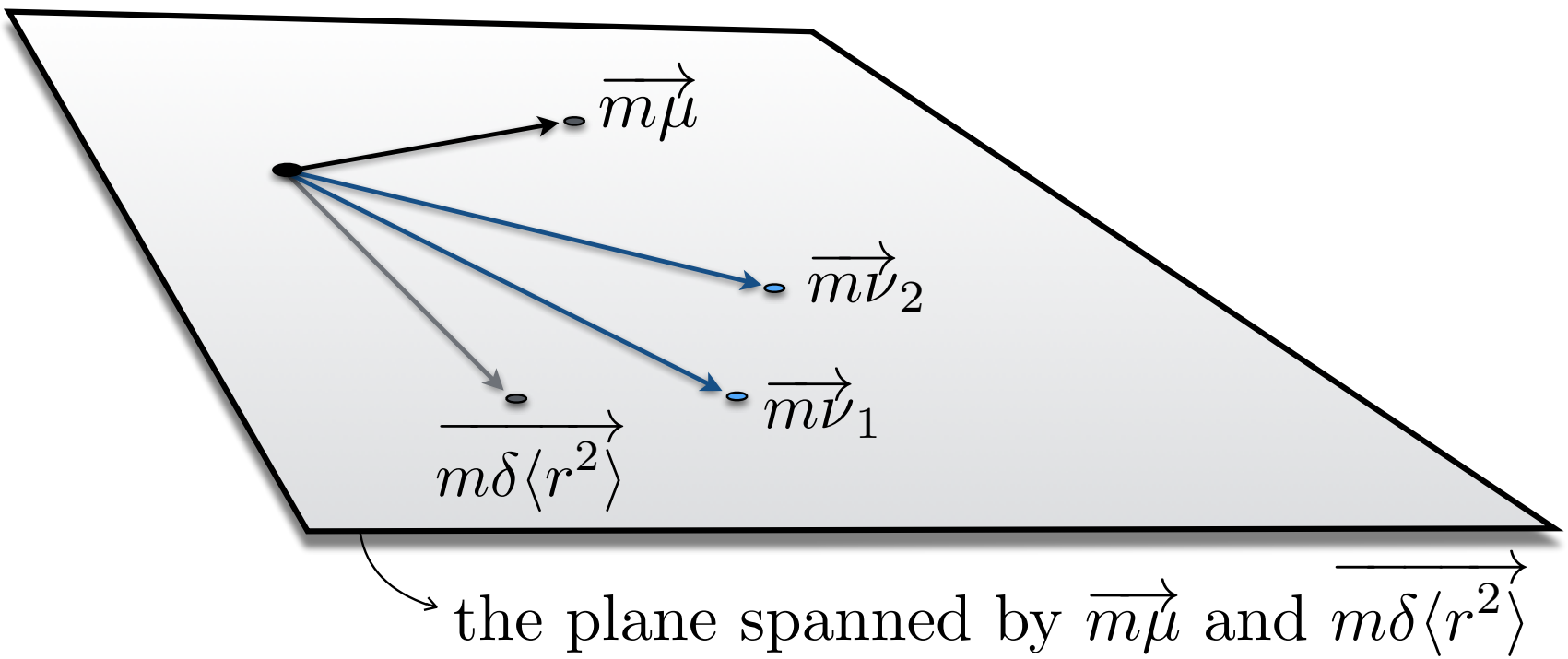
$$\mathrm{NL} = \frac{1}{2} \left| (\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu} \right|$$

Nonlinearity: King triangle



Measured IS in linearity plane of FS, MS

King linearity $\Leftrightarrow$ coplanarity of IS with FS, MS



NP contribution to King linearity violation (KLV)

- coupling of ϕ to e, n: $y_e y_n$
- NP electronic coefficient: X_i (m_ϕ- and model-dependent)
- NP isotope dependence: $\vec{h} \simeq -A\vec{A}'$ amu (for linear $\phi - N$ coupling)

new term in King relation

$$\overrightarrow{m\nu}_i = K_i \overrightarrow{m\mu} + F_i \overrightarrow{m\delta\langle r^2\rangle} + y_e y_n X_i \vec{h}\,,$$
$$\overrightarrow{m\nu}_2 = K_{21}\overrightarrow{m\mu} + F_{21}\overrightarrow{m\nu}_1 + y_e y_n \vec{h}\,(X_2 - X_1 F_{21})$$

NP can break linearity: non-linearity measure $\mathrm{NL_{NP}}$

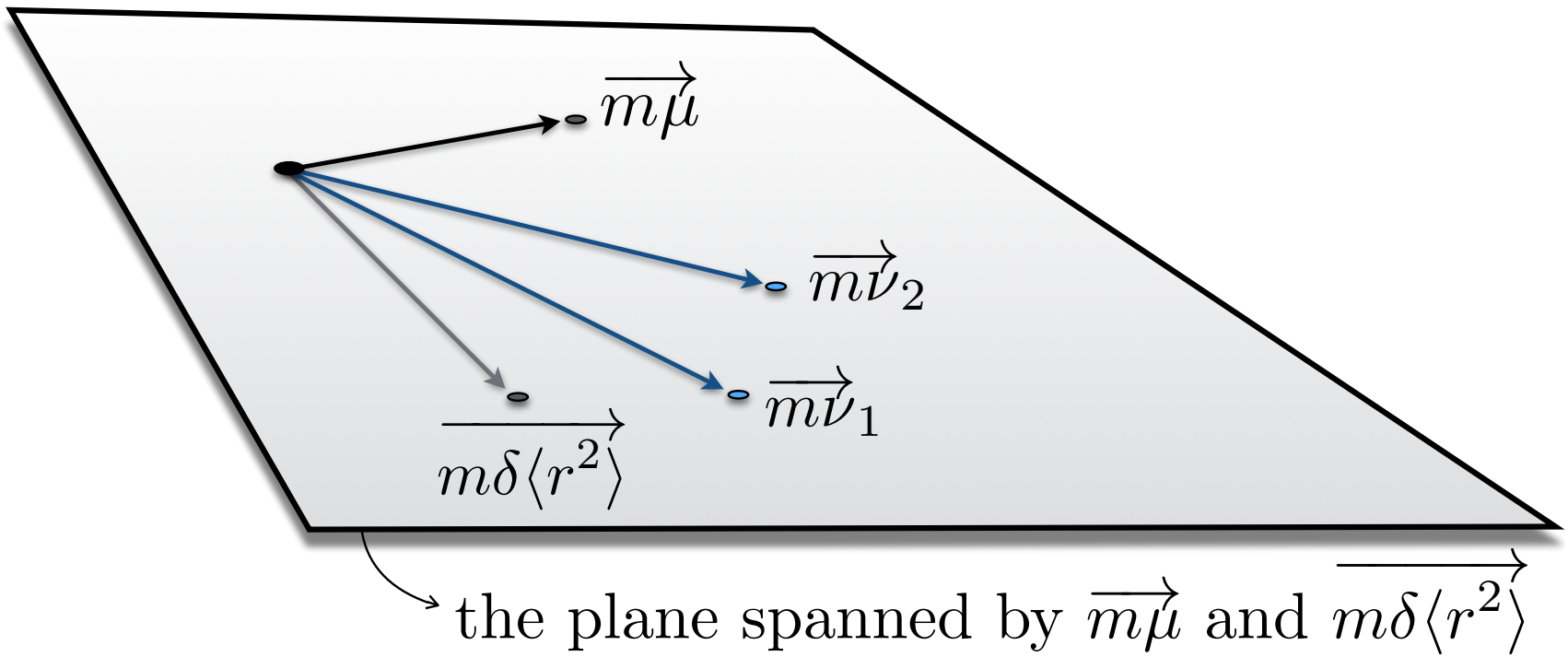
$$\mathrm{NL_{NP}} = [\overrightarrow{m\mu} \times (X_2 - F_{21}X_1)\ \overrightarrow{m\nu}_1] \cdot \vec{h}$$

$\mathrm{NL_{NP}} = 0$ if

(i) $X_i \propto F_i$ (heavy m_ϕ)

(ii) $\vec{h} || \overrightarrow{m\mu}$ or $\overrightarrow{m\delta\langle r^2\rangle}$

NP orthogonal to linearity plane



NP component out of the linearity plane

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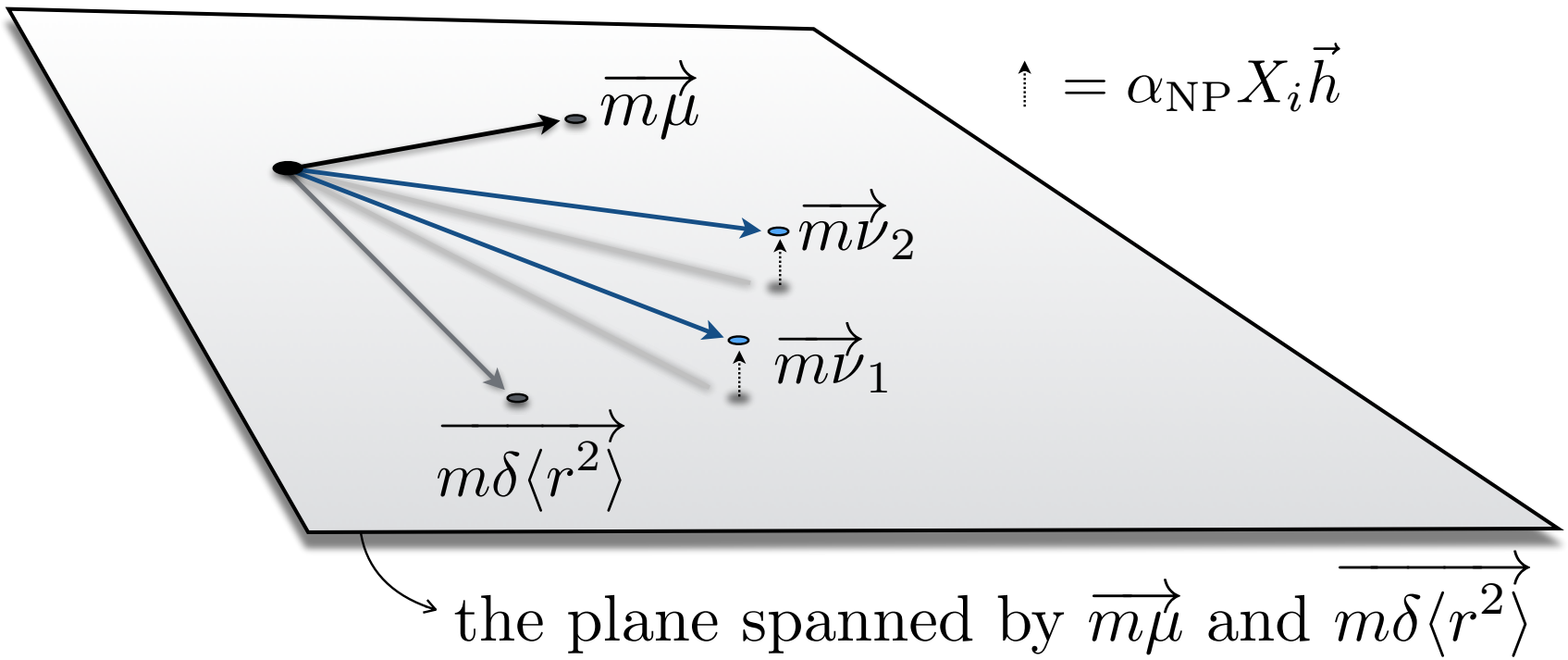
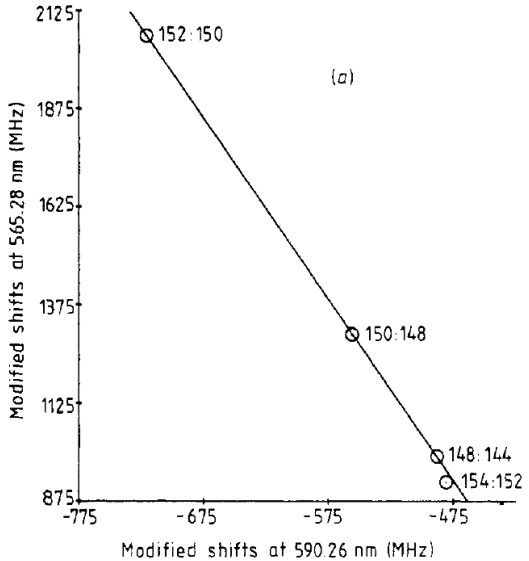


