LHCphenonet, Valencia

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"Exclusive processes in the Regge limit"

1. Quasi–Multi–Regge kinematics
2. Monte Carlo event generator
3. Collinear improvements
4. Multijet events

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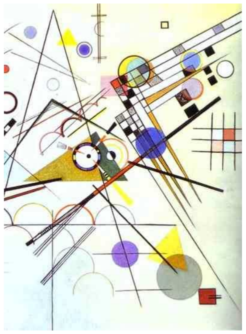
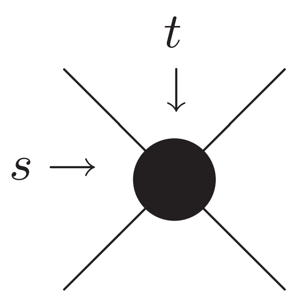
1. Quasi–Multi–Regge kinematics



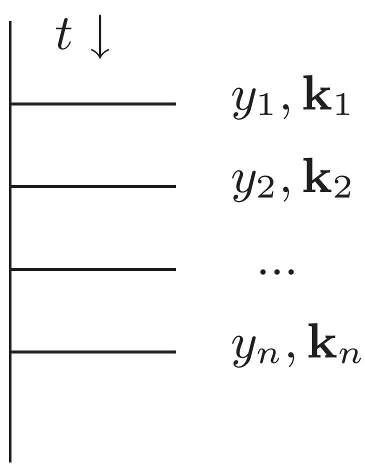
High energy limit of scattering amplitudes in perturbative QCD:



$s >> |t|\,, Q^2$
$\alpha_s(Q^2) << 1$

Large logarithms in s compensate small α_s: $\alpha_s \ln s \sim 1$

All orders resummation in multi–Regge kinematics: <u>LL BFKL</u> (70's)

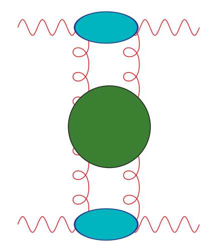


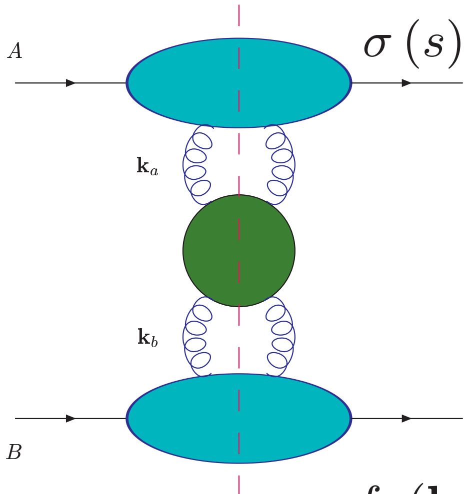
$$Y \sim \ln s$$

$$\mathbf{k}_i \sim \mathbf{k}_{i+1}$$

$$y_i << y_{i+1}$$

$$\alpha_s^n \int_0^Y dy_1 \int_0^{y_1} dy_2 ... \int_0^{y_{n-1}} dy_n \sim \frac{(\alpha_s Y)^n}{n!}$$

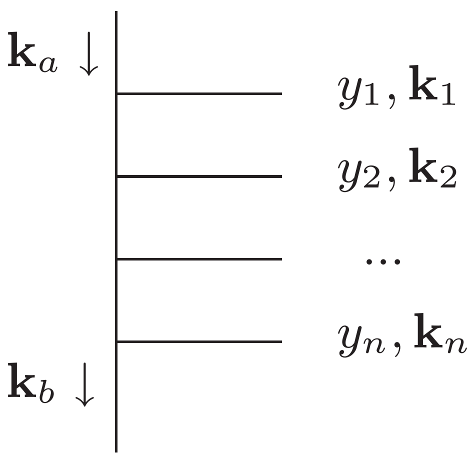




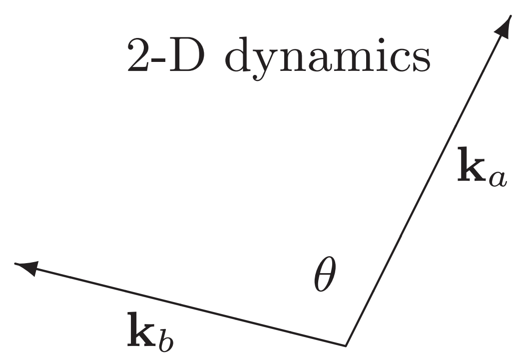
$$\sigma\left(s\right) = \int \frac{d^2\mathbf{k}_a}{\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{\mathbf{k}_b^2} \Phi_A\left(\mathbf{k}_a\right) \Phi_B(\mathbf{k}_b) f\left(\mathbf{k}_a, \mathbf{k}_b, \mathrm{Y} = \ln \frac{s}{s_0}\right)$$

$$f\left(\mathbf{k}_a, \mathbf{k}_b, \mathrm{Y}\right) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} d\omega\, e^{\omega \mathrm{Y}} f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right)$$

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^2\mathbf{k}\, \mathcal{K}(\mathbf{k}_a, \mathbf{k}) f_\omega(\mathbf{k}, \mathbf{k}_b)$$

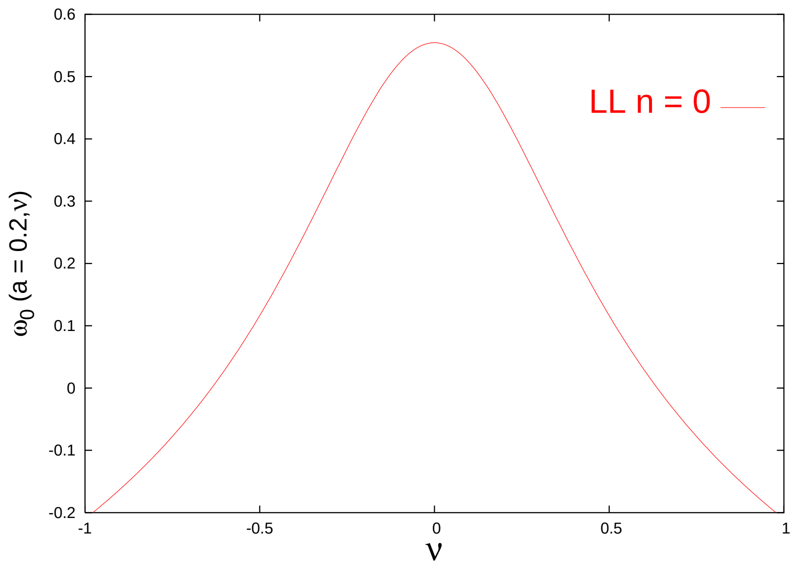


2-D dynamics



$$f\left(\mathbf{k}_a, \mathbf{k}_b, Y\right) \sim \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi i} e^{\omega Y} \int \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma-\frac{1}{2}} \frac{e^{in\theta}}{\omega-\omega_n(\bar{\alpha}_s,\gamma)}$$

$$\omega_n = \bar{\alpha}_s \left(2\Psi(1) - \Psi\left(\gamma + \frac{|n|}{2}\right) - \Psi\left(1-\gamma+\frac{|n|}{2}\right)\right)$$

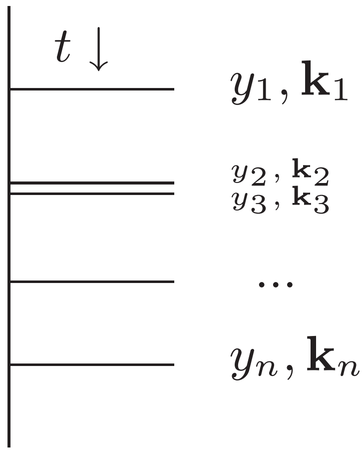


$n = 0$ dominates Cross Sections: Saddle point @ $\gamma = \frac{1}{2} + \mathrm{i}\,\nu$ ...

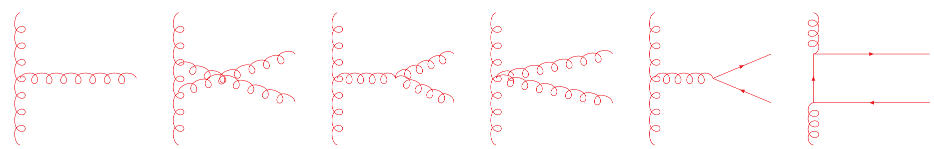
Large s behaviour: $\boxed{\sigma \sim s^{\lambda}}$ $\lambda = \frac{\alpha_s N_c}{\pi} 4 \ln 2 \sim 0.5$ for $\alpha_s = 0.2$.

HARD or LL BFKL POMERON intercept

To run the coupling & fix the energy scale in Y: QMRK



$$\mathbf{k}_i \sim \mathbf{k}_{i+1}$$
$$y_i << y_{i+1}$$
$$y_2 \sim y_3$$



NLL BFKL Equation: $(\alpha_S \mathrm{Y})^n + \alpha_S(\alpha_S \mathrm{Y})^n$

$$\sigma(s) = \int \frac{d^2\mathbf{k}_a}{\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{\mathbf{k}_b^2} \Phi_A(\mathbf{k}_a)\,\Phi_B(\mathbf{k}_b)\,f(\mathbf{k}_a,\mathbf{k}_b,\mathrm{Y})$$

$$\omega f_\omega(\mathbf{k}_a,\mathbf{k}_b) = \delta^{(2+2\epsilon)}(\mathbf{k}_a-\mathbf{k}_b) + \int d^{2+2\epsilon}\mathbf{k}\;\mathcal{K}(\mathbf{k}_a,\mathbf{k})\,f_\omega(\mathbf{k},\mathbf{k}_b)$$

2. Monte Carlo event generator

In dimension regularisation the equation reads

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}'\,\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}'\right)f_\omega\left(\mathbf{k}',\mathbf{k}_b\right)$$

with kernel

$$\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}\right)=2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\,\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)+\mathcal{K}_r\left(\mathbf{k}_a,\mathbf{k}\right)$$

integration in terms of emitted momenta:

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)=\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}\,2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)f_\omega\left(\mathbf{k},\mathbf{k}_b\right)$$

$$+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)+\int d^2\mathbf{k}\,\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)$$

Introduce a phase space slicing parameter λ ...

$$\omega f_\omega(\mathbf{k}_a,\mathbf{k}_b) = \delta^{(2+2\epsilon)}(\mathbf{k}_a-\mathbf{k}_b) + \int d^{2+2\epsilon}\mathbf{k}\, 2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}(\mathbf{k}_a-\mathbf{k})\, f_\omega(\mathbf{k},\mathbf{k}_b)$$

$$+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k})\,\underline{\left(\theta\left(\mathbf{k}^2-\lambda^2\right)+\theta\left(\lambda^2-\mathbf{k}^2\right)\right)}\, f_\omega(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b)$$

$$+\int d^{2+2\epsilon}\mathbf{k}\,\widetilde{\mathcal{K}}_r(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k})\, f_\omega(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b)$$

$f_\omega(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b) \simeq f_\omega(\mathbf{k}_a,\mathbf{k}_b)$ for $|\mathbf{k}|<\lambda$ is valid for large $|\mathbf{k}_a|$ & small λ.

$$\begin{aligned}\omega f_\omega(\mathbf{k}_a,\mathbf{k}_b) &= \delta^{(2+2\epsilon)}(\mathbf{k}_a-\mathbf{k}_b)\\ &+\left\{2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k})\,\underline{\theta\left(\lambda^2-\mathbf{k}^2\right)}\right\} f_\omega(\mathbf{k}_a,\mathbf{k}_b)\\ &+\int d^{2+2\epsilon}\mathbf{k}\left\{\mathcal{K}_r^{(\epsilon)}(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k})\,\underline{\theta\left(\mathbf{k}^2-\lambda^2\right)}+\widetilde{\mathcal{K}}_r(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k})\right\} f_\omega(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b)\end{aligned}$$

What about the ϵ poles? ...

