Hierarchies in $SU(2)_L \times SU(2)_R \times U(1)_X$ effective potential models

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Introduction

- **Motivation**: no new particles have been observed yet at current accelerators energy scale $\Rightarrow$ to introduce new scalar particles, we need to have a mass hierarchy
- Is it possible to obtain hierarchy in a *natural* way? (i.e. general model/models & wide range of parameters)

- **Problem**: we need to produce new particles while recovering the SM at low energies (masses, couplings, θ_W, ...)

- **Proposal**: start from a massless lagrangian and make use of the Coleman & Weinberg mechanism to obtain SSB; effective potential formalism (1-loop)
- This does not work for the SM alone, but we can introduce a new symmetry and scalar field

The model

$SU(2)_L \times SU(2)_R \times U(1)_X$

$$\mathcal{L} = \frac{1}{2}(\mathrm{D}^\mu_L \phi_L)^y (\mathrm{D}_{L\mu}\phi_L) + \frac{1}{2}(\mathrm{D}^\mu_R \phi_R)^y (\mathrm{D}_{R\mu}\phi_R) - V(\phi_L, \phi_R)$$

$$\mathrm{D}^\mu_{L,R} = \partial^\mu - \frac{i}{2} g_{L,R} \sigma_a W^{a\mu}_{L,R} - \frac{i}{2} g_X Q_{L,R} X^\mu$$

$$V(\phi_L, \phi_R) = \frac{1}{4!}\lambda_L \phi_L^4 + \frac{1}{4!}\lambda_R \phi_R^4 + \frac{1}{4!}\lambda_{LR}\phi_L^2\phi_R^2$$

Classical fields

$$\phi_L \longrightarrow \begin{pmatrix} 0 \\ \varphi \end{pmatrix} \sim \mathrm{SM} \,, \qquad \phi_R \longrightarrow \begin{pmatrix} 0 \\ \eta \end{pmatrix}$$

From $\mathcal{L}$ we obtain the mass matrix $G(\varphi, \eta)$ of the model $\rightarrow$ eigenvalues τ_i and eigenvectors v_i

Particle content of the model: $W_{L,R}$, $Z_{L,R}$, h, H

$$G(\varphi,\eta) = \frac{1}{4}\begin{pmatrix} g_L^2\varphi^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_L^2\varphi^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_L^2\varphi^2 & 0 & 0 & 0 & g_L g_X Q_L \varphi^2 \\ 0 & 0 & 0 & g_R^2\eta^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2\eta^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_R^2\eta^2 & g_R g_X Q_R \eta^2 \\ 0 & 0 & g_L g_X Q_L \varphi^2 & 0 & 0 & g_R g_X Q_R \eta^2 & g_X^2(Q_L^2\varphi^2 + Q_R^2\eta^2) \end{pmatrix}$$

$$\tau_L(\varphi) = \frac{1}{4} g_L^2 \varphi^2 \qquad (\quad 2)\ , \qquad \tau_R(\eta) = \frac{1}{4} g_R^2 \eta^2 \qquad (\quad 2)\ , \qquad \tau_0 = 0\ ,$$

$$\tau\quad(\varphi,\eta) = \frac{1}{8}\Big\{(g_L^2 + g_X^2 Q_L^2)\varphi^2 + (g_R^2 + g_X^2 Q_R^2)\eta^2 \quad \sqrt{\left[(g_L^2 + g_X^2 Q_L^2)\varphi^2 + (g_R^2 + g_X^2 Q_R^2)\eta^2\right]^2 \quad 4\left[g_L^2 g_R^2 + g_X^2(g_L^2 Q_R^2 + g_R^2 Q_L^2)\right]\varphi^2\eta^2)}\Big\}$$

$$v_L^\mu = W_L^{1\mu},\ W_L^{2\mu} \qquad ! \quad \mathbf{W_L}$$

$$v_R^\mu = W_R^{1\mu},\ W_R^{2\mu} \qquad ! \quad \mathbf{W_R}$$

$$N_0 v_0^\mu = g_X \frac{Q_L}{g_L} W_L^{3\mu} + g_X \frac{Q_R}{g_R} W_R^{3\mu} \quad X^\mu \qquad ! \quad \textbf{photon}$$