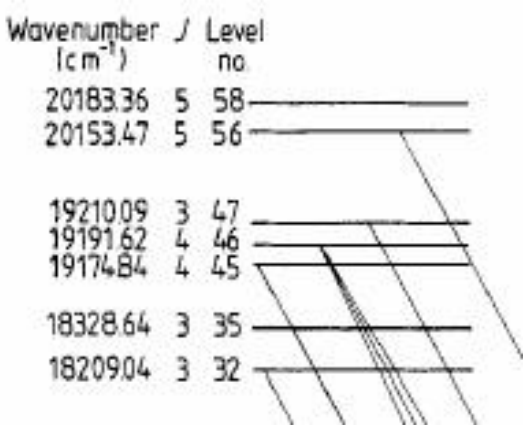
NP component out of the linearity plane

CAVEAT: SM breaks linearity at higher order

SM nonlinearity

- mixing of degenerate E levels, e.g. in Sm
  [Griffith, Isaak, New, Rall, 1981]
  [Palmer, Stacey, 1981]
- NLO FS





Standard Model contributions to King nonlinearity calculated
[Flambaum, Gerdes, Viatkina, 2018]

- for Ca^+, Sr^+, Ba^+, Yb^+, Hg^+

strategy: subtract SM NL and constrain residual NL

if linear $\Rightarrow$ set bounds on $y_e y_n$

"linear": $\mathrm{NL} \leq \sigma_{\mathrm{NL}}$ (uncertainty)

theory input needed:
perturbative relativistic many-body solution (CI+MBPT)

$$X_i = \int d^3r \frac{e^{-m_\phi r}}{4\pi r} \left[|\Psi_b(r)|^2 - |\Psi_a(r)|^2 \right]$$

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for 3 isotope pairs, 2 transitions: solve equation system

- 3 equations $\overrightarrow{m\nu}_2(\overrightarrow{m\nu}_1^{\mathrm{exp}}) \stackrel{!}{=} \overrightarrow{m\nu}_2^{\mathrm{exp}}$
- 3 unknowns: $K_{21}, F_{21}, y_e y_n$ (X_{21} theory input)

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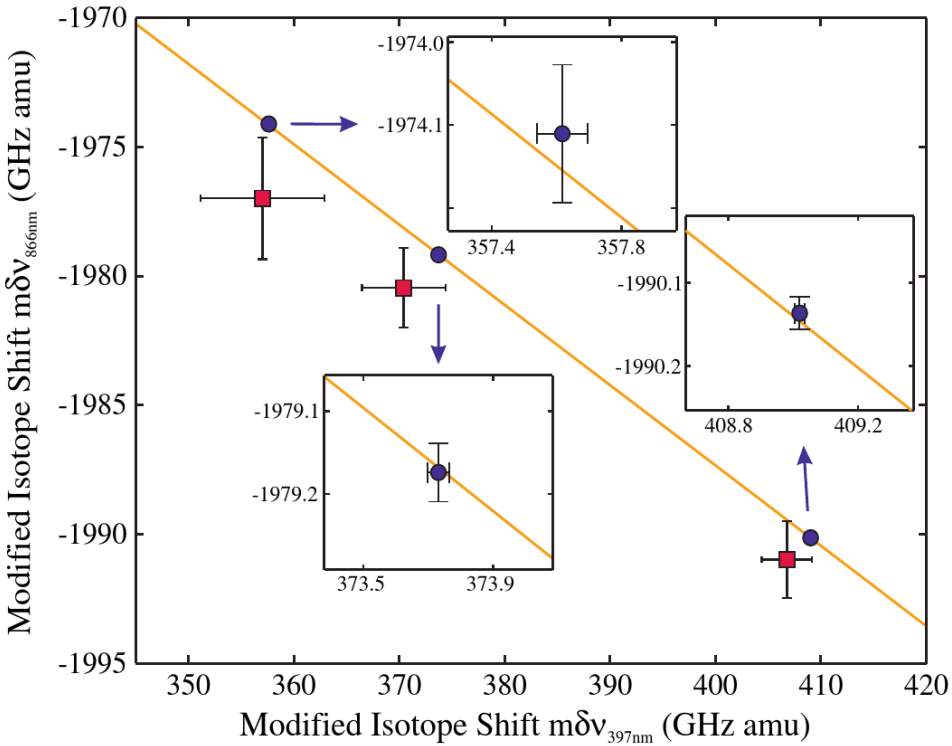
$$y_e y_n = \frac{(\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu}}{(\overrightarrow{m\mu} \times \vec{h}) \cdot (X_1 \overrightarrow{m\nu}_2 - X_2 \overrightarrow{m\nu}_1)}$$

data
theory
NP assumption

uncertainty: error propagation from frequency (and mass) measurements

New interaction between electrons and neutrons?

King plot: 2 electronic transitions in $\geq$ 3 isotope pairs



[Gebert, Wan, Wolf, Angstmann, Berengut, Schmidt; PRL 115, 053003 (2015)]

[update: see Drewsen et al]

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- very precise measurement of isotope shifts (IS) in Ca^+
  $\hookrightarrow$ King linearity as expected

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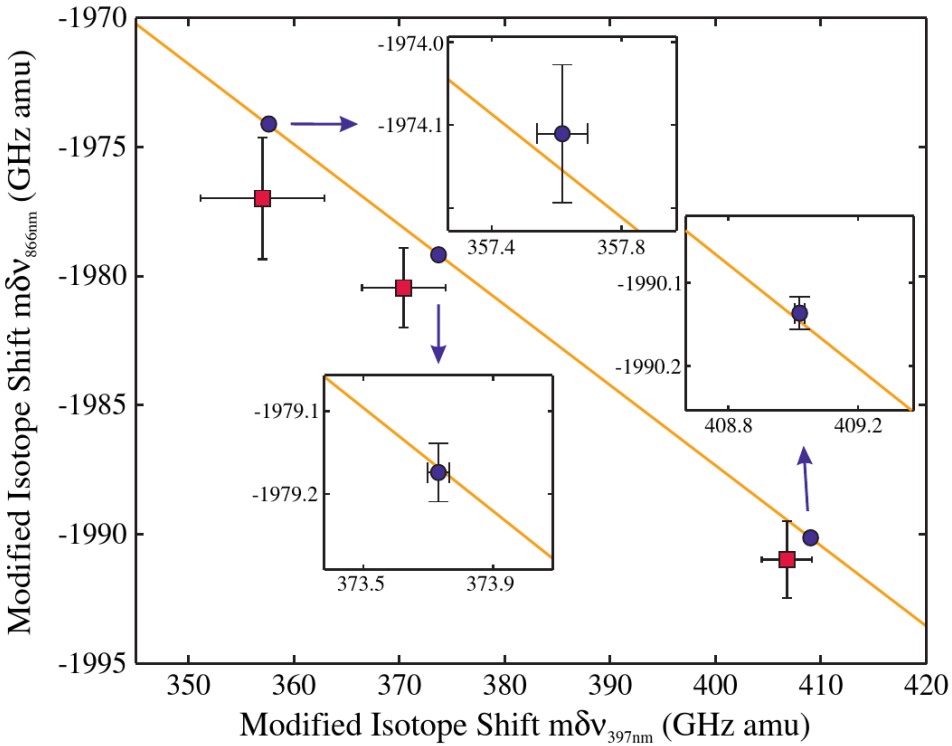

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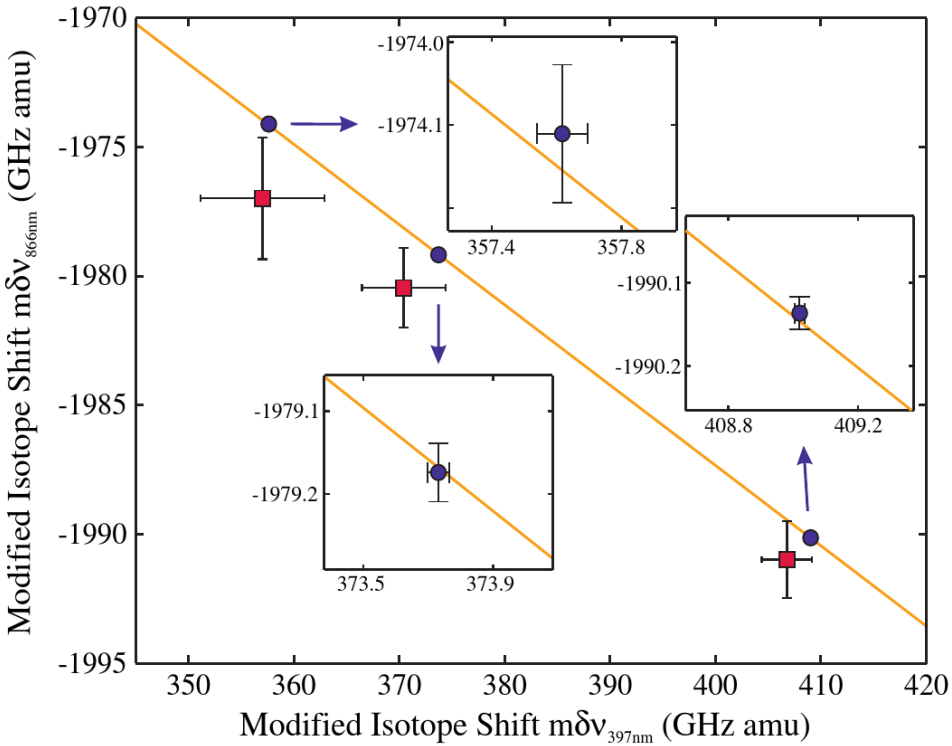

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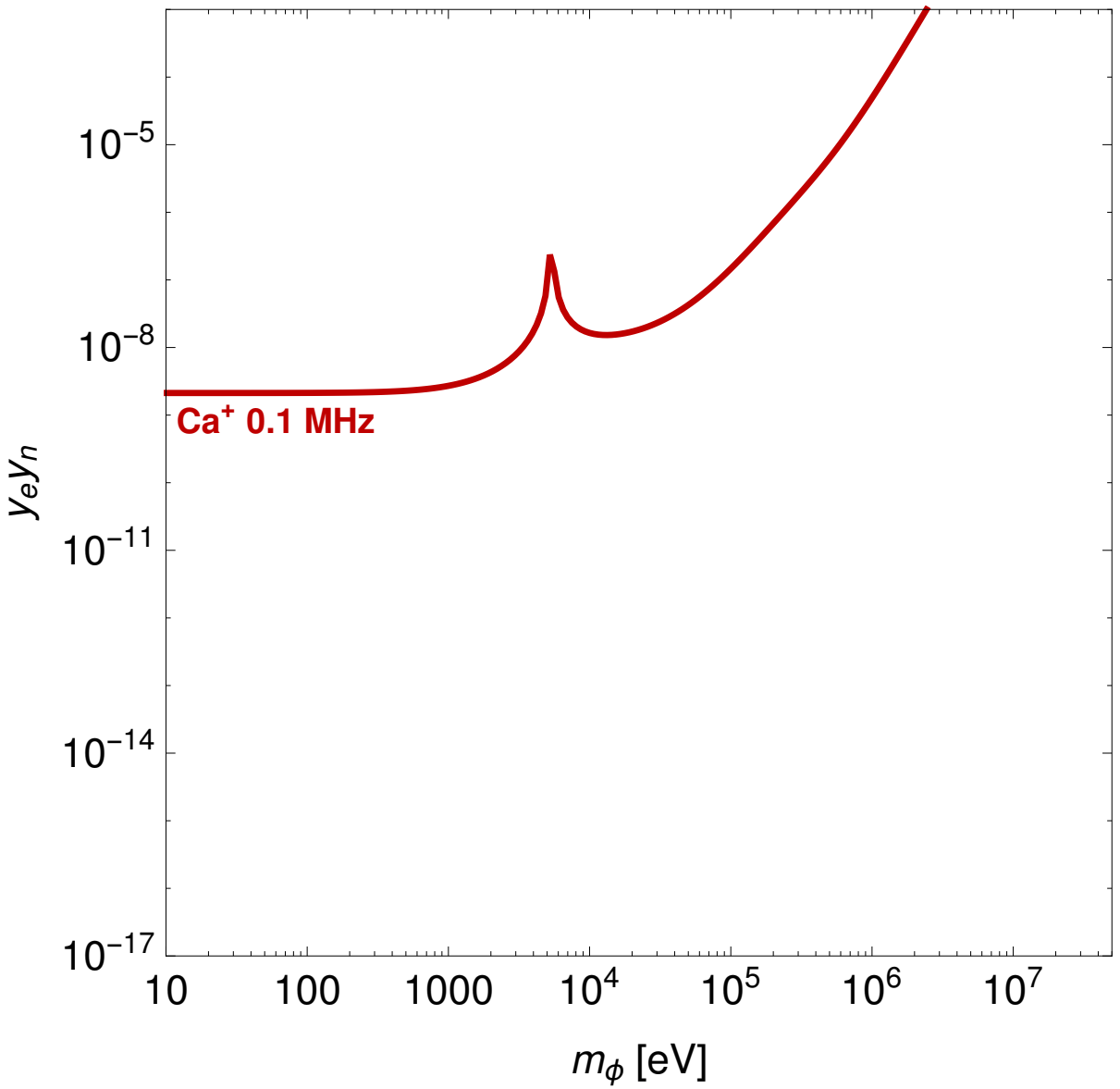
New Physics could manifest itself as nonlinearity in a King plot

