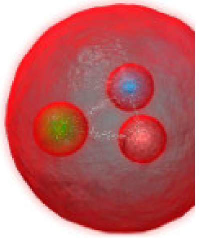
X CPAN DAYS

Salamanca, 29 - 31 October 2018



Hadron physics for neutrino oscillation experiments

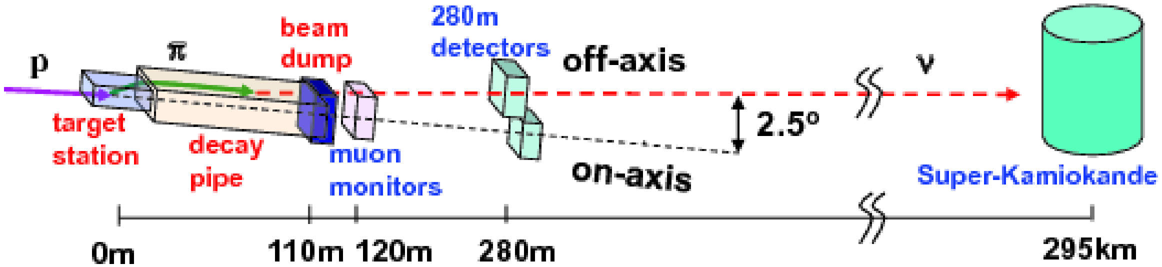
Luis Alvarez Ruso

K. Graczyk, E. Saúl Sala, M. J. Vicente Vacas, D. Yao

Introduction

- Oscillation experiments (with accelerator ν in the few-GeV region)
- Experiments with near & far detectors: T2K, NOvA, SBN, Hyper-K, DUNE

T2K



- Near detectors help to reduce systematic errors:

$$\frac{N^{far}_{events}(E_\nu)}{N_{events}(E_\nu)} = \frac{\int \sigma(E'_\nu)\Phi(E'_\nu)P(E_\nu|E'_\nu)P_{osc}(E'_\nu)dE'_\nu}{\int \sigma(E'_\nu)\Phi(E'_\nu)P(E_\nu|E'_\nu)dE'_\nu}$$

F. Sanchez @ NuPhys2015

but cross section uncertainties do not cancel (exactly) in the ratio

- exposed to different fluxes with different flavor composition
- different geometry, acceptance and targets

Introduction

- ν cross sections are crucial to achieve the precision goals of oscillation experiments

$$\frac{N^{far}_{events}(E_\nu)}{N_{events}(E_\nu)} = \frac{\int \sigma(E'_\nu)\Phi(E'_\nu)P(E_\nu|E'_\nu)P_{osc}(E'_\nu)dE'_\nu}{\int \sigma(E'_\nu)\Phi(E'_\nu)P(E_\nu|E'_\nu)dE'_\nu}$$

F. Sanchez @ NuPhys2015

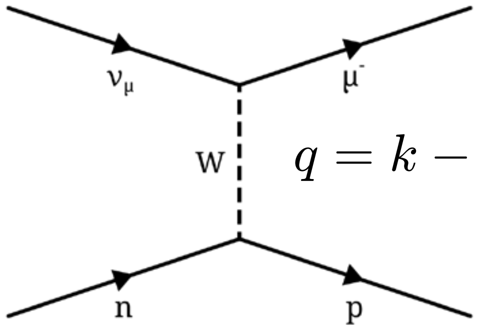
- Need for theory?
  - Measurements are not (cannot be) perfect
  - (partially) rely on simulations $\approx$ theory to determine
    - efficency, acceptance
    - (irreducible) background subtraction
  - E_ν is not known: reconstructed using kinematics and/or calorimetry
  - $\sigma(\nu_\mu)$ to $\sigma(\nu_e)$ extrapolations

- Neutrino c.s. mismodeling could lead to unacceptably large systematic uncertainties or biased measurements Coloma, Huber, PRL 111 (2013)

Nucleon axial form factor

- Fundamental nucleon property
- Main source of uncertainty for QE scattering on nucleons:

$$\begin{aligned} \text{CCQE}: \nu(k) + n(p) &\rightarrow l^-(k') + p(p') \\ \bar{\nu}(k) + p(p) &\rightarrow l^+(k') + n(p') \\ \text{NCE}: \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\ \bar{\nu}(k) + N(p) &\rightarrow \bar{\nu}(k') + N(p') \end{aligned}$$



$$q = k - k' = p' - p$$

- Largest contribution at T2K, MicroBooNE

$$\mathcal{M} = \frac{G_F \cos\theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$$

$$J_\alpha = \bar{u}(p')\left[\gamma_\alpha F_1^V + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^V + \gamma_\mu\gamma_5 F_A + \frac{q_\mu}{M}\gamma_5 F_P\right]u(p)$$

