On the rational part of 1-loop amplitudes

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Kick-off meeting of the LHCPhenoNet ITN,
Valencia, January 31th - February 4th, 2011

Abstract

The method proposed for the computation of the Rational part of 1-loop amplitudes, in particular the R_2 class of terms, in the framework of the OPP technique, is reviewed in this talk. The properties of Rational terms under gauge trasformations, and the status of the implementation in Monte Carlo event generators for the evaluation of cross-sections for multiparticle production, are discussed.

Master equation for 1-loop amplitudes

$$\mathcal{M}_{1-loop} = \underbrace{\sum_i d_i D_i}_{boxes} + \underbrace{\sum_i c_i C_i}_{triangles} + \underbrace{\sum_i b_i B_i}_{bubbles} + \underbrace{\sum_i a_i A_i}_{tadpoles} + \underbrace{R}_{Rational}$$

This expression is valid independently of the way one computes 1-loop amplitudes (Tensor Reduction / Unitarity and Generalized Unitarity inspired approaches in 4 integer dimensions).

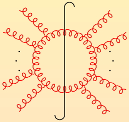
In (Generalized) Unitarity inspired methods, the first part is Cut-Constructible (CC), i.e. it can be obtained by (multiple) cuts of 1-loop amplitudes:

$$\mathcal{M}_{1-loop} = \underbrace{CC}_{Cut-Constructible} + \underbrace{R}_{Rational}$$

Unitarity methods

$$S = 1 + iT\,, \quad S^{\dagger}S = 1 \quad \Rightarrow \quad 2\mathrm{Im}(T) = T^{\dagger}T$$

$$\Rightarrow \mathrm{Im}\,\mathcal{A}^{1-loop} \sim \sum_{cuts} \int dPS_{cut}$$



- Original Unitarity formulation [Bern, Dixon, Dunbar, Kosower Nucl. Phys. B 425 (1994) 217]: only 1 cut
- Generalized Unitarity [Britto, Cachazo, Feng Nucl. Phys. B 725 (2005) 275]: multiple (quadruple) cuts, first introduced to determine the d_i coefficients of boxes in $\mathcal{N} = 4$ Super-Yang-Mills theories.

Coefficients of the CC part in Unitarity inspired approaches

- OPP method [G. Ossola, C.G. Papadopoulos, R. Pittau, Nucl. Phys. B 763 (2007), 147]: multiple cuts $\equiv$ putting on shell loop propagators (with denominator $D_i = (q + \sum_{k=1}^{i} p_k)^2 - m_i^2$)

$$
\begin{aligned}
D_i = D_j = D_k = D_l = 0 &\Rightarrow d_{ijkl}\ \ coeff.\ of\ \ boxes \\
D_i = D_j = D_k = 0 &\Rightarrow c_{ijk}\ \ coeff.\ of\ \ triangles \\
D_i = D_j = 0 &\Rightarrow b_{ij}\ \ coeff.\ of\ \ bubbles \\
D_i = 0 &\Rightarrow a_i\ \ coeff.\ of\ \ tadpoles
\end{aligned}
$$

- Further developments of 4-dim Generalized Unitarity (see BlackHat [Berger, Bern, Dixon et al., Phys. Rev. D 78 (2008), 036003]): multiple on-shell sub-amplitudes are glued together

N.B. Scalar functions up to 4-points are enough because each loop momentum has 4-components. Each cut imposes one constraint on the loop momentum, the maximum number of cuts which avoids to overconstrain the system of equations that determine the momentum is four.

Scalar integrals: D_i, C_i, B_i, A_i

Beside older results (e.g. FF [van Oldenborgh]), recent libraries are also available on the web:

- QCDLoop [K. Ellis and G. Zanderighi, 2008]
- OneLOop [A. van Hameren, arXiv:1007.4716[hep-ph]]

The problem of computing CC can be considered solved (after testing the numerical stability of the libraries......)!

4-dim cuts allow to determine the (poly)logarithmic structure of the amplitude......

And what about the Rational part R ?