Constraints on $y_e y_n$



1. massless limit (0 - inverse atomic size)
2. accidental cancellation
3. weaker for $m_\phi \to r_N^{-1}$

- measured Ca^+ with 100 kHz

[Berengut, Budker, Delaunay, Flambaum, Frugiuele, EF, Grojean, Harnik, Ozeri, Perez, Soreq '17]

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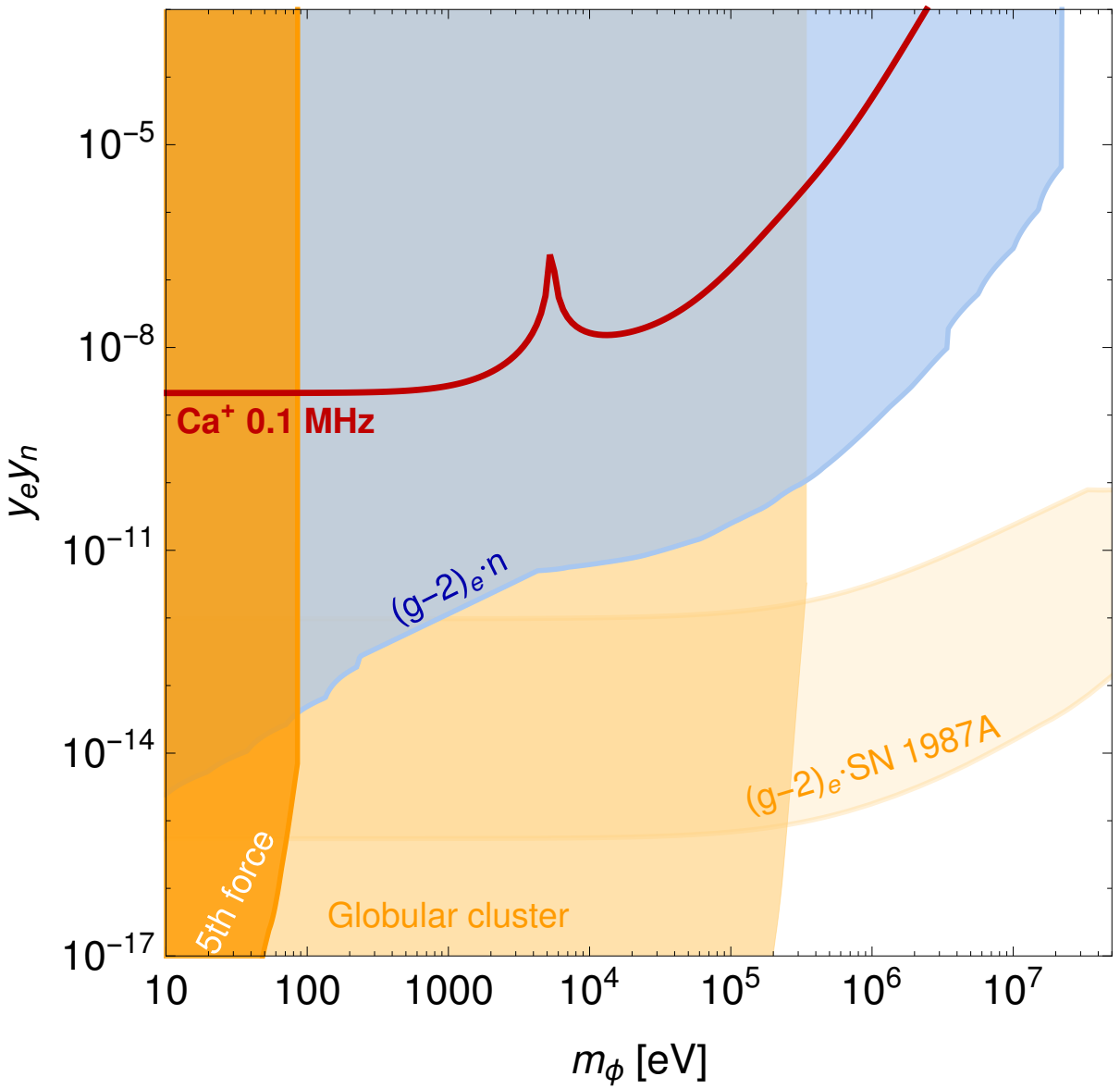

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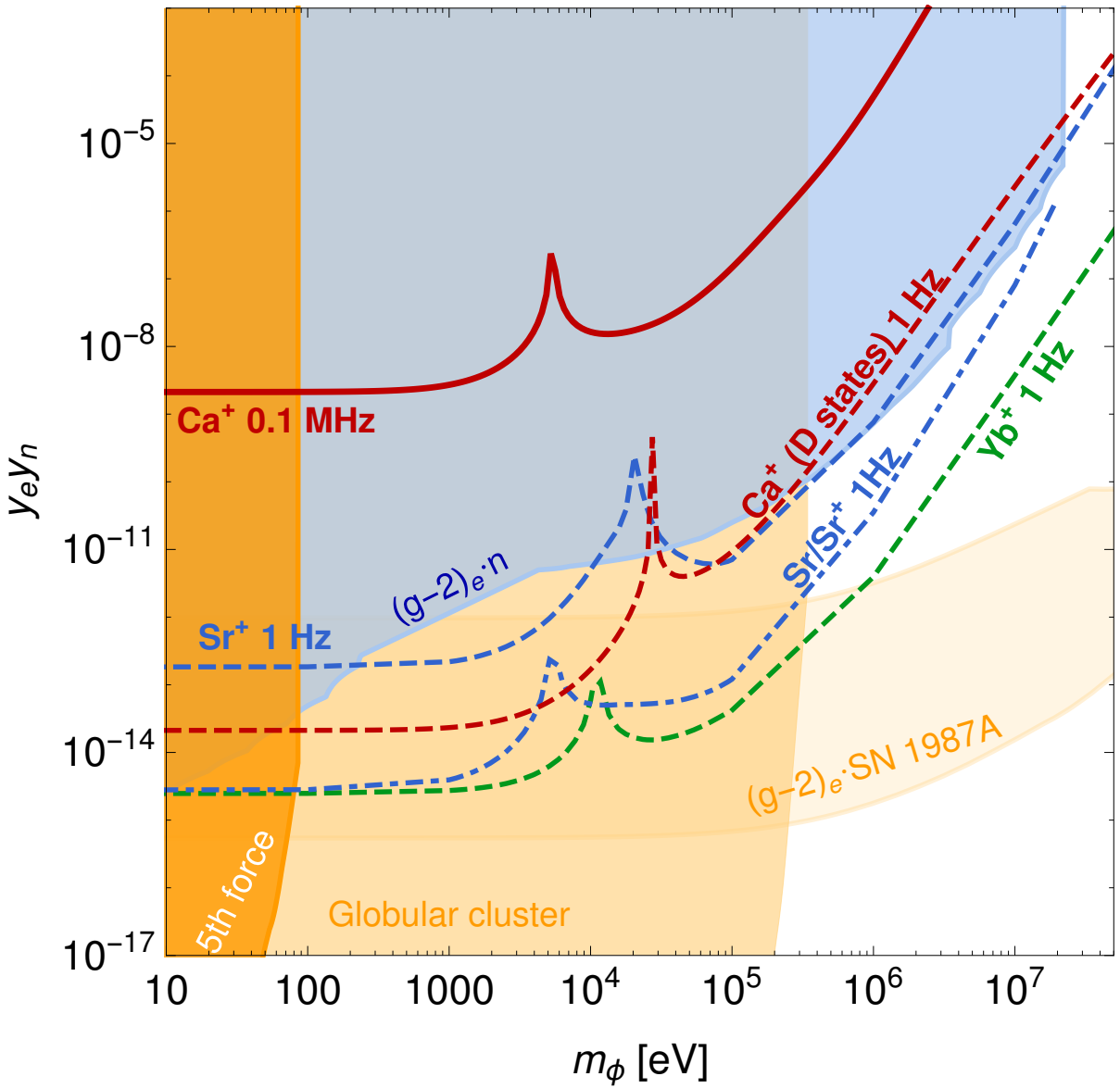

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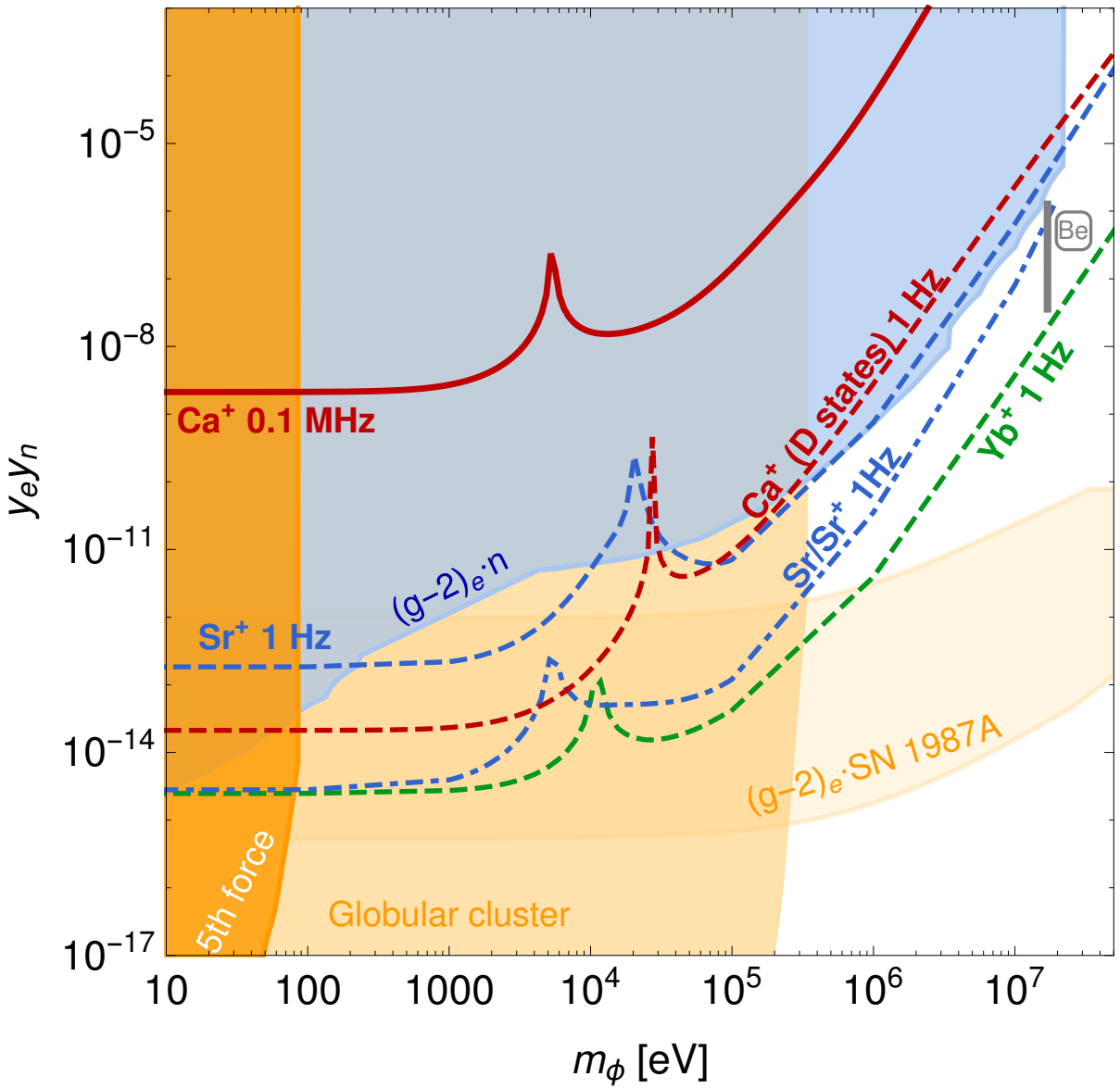