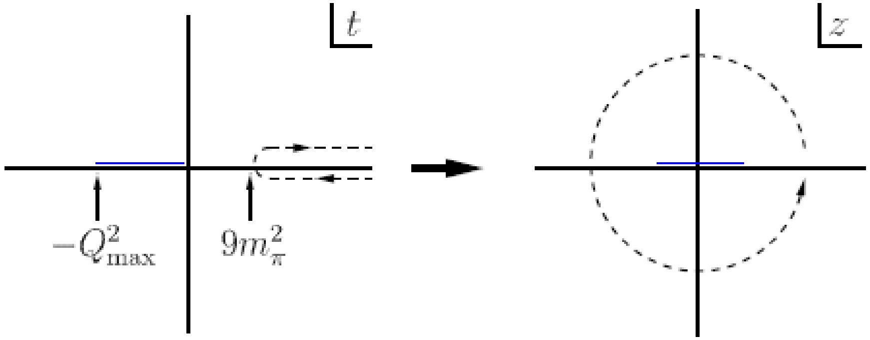
Nucleon axial form factor

- What is known:
  - $F_A(0) = g_A \leftarrow \beta$ decay
  - $F_A(\infty) \sim Q^{-4} \leftarrow$ QCD
- Main source of information: bubble chamber (ANL, BNL, FNAL) data

- Dipole ansatz: Bodek et al., EPJC 53 (2008)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

- Axial-meson dominance: Masjuan et al., PRD 87 (2013)
- z-expansion: Meyer et al., PRD 93 (2016)

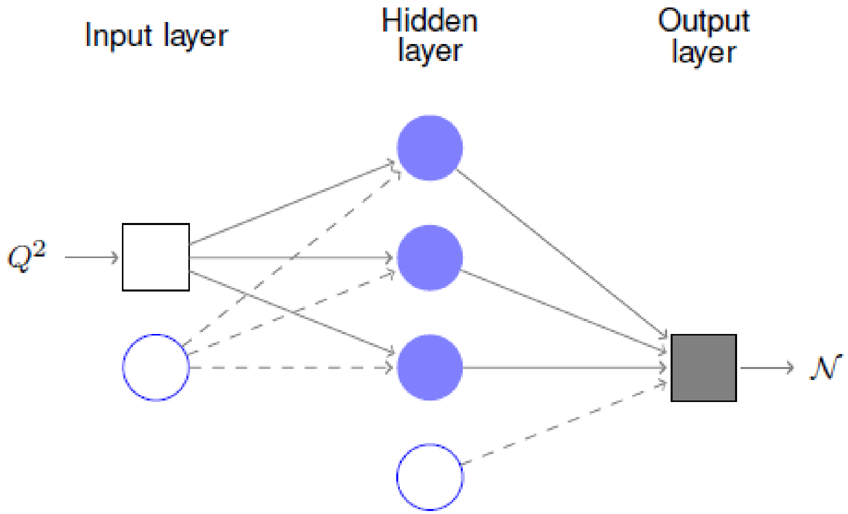


$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k z(Q^2)^k$$

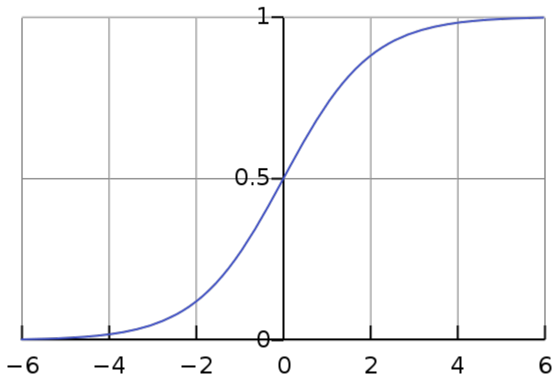
- Neural networks + Bayesian statistics: LAR, Graczyk, Saúl-Sala, arXiv:1805.00905

Neural Networks for F_A

- Feed-forward NN in multilayer perceptron (MLP) configurations
- Non-linear map $\mathcal{N}$: $R^{in} \to R^{out}$



- For every unit: $y_j = f_{\text{act}}\left(\sum_{i \in \text{prev. layer}} w_i\, y_i\right)$



- Activation function: $f(x) = \dfrac{1}{1+\exp(-x)}$
  - except in bias units: $f(x) = 1$ and output: $f(x) = x$

Neural Networks for F_A

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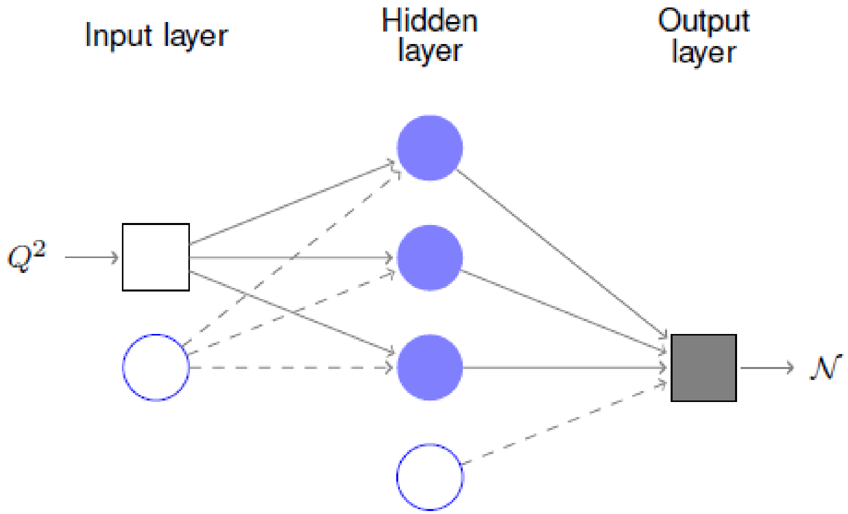
- Non-linear map $\mathcal{N}: R^{in} \rightarrow R^{out}$



$$\mathcal{N}_M(Q^2;\{w_j\}) = \sum_{n=1}^{M} w_{2M+n}\, f\left(w_n Q^2 + w_{M+n}\right) + w_{3M+1}.$$

$$F_A(Q^2) = F_A^{\mathrm{dipole}}(Q^2) \times \mathcal{N}_M(Q^2;\{w_i\})$$ $\leftarrow$ function of W=3 M +1 weights and Q^2