The gluon Regge trajectory reads

$$
\begin{aligned}
2\,\omega^{(\epsilon)}\left(\mathbf{q}^2\right) \;=\; & -\bar{\alpha}_s \frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}} \left(\frac{1}{\epsilon} + \ln\frac{q^2}{\mu^2}\right) - \frac{\bar{\alpha}_s^2}{8}\frac{\Gamma^2(1-\epsilon)}{(4\pi)^{2\epsilon}} \left\{ \frac{\beta_0}{N_c}\left(\frac{1}{\epsilon^2} + \ln^2\frac{q^2}{\mu^2}\right) \right. \\
& \left. + \left(\frac{4}{3} - \frac{\pi^2}{3} + \frac{5}{3}\frac{\beta_0}{N_c}\right)\left(\frac{1}{\epsilon} + 2\ln\frac{q^2}{\mu^2}\right) - \frac{32}{9} + 2\zeta(3) - \frac{28}{9}\frac{\beta_0}{N_c} \right\}
\end{aligned}
$$

$\beta_0 \equiv \frac{11}{3}N_c - \frac{2}{3}n_f$, $\bar{\alpha}_s \equiv \frac{\alpha_s(\mu)N_c}{\pi}$, μ is the $\overline{\mathrm{MS}}$ scale.

Integrating ϵ-dependent real emission ...

$$
\begin{aligned}
\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}(\mathbf{q},\mathbf{q}+\mathbf{k})\,\theta\left(\lambda^2 - \mathbf{k}^2\right) \;=\; & \frac{1}{\Gamma(1+\epsilon)}\frac{\bar{\alpha}_s}{(4\pi)^{\epsilon}}\frac{1}{\epsilon}\left(\frac{\lambda^2}{\mu^2}\right)^{\epsilon} \\
\left\{ 1 + \frac{\bar{\alpha}_s}{4}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left[\frac{\beta_0}{N_c}\frac{1}{\epsilon}\left(1 - \frac{1}{2}\left(\frac{\lambda^2}{\mu^2}\right)^{\epsilon}\left(1 - \epsilon^2\frac{\pi^2}{6}\right)\right) \right.\right. & \\
\left.\left. + \frac{1}{2}\left(\frac{\lambda^2}{\mu^2}\right)^{\epsilon}\left(\frac{4}{3} - \frac{\pi^2}{3} + \frac{5}{3}\frac{\beta_0}{N_c} + \epsilon\left(-\frac{32}{9} + 14\zeta(3) - \frac{28}{9}\frac{\beta_0}{N_c}\right)\right)\right]\right\} &
\end{aligned}
$$

We can now combine these two results ...

$$\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}\right)=2\,\omega^{(\epsilon)}\left(\mathbf{k}_a\right)\,\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)+\mathcal{K}_r\left(\mathbf{k}_a,\mathbf{k}\right)$$

Regularise the gluon Regge trajectory as

$$\omega_\lambda\left(\mathbf{q}\right) \equiv \lim_{\epsilon\to 0}\left\{2\,\omega^{(\epsilon)}\left(\mathbf{q}\right)+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{q},\mathbf{q}+\mathbf{k}\right)\theta\left(\lambda^2-\mathbf{k}^2\right)\right\} =$$

$$-\bar{\alpha}_s\left\{\ln\frac{\mathbf{q}^2}{\lambda^2}+\frac{\bar{\alpha}_s}{4}\left[\frac{\beta_0}{2N_c}\ln\frac{\mathbf{q}^2}{\lambda^2}\ln\frac{\mu^4}{\mathbf{q}^2\lambda^2}+\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}\right)\ln\frac{\mathbf{q}^2}{\lambda^2}-6\zeta(3)\right]\right\}$$

Very simple:

$$\omega_\lambda\left(\mathbf{q}\right) = -\int_{\lambda^2}^{\mathbf{q}^2}\frac{d\mathbf{k}^2}{\mathbf{k}^2}\underbrace{\left(\bar{\alpha}_s\left(\mathbf{k}^2\right)+\bar{\alpha}_s^2\mathcal{S}\right)}_{\text{cusp anomalous dimension}} +\text{constant}$$

$$\mathcal{S} = \tfrac{1}{3}-\tfrac{\pi^2}{12}+\tfrac{5}{12}\tfrac{\beta_0}{N_c} \qquad\qquad \text{constant} = \bar{\alpha}_s^2\tfrac{3}{2}\zeta(3)$$

The NLL BFKL equation then reads

$$\begin{aligned}(\omega - \omega_\lambda(\mathbf{k}_a)) f_\omega(\mathbf{k}_a, \mathbf{k}_b) &= \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \\ &+ \int d^2\mathbf{k} \left(\frac{\Gamma_{\rm cusp}\left(\mathbf{k}^2\right)}{\pi \mathbf{k}^2} \theta\left(\mathbf{k}^2 - \lambda^2\right) + \widetilde{\mathcal{K}}_r(\mathbf{k}_a, \mathbf{k}_a + \mathbf{k}) \right) f_\omega(\mathbf{k}_a + \mathbf{k}, \mathbf{k}_b)\end{aligned}$$

where $\Gamma_{\rm cusp}(\mathrm{X}) = \bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left(\frac{4}{3} - \frac{\pi^2}{3} + \frac{5}{3}\frac{\beta_0}{N_c} - \frac{\beta_0}{N_c}\ln\frac{\mathrm{X}}{\mu^2}\right)$

In this notation:

$$\omega_\lambda(\mathbf{q}) = -\int_{\lambda^2}^{\mathbf{q}^2} \frac{d\mathbf{k}^2}{\mathbf{k}^2} \Gamma_{\rm cusp}\left(\mathbf{k}^2\right) + \text{constant}$$

Ensuring the λ–independence of the equation...

The equation can be iterated using the initial condition:

$$
\begin{aligned}
f_\omega(\mathbf{k}_a,\mathbf{k}_b) \quad = \quad & \frac{\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)}{\omega-\omega_\lambda(\mathbf{k}_a)} + \int d^2\mathbf{k}_1 \frac{\widehat{\mathcal{K}}_r^\lambda(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}_1)}{\omega-\omega_\lambda(\mathbf{k}_a)} \frac{\delta^{(2)}(\mathbf{k}_a+\mathbf{k}_1-\mathbf{k}_b)}{\omega-\omega_\lambda(\mathbf{k}_a+\mathbf{k}_1)} \\
& + \int d^2\mathbf{k}_1 \frac{\widehat{\mathcal{K}}_r^\lambda(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}_1)}{\omega-\omega_\lambda(\mathbf{k}_a)} \\
& \quad \int d^2\mathbf{k}_2 \frac{\widehat{\mathcal{K}}_r^\lambda(\mathbf{k}_a+\mathbf{k}_1,\mathbf{k}_a+\mathbf{k}_1+\mathbf{k}_2)}{\omega-\omega_\lambda(\mathbf{k}_a+\mathbf{k}_1)} \frac{\delta^{(2)}(\mathbf{k}_a+\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_b)}{\omega-\omega_\lambda(\mathbf{k}_a+\mathbf{k}_1+\mathbf{k}_2)} \\
& + \cdots
\end{aligned}
$$

and Mellin transform back into energy space:

$$
f(\mathbf{k}_a,\mathbf{k}_b,\mathrm{Y}) \quad = \quad \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} d\omega \; e^{\omega \mathrm{Y}} f_\omega(\mathbf{k}_a,\mathbf{k}_b)
$$

finally ...

... the NLL BFKL gluon Green's function reads

$$f(\mathbf{k}_a, \mathbf{k}_b, \mathrm{Y}) = e^{\omega_\lambda(\mathbf{k}_a)\mathrm{Y}} \Bigg\{ \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b)$$

$$+ \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d^2\mathbf{k}_i \left[\frac{\theta\left(\mathbf{k}_i^2 - \lambda^2\right)}{\pi \mathbf{k}_i^2} \Gamma_{\mathrm{cusp}}\left(\mathbf{k}_i^2\right) + \widetilde{\mathcal{K}}_r \left(\mathbf{k}_a + \sum_{l=0}^{i-1} \mathbf{k}_l, \mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l \right) \right]$$

$$\times \int_0^{y_{i-1}} dy_i \; e^{\left(\omega_\lambda\left(\mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l\right) - \omega_\lambda\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l\right)\right) y_i} \delta^{(2)} \left(\sum_{l=1}^{n} \mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b \right) \Bigg\}$$

with $y_0 \equiv \mathrm{Y}$.

Correct LL limit:

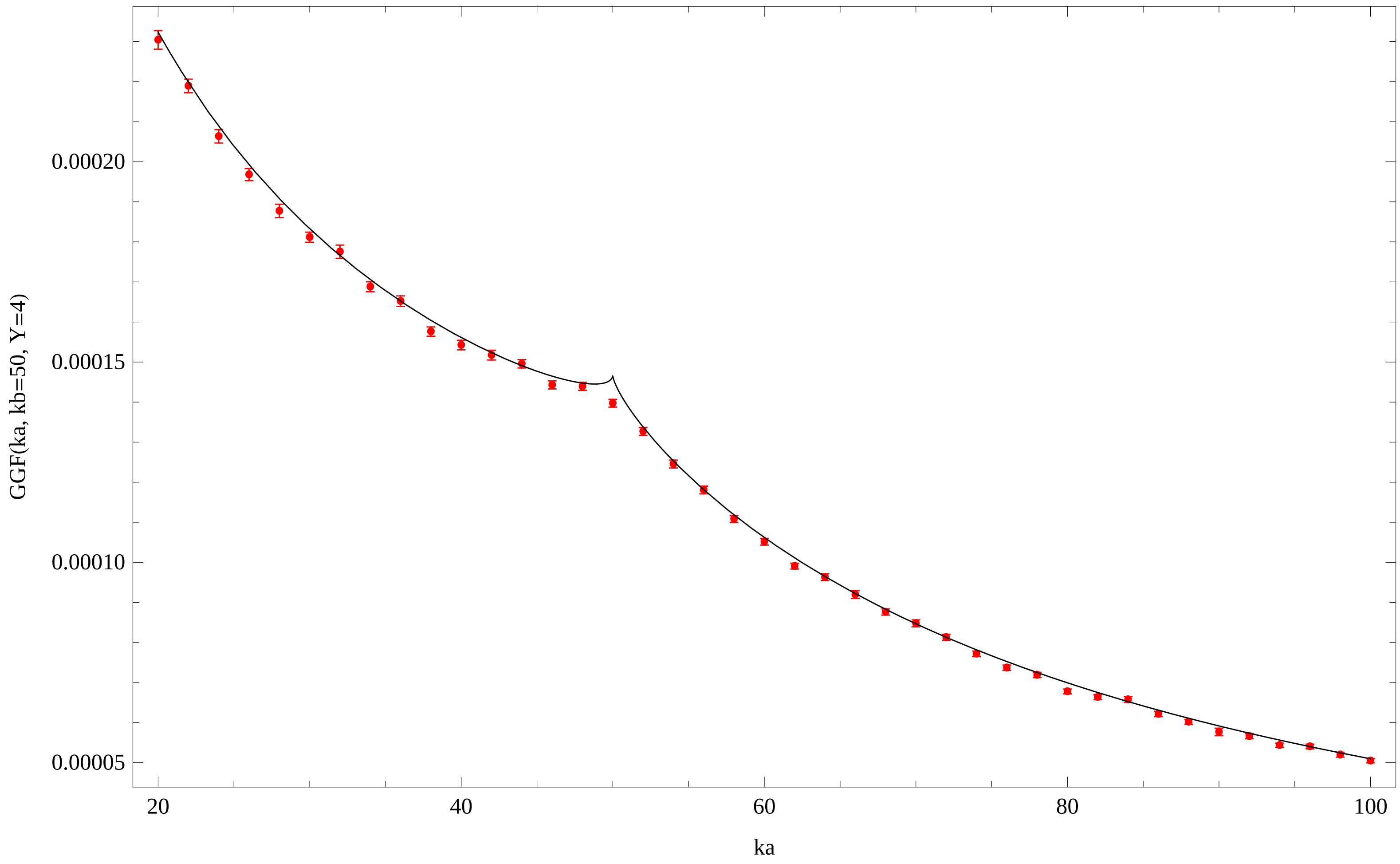
$$\begin{aligned}
\omega_0(\mathbf{q}^2,\lambda) &= -\bar{\alpha}_s \ln \frac{\mathbf{q}^2}{\lambda^2} \\
\Gamma_{\rm cusp} &= \bar{\alpha}_s \\
\eta &= 0 \\
\widetilde{\mathcal{K}}_r\left(\mathbf{q},\mathbf{q}'\right) &= 0
\end{aligned}$$

and the solution is very simple

$$f(\mathbf{k}_a,\mathbf{k}_b,\mathrm{Y}) = \left(\frac{\lambda^2}{k_a^2}\right)^{\bar{\alpha}_s\,\mathrm{Y}} \Bigg\{ \delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)$$

$$+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\bar{\alpha}_s\int d^2\mathbf{k}_i\,\frac{\theta\left(\mathbf{k}_i^2-\lambda^2\right)}{\pi\mathbf{k}_i^2}$$

$$\int_0^{y_{i-1}} dy_i \left(\frac{\left(\mathbf{k}_a+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2}{\left(\mathbf{k}_a+\sum_{l=1}^{i}\mathbf{k}_l\right)^2}\right)^{\bar{\alpha}_s\,y_i} \delta^{(2)}\left(\sum_{l=1}^{n}\mathbf{k}_l+\mathbf{k}_a-\mathbf{k}_b\right)\Bigg\}$$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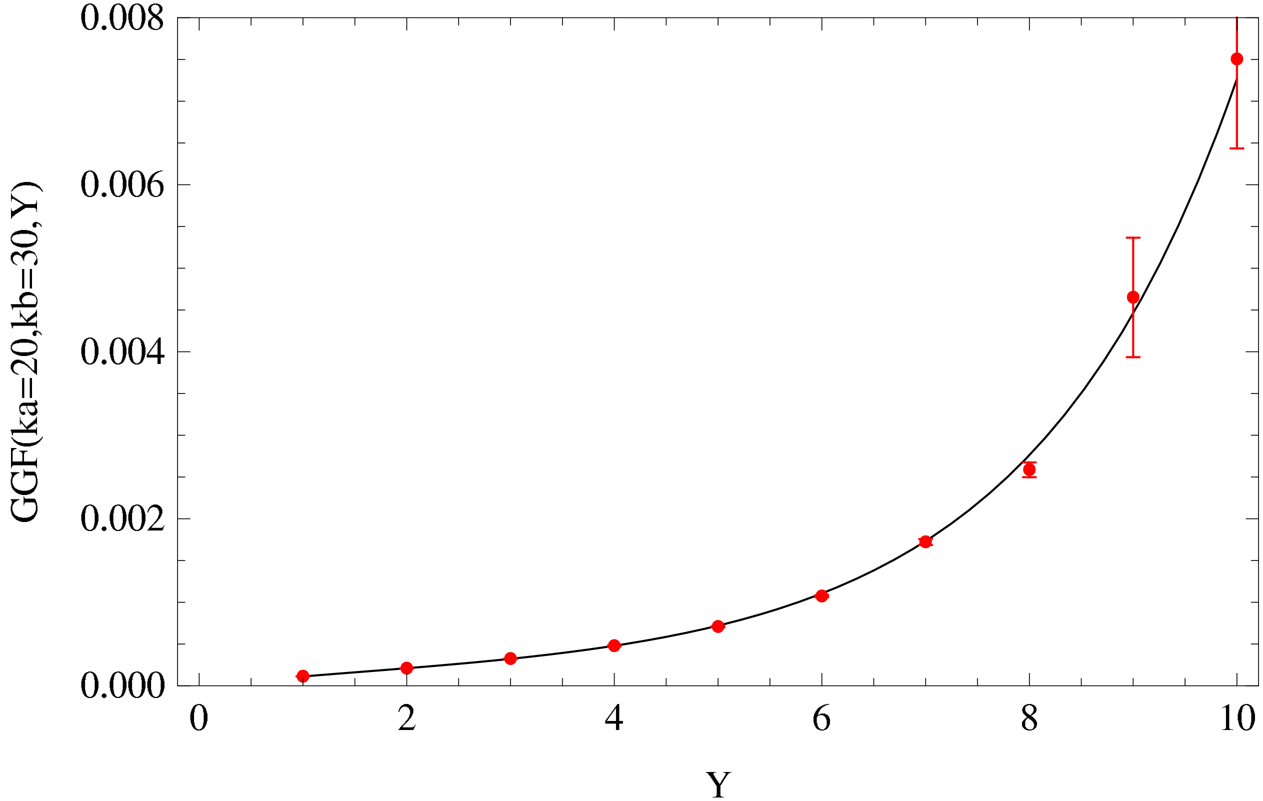
0.00020
0.00015
0.00010
0.00005
20
40
60
80
100
ka



- **Generate F_2 fits of HERA data.**
- **Central heavy quark production at LHC.**

Non–forward NLL BFKL kernel can be studied in the same way ...

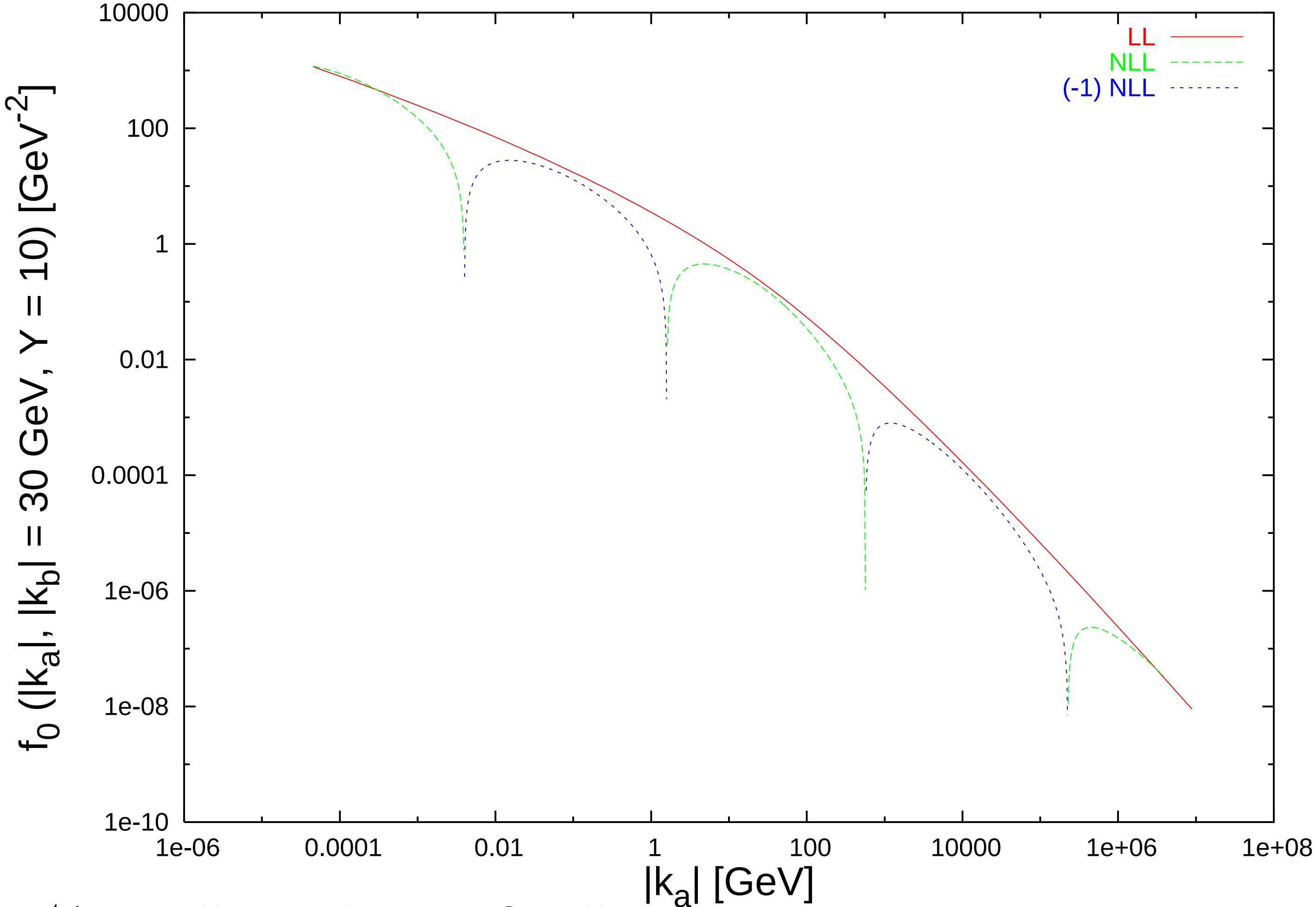
$$\omega_\lambda\left(\mathbf{k}_a,\mathbf{q}\right) = -\frac{1}{2}\left\{\bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left[\mathcal{S} - \frac{\beta_0}{2N_c}\ln\frac{\mathbf{k}_a^2\lambda^2}{\mu^4}\right]\right\}\ln\frac{\mathbf{k}_a^2}{\lambda^2} + 3\zeta(3)\bar{\alpha}_s^2$$
$$-\frac{1}{2}\left\{\bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left[\mathcal{S} - \frac{\beta_0}{2N_c}\ln\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2\lambda^2}{\mu^4}\right]\right\}\ln\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2}{\lambda^2}$$

$$\xi\left(\mathbf{k}_a,\mathbf{k},\mathbf{q}\right) = \frac{\bar{\alpha}_s}{2}\left(1 + \frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2\left(\mathbf{k}_a+\mathbf{k}\right)^2 - \mathbf{q}^2\mathbf{k}^2}{\mathbf{k}_a^2\left(\mathbf{k}_a-\mathbf{q}+\mathbf{k}\right)^2}\right)\left\{1 + \frac{\bar{\alpha}_s}{4}\left(\mathcal{S} - \frac{\beta_0}{N_c}\ln\frac{\mathbf{k}^2}{\mu^2}\right)\right\}$$