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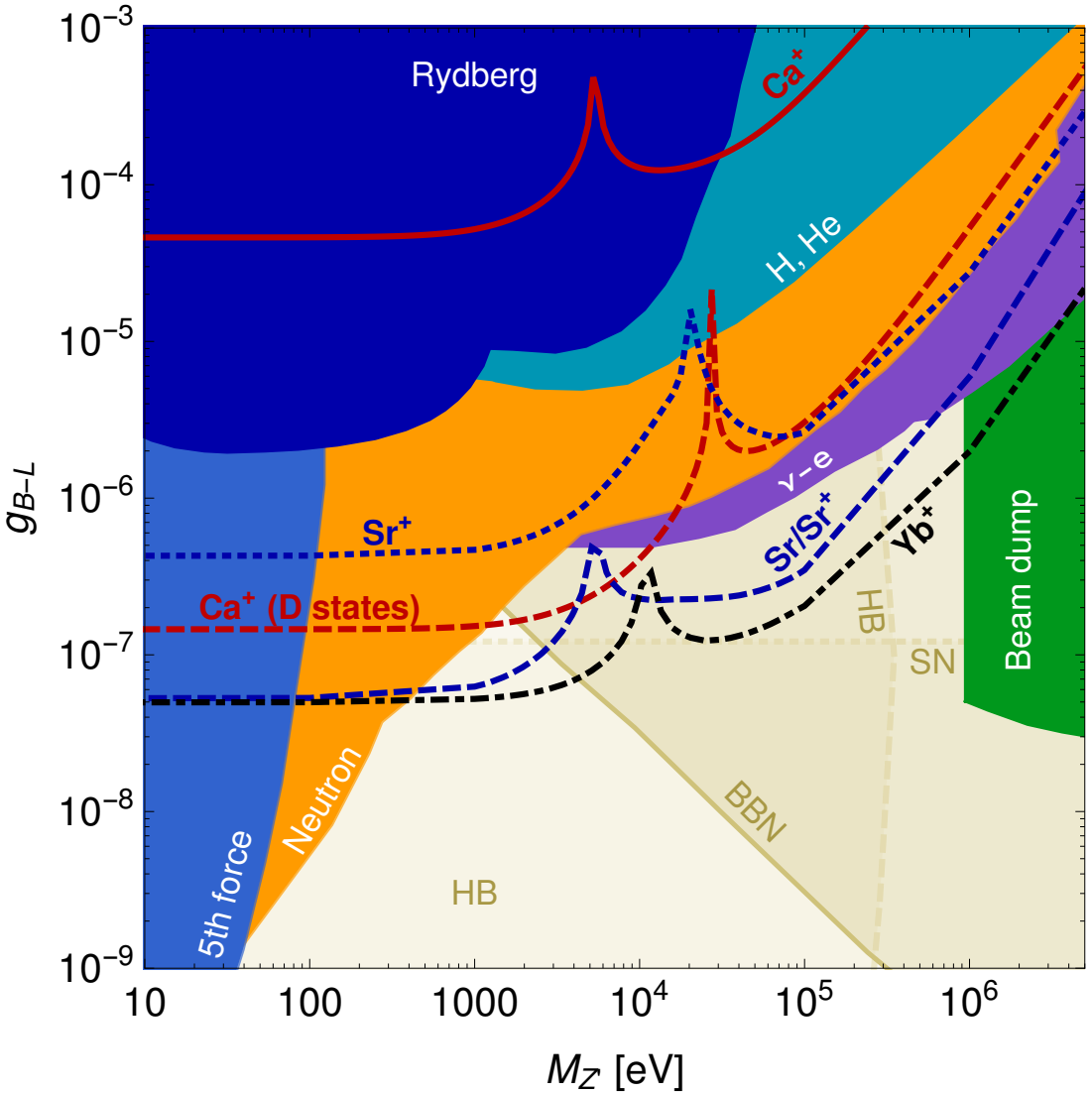
- probe Be anomaly

[Berengut, Budker, Delaunay, Flambaum, Frugiuele, EF, Grojean, Harnik, Ozeri, Perez, Soreq '17]

Z' of $U(1)_{B-L}$ (gauged baryon – lepton number)



$B-L$ charges

$$z_l = -1,\ z_q = \tfrac{1}{3},$$
$$z_{n,p} = 1,\ z_A = A$$

$\Rightarrow$ high sensitivity to

$$g_e^V \cdot g_n^V = -g_{B-L}^2$$

[Frugiuele, EF, Perez, Schlaffer '17]

[B-L bounds: e.g. Harnik, Kopp, Machado '12; Heeck '13]

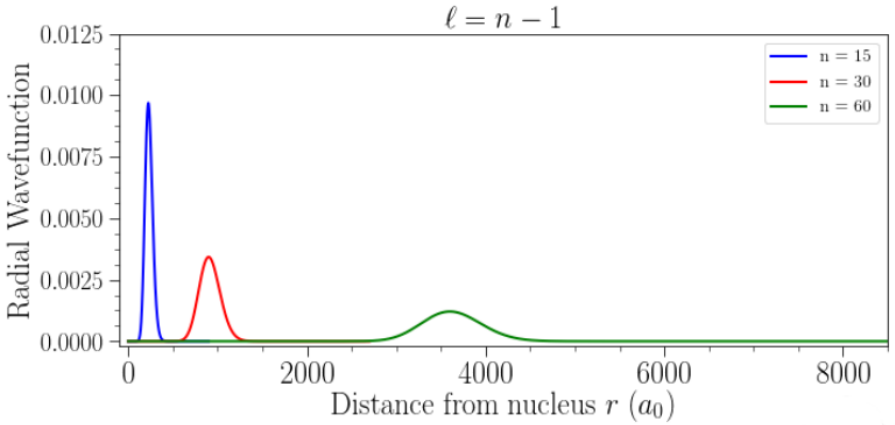
strong astro bounds, KLV potentially the strongest lab probe

Rydberg states

Duque-Mesa, Firstenberg, EF, Perez, Shpilman (work in progress)

Rydberg states (highly excited)

- high $n, l \rightsquigarrow$ advantages:
  - valence electron far and less affected by nucleus
  - long lifetimes
- quantum defects modify energy levels: $n \to n - \delta_{n,l}$

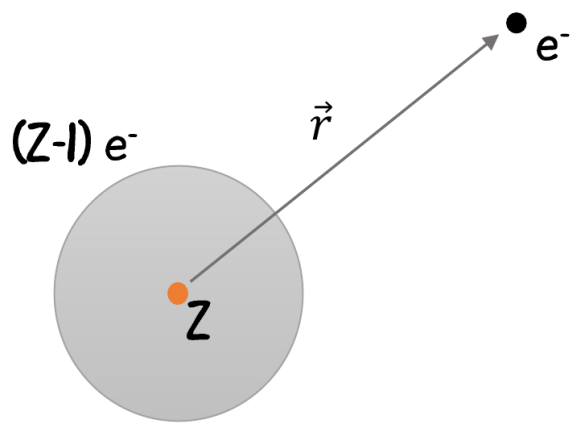


Multi-electron effects

- shielding: $V_{\text{penetration}} = -\frac{1}{4\pi\epsilon_0}\frac{Z_{\text{eff}}(r)e^2}{r}$,
  $Z_{\text{eff}}(r)$ is effective nuclear charge
- polarizability: $V_{\text{pol}} = -\sum_{k=1}^{\infty}\frac{\alpha^{E_k}}{2r^{2(k+1)}} \approx -\frac{C_4}{r^4} - \frac{C_6}{r^6}$,
  α^{E_k} is electric multipole polarizability

factorise wavefunction $\psi = \psi^{\text{core}}\psi^{\text{Rydberg}-\text{valence}}$

[courtesy: S. Duque-Mesa]



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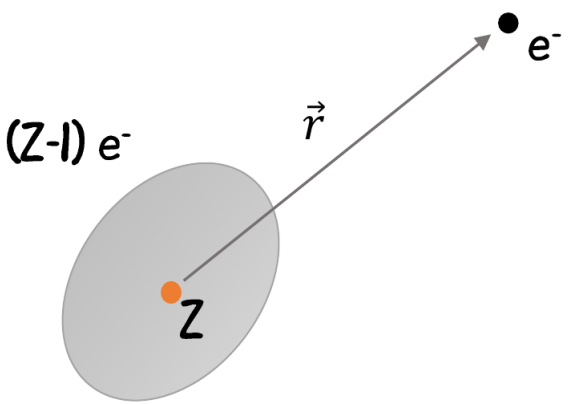
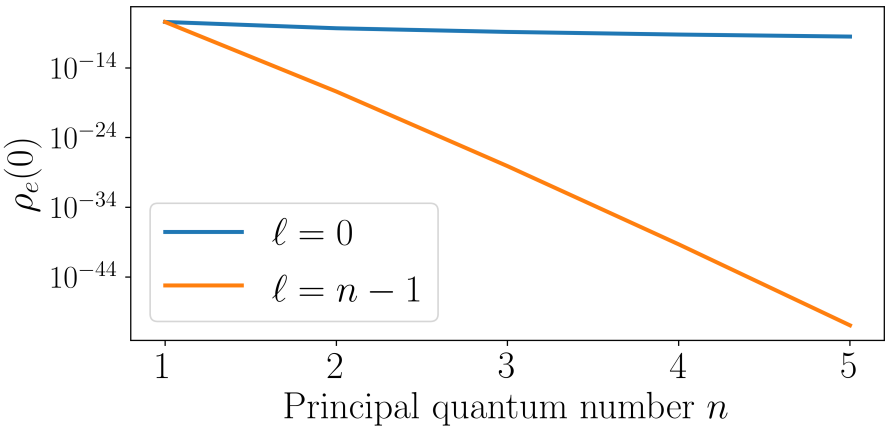


Rydberg isotope shifts

[Sebastián Duque-Mesa, master thesis]



$\Rightarrow$ field shift vanishes

- long-range polarization factorised into electronic factor P_i and $\mu_{AA'}$
  $\Rightarrow$ absorbed in mass shift
- isotope dependence of polarizability negligible if below exp. resolution

$$\nu_i^A - \nu_i^{A'} = K_i\,\mu_{AA'} + \underbrace{F_i}_{\to 0}\,\lambda^{AA'} + X_i\,\frac{y_e y_n}{4\pi}\gamma^{AA'}$$

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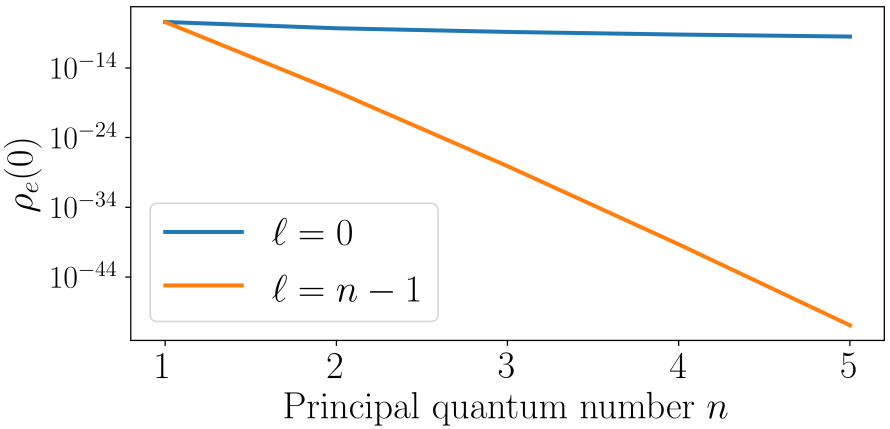

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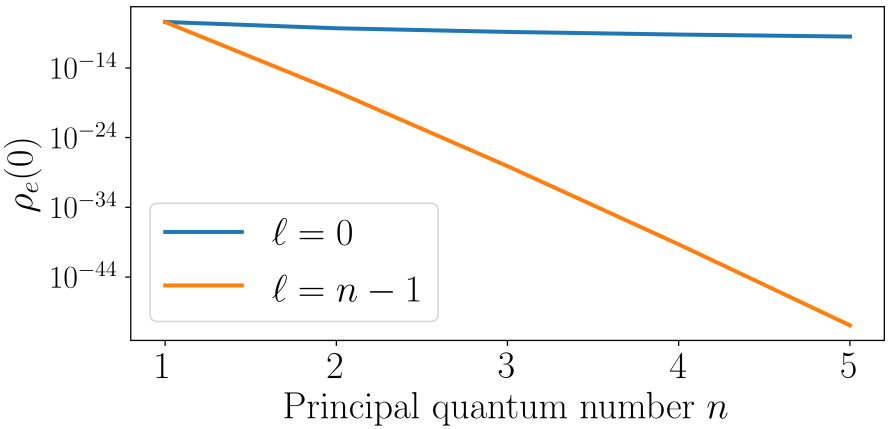