- This still remains the most controversial part of the computation of 1-loop amplitudes.....
- While staying in 4-dimensions, it is impossible to compute it completely by means of cuts! Thus, different methods were proposed:
  - bootstrapping techniques/on-shell recursion relations [Bern, Dixon, Kosower, Phys. Rev. D 71 (2005) 105013], adopted in BlackHat
  - decomposition in two parts, according to the OPP approach: $R = R_1 + R_2$, and effective Feynman Rules for R_2

  Only a piece (the overlapping term or R_1) can be reconstructed from the CC part.....
- Only by going in an higher number of integer dimensions, it is possible to put R on the same footing as the CC part:
  - Generalized Unitarity in d-dimensions [W. Giele, Z. Kunszt, K. Melnikov, JHEP 0804 (2008) 049] adopted in Rocket [W. Giele, G. Zanderighi, JHEP 0806 (2008) 038]
  - Samurai approach [P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano, JHEP 1008(2010), 080] (hybrid algorithm: extension of OPP, allowing to perform the rational term evaluation in d-dim).

R computation: d-dim vs. 4-dim techniques

- Advantages of d-dim methods with respect to the 4-dim ones:
  - elegance, due to the unified treatment of CC and R
  - automation
- Disadvantages of d-dim methods with respect to the 4-dim ones:
  - d-dim wavefunctions and d-dim loop momenta are required.
  - cumbersome (besides boxes, even penthagons have to be included, since loop momenta are d-dimensional. External momenta remain 4-dim, thus it is not necessary to include exagons, etc.....).
  - more time consuming (?)

R staying in 4 integer dimensions

OPP approach: $R = R_1 + R_2$ [Ossola, Papadopoulos, Pittau, JHEP 0805 (2004), 004]

Both R_1 and R_2 are the residuals of the dimensional regularization procedure introduced to compute 1-loop integrals.

To understand their origin, one can start from the integrand of a generic m-point 1-loop amplitude:

$$A(q) = \frac{N(q)}{D_0 D_1 .... D_{m-1}}$$

$N(q)$ includes the information concerning the interaction vertices of the Theory, and the numerators of the propagators.
D_i are the denominators of the propagators of the loop particles.

$N(q)$ is then recast according to the Universal OPP decomposition.

Universal OPP decomposition of $N(q)$ in 4 dim

$$
\begin{aligned}
N(q) \; = \; & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left(d(i_0,i_1,i_2,i_3) + \tilde{d}(q,i_0,i_1,i_2,i_3) \right) \prod_{i \neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
+ \; & \sum_{i_0<i_1<i_2}^{m-1} \left(c(i_0,i_1,i_2) + \tilde{c}(q,i_0,i_1,i_2) \right) \prod_{i \neq i_0,i_1,i_2}^{m-1} D_i \\
+ \; & \sum_{i_0<i_1}^{m-1} \left(b(i_0,i_1) + \tilde{b}(q,i_0,i_1) \right) \prod_{i \neq i_0,i_1}^{m-1} D_i \\
+ \; & \sum_{i_0}^{m-1} \left(a(i_0) + \tilde{a}(q,i_0) \right) \prod_{i \neq i_0}^{m-1} D_i \\
+ \; & P(\tilde{q}) \prod_{i}^{m-1} D_i
\end{aligned}
$$

R_1 and R_2 contributions: origin of R_1

Going to d-dimensions:

$$q \to q + \tilde{q} \ , \quad \gamma_\mu \to \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}} \ , \quad g_{\mu\nu} \to g_{\mu\nu} + \tilde{g}_{\tilde{\mu}\tilde{\nu}}$$

$\Rightarrow$

$$A(q) \to \bar{A}(\bar{q}) \ = \ \frac{N(q) + \tilde{N}(q, \tilde{q}^2, \epsilon)}{\bar{D}_0 \bar{D}_1 .... \bar{D}_{m-1}} \tag{1}$$

The first piece of (1) can be recast in a different way by taking into account that

$$\frac{1}{\bar{D}_i} = \frac{D_i}{\bar{D}_i D_i} = \frac{\bar{D}_i - \tilde{q}^2}{\bar{D}_i D_i} = (\frac{1}{D_i} - \frac{\tilde{q}^2}{\bar{D}_i D_i}) \tag{2}$$

The first piece of (2) gives rise to the CC part of 1-loop amplitudes, whereas the remaining piece gives rise to the R_1 terms:

$$R_1 \equiv \int d^d q \frac{f(q, \tilde{q}^2)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 ...... \bar{D}_{m-1}} \, .$$

R_1 and R_2 contributions: origin of R_2

The second piece of (2) gives rise to the R_2 terms:

$$\mathrm{R}_2 \quad \equiv \int d^d\bar{q} \frac{\tilde{N}(q,\tilde{q}^2,\epsilon)}{\bar{D}_0\bar{D}_1\bar{D}_2\ldots\ldots\bar{D}_{m-1}} \,.$$

R_2 has a UV nature [Binoth, Guillet, Heinrich, JHEP 0702 (2007), 013; Bredenstein, Denner, Dittmaier, Pozzorini, JHEP 0808 (2008) 108]

$\Rightarrow$ it is enough to compute once and for all R_2 effective vertices by considering all possible 1-loop 1-particle irreducible diagrams, up to four external legs, for the theory at hand and then build the contribution to any 1-loop amplitude by considering all contributing diagrams including one and only one effective vertex and all other standard vertices.