- **Study of NLL non-forward kernel in simpler k_t representation**

3. Collinear improvements

Forward case: Behaviour for small/large $\frac{k_a}{k_b}$ ratios:



Collinear/Anticollinear limits: Oscillations

Origin of the oscillations: $\chi_0(\gamma) = \bar{\alpha}_s \left(2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)\right)$

$$f \sim \int \left(\frac{s}{k_a k_b}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma-\frac{1}{2}} \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma)} = \int \left(\frac{s}{k_a^2}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma-\frac{1}{2}} \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma - \frac{\omega}{2})}$$

In collinear limit $\gamma \sim 0$: $\omega(\gamma) \sim \frac{\bar{\alpha}_s}{\gamma}$

$$\omega \sim \frac{\bar{\alpha}_s}{\gamma - \frac{\omega}{2}} \longrightarrow \omega \sim \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s^2}{2\gamma^3} + \sum_{n=2}^{\infty} \frac{(2n)!}{2^n n!(n+1)!} \frac{\bar{\alpha}_s^{n+1}}{\gamma^{2n+1}}$$

Not allowed by DGLAP. Only the second one cancelled by NLL kernel.

The remaining terms are numerically large.

Proposal: $\chi_0^{\text{new}}(\gamma) \equiv \chi_0\left(\gamma + \frac{\omega}{2}\right)$

$$f \sim \int \left(\frac{s}{k_a k_b}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma-\frac{1}{2}} \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma + \frac{\omega}{2})} = \int \left(\frac{s}{k_a^2}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma-\frac{1}{2}} \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma)}$$

Collinear limit free from unphysical double logs.

$$\int d^2\mathbf{q}_2\,\mathcal{K}\left(\mathbf{q}_1,\mathbf{q}_2\right)\left(\frac{\bar{\alpha}_s\left(q_2^2\right)}{\bar{\alpha}_s\left(q_1^2\right)}\right)^{-\frac{1}{2}}\left(\frac{q_2^2}{q_1^2}\right)^{\gamma-1}=\bar{\alpha}_s\left(q_1^2\right)\chi_0\left(\gamma\right)+\bar{\alpha}_s^2\chi_1\left(\gamma\right)$$

$$\chi_0\left(\gamma\right)\quad\simeq\quad\frac{1}{\gamma}+\left\{\gamma\rightarrow 1-\gamma\right\},\quad\chi_1\left(\gamma\right)\quad\simeq\quad\frac{\mathrm{a}}{\gamma}+\frac{\mathrm{b}}{\gamma^2}-\frac{1}{2\gamma^3}+\left\{\gamma\rightarrow 1-\gamma\right\}$$

$$\mathrm{a}\quad=\quad\frac{5}{12}\frac{\beta_0}{N_c}-\frac{13}{36}\frac{n_f}{N_c^3}-\frac{55}{36},\quad\mathrm{b}\quad=\quad-\frac{1}{8}\frac{\beta_0}{N_c}-\frac{n_f}{6N_c^3}-\frac{11}{12}.$$

Renormalization–group–improved kernel:

$$\begin{aligned}\omega\quad&=\quad\bar{\alpha}_s\left(1+\left(\mathrm{a}+\frac{\pi^2}{6}\right)\bar{\alpha}_s\right)\left(2\psi(1)-\psi\left(\gamma+\frac{\omega}{2}-\mathrm{b}\,\bar{\alpha}_s\right)-\psi\left(1-\gamma+\frac{\omega}{2}-\mathrm{b}\,\bar{\alpha}_s\right)\right)\\&+\quad\bar{\alpha}_s^2\left(\chi_1\left(\gamma\right)+\left(\frac{1}{2}\chi_0\left(\gamma\right)-\mathrm{b}\right)\left(\psi'(\gamma)+\psi'(1-\gamma)\right)-\left(\mathrm{a}+\frac{\pi^2}{6}\right)\chi_0(\gamma)\right)\end{aligned}$$

Is there a simple representation in k_t–rapidity space?

Main idea: the solution to

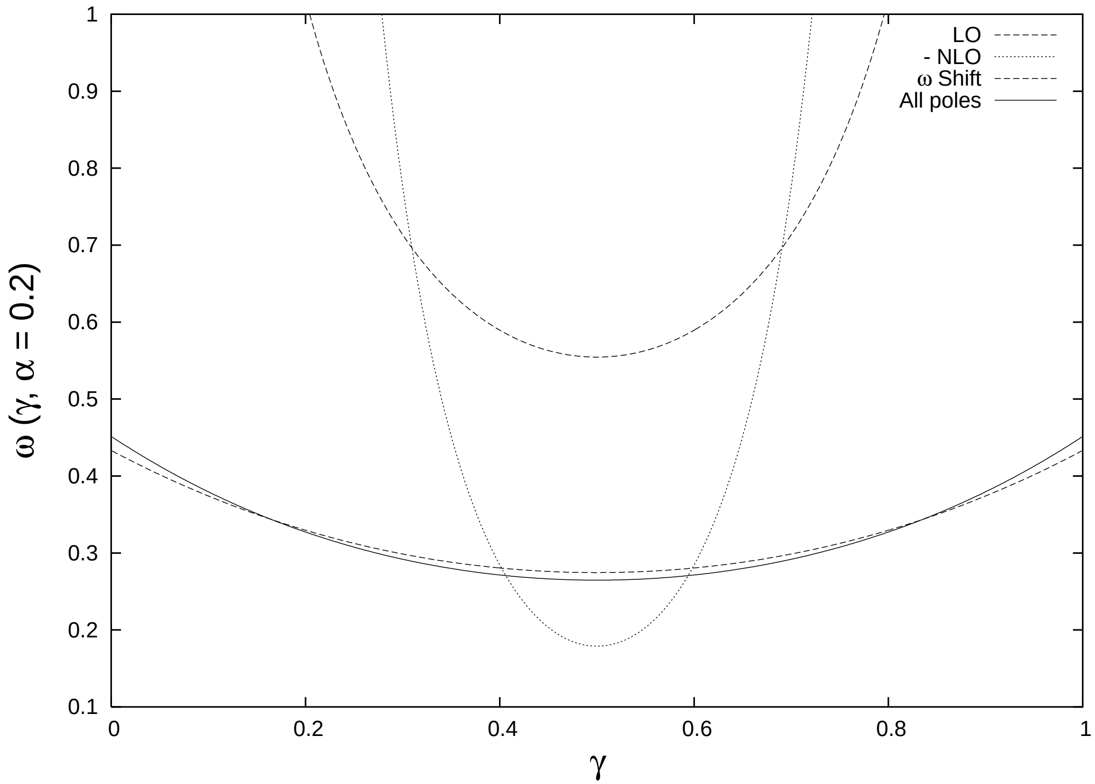
$$\omega = \bar{\alpha}_s \left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1 - \gamma + \frac{\omega}{2}\right)\right)$$

to a high accuracy is

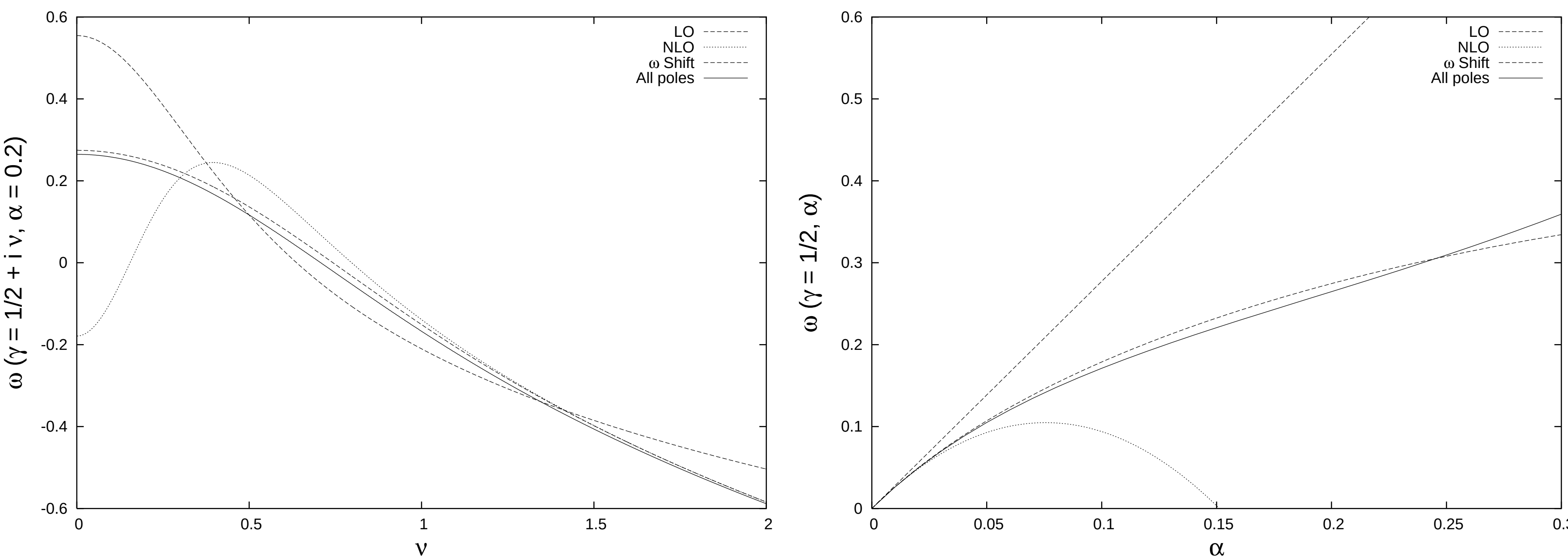
$$\omega \quad = \quad \int_0^1 \frac{dx}{1-x} \left\{ \left(x^{\gamma-1} + x^{-\gamma}\right) \sqrt{\frac{2\bar{\alpha}_s}{\ln^2 x}} J_1\left(\sqrt{2\bar{\alpha}_s \ln^2 x}\right) - 2\bar{\alpha}_s \right\}$$

