The high energy limit of QCD: k_T-factorization and exclusive production cross-sections at the LHC

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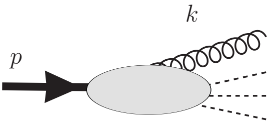
Based on results obtained with
G. Chachamis, A. Sabio Vera & C. Salas and J.Bartels & L.N. Lipatov

Outline

Motivation



- collinear factorization: $k = x \cdot p$ -collinear
  $k^2 = 0$ - onshell
- k_T factorization: $k = x \cdot p + \boldsymbol{k}$ - add k_T,
  $k^2 = -\boldsymbol{k}^2$ off-shell

- natural @ small x: center of mass frame: $x\sqrt{s} \simeq |\boldsymbol{k}|$
  ➡ arises from high energy factorization
  & for BFKL evolution (small x resummation)

Why?

- take into account k_T of gluon
  ➡ more accurate kinematics - exclusive!
  collinear factorization: only NLO and beyond
- direct connection to small x
  ➡ appropiate for high center of mass energies!
- direct test of BFKL-amplitudes, access to QCD-ReggeonFieldTheory, saturation physics, ...

Different realizations of k_T factorization

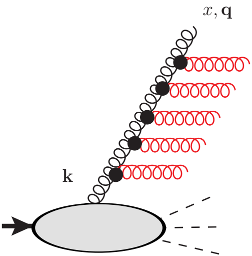
LO order approaches:

- LO BFKL evolution: scaleless, running coupling doesn't run
- DGLAP: KMR unintegrated parton distributions (and extensions thereof): based on standard DGLAP evolution, generate k_T in the last evolution step
- CCFM equation: based on color coherence, interpolation between BFKL and DGLAP, implemented into CASCADE Monte Carlo

Here: construct approach based on NLO BFKL evolution:

- reduce LO scale dependence
- running coupling: naturally incorporated
- complete small x resummation
- gluon k_T build up through the complete evolution

NLO unintegrated gluon density



Gluon density: convolution of BFKL Green's function f_{BFKL} & proton impact factor $\phi_P(\boldsymbol{k})$

$$g(x, \boldsymbol{q}^2) = \int \frac{d^2\boldsymbol{k}}{2\pi} f_{\text{BFKL}}(x, \boldsymbol{q}, \boldsymbol{k}) \frac{\phi_P(\boldsymbol{k})}{\boldsymbol{k}^2}$$

ϕ_P: non-perturb.: model & fit to data (HERA)

$$\phi_P(\boldsymbol{k}; Q_0, \alpha, b_0, A) = A \cdot \left(\frac{\boldsymbol{k}^2}{Q_0^2 + \boldsymbol{k}^2}\right)^{\alpha} e^{-b_0 \cdot \boldsymbol{k}^2}$$

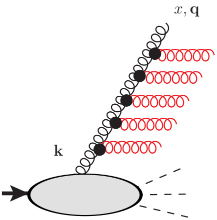
Cross-section: need off-shell partonic cross-section $\hat{\sigma}(x, \boldsymbol{q}^2, \ldots)$

$$d\sigma = \int_\tau^1 \frac{dx}{x} \int_0^\infty d\boldsymbol{q}^2\, g(x, \boldsymbol{q}^2) \cdot d\hat{\sigma}(x, \boldsymbol{q}^2, \ldots)$$

Here: DIS-like ($\gamma^* p \to X$) example

The NLO BFKL Green's function

Use 2 methods in parallel: (A) '**analytic**'



$$f(x, \boldsymbol{q}, \boldsymbol{k}) = \frac{1}{\sqrt{\boldsymbol{q}^2 \boldsymbol{k}^2}} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi^2 i}\, x^{-\chi_{\text{BFKL}}(\gamma)} \left(\frac{\boldsymbol{q}^2}{\boldsymbol{k}^2}\right)^{\gamma-1/2}$$

$$\chi_{\text{BFKL}}(\gamma) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) + \bar{\alpha}_s^3 \chi_{\text{RG}}(\gamma)$$

representation of BFKL kernel in γ-space ➡ exhibits (anti-)collinear singularities of kernel