Advantages of this method:

- It is fast: a 1-loop computation is reduced to a tree-level computation with the same number of external legs
- No additional integer dimensions are required

Disadvantages of this method:

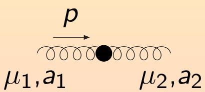
- It is theory dependent: the computation should be repeated for any give theory (QED, QCD, EWSM....).
- A dedicated numerical implementation in MC event generators for the computation of NLO cross-sections for multiparticle production is required.

R_2 effective Feynman rules

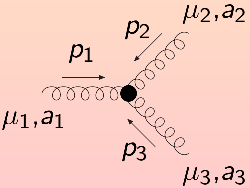
Analytical formulas of the R_2 effective vertices have been computed for the following theories:

- QED corrections [Ossola, Papadopoulos, Pittau, JHEP 0805 (2004), 004]
- QCD corrections to SM processes [Draggiotis, Garzelli, Papadopoulos, Pittau, JHEP 0904 (2009), 072]
- EW corrections to SM processes [Garzelli, Malamos, Pittau, JHEP 1001 (2010), 040]

* pure QCD examples:



$$= \frac{ig^2 N_{col}}{48\pi^2} \delta_{a_1 a_2} \left[\frac{p^2}{2} g_{\mu_1\mu_2} + \lambda_{HV} \left(g_{\mu_1\mu_2} p^2 - p_{\mu_1} p_{\mu_2} \right) + \frac{N_f}{N_{col}} (p^2 - 6\, m_q^2)\, g_{\mu_1\mu_2} \right]$$



$$= -\frac{g^3 N_{col}}{48\pi^2} \left(\frac{7}{4} + \lambda_{HV} + 2\frac{N_f}{N_{col}} \right) f^{a_1 a_2 a_3}\, V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$

Some simplification in R_2 computation: ghosts never enter!

Example from QCD:

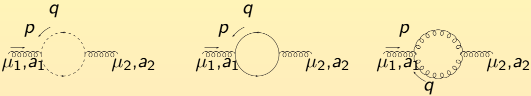


Figure: *Diagrams contributing to the gluon self-energy.*

As for the ghost loop with 2 external gluons, we can write the numerator as

$$\bar{N}(\bar{q}) = \frac{g^2}{(2\pi)^4} f^{a_1 bc} f^{a_2 cb} (p + \bar{q})^{\mu_1} \bar{q}^{\mu_2} \, .$$

When a D-dimensional index is contracted with a 4-dimensional (observable) vector v_μ, the 4-dimensional part is automatically selected. For example,

$$\bar{q} \cdot v \equiv (q + \tilde{q}) \cdot v = q \cdot v \quad \text{and} \quad \bar{\not{v}} \equiv \bar{\gamma}_{\bar{\mu}} v^{\mu} = \not{v} \, . \tag{3}$$

Since μ_1 and μ_2 are external Lorentz indices, that are eventually contracted with 4-dimensional external currents, their ϵ-dimensional component is killed due to eq. 3. Therefore, $\mathrm{R}_2 = 0$ for this diagram, being the ϵ-dependent part of the numerator $\tilde{N}(\tilde{q}^2, q, \epsilon) = 0$.

.....However, ghosts enter indeed in the analytical computation of the R_1 effective vertices.