Including all terms and matching at NLL: No γ/ω mixing, $\omega = \omega(\gamma)$

$$\omega \quad = \quad \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^n n! (n+1)!} \frac{\left(\bar{\alpha}_s + \mathrm{a}\, \bar{\alpha}_s^2\right)^{n+1}}{\left(\gamma + m - \mathrm{b}\, \bar{\alpha}_s\right)^{2n+1}} - \mathrm{M\ T}$$



$$\omega = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^n n! (n+1)!} \frac{\left(\bar{\alpha}_s + \mathrm{a}\, \bar{\alpha}_s^2\right)^{n+1}}{(\gamma + m - \mathrm{b}\, \bar{\alpha}_s)^{2n+1}} - \mathrm{M\ T}$$



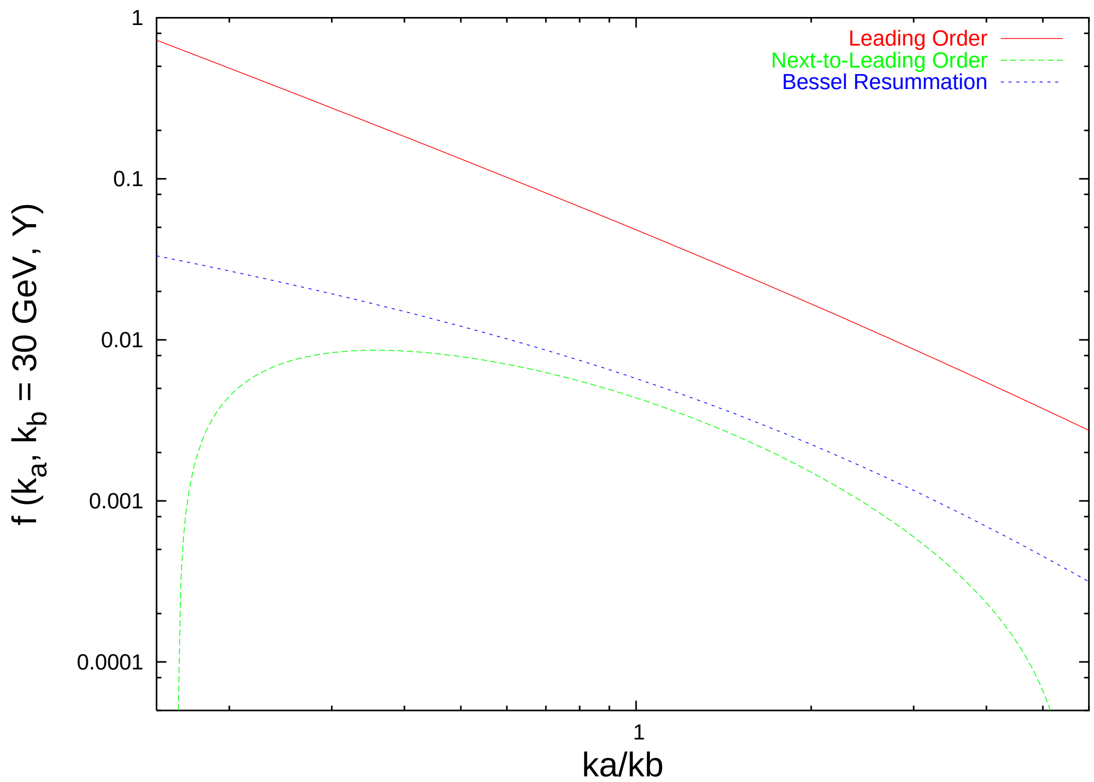
Prescription: Remove $-\frac{\bar{\alpha}_s^2}{4}\frac{1}{(\mathbf{q}-\mathbf{k})^2}\ln^2\left(\frac{q^2}{k^2}\right)$ from kernel and replace it with

$$\frac{1}{(\mathbf{q}-\mathbf{k})^2}\left(\frac{q^2}{k^2}\right)^{-\mathrm{b}\bar{\alpha}_s\frac{|k-q|}{k-q}}\sqrt{\frac{2\left(\bar{\alpha}_s+\mathrm{a}\,\bar{\alpha}_s^2\right)}{\ln^2\left(\frac{q^2}{k^2}\right)}}J_1\left(\sqrt{2\left(\bar{\alpha}_s+\mathrm{a}\,\bar{\alpha}_s^2\right)\ln^2\left(\frac{q^2}{k^2}\right)}\right)-\mathrm{M\ T}$$

$$J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right)\quad\simeq\quad\sqrt{\frac{\bar{\alpha}_s}{2}\ln^2\left(\frac{q^2}{k^2}\right)}$$

$$J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right)\simeq\left(\frac{2}{\pi^2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}\right)^{\frac{1}{4}}\cos\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{q^2}{k^2}\right)}-\frac{3\pi}{4}\right)$$

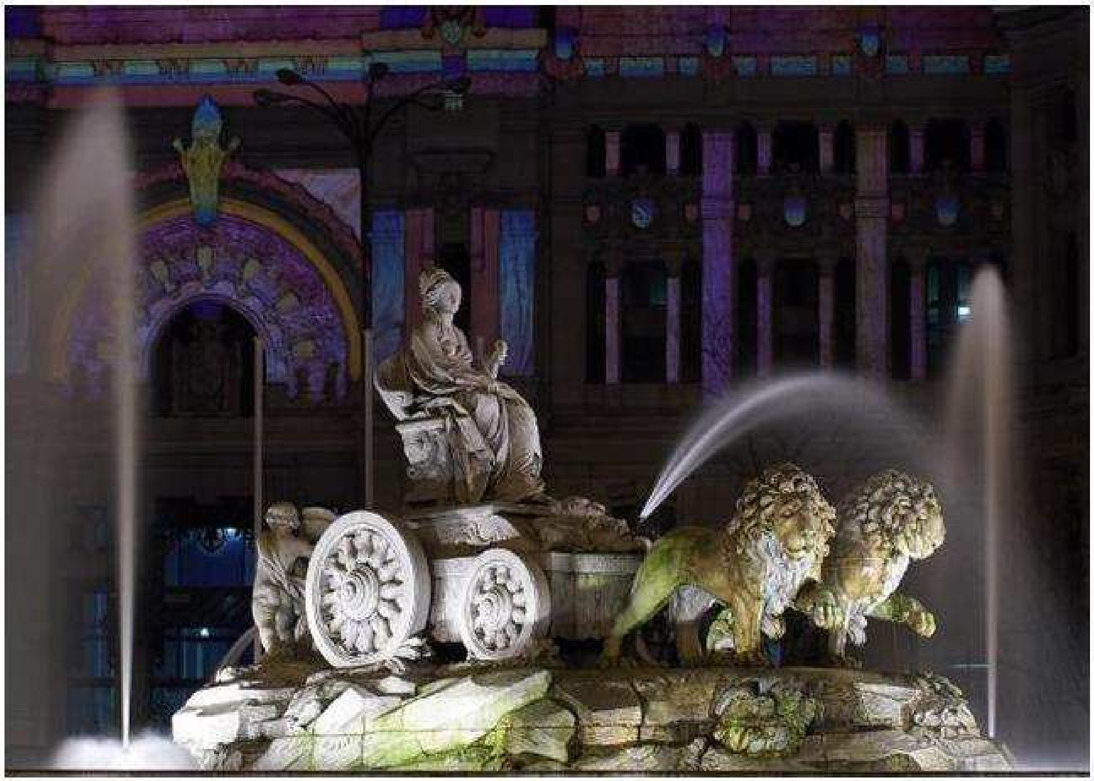
This generates the correct collinear behaviour ...



- **Interpretation of Bessel function J_1 in coordinate space?**
- **Is the collinear improvement needed in the non–forward case?**

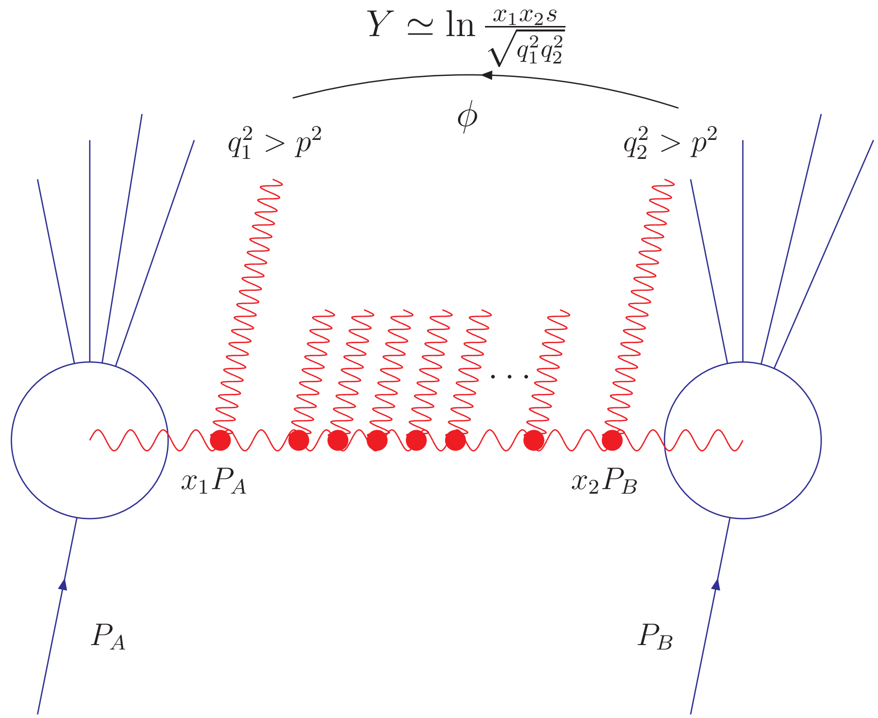
4. Multijet events

Two forward jets with similar p_t and large rapidity separation ...