- Cybenko's theorem: for large enough M, can map arbitrarily well any continuous function and its derivative

Bayesian inference for NN

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

1. For a given $\mathcal{N}$: $\mathcal{P}(\{w_j\} \mid \mathcal{D}, \mathcal{N}) = \dfrac{\mathcal{P}(\mathcal{D} \mid \{w_j\}, \mathcal{N}) \mathcal{P}(\{w_j\} \mid \mathcal{N})}{\mathcal{P}(\mathcal{D} \mid \mathcal{N})}$,

- Likelihood in terms of χ^2 :

$$\mathcal{P}(\mathcal{D} \mid \{w_j\}, \mathcal{N}) = \frac{1}{N_L} \exp(-\chi^2)$$

- Prior: weights w_j are Gaussian distributed

$$\mathcal{P}(\{w_j\}, \mathcal{N}) = \frac{1}{N_w} \exp\left(-\alpha \frac{1}{2} \sum_{i=1}^{W} w_i^2\right)$$

$\alpha \leftarrow$ regularizer

- Algorithm to find the optimal: $(\{w_j\}_{MP}, \alpha_{MP})$

Bayesian inference for NN

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

1. For a given $\mathcal{N}$: $\mathcal{P}(\{w_j\} \mid \mathcal{D}, \mathcal{N}) = \dfrac{\mathcal{P}(\mathcal{D} \mid \{w_j\}, \mathcal{N})\mathcal{P}(\{w_j\} \mid \mathcal{N})}{\mathcal{P}(\mathcal{D} \mid \mathcal{N})},$
2. For $\mathcal{N}_{\text{1-M}}$: $\mathcal{P}(\mathcal{N}_i \mid \mathcal{D}) = \dfrac{\mathcal{P}(\mathcal{D} \mid \mathcal{N}_i)\mathcal{P}(\mathcal{N}_i)}{\mathcal{P}(\mathcal{D})}$

- Assuming all NN configurations are equally suited to describe data:

  $\mathcal{P}(\mathcal{N}_1) = \mathcal{P}(\mathcal{N}_2) = \ldots = \mathcal{P}(\mathcal{N}_M)$ then $\mathcal{P}(\mathcal{N} \mid \mathcal{D}) \propto \mathcal{P}(\mathcal{D} \mid \mathcal{N})$

- In the Hessian approximation:

$$\mathcal{P}(\mathcal{D} \mid \mathcal{N}) = \int dw_1 \cdots dw_W \mathcal{P}(\mathcal{D} \mid \{w_j\}, \mathcal{N})\mathcal{P}(\{w_j\} \mid \mathcal{N})$$

$$\ln \mathcal{P}\left(\mathcal{D} \middle| \mathcal{N}\right) \approx -\chi^2 - \alpha_{MP} \frac{1}{2} \sum_{i=1}^{W} \{w_i\}^2_{MP} - (\text{Occam's factor})$$

large for models with many parameters

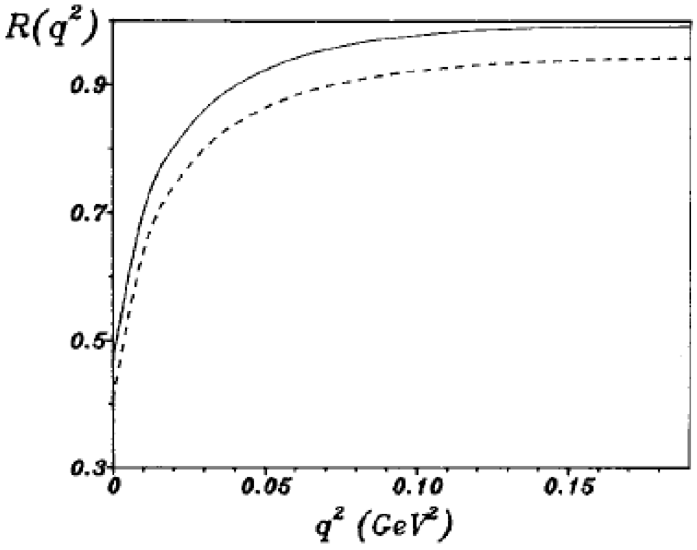
Bayesian NN analysis of ANL data

- LAR, Graczyk, Saúl-Sala, arXiv:1805.00905

$$\frac{d\sigma_{\nu n}}{dQ^2} = \frac{G_F^2 m_N^2}{8\pi E_\nu^2}\left[A(Q^2) + B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right]$$

- A, B, C are functions of $F^V_{1,2}$ and $F_{A,P}$
- $F^V_{1,2}$ assumed exact; F_P given in terms of F_A

$$\frac{d\sigma_{\nu d}}{dQ^2} = R(Q^2)\,\frac{d\sigma_{\nu n}}{dQ^2}$$



Singh, Arenhövel, Z. Phys. A 324 (1986)