- (anti-)collinear singularities: poles of χ_{BFKL} for $\gamma \to 0, 1$
- resummation by higher order term $\chi_{RG}(\gamma)$ ➡ BFKL kernel in agreement with DGLAP limit ➡ RG-improved BFKL

Some aspects of the running coupling

- NLO kernel $\chi_1(\gamma)$ contains running coupling term

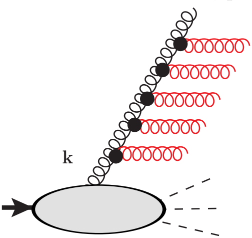
$$-\frac{\alpha_s(\mu^2)\beta_0}{8N_c}\left[-i\overleftarrow{\partial}_\gamma\chi_0(\gamma)+\chi_0(\gamma)i\overrightarrow{\partial}_\gamma-2\ln\mu^2\right]$$

- operator: acts on Mellin transforms of ϕ_P and $d\hat{\sigma}$

$$\sigma(Q^2,\tau)\sim\int\limits_{1/2-i\infty}^{1/2+i\infty}\frac{d\gamma}{2\pi i}\tilde{\phi}_P(\gamma;A,\alpha,b,Q_0^2)\cdot\tilde{f}_{\text{BFKL}}(\tau,\gamma)\cdot\tilde{\hat{\sigma}}(\gamma,\ldots)\left(\frac{Q^2}{Q_0^2}\right)^\gamma$$

- running coupling depends on external scales + structure of impact factors
- running coupling term: (anti-)collinear singularities ➡ requires modification of $\chi_{RG}(\gamma)$
- pp-scattering: complication due to 2^{nd} gluon density

The NLO BFKL Green's function



(B) '**Monte Carlo**' [Andersen, Sabio Vera, 03-04]
$\rightarrow$ see talks by M. Deak, A. Sabio Vera

- iterative solution of BFKL Green's function in (x, k_T) space
- close connection to n-gluon production
- inclusive $\rightarrow$ exclusive straight forward

$$f(x,\mathbf{q},\mathbf{k}) = x^{-\omega_\lambda(\mathbf{q})}\Big\{\delta^{(2)}(\mathbf{q}-\mathbf{k}) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{l}_i \Big[K_\lambda^{\text{real}}\Big(\mathbf{q}+\sum_j^{i-1}\mathbf{l}_j, \mathbf{q}+\sum_j^{i}\mathbf{l}_j\Big)$$

$$\times \int_{x_{i-1}}^{1} \frac{dx_i}{x_i} x_i^{-\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i}\mathbf{l}_j)+\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i-1}+\mathbf{l}_j)}\Big]\delta^{(2)}\Big(\mathbf{q}+\sum_{j=1}^{n}\mathbf{l}_j-\mathbf{k}\Big),\Big\}$$

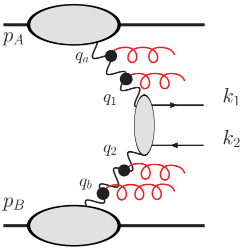
- RG-improved BFKL kernel in k_T-space known, once singularity structure understood within γ-representation
- use both approaches as cross-check

A study of a LHC process

Completely differential $b\bar{b}$-production:

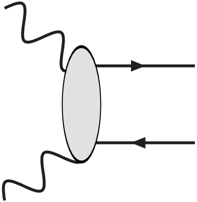
$$\frac{d\sigma^5}{d\eta_1 d\eta_2 d\boldsymbol{k}_1^2 d\boldsymbol{k}_2^2 d\phi_{Q\bar{Q}}} = \int d^2\boldsymbol{q}\, g\left(x_1, \boldsymbol{q}^2\right) \cdot V_{Q\bar{Q}}^{g^* g^*}(\boldsymbol{q}, \boldsymbol{k}_1, \boldsymbol{k}_2, \Delta\eta) \cdot g\left(x_2, (\boldsymbol{q} - \boldsymbol{k}_{12})^2\right)$$

$$x_{1,2} = \sqrt{\frac{M^2 + \boldsymbol{k}_1^2}{s}} e^{\pm\eta_1} + \sqrt{\frac{M^2 + \boldsymbol{k}_2^2}{s}} e^{\pm\eta_2}$$