... and soft radiation in the middle generated by the BFKL kernel

Another representation ...



$$\frac{d\hat{\sigma}\left(\alpha_s, \mathrm{Y}, p_{1,2}^2\right)}{d\phi} = \frac{\pi^2 \bar{\alpha}_s^2}{4\sqrt{p_1^2 p_2^2}} \sum_{n=-\infty}^{\infty} e^{in\phi}\, \mathcal{C}_n\left(\mathrm{Y}\right),$$

with

$$\mathcal{C}_n\left(\mathrm{Y}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d\nu}{\left(\frac{1}{4}+\nu^2\right)} \left(\frac{p_1^2}{p_2^2}\right)^{i\nu} e^{\chi\left(|n|, \frac{1}{2}+i\nu, \bar{\alpha}_s(p_1 p_2)\right)\mathrm{Y}},$$

and

$$\chi\left(n, \gamma, \bar{\alpha}_s\right) \equiv \bar{\alpha}_s \chi_0\left(n, \gamma\right) + \bar{\alpha}_s^2 \left(\chi_1\left(n, \gamma\right) - \frac{\beta_0}{8N_c} \frac{\chi_0\left(n, \gamma\right)}{\gamma\left(1-\gamma\right)}\right).$$

$$\chi_0(n,\gamma) = 2\psi(1) - \psi\left(\gamma + \frac{n}{2}\right) - \psi\left(1 - \gamma + \frac{n}{2}\right),$$

The eigenvalue of the scale invariant NLO sector is

$$\begin{aligned}
\chi_1(n,\gamma) &= \mathcal{S}\chi_0(n,\gamma) + \frac{3}{2}\zeta(3) - \frac{\beta_0}{8N_c}\chi_0^2(n,\gamma) \\
&+ \frac{1}{4}\left[\psi''\left(\gamma + \frac{n}{2}\right) + \psi''\left(1 - \gamma + \frac{n}{2}\right) - 2\,\phi(n,\gamma) - 2\,\phi(n, 1-\gamma)\right] \\
&- \frac{\pi^2\cos(\pi\gamma)}{4\sin^2(\pi\gamma)(1-2\gamma)}\left\{\left[3 + \left(1 + \frac{n_f}{N_c^3}\right)\frac{2 + 3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right]\delta_n^0 \right. \\
&\qquad \left. - \left(1 + \frac{n_f}{N_c^3}\right)\frac{\gamma(1-\gamma)}{2(3-2\gamma)(1+2\gamma)}\delta_n^2\right\},
\end{aligned}$$

The full cross section corresponds to the integration over the azimuthal angle and it only depends on the $n=0$ component:

$$\hat{\sigma}\left(\alpha_s, \mathrm{Y}, p_{1,2}^2\right) = \frac{\pi^3 \bar{\alpha}_s^2}{2\sqrt{p_1^2 p_2^2}} \mathcal{C}_0\left(\mathrm{Y}\right).$$

We are interested in distributions sensitive to higher conformal spins. The average of the cosine of the azimuthal angle times an integer projects out the contribution from each of these angular components:.

$$\langle \cos\left(m\phi\right)\rangle = \frac{\mathcal{C}_m\left(\mathrm{Y}\right)}{\mathcal{C}_0\left(\mathrm{Y}\right)}.$$

The associated ratios

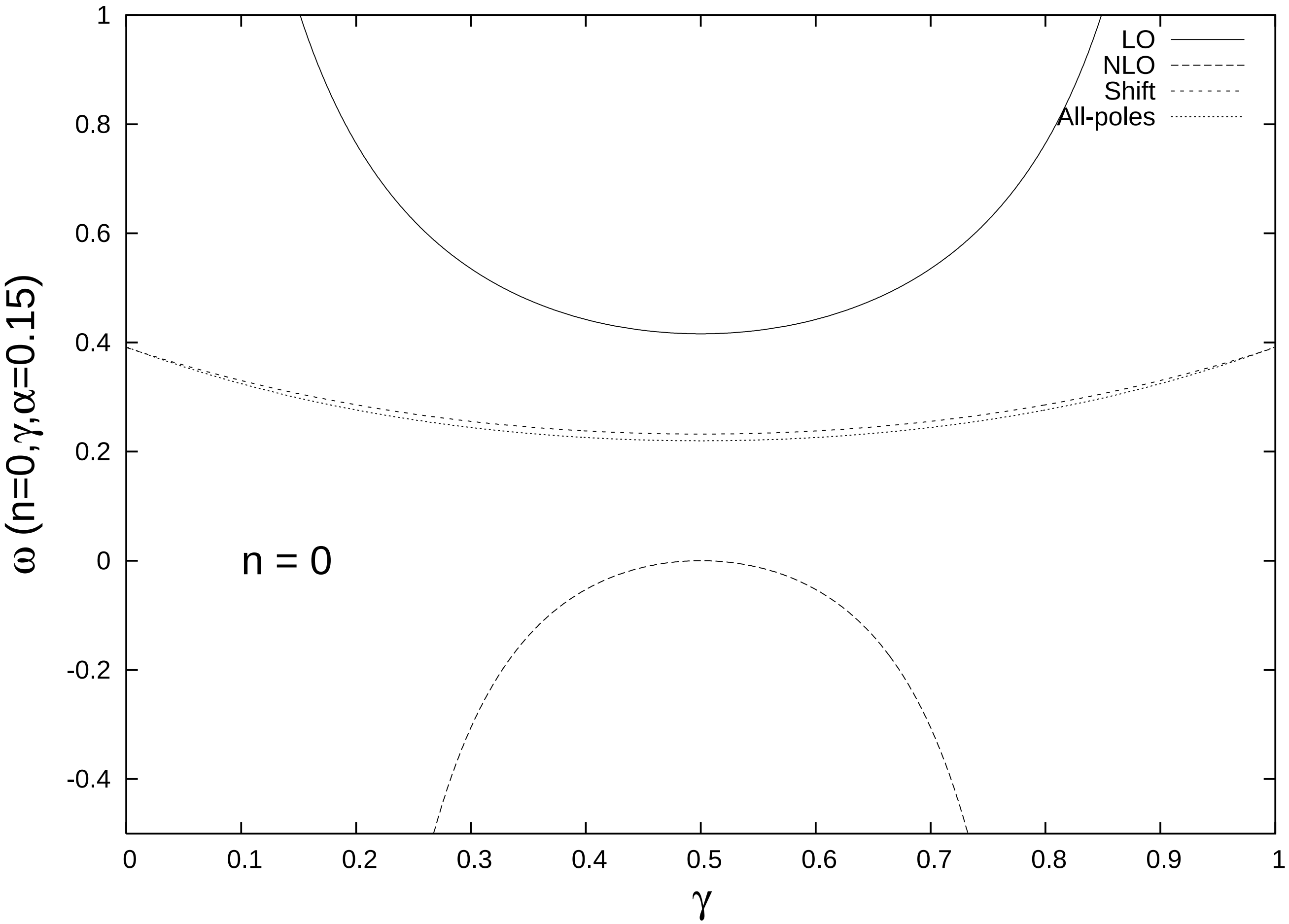
$$\frac{\langle \cos\left(m\phi\right)\rangle}{\langle \cos\left(n\phi\right)\rangle} = \frac{\mathcal{C}_m\left(\mathrm{Y}\right)}{\mathcal{C}_n\left(\mathrm{Y}\right)}$$

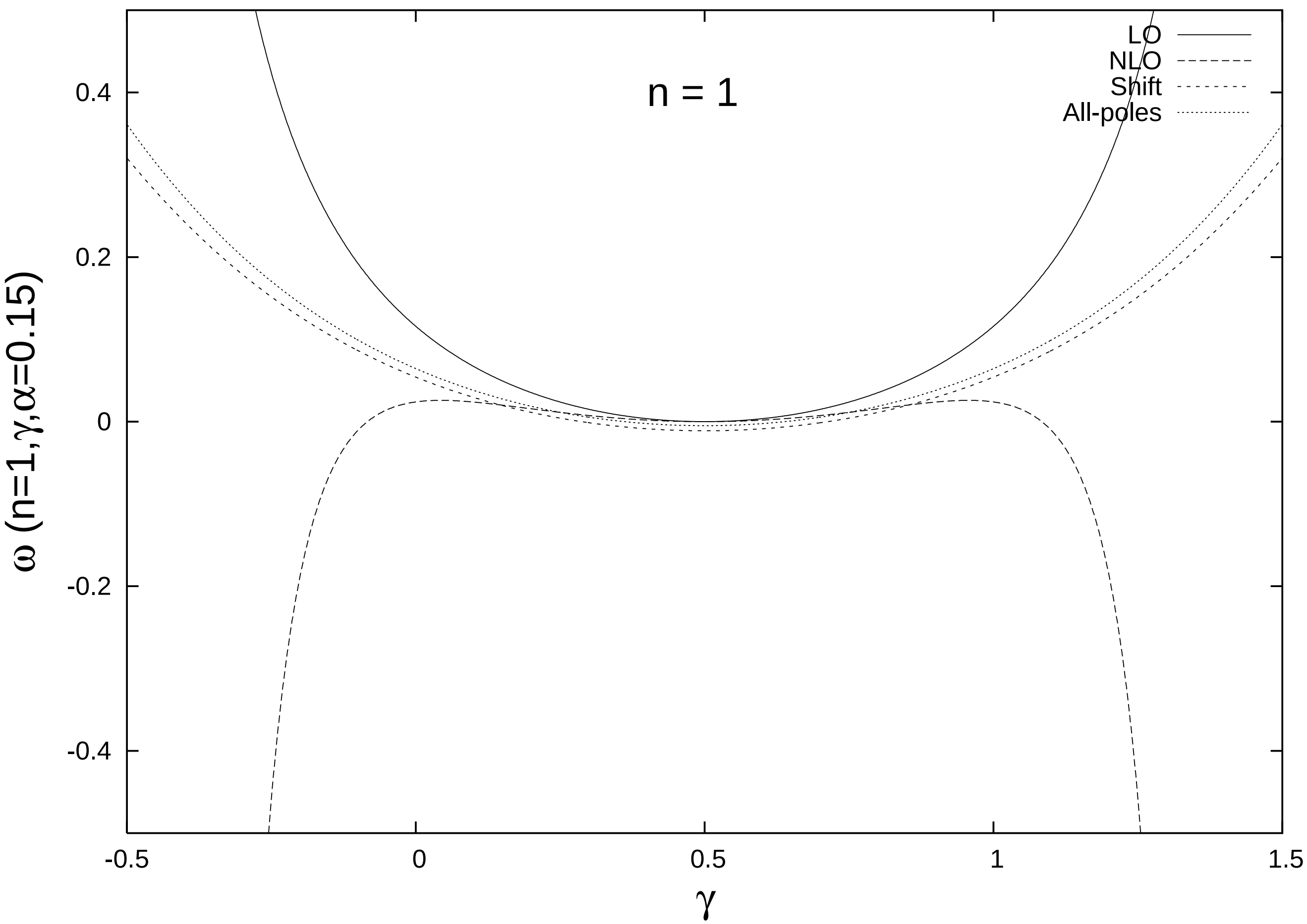
can be used to remove the uncertainty associated to the $n=0$ component.