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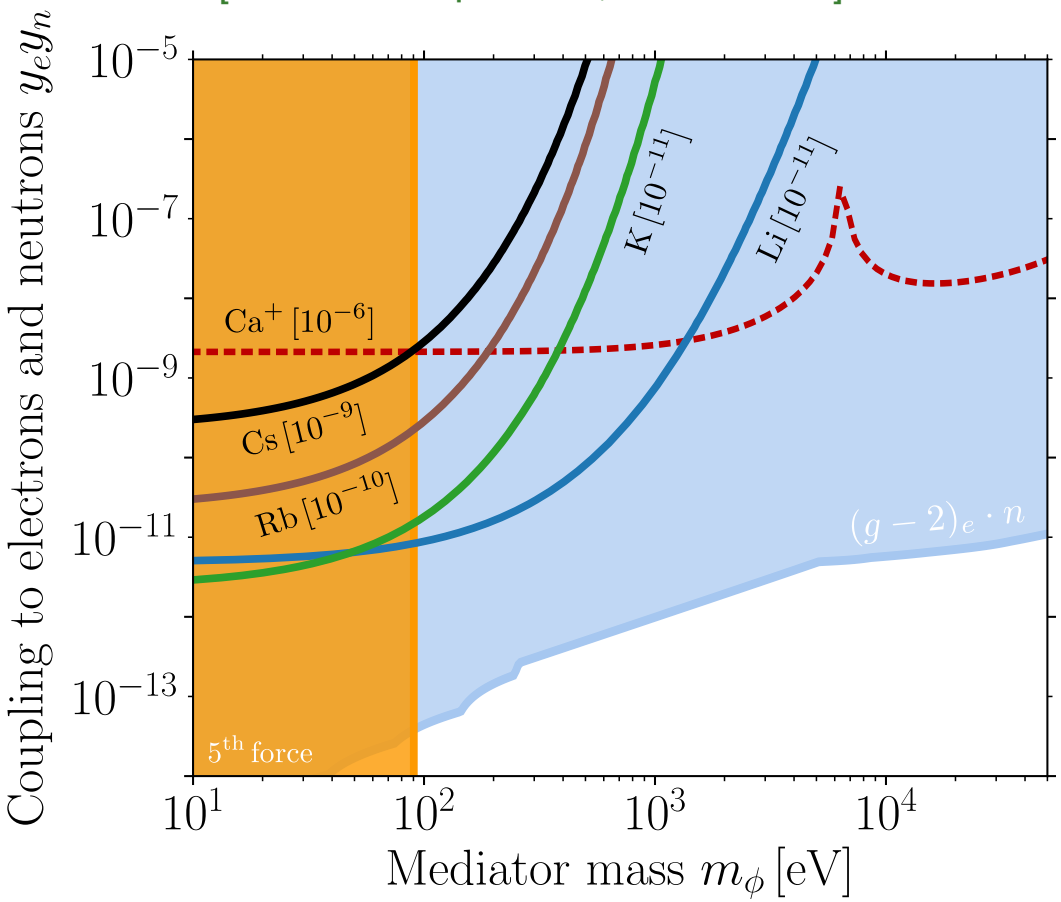
  $\Rightarrow$ absorbed in mass shift
- isotope dependence of polarizability negligible if below exp. resolution

$$\nu_i^A - \nu_i^{A'} = (K_i + P_i)\,\mu_{AA'} + X_i\,\frac{y_e y_n}{4\pi}\gamma^{AA'}$$

1 transition, 3 isotopes: linearity check of $\nu_{AA'}$ vs $(A - A')/(AA')$

Alkali Rydberg bounds vs King plot

[Sebastián Duque-Mesa, master thesis]



- Rydberg bounds strong for low m_ϕ
- can exceed King bound for m_ϕ <keV
- $(g-2)_e$ by scalar = -3 times by pseudoscalar $\Rightarrow$ cancellation possible!
- then Rydberg strongest laboratory bound

Rydberg bounds complementary to King plot: lower m_ϕ

Dark Matter constrained by Dynamical Decoupling

1902.02788

Aharony, Akerman, Ozeri, Perez, Savoray, Shaniv

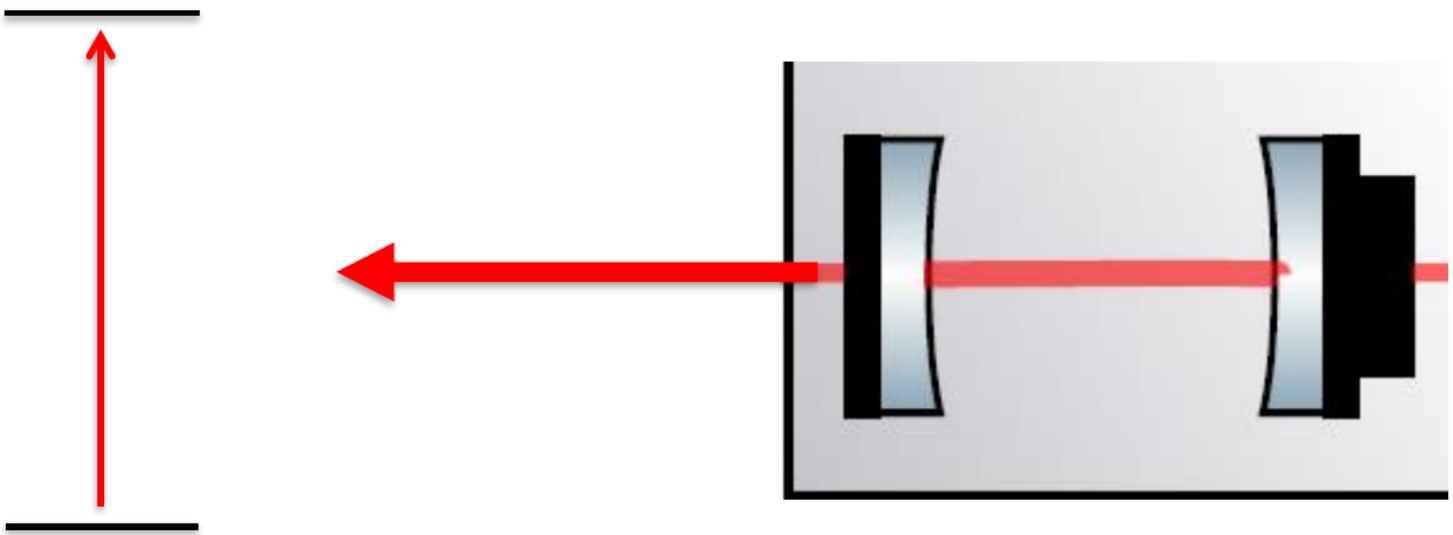
1902.08212

Banerjee, Budker, Eby, Kim, Perez

Dynamical Decoupling

- aim: probe variation of α and m_e due to sub-eV scalar DM
- general idea: amplify signal in Hz- MHz bandwidth in noisy environment
- narrow optical clock transition
- low frequencies: only sensitive to α variation because cavity stiff; at high frequency to both α and m_e

[talk by Ozeri, NPIK 2019]



$R_\infty \propto \alpha^2 m_e$

$a_0 \propto (m_e \alpha)^{-1}$

Dynamical Decoupling constrains light DM

Consider

$$\mathcal{L}\frac{g_{\phi\gamma}}{4}\phi F^{\mu\nu}F_{\mu\nu} - g_{\phi e}\phi\bar{e}e$$

modification of constants $X = X^{\rm SM} + \delta X$, $X = \alpha, m_e$:

$$\delta m_e = g_{\phi e}\,\langle\phi(t, \overrightarrow{x})\rangle$$
$$\delta\alpha = g_{\phi\gamma}\,\langle\phi(t, \overrightarrow{x})\rangle\,\alpha^{\rm SM}$$

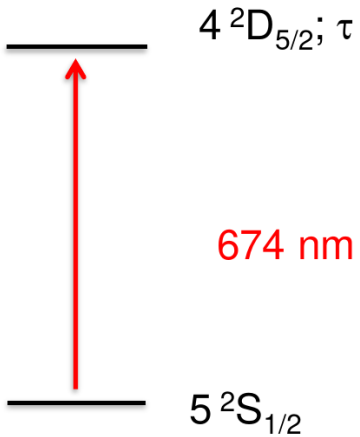
if ϕ DM candidate, will oscillate ($\rho_{\rm DM}$ is DM density)

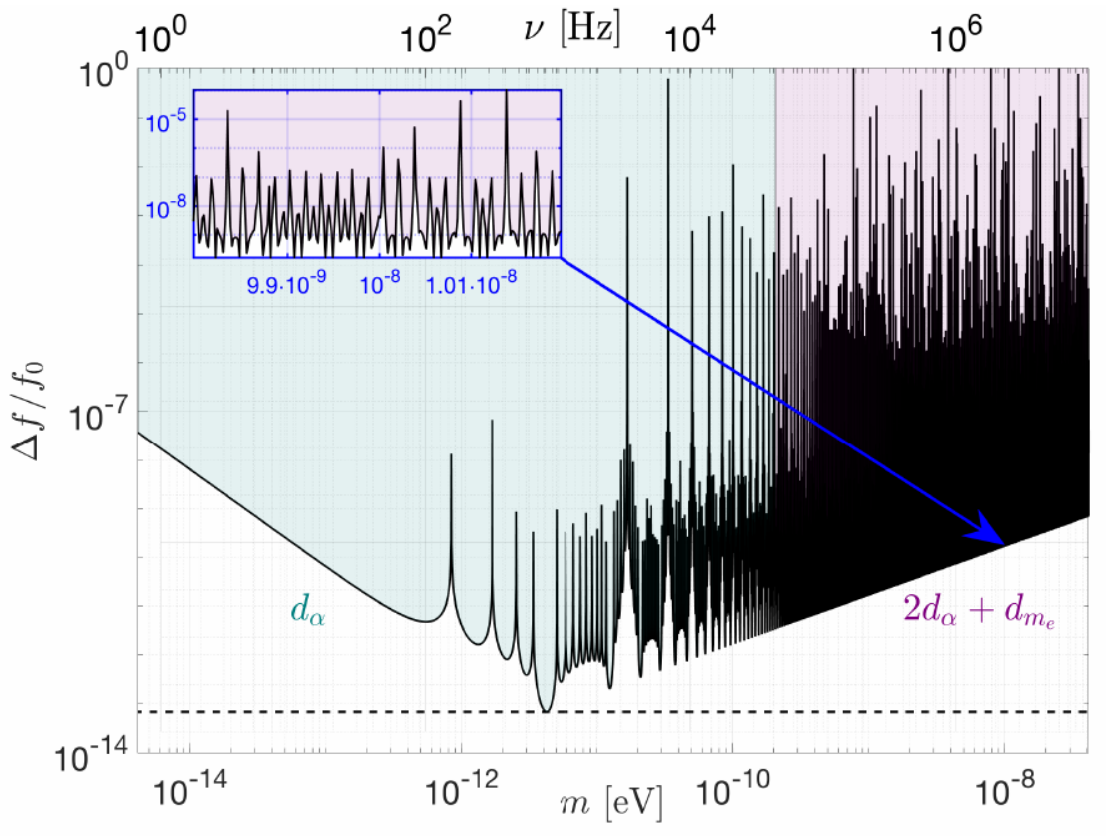
$$\langle\phi(t, \overrightarrow{x})\rangle \simeq \frac{\sqrt{2\rho_{\rm DM}}}{m_\phi}\cos\left(m_\phi(t - \overrightarrow{v}\cdot\overrightarrow{x})\right)$$

oscillation frequency = mass

DD: experimental precision [Aharony, Akerman, Ozeri, Perez, Savoray, Shaniv]

[talk by Ozeri, NPIK 2019]