- $4m_b^2/s \sim 1.4 \cdot 10^{-6}$: BFKL kinematics
- $d\sigma$ differential in 5 variables ➡ direct access on k_T of gluons
- test and constrain ugds @ LHC
- construction of large variety of distributions possible ➡ contain intrinsic gluon k_T and resummation
- study differences to collinear factorized expressions

Matrix elements within k_T factorization



- The LO heavy quark production vertex $V_{Q\bar{Q}}^{g^* g^*}$ is known [Catani,Ciafaloni,Hautmann, 91]
- Main corrections from Green's function: study with LO vertex meaningful
- complete NLO treatment → NLO matrix elements

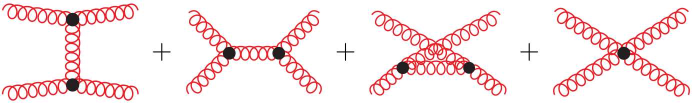
- only a limited # of NLO matrix elements known
- determination complicated due to off-shellness → gauge invariance
- promising method: gauge invariant effective action of high energy QCD [Lipatov, '95]

The effective action of high energy QCD and k_T factorized matrix elements

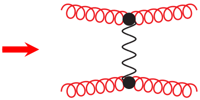
High energy factorization and the reggeized gluon

→ High energy (or Regge) factorization of QCD scattering amplitudes

gluon scattering at LO:



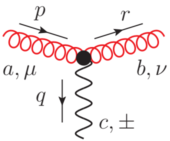
high energy limit: light-cone momenta of gluons strongly ordered



$s \gg |t|$, $t = q^2$

- effective d.o.f.: **reggeized gluon**
- couples to QCD partons by **effective vertex**
- on the mass-shell $q^2 = 0$: reggeized gluon → gluon
- polarization tensor $(n^-)^\mu (n^-)^\nu$ → leading power in s
- $q^2 = -\boldsymbol{q}^2$ only transverse,

Effective vertices for the reggeized gluon



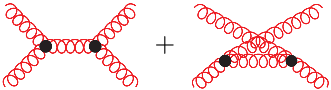
$= g f^{abc} \Gamma^{\mu\nu}_{\pm}(p, r)$

contains projection of QCD 3-gluon vertex on 'reggeized gluon state':

* polarization $n^{\pm}$ and 'kinematical constraint' $k^{\pm} = 0$
* high energy limit of QCD t-channel diagram

additional induced term $\sim \boldsymbol{q}^2/p^{\pm}$:

* collects non-suppressed parts of s- and u-channel diagrams
* $1/p^{\pm}$ identified with expansion of s/u-channel propagator



Ward ID:

$$\epsilon_{\mu}(p) \cdot \Gamma^{\mu\nu}_{\pm}(p, r) \cdot r_{\nu} = 0$$

- vanishes for real on-shell gluon
- gauge invariant reggeized gluon
  → factorization and formulation of effective action!

The gauge invariant effective action

[Lipatov, 1995]: systemative determination of effective vertices

$$S_{\text{eff}} = \int d^4x \, [\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\mu, A_+, A_-)]$$

- interaction of reggeized gluons $(A^a_\pm)$ with quark- (ψ) and gluon-fields (v_μ)
- reggeized gluon $A^a_\pm$: invariant under local gauge transformations, but global $SU(N_c)$ matrix $A_\pm = A^a_\pm \cdot t^a$
- kinematical condition $\partial_\pm A_\mp = 0$ (rapidity ordering!)
- coupling of $A^a_\pm$ to QCD fields described by induced term