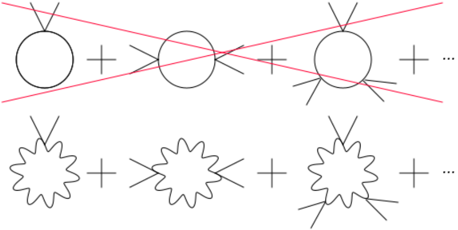
$$N \quad v^\mu = \frac{g_L(4\tau\quad(\varphi,\eta) \quad g_L^2\varphi^2)}{g_X Q_L (g_L^2\varphi^2 \quad g_R^2\eta^2)} W_L^{3\mu} + \frac{g_X g_R Q_R \eta^2}{4\tau\quad(\varphi,\eta) \quad g_R^2\eta^2} W_R^{3\mu} + X^\mu \qquad ! \quad \mathbf{Z_{L,R}}$$

Effective potential

Tree level + 1-loop:

$$V(\varphi,\eta) = \frac{1}{4!}\lambda_L\varphi^4 + \frac{1}{4!}\lambda_R\eta^4 + \frac{1}{4!}\lambda_{LR}\varphi^2\eta^2 +$$
$$+\frac{3}{64\pi^2}\left\{2\left\{\tau_L^2(\varphi)\left[\log\left(\frac{\tau_L(\varphi)}{M^2}\right)-\frac{25}{6}\right]+\tau_R^2(\eta)\left[\log\left(\frac{\tau_R(\eta)}{M^2}\right)-\frac{25}{6}\right]\right\}+\right.$$
$$\left.+\tau_+^2(\varphi,\eta)\left[\log\left(\frac{\tau_+(\varphi,\eta)}{M^2}\right)-\frac{25}{6}\right]+\tau^2(\varphi,\eta)\left[\log\left(\frac{\tau\ \ (\varphi,\eta)}{M^2}\right)-\frac{25}{6}\right]\right\}.$$

Coleman & Weinberg hypothesis $\rightarrow \lambda \ll g_R^2$



Coleman, Weinberg, Phys. Rev. D7 1888 (1973)

Casas, Espinosa, Quirós, arXiv:hep-ph/9409458v1

Renormalization Group Equations (1-loop); $g_X = 0, \lambda_{LR} = 0$

$$g_i(\mu), \lambda_i(\mu) \xrightarrow{?} V(\mu), \mathrm{R}(\mu), m^2(\mu) \qquad (\mathrm{R}{=}m^2_{W_R}/m^2_{W_L})$$

$$\frac{\partial V}{\partial \log \mu^2}, \frac{\partial \mathrm{R}}{\partial \log \mu^2}, \frac{\partial m^2_W}{\partial \log \mu^2} \longrightarrow \mathcal{O}\left(\frac{\lambda}{g^2}\right) + \mathcal{O}\left(\frac{g^2}{(4\pi)^2}\right)$$

RGEs for simplest case $g_X = 0, \lambda_{LR} = 0$

$$\frac{\partial g^2}{\partial x} = \hat{\beta} g^4 \;\longrightarrow\; g^2(x) = \frac{g_0^2}{1 - \hat{\beta} g_0^2 x}$$

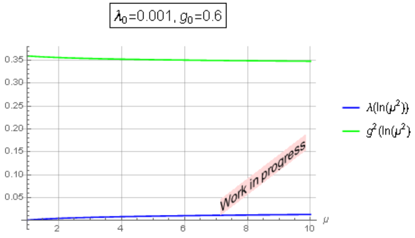
$$\frac{\partial \lambda}{\partial x} = \hat{\delta} g^4 \;\longrightarrow\; \lambda(x) = \lambda_0 + \frac{\hat{\delta}}{\hat{\beta}}[g^2(x) - g_0^2]$$

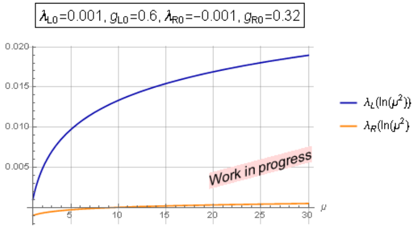
$$x = \log \mu^2$$

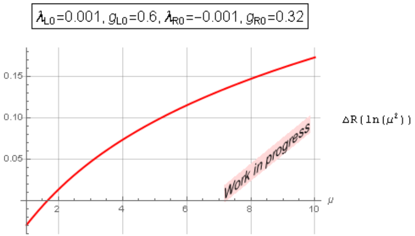
For cases with $g_X \neq 0, \lambda_{LR} \neq 0$: mixed RGEs

Antipin, et al., arXiv:1306.3234 [hep-ph]   Butazzo, et al., arXiv:1307.3536v4 [hep-ph]

Renormalization Group Equations (1-loop); $g_X = 0, \lambda_{LR} = 0$







Phenomenology: hierarchy?

