α'-corrected Black Holes

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Motivation

- The action of String Theory is believed to be a double expansion in both the string coupling constant g_s and $\alpha' = \ell_s^2$

$$S = \sum_{n,i} S_{(n,i)} \, g_s^n \, \alpha'^i$$

- At low energies, ST predicts GR coupled to certain matter (ten dimensional supergravity theories)

$$S^{com}_{(0,0)} \sim \int d^{10}x \sqrt{|g|} \, e^{-2\phi} \, \left[R - 4(\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 \right]$$

- Corrections to $S_{(0,0)}$ contain **higher-curvature terms**. Only a few have been constructed...
- The *supergravity approximation* is reliable when the curvature is low as compared to the string scale

Black Holes in String Theory

- Black holes are thought to be a dual description of systems of strings and branes

- This was confirmed for a certain class of **extremal** black holes by Strominger and Vafa. They showed that the (macroscopic) Bekenstein-Hawking entropy matched the degeneracy of microscopic states when the number of branes is large (low curvature)

- Ever since, people have tried to go beyond this approximation. From a macroscopic point of view, this means that one must take into account higher-curvature corrections, which translate into $\mathcal{O}(1/N)$ corrections to the BH entropy

Plan of the talk

- In this talk, I will consider corrections to extremal 4-charge **heterotic** black holes with large horizon area
- I will show the corrections to the mass and to the BH entropy
- Finally, as a particular case, I will discuss the case of **small black holes**, which in the supergravity approximation have vanishing area

The talk is based on ...

- *The small black hole illusion*, P. A. Cano, P. F.Ramírez and AR, arXiv:1808.10449[hep-th]
- *Beyond the near-horizon limit: Stringy corrections to Heterotic Black Holes*, P. A. Cano, S. Chimento, P. Meessen, T. Ortín, P. F.Ramírez and AR, arXiv:1808.03651[hep-th]
- *On a family of α'-corrected solutions of the Heterotic Superstring effective action*, S. Chimento, P. Meessen, T. Ortín, P. F.Ramírez and AR, arXiv:1803.04463[hep-th], JHEP 1807(2018)080.

The effective action of the Heterotic Superstring at $\mathcal{O}(\alpha')$

- Field content: $\underbrace{(g_{\mu\nu}, B_{\mu\nu}, \phi)}_{\text{common sector}} + A^A_\mu$

- To deal with α'-corrections, it is convenient to define the **torsionful spin-connection** as $\Omega_{(-)}{}^a{}_b = \omega^a{}_b - \frac{1}{2} H_\mu{}^a{}_b \, dx^\mu$

- 3-form field strength: $H = dB + \underbrace{\frac{\alpha'}{4}\left(\omega^{\text{YM}} + \omega^L_{(-)}\right)}_{\text{Chern-Simons terms}}$, where

$$\omega^{YM} = dA^A \wedge A^A + \frac{1}{3}\epsilon^{ABC} A^A \wedge A^B \wedge A^C$$

$$\omega^{\text{L}}_{(-)} = d\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_a - \frac{2}{3}\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_c \wedge \Omega_{(-)}{}^c{}_a$$

The effective action of the Heterotic Superstring at $\mathcal{O}(\alpha')$

To recover supersymmetry, we must add **higher-curvature terms** to the zeroth-order action

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 - \tfrac{\alpha'}{8} \left(F^A{}_{\mu\nu} F^{A\mu\nu} + R_{(-)\mu\nu}{}^a{}_b R_{(-)}{}^{\mu\nu\, b}{}_a \right) \right\}$$

[Bergshoeff and de Roo, '89]

where

$$\begin{aligned} R_{(-)}{}^a{}_b &= d\Omega_{(-)}{}^a{}_b - \Omega_{(-)}{}^a{}_c \wedge \Omega_{(-)}{}^c{}_b \\ F^A &= dA^A + \frac{1}{2}\epsilon^{ABC} A^B \wedge A^C \end{aligned}$$

EOMs

$$R_{\mu\nu} - 2\nabla_\mu \partial_\nu \phi + \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} = \frac{\alpha'}{4} \left(F^A{}_{\mu\rho} F^A{}_\nu{}^\rho + R_{(-)\mu\rho}{}^a{}_b R_{(-)\nu}{}^{\rho\, b}{}_a \right)$$

$$(\partial\phi)^2 - \frac{1}{2} \nabla^2 \phi - \frac{1}{4 \cdot 3!} H^2 = -\frac{\alpha'}{32} \left(F^A{}_{\mu\nu} F^{A\mu\nu} + R_{(-)\mu\nu}{}^a{}_b R_{(-)}{}^{\mu\nu\, b}{}_a \right)$$

$$d\left(e^{-2\phi} \star H \right) = 0$$

$$\alpha' \left[d\left(e^{-2\phi} \star F^A \right) + \epsilon^{ABC} A^B \wedge \star F^C + \star H \wedge F^A \right] = 0$$