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$$\frac{d\sigma_{\nu n}}{dQ^2} = \frac{G_F^2 m_N^2}{8\pi E_\nu^2}\left[A(Q^2) + B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right]$$

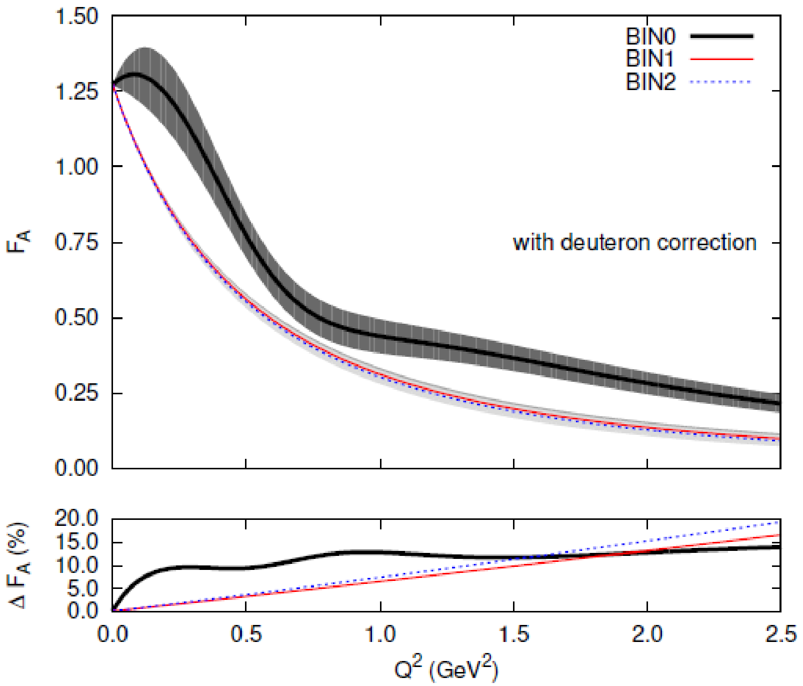
- A, B, C are functions of $F^V_{1,2}$ and $F_{A,P}$
- $F^V_{1,2}$ assumed exact; F_P given in terms of F_A
- Events: $N^{th}(Q^2) = \int_0^\infty dE_\nu \frac{d\sigma}{dQ^2}(E_\nu, F_A, Q^2)\phi(E_\nu)$
- Neutrino flux: $\phi(E_\nu) = p\,\frac{1}{\sigma(E_\nu, F_A)}\frac{dN}{dE_\nu}$

$\frac{dN}{dE_\nu}$ ← experimental E_ν distribution of observed events
Barish et al., PRD19 (1979)

- $$\chi^2 = \left(\frac{F_A(0) - g_A}{\Delta g_A}\right)^2 + \sum_{i=k}^{n_{\mathrm{ANL}}} \frac{\left(N_i - N_i^{th}\right)^2}{N_i} + \left(\frac{1-p}{\Delta p}\right)^2 \qquad \Delta p = 20\,\%$$

Bayesian NN analysis of ANL data

- Results:

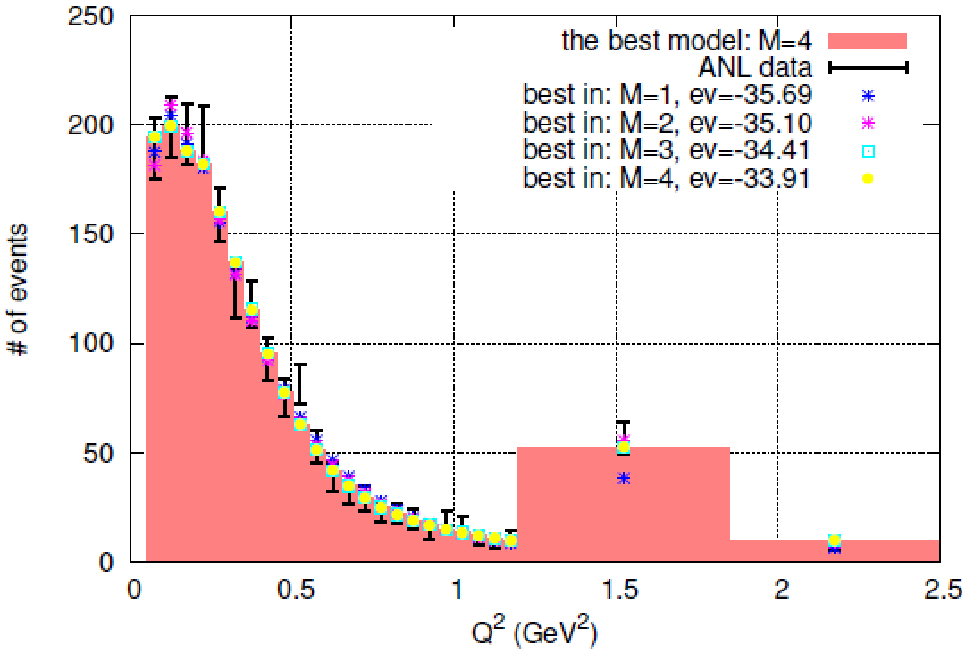


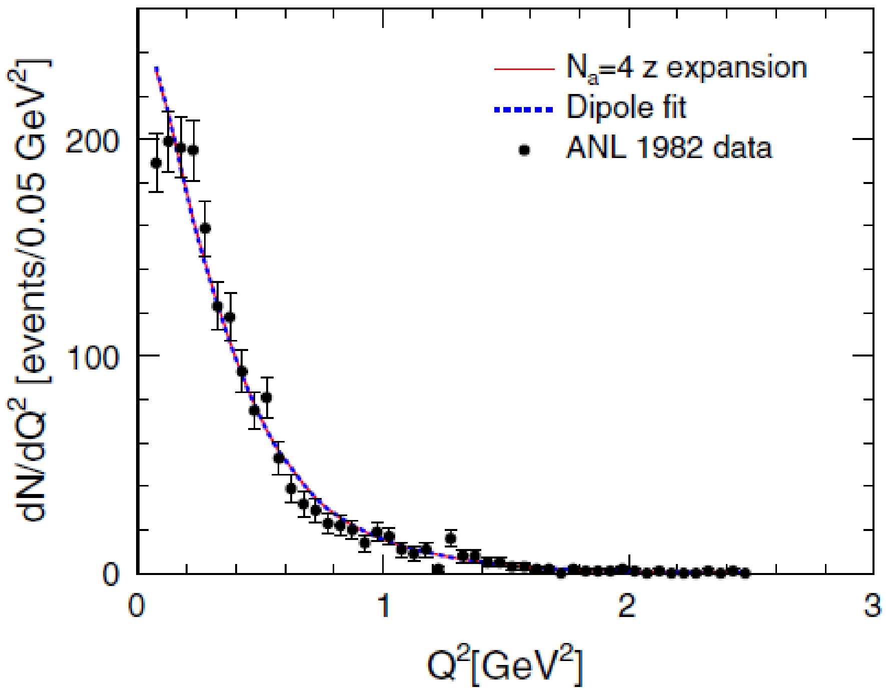
- BIN0 results inconsistent with z-exp ones:
  - r_A^2 = -1.61 $\pm$ 0.24 fm^2 vs 0.46(22) fm^2 [νd] & 0.43(24) fm^2 [μ-capt.]

Meyer et al., PRD93 (2016)
Hill et al., arXiv:1708.08462

Bayesian NN analysis of ANL data

- Results:



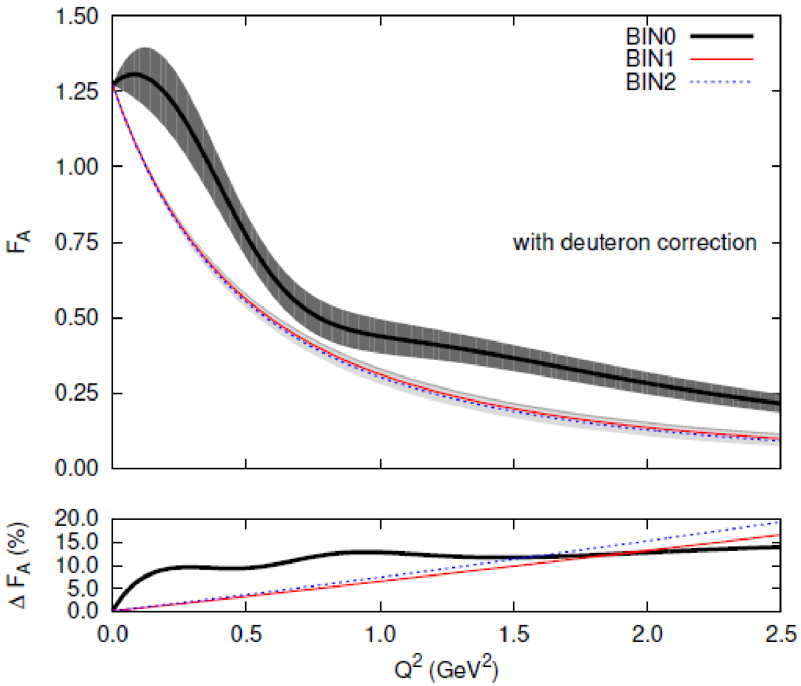


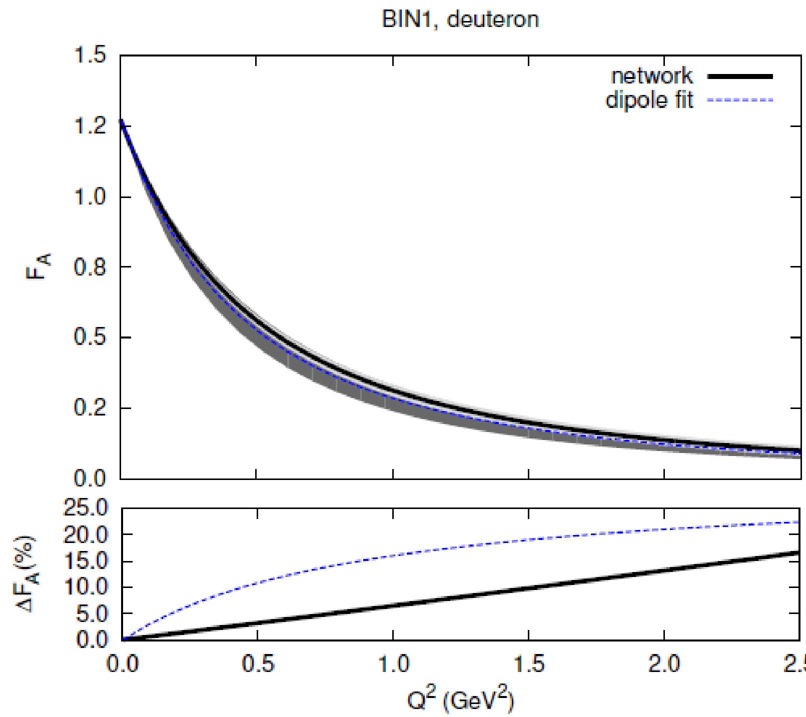
Meyer et al., PRD93 (2016)