We can define the hierarchy between the L and R sectors as:

$$\mathrm{R} = \frac{m^2_{W_R}}{m^2_{W_L}} = \frac{g_R^2 \langle \eta \rangle^2}{g_L^2 \langle \varphi \rangle^2}$$

- L: SM particles $\longrightarrow m^2_{W,Z_L} \leftrightarrow m^2_{W,Z_{SM}}$
- R: new particles at a different mass scale m^2_{W,Z_R}
- If R $\gg 1 \longrightarrow$ many expressions are simplified
- Take into account θ_W for SM particles + hierarchy ($\mathfrak{R}$ 1):
  $\frac{1}{g_X^2} = \cot^2\theta_W \frac{Q_L^2}{g_L^2} - \frac{Q_R^2}{g_R^2}$
- $\frac{m_H^2}{m_h^2} \propto$ R for $\lambda_{LR} = 0 \longrightarrow \lambda_{LR} \neq 0$?

Possible hierarchy cases

1. $g_X = 0, \;\; \lambda_{LR} = 0$

$$\text{R} = e^{\frac{128\pi^2}{27}\left(\frac{\lambda_L}{g_L^4} \quad \frac{\lambda_R}{g_R^4}\right)}$$

- analitical expressions
- $g_L \neq g_R, \lambda_L \neq \lambda_R$ needed for R $\neq 1$

2. $g_X \neq 0, \;\; \lambda_{LR} = 0$

$$\text{R} = e^{\frac{99 g_R^4 + 57 g_R^2 g_X^2 Q_R^2 \quad 128\pi^2 \lambda_R}{54 g_R^4 + 36 g_R^2 g_X^2 Q_R^2}} \quad (R \to L)$$

- $g_X << 1 \;\Rightarrow$ analitical expressions
- R $\neq 1$ for $g_L = g_R, \lambda_L = \lambda_R$ with $Q_L \neq Q_R$

3. $g_X = 0, \;\; \lambda_{LR} \neq 0$

R =?

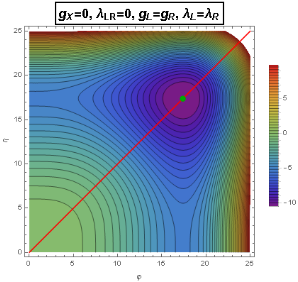
- $g_L \neq g_R, \lambda_L \neq \lambda_R$ needed for R $\neq 1$
- but: no analitical expressions
- problems with the behaviour of the potential: critical points on the axes

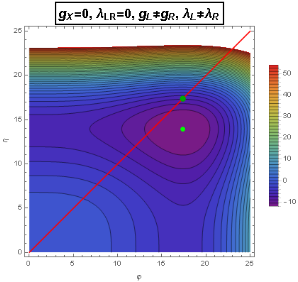
4. $g_X \neq 0, \;\; \lambda_{LR} \neq 0$

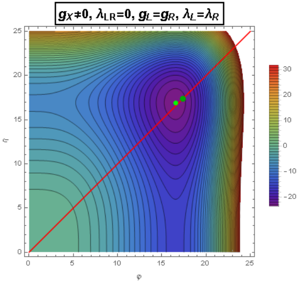
R =?