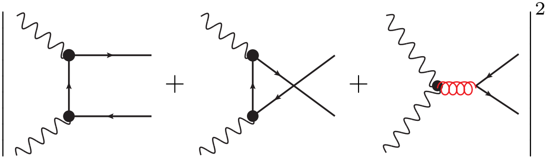
$$\mathcal{L}_{\text{ind}} = -\text{tr}[\frac{1}{g}\partial_- U(v_-)\partial^2_\sigma A_+] - \text{tr}[\frac{1}{g}\partial_+ U(v_+)\partial^2_\sigma A_-] - 2\text{tr}[\cdot A_+ \partial^2_\sigma A_-]$$

- all interaction by definition 'local' in rapidity space:

implied but not contained

Matrix elements from the effective action

LO matrix elements straight forward:
e.g. central heavy quark production



$= c \cdot V_{g^* g^* Q\bar{Q}}$

- Loop corrections show however divergencies, not present in QCD
- Natural for effective theory: expansion in certain energy ratios
  ➡ lose regulation present in full theory

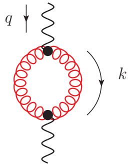
classical example: collinear factorization: $\frac{\boldsymbol{k}^2}{Q^2} \to 0 \Rightarrow$ collinear divergence

$\longrightarrow$ regularize by dimensional regularization

$\longrightarrow$ renormalization of parton densities: subtract divergencies and absorb them into parton densities

High energ divergencies of the effective action

effective action: loop correction to reggeized gluon propagator

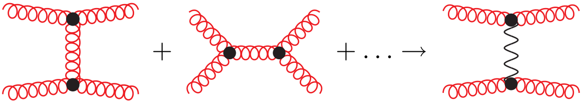


$$\sim \underbrace{\omega(\boldsymbol{q}^2)}_{\text{gluon trajectory}} \cdot \underbrace{\int_0^\infty \frac{dk^-}{k^-}}_{\text{crude cutoff}=ln s}$$

gluon trajectory
virtual part of BFKL kernel

crude cutoff=lns

reason: underlying approximation *e.g.*



only valid for $s \gg \Lambda^2$

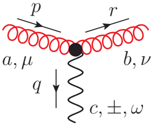
- Λ^2: scale of the order, but bigger than all transverse scale
- factorization scale: separates longitudinal from transverse dynamics

'ω'-regularization

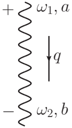
Constraint $s > \Lambda^2$ can be rewritten as Mellin integral

$$\Theta(|s| - \Lambda^2) = \lim_{\nu \to 0^+} \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{\omega + \nu} \left(\frac{s}{\Lambda^2}\right)^{\omega}$$

'ω'-regularization: factorize Θ-function ➡ modified Feynman rules



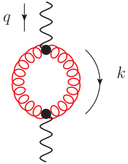
$$= 2g f^{abc} \Gamma^{\mu\nu}_{\pm}(p, r) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{p^{\pm}}{\Lambda}\right)^{\omega} \xi^{(-)}(\omega)$$



$$= \frac{2\pi i \delta(\omega_1 - \omega_2)}{\omega_1 + \nu} \frac{i/2}{\boldsymbol{q}^2 \xi^{(-)}(\omega)}$$

signature factor $\xi^{(-)}(\omega) = \frac{e^{-i\pi\omega} + 1}{2}$ ➡ $i\epsilon$ prescription of branch-cuts

The 1-loop propagator of the reggeized gluon



$$= 2\pi i \delta(\omega_1 - \omega_2) \xi^{(-)}(\omega_1) f\left(\omega_1, \frac{\boldsymbol{q}^2}{\Lambda^2}; \epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right)$$