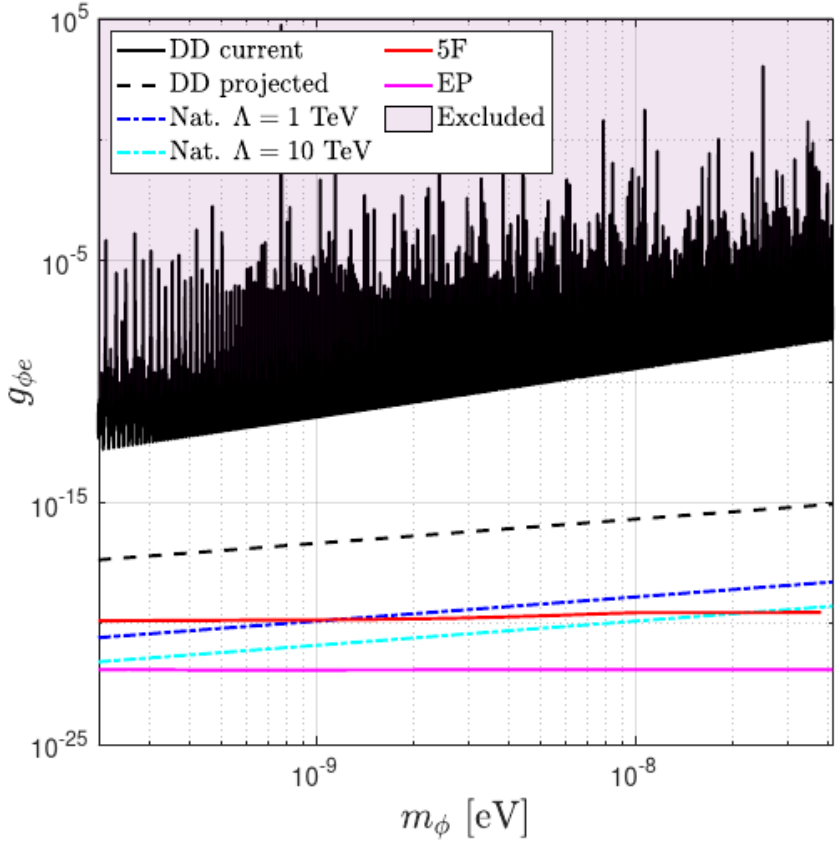


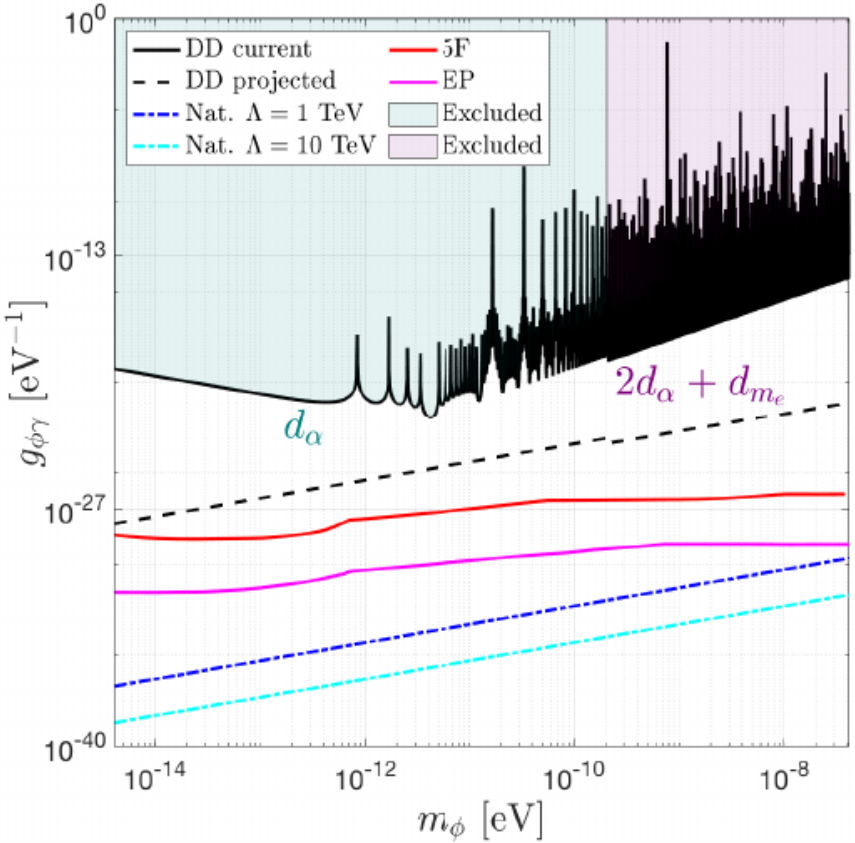


overall sensitivity can be achieved in scan (here fixed $\nu_\mu = 1013\,\mathrm{Hz}$)

Constraints on $g_{\phi e}$ and $g_{\phi\gamma}$

[Aharony, Akerman, Ozeri, Perez, Savoray, Shaniv]

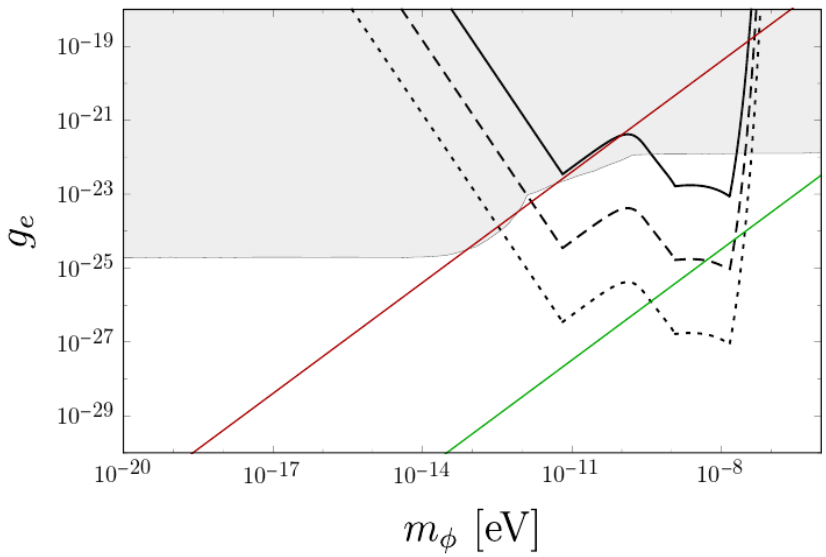


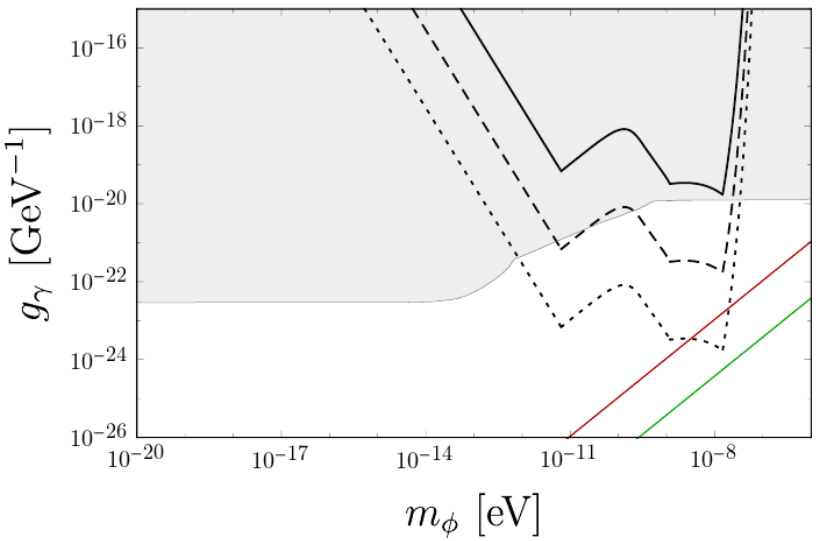


significant experimental improvement realistic

Dynamical Decoupling constrains relaxion halo

- very light relaxion can constitute coherently oscillating DM [Banerjee, Kim, Perez '18] (for keV relaxion DM see [Fonseca, Morgante])
- can form *relaxion stars* or trapped Earth or Sun *relaxion halo*: denser than local DM density [Banerjee, Budker, Eby, Kim, Perez]
- applicable to other light-DM models





experimental sensitivities $\delta\alpha/\alpha = \delta m_e/m_e = 10^{-14}, 10^{-16}, 10^{-18}$

Earth halo probes natural and physical relaxion region

Direct comparison of precise data and theory

1602.04822, PRD D96 (2017) no.11, 115002

Delaunay, Frugiuele, EF, Soreq

Few-electron systems: exploiting precision

- King plots:
  - \+ data-driven
  - \+ eliminate unknown $\delta \langle r^2 \rangle$
  - − but need 4 isotopes, 2 transitions

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  - \+ 1 isotope, 1 transition sufficient
  - \+ applicable to isotope shifts and absolute frequencies
  - \+ couplings $y_e\, y_n,\ y_e\, y_p,\ y_e^2$
  - − restricted to light elements: H/D, He, Li, N, Ps,...
  - − relies on theory control
  - − proton radius puzzle $\rightsquigarrow$ IS instead of absolute frequencies for H, D

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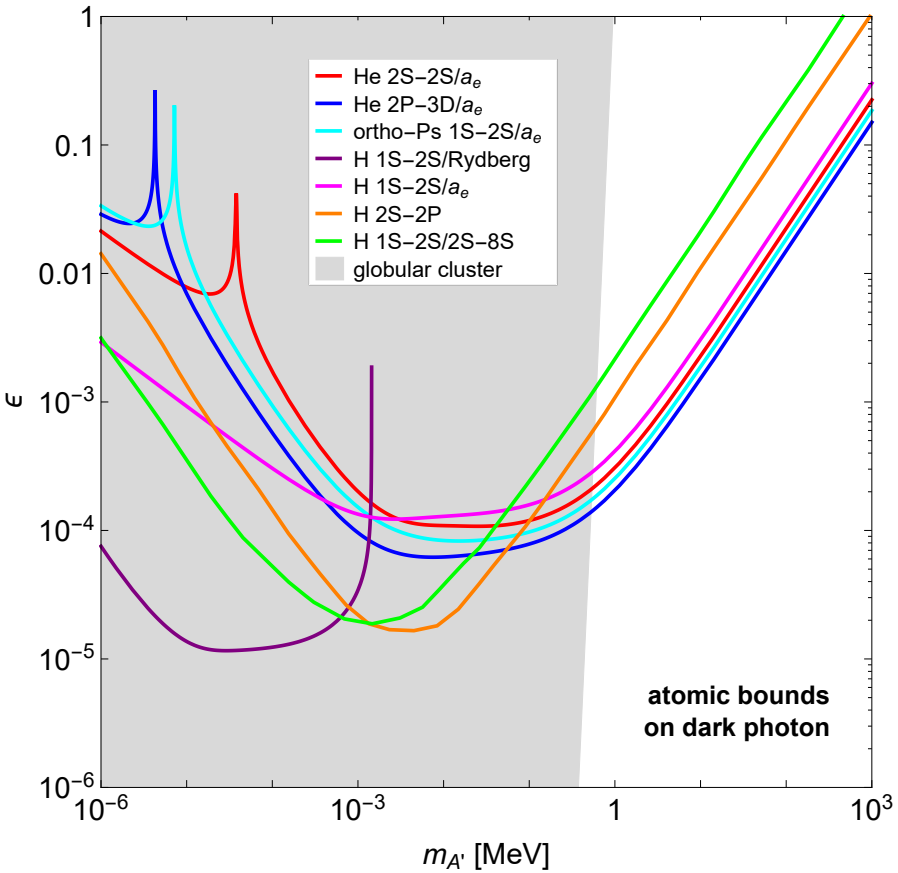
element	transition i	$\nu_i^{\rm exp}$ [kHz]	ref.	$\nu_i^{\rm th}$ [kHz]	ref
H	$2S_{1/2} - 12D_{3/2,5/2}$	799 191 727 402.8 ± 6.7	[83]	799 191 727 409.1 ± 3.0	[103]
D		799 409 184 967.6 ± 6.5		799 409 184 973.4 ± 3.0	

e.g. [Beauvoir et al 2000, Jentschura et al 2005]

direct comparison between predictions and data: see if room for NP

Application to Dark Photon model



[Delaunay, Frugiuele, EF, Soreq '17]

- Dark photon A' with kinetic mixing ϵ
- He, positronium with a_e
- H spectroscopy $1S - 2S$
  - with Rydberg states or a_e [Karshenboim 2010, 2010] [Pospelov 2009]
  - combine 2 transitions [Roy, Jaeckel 2010]

Conclusions: atomic precision probes of light NP

- **King linearity** probes new particle interacting between e and n
  - IS: sensitive to $y_e y_n$ for $0 \leq m_\phi \lesssim \mathcal{O}(10)\,\mathrm{MeV}$
  - future measurements sensitive to unexplored parameter space
- **Rydberg** complementarity to King plots, less isotopes, 1 transition
  - less affected by nuclear uncertainties
  - sensitivity determined by precision of polarizability effects
- **Dynamical Decoupling** tests time-variation of m_e, α and thereby oscillating DM
- **Direct theory-experiment comparison** for few-electron systems
- **Outlook**
  - Ca^+ D-state isotope shifts improve King plot bound [Drewsen et al]
  - highly-charged ions: explore suitable systems for constraints on slightly larger masses

THANK YOU

APPENDIX

Considered transitions

	transition 1 [nm]	transition 2 [nm]	accuracy σ_i	$y_e y_n$ bound $(m_\phi = 0)$
Ca^+	397.0	866.5	$0.1\,\mathrm{MHz}$	$2 \cdot 10^{-9}$
Ca^+	729.3	732.6	$1\,\mathrm{Hz}$	$2 \cdot 10^{-14}$
Sr^+	674.0	687.0	$1\,\mathrm{Hz}$	$2 \cdot 10^{-13}$
Sr/Sr^+	698.4	674.0	$1\,\mathrm{Hz}$	$3 \cdot 10^{-15}$
Yb^+	435.5	466.9	$1\,\mathrm{Hz}$	$2 \cdot 10^{-15}$

The 95 % CL bounds on $y_e y_n$ for a massless mediator ϕ from Ca^+ data and 95 % CL projections for Ca^+, Sr^+, Sr/Sr^+ and Yb^+ assuming on error of $\sigma_i = 1\,\mathrm{Hz}$.