- BIN0 results inconsistent with z-exp ones:
  - r_A^2 = -1.61 $\pm$ 0.24 fm^2 vs 0.46(22) fm^2 [νd] & 0.43(24) fm^2 [μ-capt.]
  - Deuteron corrections (important in the 1st,2nd bins)
  - Experimental efficiency issues at low Q^2

Bayesian NN analysis of ANL data

- Results:



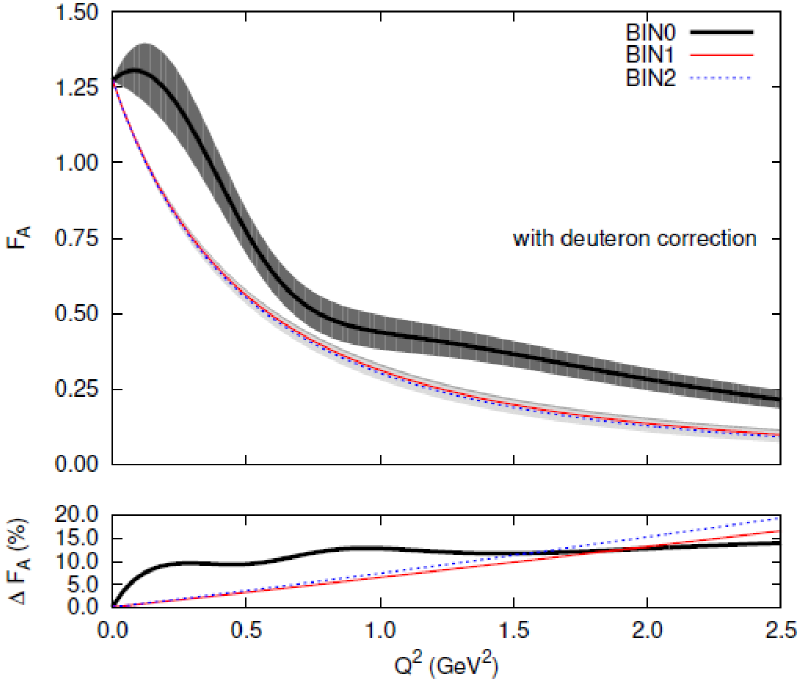


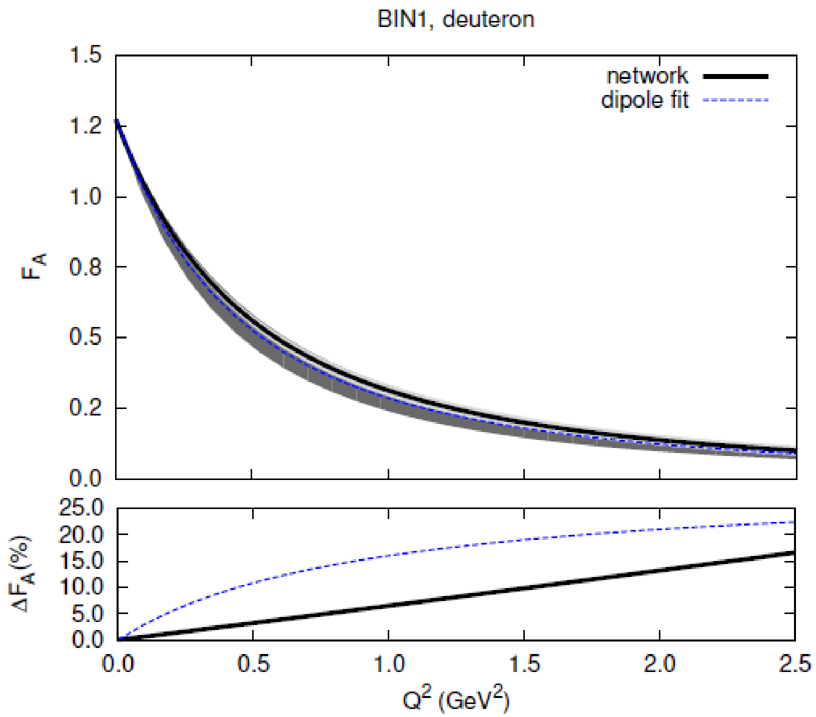
- BIN1 results consistent with z-exp (and dipole) ones:
  - $r_A^2 = 0.471 \pm 0.015$ fm² vs 0.46(22) fm² [νd] & 0.43(24) fm² [μ-capt.]