$$f\left(\omega, \frac{\boldsymbol{q}^2}{\Lambda^2}; \epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = (-2i\boldsymbol{q}^2) \frac{-\alpha_s N_c (4\pi)^\epsilon}{2\pi} \left(\frac{\boldsymbol{q}^2}{\Lambda^2}\right)^\omega \left(\frac{\mu^2}{\boldsymbol{q}^2}\right)^\epsilon$$
$$\times \left[\frac{\Gamma^2(\omega - \epsilon)\Gamma(1 - \omega + \epsilon)}{2\Gamma(1 - \omega)\Gamma(2\omega - 2\epsilon)} - \frac{(2 - \epsilon)}{2}\left(\frac{\Gamma(-\omega - 1 + \epsilon)\Gamma(2 + \omega - \epsilon)\Gamma(1 - \omega)}{\Gamma^2(-\omega)\Gamma(4 + 2\omega - 2\epsilon)} - \frac{\Gamma(-\omega + \epsilon)\Gamma(2 + \omega - \epsilon)\Gamma(1 + \omega - \epsilon)}{\Gamma(-\omega)\Gamma(3 + 2\omega - 2\epsilon)}\right) - 2\frac{\Gamma(-\omega + \epsilon)\Gamma^2(1 + \omega - \epsilon)}{\Gamma(-\omega)\Gamma(2 + 2\omega - 2\epsilon)}\right]$$

interesting limit $\omega \to 0$:

$$f\left(\omega = 0, \frac{\boldsymbol{q}^2}{\Lambda^2}; \epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = -2i\boldsymbol{q}^2 \cdot \omega\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) \qquad \text{gluon trajectory}$$

The 1-loop propagator of the reggeized gluon

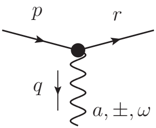
Combine with reggeized gluon propagators & GGR-vertices

$$\sim \omega\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{\omega^2} \left(\frac{s}{\Lambda^2}\right)^{\omega} \xi^{(-)}(\omega) = \frac{\ln(-s) + \ln s}{2} \omega\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right)$$

logharithmically enhanced contribution, complete $f(\omega)$?

- remainder $\sim \omega$ ➡ technically higher order (next-to-logharithmic)
- artefact of regularization, NLL correction something else
- use scheme where $f(\omega)$ is evaluated at $\omega = 0$

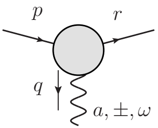
Example: Quark-Quark-Reggeized gluon (QQR) vertex



$$= \underbrace{igt^a (\not{n})_{\alpha\alpha'}}_{\text{eff. action}} \cdot \underbrace{\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{p^\pm}{\Lambda}\right)^{\omega} \xi^{(-)}(\omega)}_{\text{regularization}}$$

The 1-loop quark-reggeized gluon (QQR) vertex

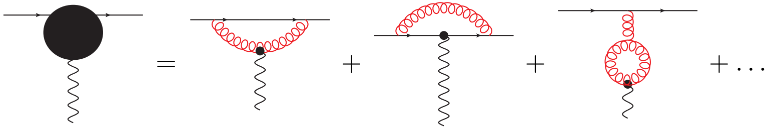
1-loop structure of following type (here: $p^2 = r^2 = 0$):



$$= \underbrace{\int\limits_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{p^\pm}{\Lambda}\right)^\omega \xi^{(-)}(\omega)}_{\text{regularization}} \cdot \underbrace{G^a_{\alpha\alpha'}\left(\omega; \frac{\boldsymbol{q}^2}{\Lambda^2}, \epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right)}_{\text{1-loop vertex}}$$

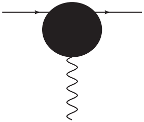
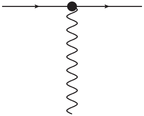
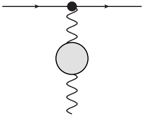
Idea: evaluate QQR-vertex at $\omega = 0$,
but keep leading order ω-structure
$\equiv$ use $G(\omega = 0)$.

The 1-loop correction can be calculated:



find: $G^\mu(\omega; r, p)$ not finite for $\omega \to 0$!