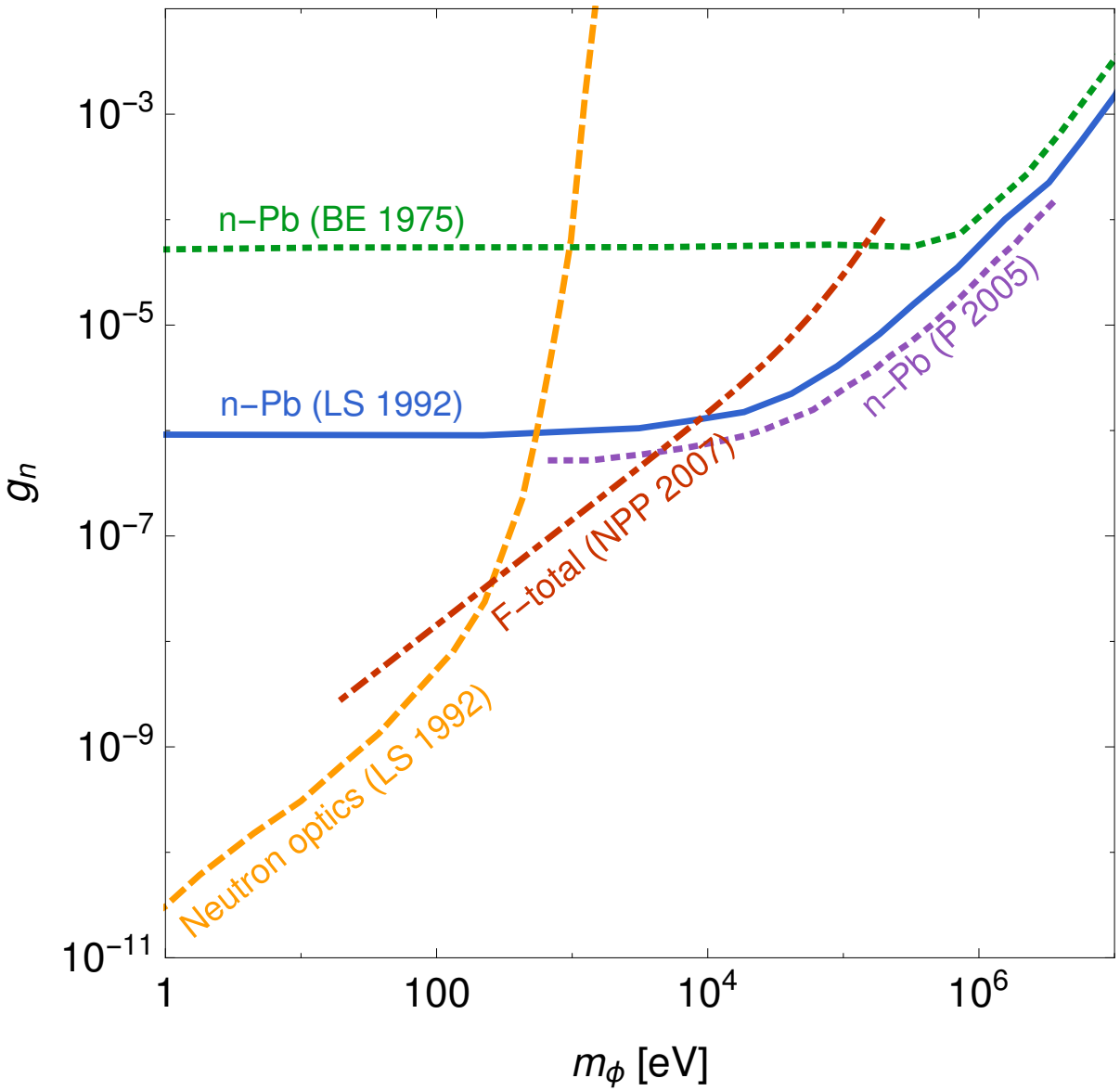
SM contribution to nonlinearity

Table IV. Estimates for the non-linearities of King plot (defined in (23)). Methods 1 and 2 are based on the mean-field analytic expression of field isotope shift (25) found from the estimate of wave function density (9) in an uniformly charged spherical nucleus. Method 1 utilizes the liquid drop approximation for nuclear radius $R = r_0 A^{\frac{1}{3}}$ with $r_0 = 1.15$ fm and Method 2 uses experimental data [18, 19] for mean squares of nuclear charge radii $\langle r^2 \rangle$ when finding the equivalent nuclear radius: $\langle r^2 \rangle = \frac{3}{5} R^2$. Method 3 accounts for both expression (25) and the contribution of nuclear polarizability (26), equivalent nuclear radius R is again based on the experimental data, i.e. it is the most complete calculation in this table. We have not calculated these corrections and the nuclear polarizability contribution in $s - p/d - p$ transitions in Ca^+ since they are expected to be similar to $s - d$ transitions.

Ion						Pair of transitions	Non-linearity (Hz)		
	Z	A	A_1	A_2	A_3		Method 1	Method 2	Method 3
Ca^+	20	40	42	44	48	$3p^6 4s\ ^2S_{1/2} \to 3p^6 4p\ ^2P_{1/2}$ $3p^6 3d\ ^2D_{3/2} \to 3p^6 4p\ ^2P_{1/2}$	-1.2×10^{-4}	3.0×10^{-4}	–
						$3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{3/2}$ $3p^6 4s\ ^2S_{1/2} \to 3p^6 3d\ ^2D_{5/2}$	-1.2×10^{-4}	3.0×10^{-4}	-6.6×10^{-2}
Sr^+	38	84	86	88	90	$4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{3/2}$ $4p^6 5s\ ^2S_{1/2} \to 4p^6 4d\ ^2D_{5/2}$	-1.1×10^{-3}	1.1×10^{-2}	-2.6
Ba^+	56	132	134	136	138	$5p^6 6s^1\ ^2S_{1/2} \to 5p^6 5d\ ^2D_{3/2}$ $5p^6 6s^1\ ^2S_{1/2} \to 5p^6 5d\ ^2D_{5/2}$	-3.6×10^{-3}	-3.9×10^{-2}	7.6
Yb^+	70	168	170	172	176	$4f^{14} 6s\ ^2S_{1/2} \to 4f^{13} 6s^2\ ^2F^o_{7/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$	6.1×10^{-2}	-3.1	38
						$4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{3/2}$ $4f^{14} 6s\ ^2S_{1/2} \to 4f^{14} 5d\ ^2D_{5/2}$	-6.1×10^{-2}	3.1	-18
Hg^+	80	196	198	200	204	$5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{3/2}$ $5d^{10} 6s\ ^2S_{1/2} \to 5d^9 6s^2\ ^2D_{5/2}$	5.5×10^{-1}	3.1	-14

[Flambaum, Gerdes, Viatkina, 2017]

Bounds on g_n


10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
g_n
1
100
10^4
10^6
m_ϕ [eV]
n–Pb (BE 1975)
n–Pb (LS 1992)
n–Pb (P 2005)
F–total (NPP 2007)
Neutron optics (LS 1992)

Rydberg details

Property	n-scaling
$\langle r \rangle$	n^2
E_n	n^{-2}
τ (low-ℓ)	n^3
τ (high-ℓ)	n^5
ΔE_n	n^{-3}

circular states

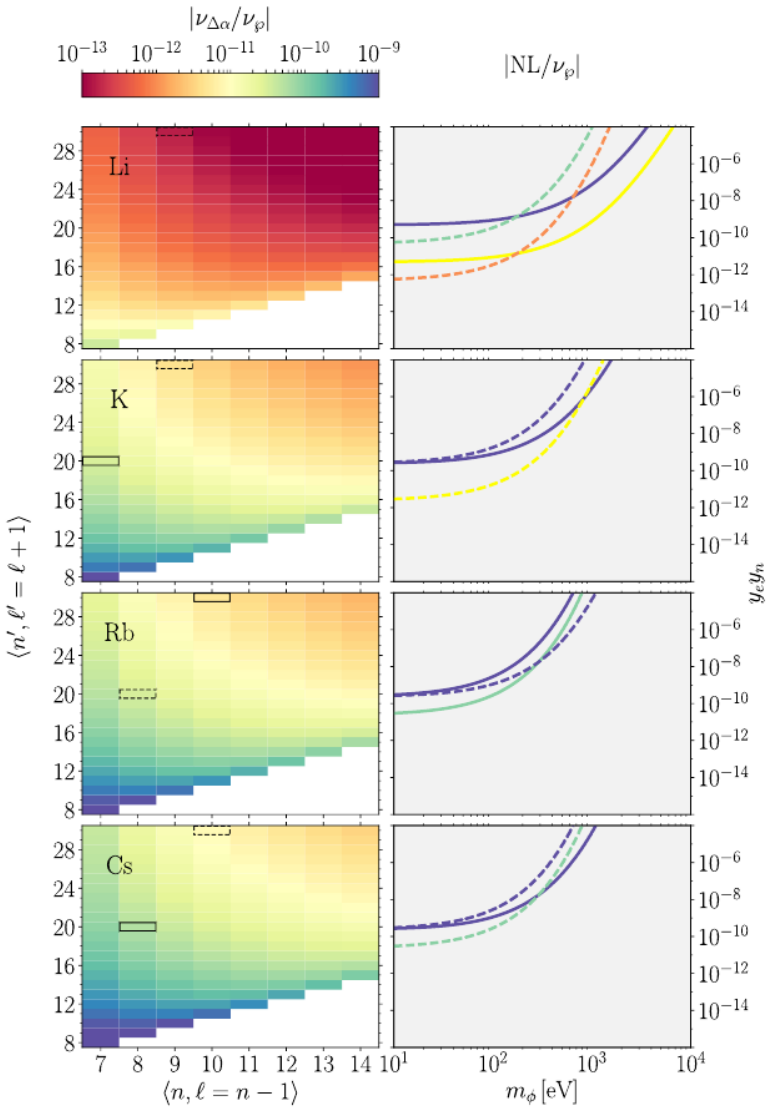
- maximal $l = n - 1$
- high density only in ring around nucleus
- long lifetime
- suppressed Stark effect

Quantum defect δ_l

- $\delta_{n,l} \approx \delta_0 + \delta_2/(n - \delta_0)^2$
- capture deviation from H (phase shift)
- determined from experimental dipole and quadrupole polarizabilities