Contributing diagrams in EWSM (generalized R_ξ gauges)

2-leg cases [Garzelli, Malamos, arXiv:1010.1248[hep-ph]]

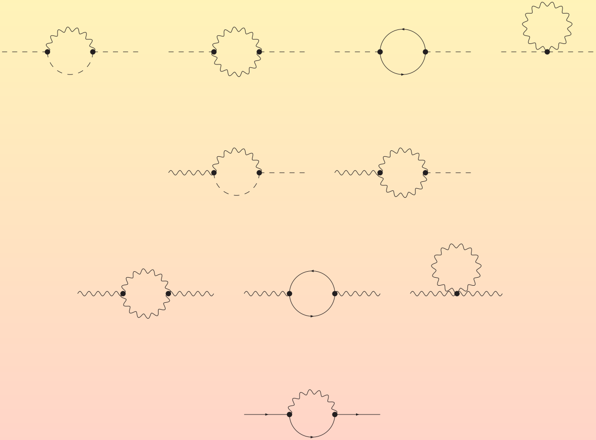


Figure: Non null contributions to the `ss`, `vs`, `vv` and `ff` R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A.

Contributing diagrams in EWSM (generalized R_ξ gauges)

3-leg cases [Garzelli, Malamos, arXiv:1010.1248[hep-ph]]

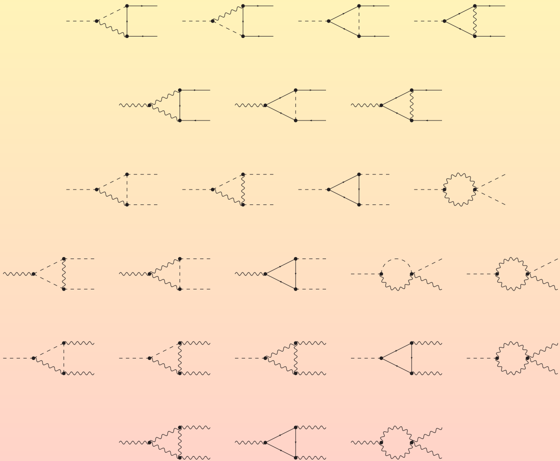


Figure: Non null contributions to the `sff`, `vff`, `sss`, `vss`, `svv` and `vvv` R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A.

Contributing diagrams in EWSM (generalized R_ξ gauges)

4-leg cases [Garzelli, Malamos, arXiv:1010.1248[hep-ph]]

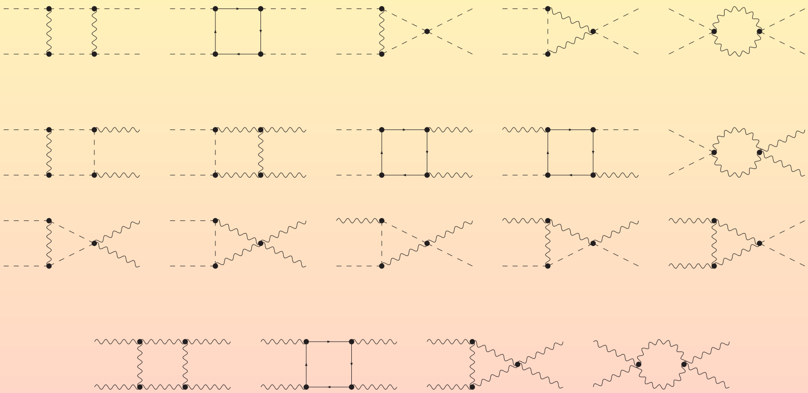


Figure: Non null contributions to the $\mathtt{ssss}$, $\mathtt{ssvv}$ and $\mathtt{vvvv}$ R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A.

Gauge choice

A gauge is fixed in order to quantize the Theory and reduce the number of degrees of freedom of the gauge fields. Covariant and non-covariant gauges can be chosen. The resulting R_2 terms are gauge dependent......which is the best gauge option ?

- We compute R_2 effective vertices in both the most generalized R_ξ gauge (three gauge parameters ξ, ξ_A and ξ_Z) and in the Unitary gauge. The formulas in the Unitary gauge turn out to be much more complicated, due to the fact that the theory in Unitary gauge is not manifestly renormalizable: the UV behaviour of the Theory appears worser than in the R_ξ gauges, due to the propagator features ($q \to \infty$):

$$P_{uni} = \frac{-i}{q^2 - m_i^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_i^2}\right) \quad (i = W,\ Z),$$

$$P_{R_\xi} \quad = \quad \frac{-i}{q^2 - m_i^2}\left(g_{\mu\nu} + (\xi_i - 1)\,\frac{q_\mu q_\nu}{q^2 - \xi_i m_i^2}\right)$$
$$\text{with } \ \xi_i = \xi, \xi_Z, \xi_A \ \ (i = W, Z, A),$$

- Of course, it also depends on the event generator.....: several LO matrix-element event generators are written in Unitary gauge, since the number of degrees of freedom is reduced (no FP ghosts, no unphysical scalars).