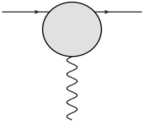
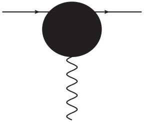
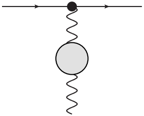
QQR vertex at 1-loop

Obtain $\lim_{\omega \to 0}$  $= \frac{1}{\omega} \cdot \omega(\boldsymbol{q}^2) \cdot$  $=$ 

collinear factorization: divergence in 1-loop X-sec $\propto$ splitting function

→ subtract divergent term

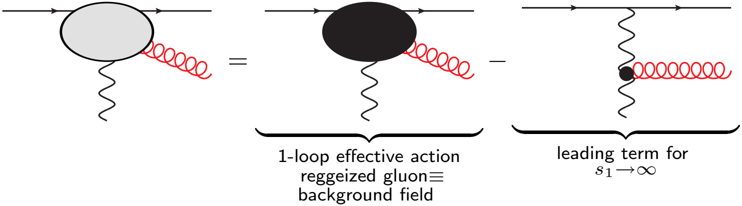
Here: "renormalize" QQR-vertex

 $=$  $-$ 

- subtraction → QQR vertex local in rapidity space
- no overcounting

Subtractions for effective action amplitudes

The same applies to real corrections:



resulting 'coefficent' local in rapidity space ➡ fulfill requirement of effective action

- subtracted term ≡ leading logharithmic term
- effective action: leading logharithmic terms ≡ diagrams with only reggeized gluon propagator

An attempt of a formal interpretation

$S_{RG}[\xi, \bar{\xi}, J_\mu, A_\pm]$: non-local reggeized gluon action

- $J_\mu, \xi, \bar{\xi}$: external currents, couple to (anti-)quarks, gluons
- $A_\pm$: reggeized gluon fields, only interacting d.o.f.

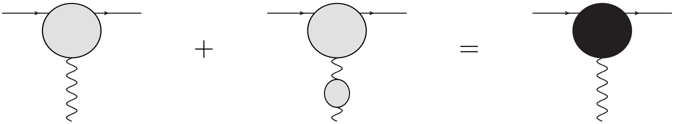
$$Z_{RG}[\xi, \bar{\xi}, J_\mu] = \int \mathcal{D}A_\pm e^{iS_{RG}[\xi, \bar{\xi}, J_\mu, A_\pm]}$$

- fix non-local couplings of S_{RG} order by order in g through matching with effective action with $A_\pm$ as background field

$$Z_{\text{eff}}[\xi, \bar{\xi}, J_\mu, A_\pm] = \int \mathcal{D}V \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{iS_{\text{eff}}[V, \psi, A_\pm] + i\int d^4x (J\cdot V + \bar{\psi}\xi + \bar{\xi}\psi)}$$

- functional derivatives of partition functions w.r.t currents: QCD correlators
- 'renormalization condition': correlator(S_{RG}) = correlator(S_{eff})
- by construction: QCD scattering amplitudes of pure QCD and reggeized gluon action S_{RG} coincide order by order!

example: QQR correlator to $\mathcal{O}(g^3)$



S_{RG} contributions S_{eff} ($A_\pm$ background)

Other important checks have been performed for effective action [J.Bartels, MH, L.N. Lipatov, in prep.]:

- BFKL equation
- s-channel discontinuity structure of production amplitudes
  ➡ Steinmann rules!
- signature conservation
- exchange of 4 reggeized gluons: triple Pomeron Vertex recalculated

Summary and conclusions

k_T **factorization**:

- more accurate kinematics through finite k_T of gluon
- take into account small x dynamics through BFKL evolution
- NLO BFKL: reduce scale dependence and increase accuracy

effective action:

- complete NLO treatment requires NLO matrix elements
- tool: high energy factorization and effective action of high energy QCD
- ω-regularization and subtractions: systematic determination (conservative: at least for 1-loop)
- important consistency conditions of the effective action have been checked

.... work in progress

reggeized gluon action S_{RG} effective action S_{eff}

$\mathcal{O}(g^0)$

$\mathcal{O}(g^1)$

$\mathcal{O}(g^2)$

$\mathcal{O}(g^3)$