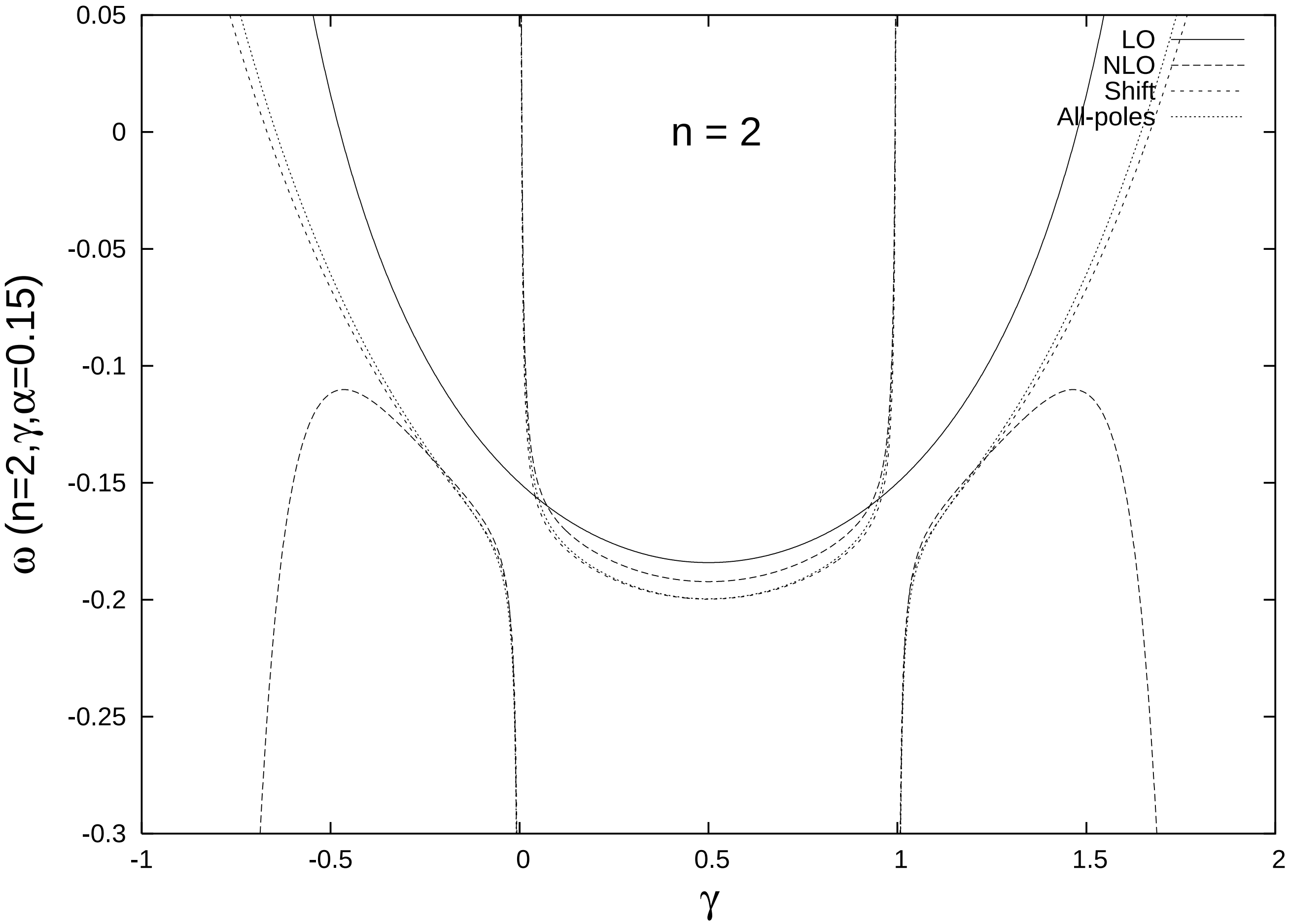
We use the collinear improvement also for conformal spins:

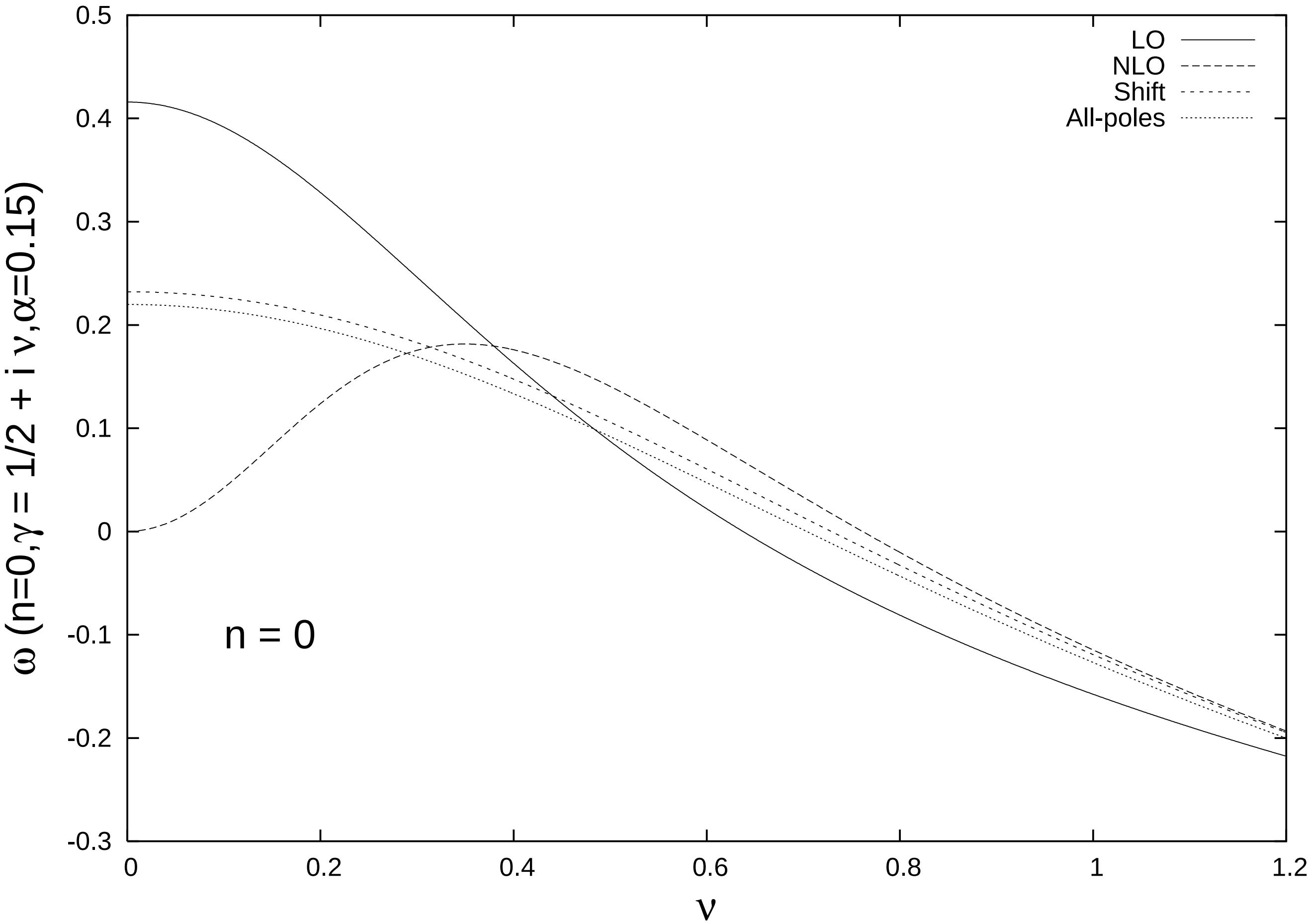
$$\begin{aligned}\omega &= \bar{\alpha}_s\left(1+\mathcal{A}_n\bar{\alpha}_s\right)\left\{2\,\psi\left(1\right)-\psi\left(\gamma+\frac{|n|}{2}+\frac{\omega}{2}+\mathcal{B}_n\bar{\alpha}_s\right)\right.\\ &\left.-\,\psi\left(1-\gamma+\frac{|n|}{2}+\frac{\omega}{2}+\mathcal{B}_n\bar{\alpha}_s\right)\right\}+\bar{\alpha}_s^2\left\{\chi_1\left(|n|\,,\gamma\right)-\frac{\beta_0}{8N_c}\frac{\chi_0\left(n,\gamma\right)}{\gamma\left(1-\gamma\right)}\right.\\ &\left.-\mathcal{A}_n\chi_0\left(|n|\,,\gamma\right)\right)+\left(\psi'\left(\gamma+\frac{|n|}{2}\right)+\psi'\left(1-\gamma+\frac{|n|}{2}\right)\right)\left(\frac{\chi_0\left(|n|\,,\gamma\right)}{2}+\mathcal{B}_n\right)\right\}\end{aligned}$$

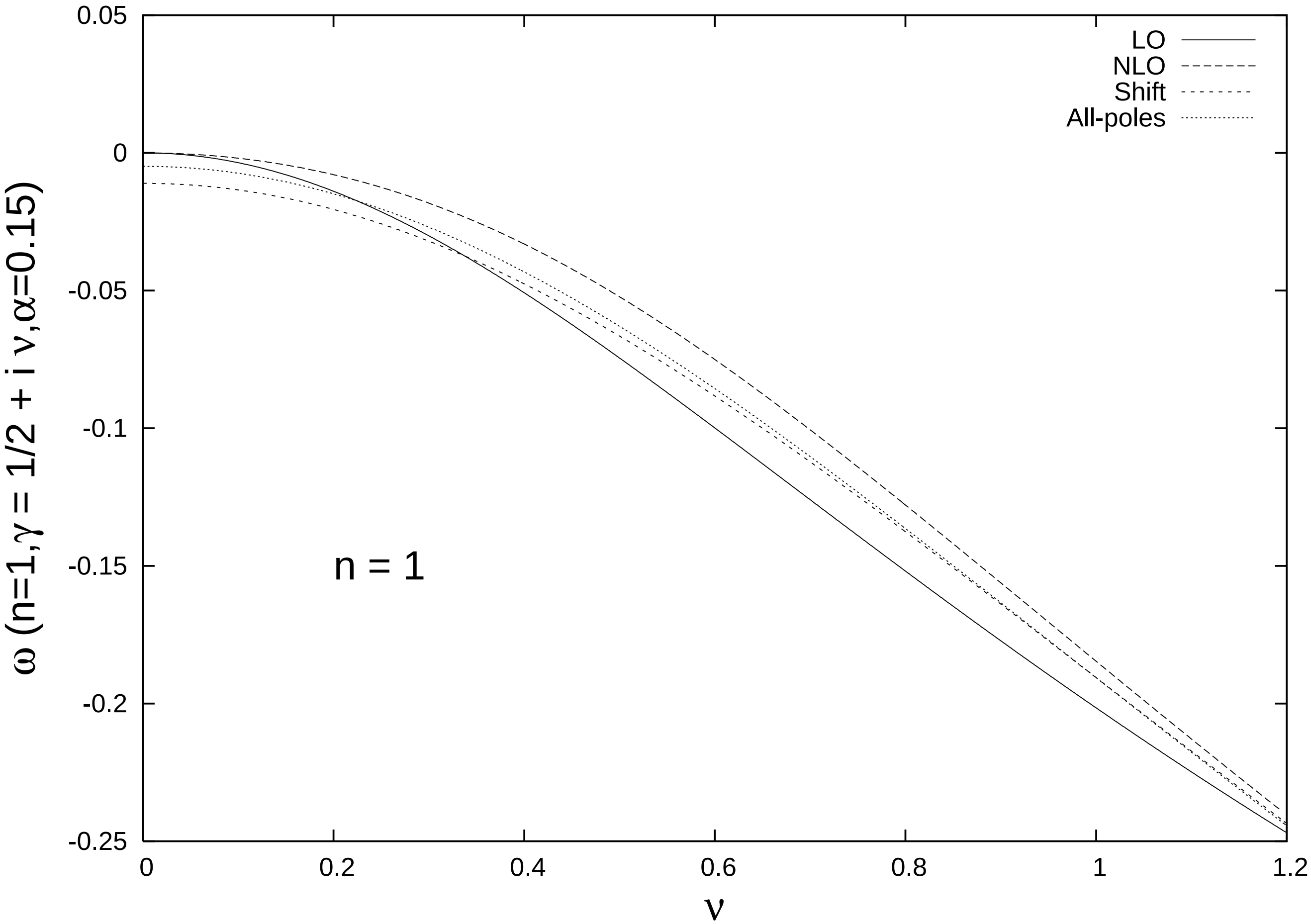
$$\begin{aligned}\mathcal{A}_n &= a_n+\psi'\left(n+1\right),\\ \mathcal{B}_n &= \frac{1}{2}\mathrm{H}_n-b_n.\end{aligned}$$