The results for $R_1 + R_2$ are gauge invariant. Check of the analytical formulas for the effective vertices by means of Ward identities.

Ward identities

- We wrote down a huge set of Ward identities [Garzelli, Malamos, Pittau, JHEP 1001 (2010) 040] by using the Background Field Method [Denner, Weiglein, Dittmaier, Nucl. Phys. B 440 (1995) 95]
- We got analytical formulas for the R_1 effective vertices.
- We verified that $R_1 + R_2$ satisfy the Ward identities.
- This is a non trivial check!:
  - Many Ward identities involve more than one effective vertex
  - The same effective vertex appears in more than one Ward identity
  - Several R_1 effective vertices have a complicated structure, involving Gram determinants.....
- Ward identities can also be implemented numerically in MC event generators and used to check the results of the computation of 1-loop amplitudes in specific phase-space points.

Computation of R_1 in the OPP framework

R_1 terms can be computed together with the CC part of 1-loop amplitudes.

Two methods:

- looking at the explicit mass dependence in the coefficients d_i, c_i, b_i, a_i of the scalar integrals [Ossola, Papadopoulos, Pittau, Nucl. Phys. B 763 (2007) 147]
- using the explicit dependence on q of the spurious coefficients $\tilde{d}_i$, $\tilde{c}_i$, $\tilde{b}_i$, $\tilde{a}_i$ [Ossola, Papadopoulos, Pittau, JHEP 0707 (2007) 085].

The second method is implemented in the CutTools code that gives in an automatic way the CC and the R_1 numerical contribution to the amplitudes [Ossola, Papadopoulos, Pittau,JHEP 0803 (2008) 042]

Interface to MC event generators

- CutTools has been interfaced to the HELAC code
- QCD R_2 effective vertices have been implemented in HELAC, that uses off-shell recursive relations to compute R_2 contributions to any given helicity amplitudes
- $\Rightarrow$ HELAC-1-loop [A. van Hameren, C.G. Papadopoulos, R. Pittau, arXiv:0903.....[hep-ph]]
- The implementation of EW R_2 vertices in HELAC-1-loop is in progress.....

- An implementation in other event generators to compute NLO cross-sections is also possible. It should work both in case of matrix-elements generated by means of Feynman diagrams (2-point effective vertices have to be added!) and in case of matrix-elements generated by means of recursion relations.

Use of HELAC-1-loop

HELAC-1-loop has already been used to compute cross-sections including NLO QCD corrections for signal and background processes of interest at LHC:

- $pp \to t\bar{t}b\bar{b}$ [Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, JHEP 0909 (2009), 109]
- $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$ [Bevilacqua, Czakon, Garzelli, Papadopoulos, Pittau, Worek, arXiv:1003.1241[hep-ph], contribution to the Higgs Cross-section Working Group]
- $pp \to t\bar{t}jj$ [Bevilacqua, Czakon, Papadopoulos, Worek, Phys. Rev. Lett. 104 (2010), 162002]
- $pp \to WWb\bar{b}$ [Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek, arXiv:1012.4230 [hep-ph]]
- ....

Conclusions and Work to be done

- In the framework of the OPP, only a class of rational terms can be reconstructed together with the CC part of the amplitude. Another class, R_2, need a dedicated computation.
- Analytical formulae corresponding to effective Feynman rules, allowing to estimate the contribution of R_2 terms to any 1-loop SM scattering amplitude have been derived in different gauges. This complete the OPP technique for the evaluation of 1-loop amplitudes in the SM.
- The separate contributions R_1 and R_2 to renormalized S-matrix elements are gauge dependent. Only the contribution of $R_1 + R_2$ to renormalized S-matrix elements is the same in all gauges considered.
- To be done: comparing more closely different methods to get R.....
- To be done: completing the numerical implementation of the EW R_2 effective vertices.....
- To be done: considering the numerical implementation in different gauges, for gauge check purposes (renormalized S-matrix elements must be gauge independent): this step also requires the extension of CutTools/OPP to the R_ξ and Unitary gauges (rank of the integrand larger than the number of denominators)

Acknowledgements

The results presented in this talk have been obtained mainly thanks to the collaboration with P. Draggiotis, I. Malamos, C.G. Papadopoulos and R. Pittau.

Thank you for your attention!