Polarizability shift and bound on NP

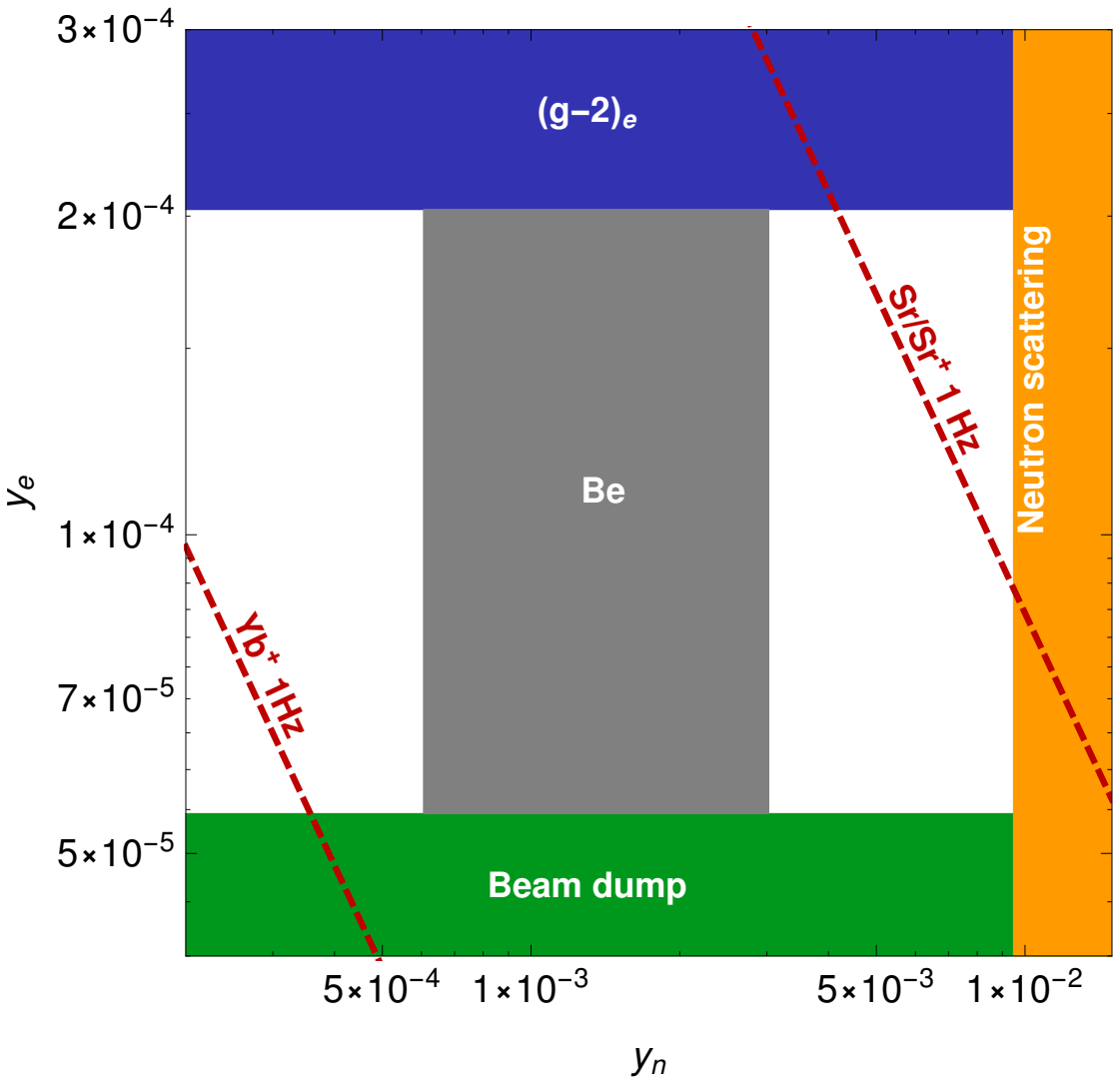
[Master thesis, S. Duque-Mesa]



polarizability contribution to isotope shift, normalized by transition frequency

- for transition $\nu = (n, l = n-1) \to (n', l' = l+1)$
- if $\nu_{\Delta\alpha}/\nu <$ exp. precision: neglect polarizability shift
- bounds on $y_e y_n$ assuming exp. precision of $\nu_{\Delta\alpha}/\nu$

^{8}Be (17 MeV) anomaly



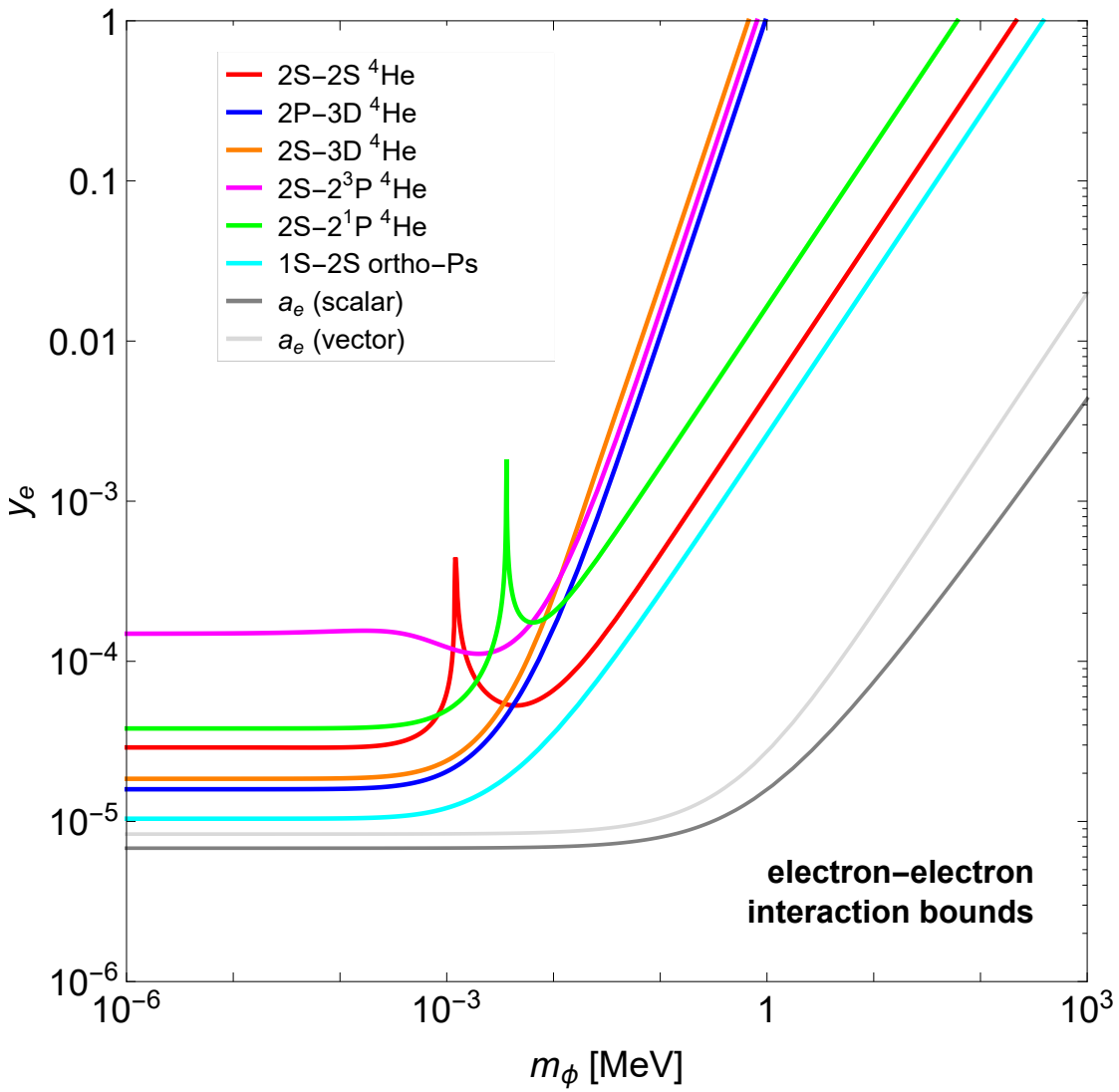
$^8\mathrm{Be}^* \to {}^8\mathrm{Be} + e^+e^-$

6.8σ excess in e^+e^- angle
[Krasznahorkay et al '15]

Protophobic interpretation:
$^8\mathrm{Be}^* \to {}^8\mathrm{Be} + \mathrm{X}$,
$X \to e^+e^-$, $m_X = 17\,\mathrm{MeV}$
[Feng et al '16 '16]

Yb^+ with 1 Hz can probe entire Be parameter space

Direct comparison of data and theory

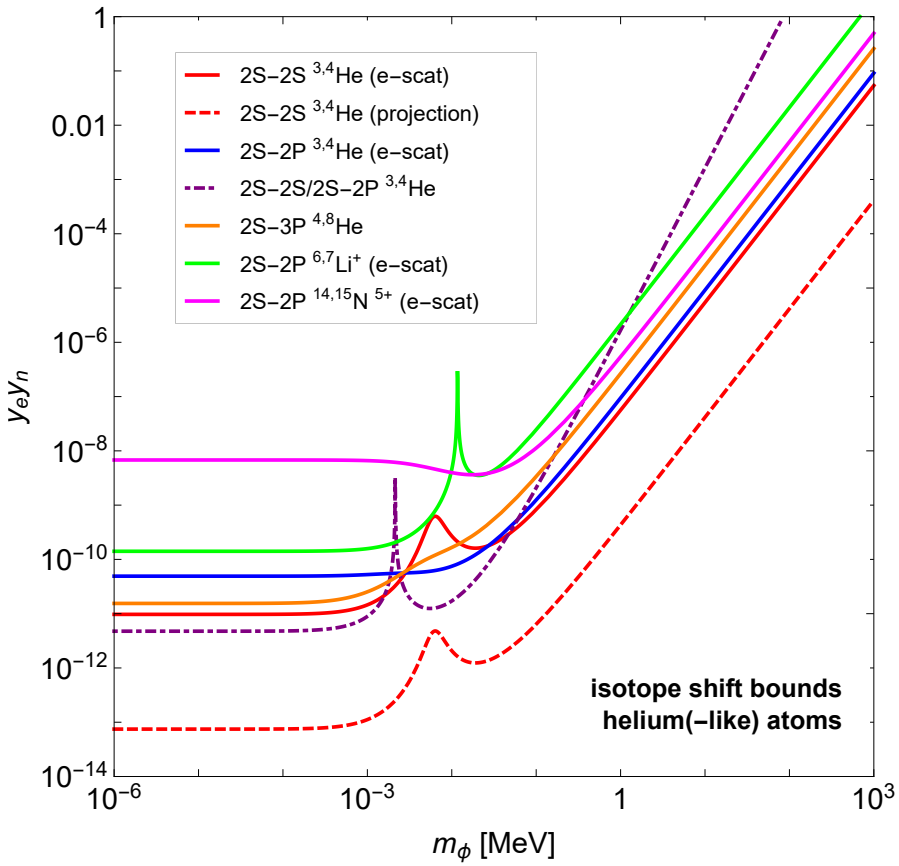


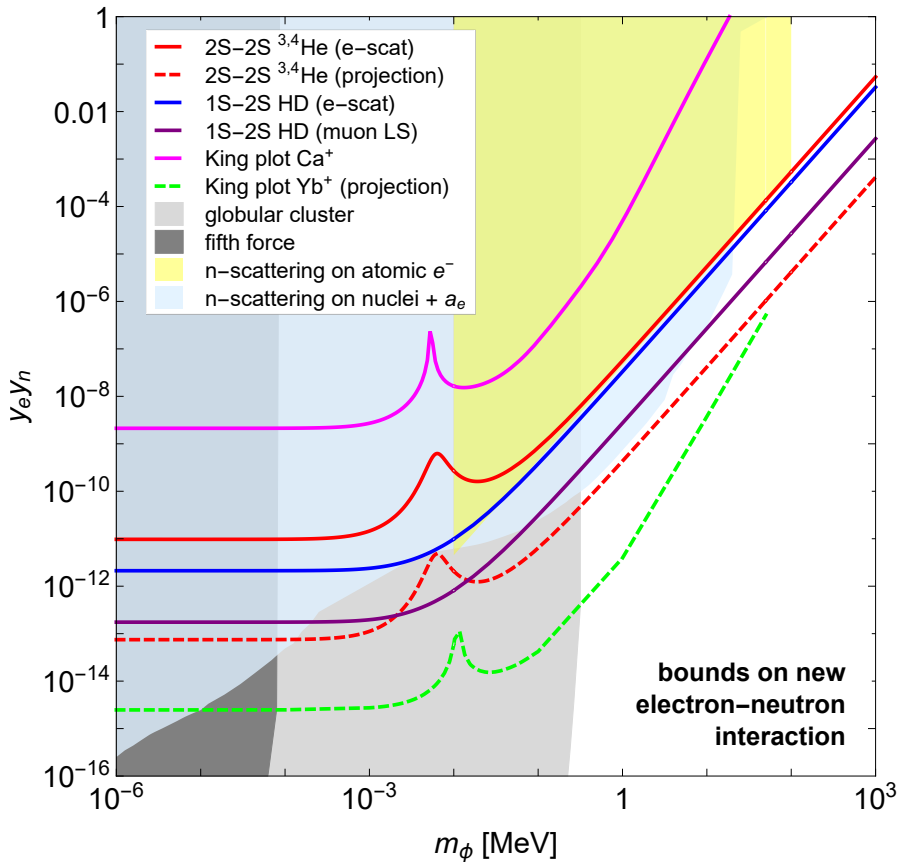
[Delaunay, Frugiuele, EF, Soreq '17]

- He, Ps spectroscopy for y_e^2
- H used for $y_e y_p$
  [Karshenboim 2010, 2010]
  [Jaeckel, Roy 2010]

positronium almost as sensitive to y_e as $(g-2)_e$

Isotope shifts without King plot





[Delaunay, Frugiuele, EF, Soreq '17]