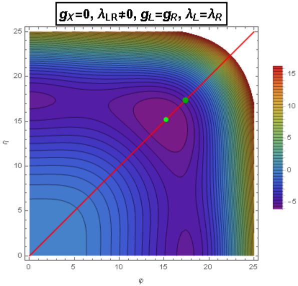
- difficult numerical analysis

Possible hierarchy cases





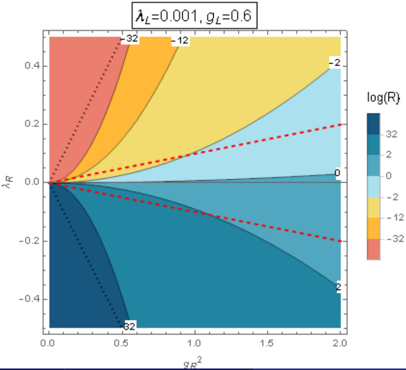


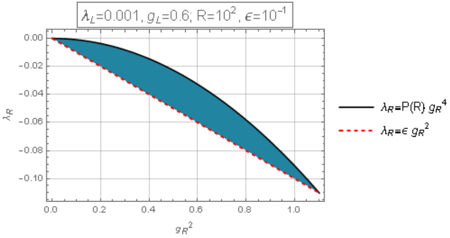


Parameter space ($g_X = 0, \lambda_{LR} = 0$)

$$\mathrm{R} = e^{\frac{128\pi^2}{27}\left(\frac{\lambda_L}{g_L^4} \quad \frac{\lambda_R}{g_R^4}\right)} \longrightarrow \lambda_R = \overbrace{\left(\frac{\lambda_L}{g_L^4} - \frac{27}{128\pi^2}\log \mathrm{R}\right)}^{P(\mathrm{R})} g_R^4$$

- $\lambda_L, g_L \rightarrow$ fixed ($\sim$ SM) $\Rightarrow \lambda_R = \mathcal{P}(\mathrm{R}) g_R^4$
- CW hypothesis: $|\lambda_R| \ll g_R^2 \Rightarrow |\lambda_R| = \varepsilon g_R^2$
- $\rightarrow$ region in the parameter space (g_R^2, λ_R), boundary $\left(\frac{\varepsilon}{P(\mathrm{R})}, \frac{\varepsilon^2}{P(\mathrm{R})}\right)$



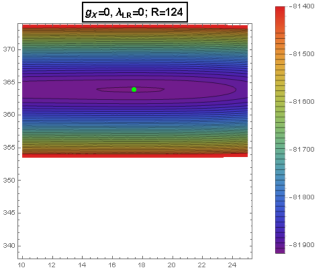


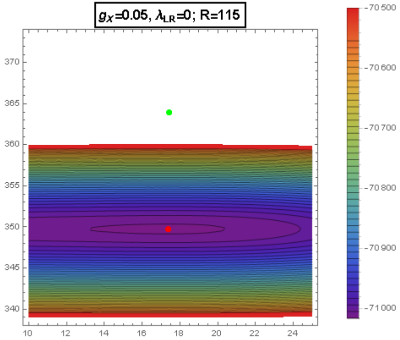
More complicated models: $\lambda_{LR} \neq 0$, $g_X \neq 0$

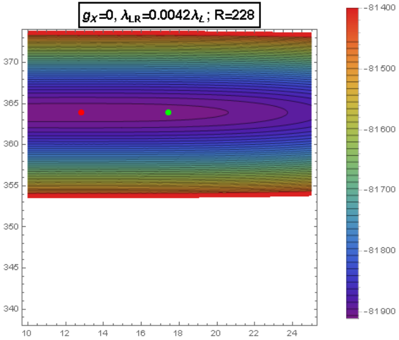
Starting from simplest case with

$g_L = 0.6, \lambda_L = 0.001$

$g_R = 0.32, \lambda_R = -0.001$







Conclusions

Summary & results:

- model with $SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry
- 2 sectors of particles with possible hierarchy between them
- hierarchy: depends on the couplings $\rightarrow g_L, g_R, \lambda_L, \lambda_R, g_X, \lambda_{LR}$
- promising results: wide region of parameter space giving place to hierarchy

Future work:

- we are studying a hierarchy$\sim$ LHC scale $\Rightarrow$ R $\sim 10^2$; but: other hierarchies?: $\sim$ gravity/inflation scale $\Rightarrow$ R $\sim 10^{32}$; neutrino mass scale $\Rightarrow$ R $\sim 10^{12}$
- include fermions in the model and study their effects; specifically: introduce $y_t \neq 0$
- thorough study of the compatibility of the model with SM!

Thank you for your attention

Possible hierarchy cases