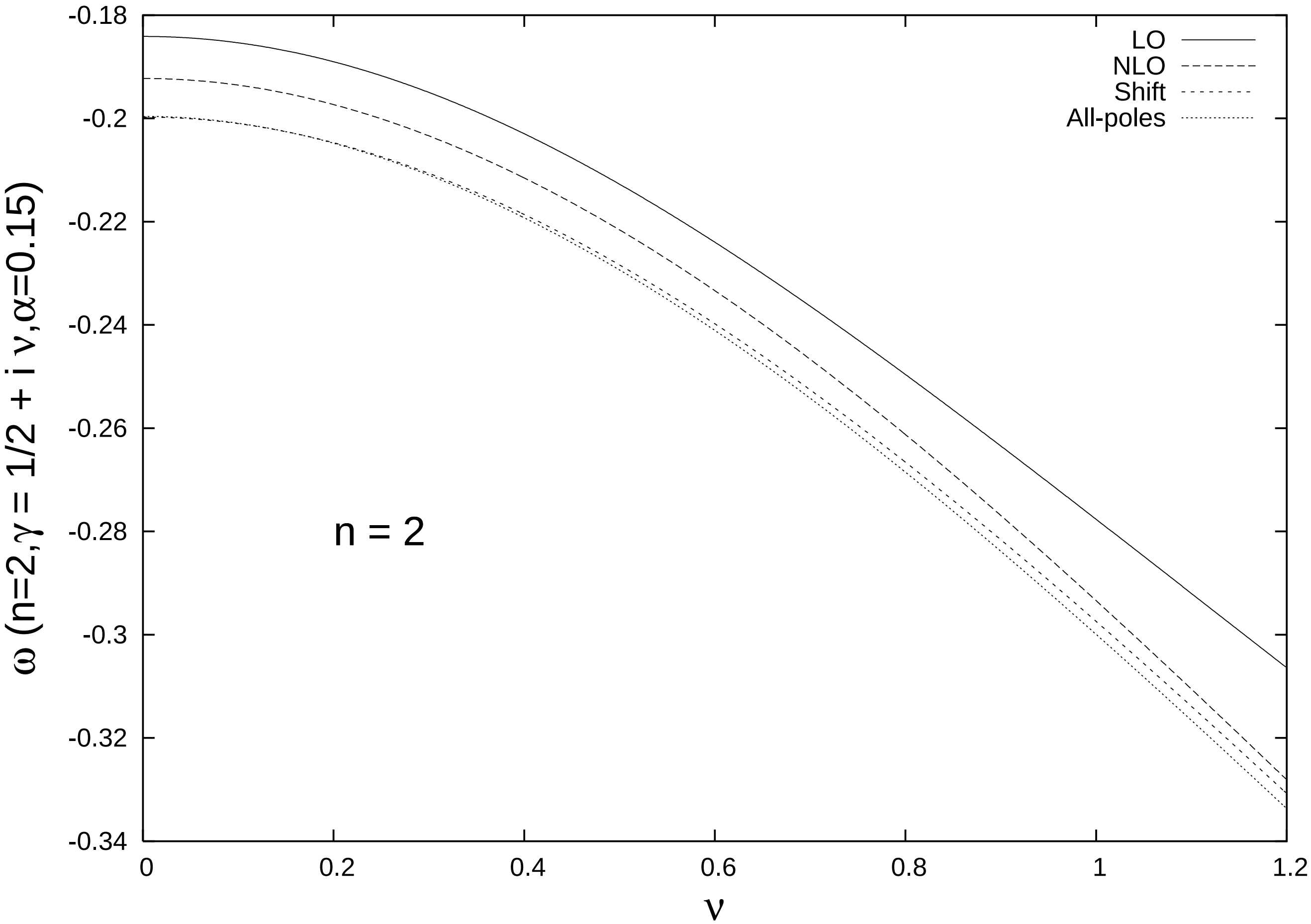

LO
NLO
Shift
All-poles
n = 0
ω (n=0,γ,α=0.15)
γ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


n = 1
LO
NLO
Shift
All-poles
ω (n=1,γ,α=0.15)
γ
0.4
0.2
0
-0.2
-0.4
-0.5
0
0.5
1
1.5

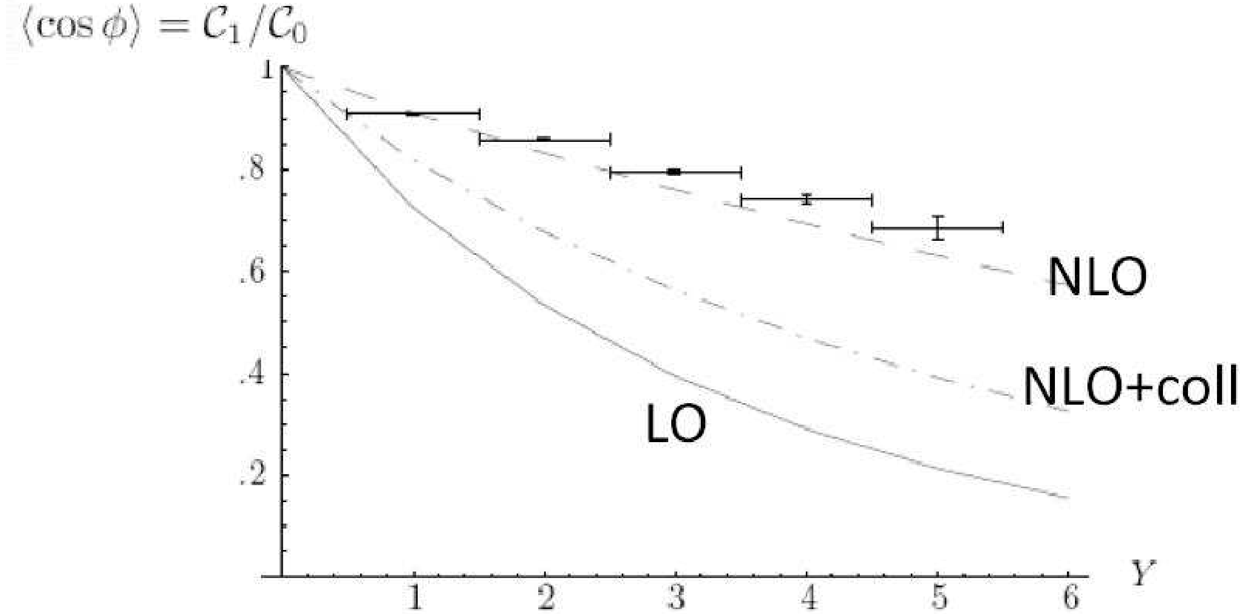

LO
NLO
Shift
All-poles
n = 2
ω (n=2,γ,α=0.15)
γ
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-1
-0.5
0
0.5
1
1.5
2


LO
NLO
Shift
All-poles
ω (n=0,γ = 1/2 + i ν,α=0.15)
n = 0
ν
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.2
0.4
0.6
0.8
1
1.2


LO
NLO
Shift
All-poles
n = 1
ω (n=1,γ = 1/2 + i ν,α=0.15)
ν
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.2
0.4
0.6
0.8
1
1.2



LO
NLO
Shift
All-poles
n = 2
ω (n=2,γ = 1/2 + i ν,α=0.15)
ν
-0.18
-0.2
-0.22
-0.24
-0.26
-0.28
-0.3
-0.32
-0.34
0
0.2
0.4
0.6
0.8
1
1.2

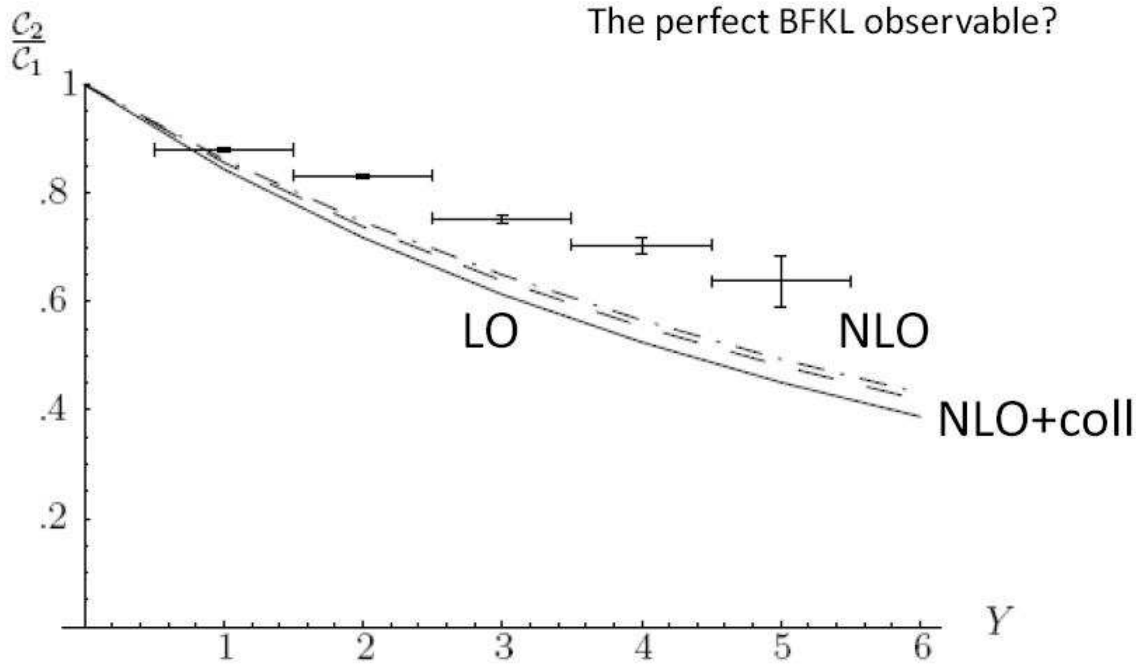


$\langle \cos\phi \rangle = \mathcal{C}_1/\mathcal{C}_0$ at a $p\bar{p}$ collider with $\sqrt{s} = 1.8$ TeV for BFKL at LO (solid), NLO (dashed) and collinear resummation (dash–dotted).

$$\frac{\langle \cos(m\phi) \rangle}{\langle \cos(n\phi) \rangle} = \frac{\mathcal{C}_m(\mathrm{Y})}{\mathcal{C}_n(\mathrm{Y})}$$

The perturbative convergence of these ratios is very fast

The perfect BFKL observable?



• **Searching for other observables which could identify the conformal structure of QCD at high energies?**