The family

A very well-known family of solutions to the zeroth-order EOMs is

$$ds^2 = \frac{2}{\mathcal{Z}_-} du \left[dv - \tfrac{1}{2}\mathcal{Z}_+ du\right] - \mathcal{Z}_0 d\sigma^2 - dy^i dy^i$$

where

$$d\sigma^2 = \mathcal{H}^{-1}(d\eta + \chi)^2 + \mathcal{H} d\vec{x} \cdot d\vec{x}\,, \quad d\chi = \star_{(3)} d\mathcal{H}$$

This solution describes a superposition of a fundamental string (F1) wrapped on $\mathbb{S}^1_u$ with winding and momentum charges, a stack of solitonic five-branes (NS5) wrapped on $\mathbb{T}^4 \times \mathbb{S}^1_u$ and a KK monopole

[Youm, Cvetic '98]

The family

The 3-form field strength is given by

$$H = d\mathcal{Z}_-{}^{-1} \wedge du \wedge dv + \star_{(4)} d\mathcal{Z}_0$$

Finally, the dilaton

$$e^{2\phi} = g_s^2 \frac{\mathcal{Z}_0}{\mathcal{Z}_-}$$

(g_s is the string coupling constant)

At **zeroth-order**, all the functions that define the solution (i.e., $\mathcal{Z}_{0,+,-}$ and $\mathcal{H}$) are **harmonic** with respect to the hyperKähler metric $d\sigma^2$

The ansatz. Metric $g_{\mu\nu}$, dilaton ϕ and 3-form H

We assume the form of the solution is unmodified, only the functions:

$$\mathcal{Z}_{0,+,-} = \mathcal{Z}^{(0)}_{0,+,-} + \alpha' \mathcal{Z}^{(1)}_{0,+,-} + \mathcal{O}(\alpha'^2)$$

- Metric (string frame)

$$ds^2 = \frac{2}{\mathcal{Z}_-} du \left[dv - \tfrac{1}{2}\mathcal{Z}_+ du\right] - \mathcal{Z}_0 d\sigma^2 - dy^i dy^i$$

- 3-form field strength H

$$H = d\mathcal{Z}_-{}^{-1} \wedge du \wedge dv + \star_{(4)} d\mathcal{Z}_0$$

- Dilaton:

$$e^{2\phi} = g_s^2 \frac{\mathcal{Z}_0}{\mathcal{Z}_-}$$

The ansatz. Gauge fields.

We activate a subgroup $\underbrace{SU(2) \times \cdots \times SU(2)}_{N_\lambda \text{ times}}$ of the full Heterotic gauge group. Our ansatz is based on a slightly generalization of the **'t Hooft ansatz**

$$A^{A_i} = \bar{\eta}^{A_i}_{mn} \partial_n \log P_i \, v^m \qquad i = 1, \ldots, N_\lambda$$

where

- $\bar{\eta}^{A_i}_{mn}$ are the 't Hooft symbols
- v^m is the four-dimensional vierbein: $d\sigma^2 = v^m v^m$
- $P(x)$ is a harmonic function wrt the hyperKähler metric ($\Rightarrow$ selfdual field strengths $F^{A_i} = + \star_{(4)} F^{A_i}$)

The general α'-corrected solution

Once a solution to the zeroth-order EOMs is given, the solution to the α'-corrected EOMs is the following

$$
\begin{aligned}
\mathcal{Z}_- &= \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2) \\
\mathcal{Z}_+ &= \mathcal{Z}_+^{(0)} - \frac{\alpha'}{2}\left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-^{(0)}}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-}\right) + \mathcal{O}(\alpha'^2) \\
\mathcal{Z}_0 &= \mathcal{Z}_0^{(0)} - \frac{\alpha'}{4}\left[\sum_{i=1}^{N_\lambda} (\partial \log P_i)^2 - \left(\partial \log \mathcal{Z}_0^{(0)}\right)^2 - (\partial \log \mathcal{H})^2\right]
\end{aligned}
$$

The inclusion of non-trivial gauge fields is crucial to cancel the corrections to $\mathcal{Z}_0$

4-charge black holes

The choice $\mathcal{Z}_{0,+,-} = 1 + \frac{q_{0,+,-}}{r}$, $\mathcal{H} = 1 + \frac{q}{r}$ describes a black hole with four charges upon compactification on $\mathbb{T}^6$

$$ds_4^2 = e^{2U} dt^2 - e^{-2U} d\vec{x} \cdot d\vec{x} \qquad e^{-2U} = \sqrt{\mathcal{Z}_0 \mathcal{Z}_+ \mathcal{Z}_- \mathcal{H}}$$

The interpretation of the charges within string theory is

- q_0 counts the number of NS5-branes: $q_0 = \frac{\ell_s^2}{2R} N$
- q_- is related to the winding number of the F1: $q_- = \frac{g_s^2 \ell_s^2}{2R} w$
- q_+ is related to the momentum carried by F1: $q_+ = \frac{g_s^2 \ell_s^4}{2RR_u^2} n$
- q is related to the KK charge: $q = \frac{R}{2} W$