Meyer et al., PRD93 (2016)
Hill et al., arXiv:1708.08462

Bayesian NN analysis of ANL data

- Results:

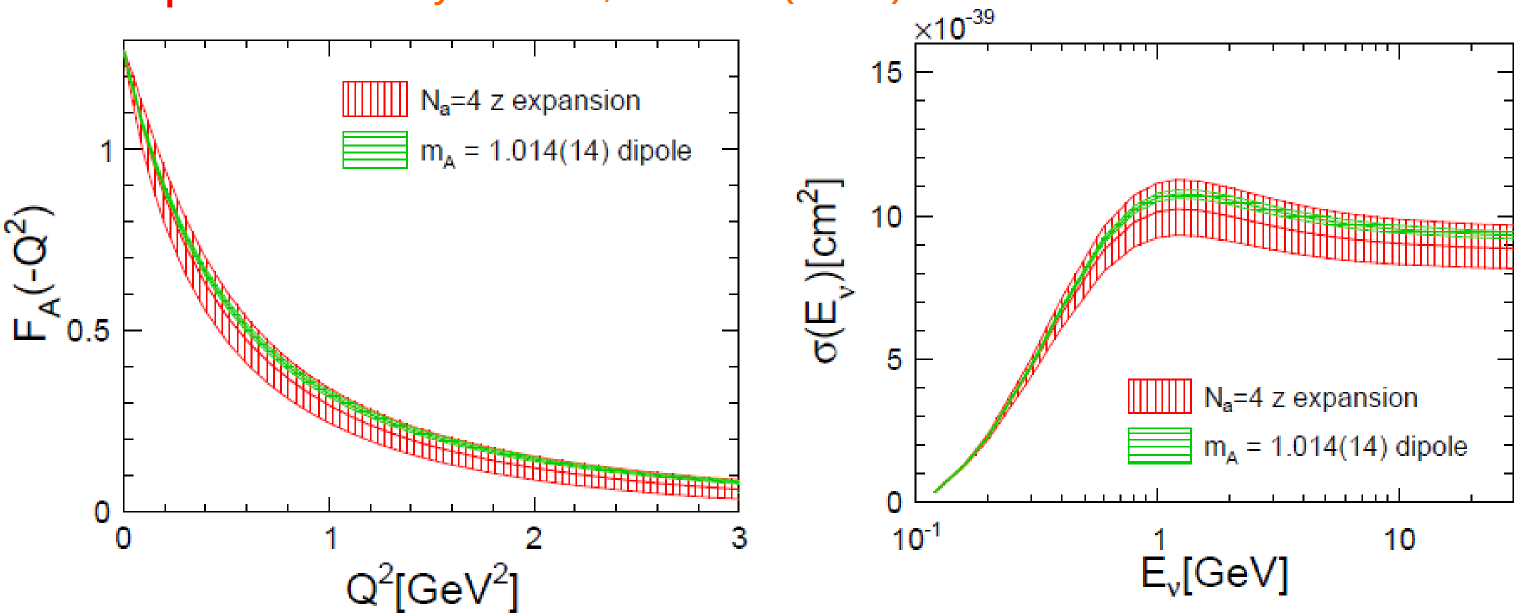




- All methods obtain similar F_A(Q2) ... but with different errors

QE scattering on the nucleon

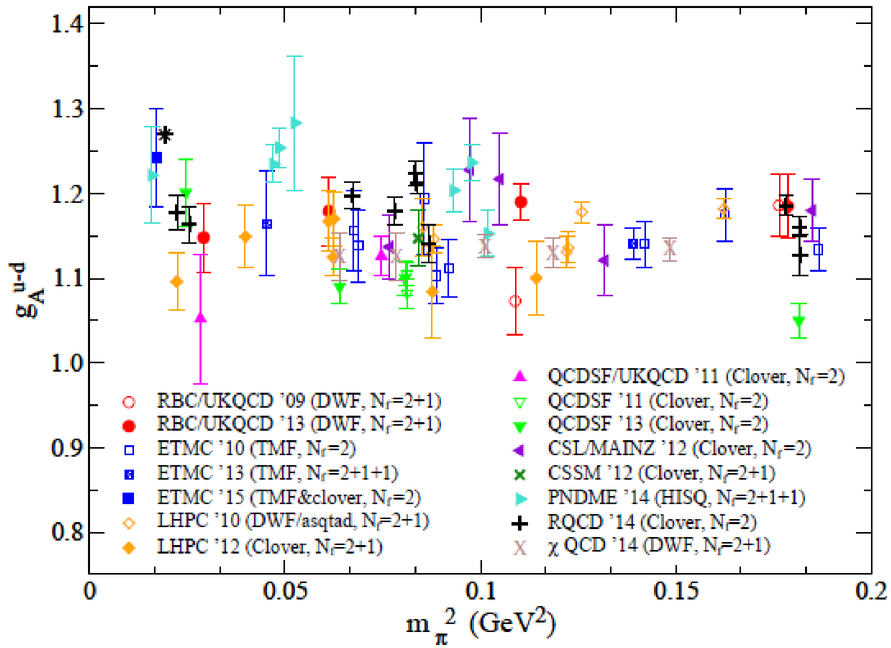
- z-expansion   Meyer et al., PRD 93 (2016)



- At $E_\nu \sim 1$ GeV σ(CCQE) has $\approx$ 10 % error
- More precise information about F_A is needed
  - Direct or indirect CCQE measurement on n/p
  - Lattice QCD