Corrections to 4-charge black holes

$$\mathcal{Z}_+ = 1 + \frac{q_+}{r} + \frac{\alpha' q_+}{2qq_0}\frac{r^2 + r(q_0 + q_- + q) + qq_0 + qq_- + q_0 q_-}{(r+q)(r+q_0)(r+q_-)} + \mathcal{O}(\alpha'^2)$$

$$\mathcal{Z}_0 = 1 + \frac{q_0}{r} + \alpha'\left[-F(r;q_0) - F(r;q) + \sum_{i=1}^{N_\lambda} F(r;\lambda_i^{-2})\right] + \mathcal{O}(\alpha'^2)$$

where

$$F(r;k) := \frac{(r+q)(r+2k)+k^2}{4q(r+q)(r+k)^2}$$

Notice that

$$\lim_{r\to 0} F(r;k) = \frac{2q+k}{4q^2 k} \sim \mathcal{O}(r^0) \qquad \lim_{r\to\infty} F(r;k) = \frac{r^{-1}}{4q} + \mathcal{O}(r^{-2})$$

Screening effect

- The S5 charge can be effectively computed by looking at the $1/r$ coefficient of $\mathcal{Z}_0$

$$\lim_{r\to 0} \mathcal{Z}_0 = \frac{q_0}{r} \qquad\qquad \lim_{r\to\infty} \mathcal{Z}_0 = \frac{q_0 - \alpha' \frac{2-N_\lambda}{4q}}{r}$$

- Higher-curvature interactions introduce **delocalized sources** of S5 charge and asymptotic and near-horizon charges do not coincide anymore. **Which one is quantized?**

- A careful analysis shows that it is again q_0 the one is related to the number of NS5-branes: $q_0 = \frac{\ell_s^2}{2R} N$

- The effective number of branes observed at infinity is

$$N^{eff} = N + \frac{N_\lambda - 2}{W}$$

Corrections to the mass and entropy

- Mass of the solution

$$M = \frac{R_u}{g_s^2 \ell_s^2}\left(N + \frac{N_\lambda - 2}{W}\right) + \frac{R_u}{\ell_s^2} n + \frac{w}{R_u}\left(1 + \frac{2}{NW}\right) + W \frac{R_z^2 R_u}{g_s^2 \ell_s^4}$$

- Wald's entropy:

$$S_{Wald} = 2\pi\sqrt{nwNW}\left(1 + \frac{1}{NW} + \dots\right) \underbrace{\approx}_{NW >> 1} 2\pi\sqrt{nw\,(NW + 2)} =$$

$$= 2\pi\sqrt{nw\,(N^{eff} W + 4 - N_\lambda)}$$

Small Black Holes (SBHs)

- Small black holes are those sourced by strings: $N = W = 0$

- It is claimed that they are the macroscopic description of the so-called Dabholkar-Harvey states

- Degeneracy of DH states

$$S_{micro} \approx 4\pi\sqrt{nw} \qquad\qquad n, w >> 1$$

- At zeroth-order in α', the horizon has zero size and therefore they are singular and the Bekenstein-Hawking entropy vanishes

- The supergravity approximation does not give a good description of SBHs and one has to take into account higher-curvature corrections

Resolution of the horizon

Higher-curvature corrections to the zeroth-order action could resolve the horizon and the Wald's entropy would match the microscopic result

[Sen '94]

By using the attractor mechanism and an effective action including only curvature squared terms, it was shown that the Wald's entropy happened to match the microscopic result to all orders!

[Dabholkar '04]
[Dabholkar, Kallosh, Maloney '04]

Why it is enough with quadratic-curvature terms?

α'-corrections to small black holes

Particularizing for the 2-charge system:

$$\mathcal{Z}_+ = 1 + \frac{q_+}{r} - \frac{\alpha' q_+ q_-}{2r^3(r+q_-)} + \mathcal{O}(\alpha'^2)$$

$$\mathcal{Z}_- = 1 + \frac{q_-}{r} + \mathcal{O}(\alpha'^2)$$

- Notice that $\lim_{r\to 0} \mathcal{Z}_+ \sim 1/r^3$, just the right behaviour to obtain a horizon with finite size in $d = 4$
- However, 10-dim metric still has a curvature singularity...
- Can we trust on this solution?

Fake resolution of small black holes

- If $N^{eff} = 0$ (asymptotic S5-charge totally screened) then the Wald's entropy happens to match the microscopic entropy associated to the DH states:

$$S_{Wald} = 2\pi\sqrt{nw(N^{eff}W + 4)} = 4\pi\sqrt{nw} = S_{micro}$$

- A misidentification of N^{eff} with the number of S5-branes would cause the illusion of a streched horizon

- However, the system under study has actually $N = \frac{2}{W}$ S5-branes and it was already regular at zeroth-order in α'

- We claim that this is exactly what happens in the resolution of the horizon previously reported in the literature of SBHs

Conclusions

- The resolution of the horizon of small black holes via addition of higher-curvature terms still remains an open problem
- Same conclusion for five-dimensional SBHs, where one cannot design a fake resolution

  [Prester, Terzic '08]

- Similarly, there are no SBHs representing excited states of a type II string (in this case, the Bianchi id. is not corrected)
- Summarizing: everything seems to point out that small black holes do not admit a perturbative description. In some sense, this would clarify the situation since it puts every small black hole at the same qualitative level

Thanks for your attention!