F_A & LQCD

- g_A : lower than experimental values have been recurrently obtained



Constantinou, PoS CD15 (2015) 009

- Recent progress:
  - improved algorithms for a careful treatement of excited states
  - low pion masses
  - A per-cent-level determination of the nucleon axial coupling from QCD

    Chang et al., Nature 558 (2018)

F_A from LQCD

- Recent determinatons of F_A (Q^2, M_π):
  - improved algorithms for a careful treatement of excited states
  - low pion masses

Alexandrou et al., Phys. Rev. D 96 (2017)
Capitani et al., arXiv:1705.06186
Gupta, Phys.Rev. D96 (2017)

F_A in BChPT

$$A^a_\alpha = \bar{u}(p')\left[\gamma_\alpha\gamma_5 F_A + \frac{q_\alpha}{m_N}\gamma_5 F_P\right]\frac{\tau^a}{2}u(p)$$

- $F_A(Q^2, M_\pi)$ calculated using covariant ChPT
  Yao, LAR, Vicente Vacas, PRD 96 (2017)
- up to leading one-loop $O(p^3)$
  - standard power counting
- with explicit $\Delta(1232)$
  - $\delta = m_\Delta - m_N \sim O(p)$
- Power-counting breaking (PCB) terms:
  - because of N,Δ with masses that do not vanish in the χ limit
  - EOMS (Extended on mass shell) scheme Gegelia & Scherer
    - PCB terms absorbed by low-energy constants (LEC)
    - Covariance and analytic properties of loops preserved.

F_A in BChPT

$$A_\alpha^a = \bar{u}(p') \left[\gamma_\alpha \gamma_5 F_A + \frac{q_\alpha}{m_N} \gamma_5 F_P \right] \frac{\tau^a}{2} u(p)$$

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$

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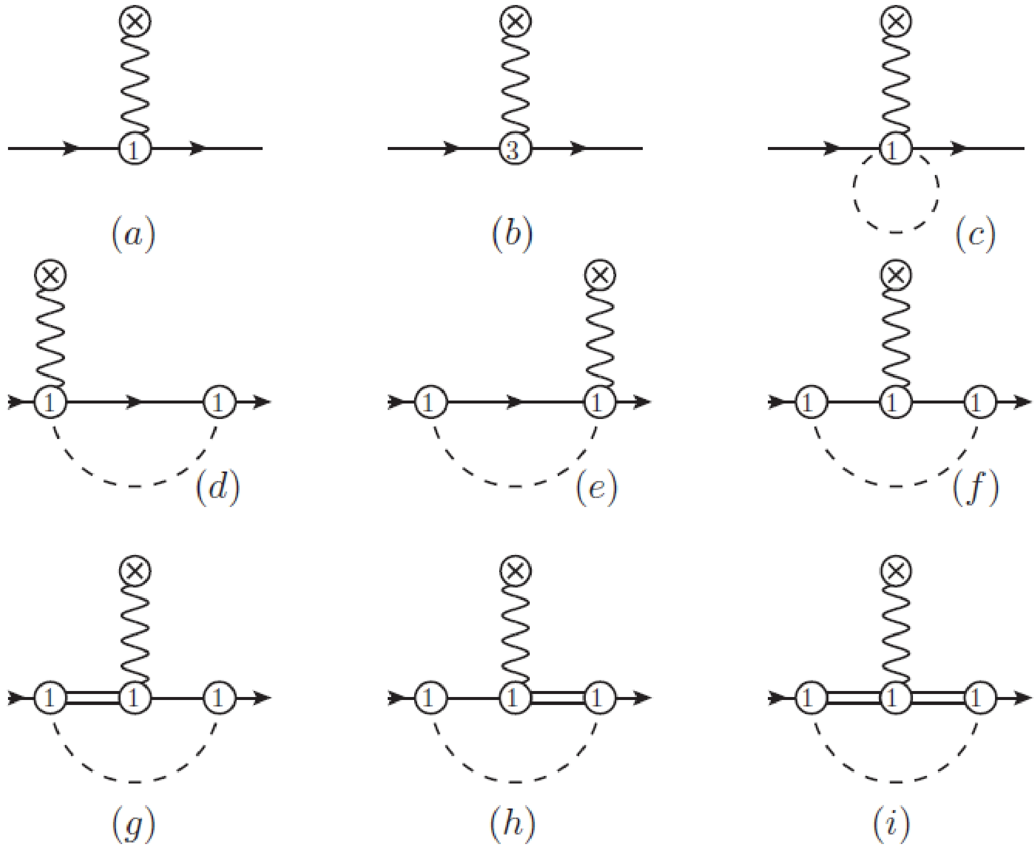

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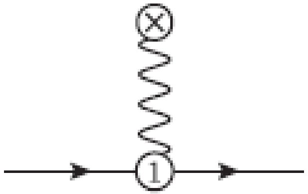
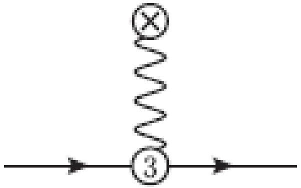


$$\mathcal{L}^{(1)}_{\pi N} \supset \bar{\Psi}\left(\frac{1}{2}g u^\mu\gamma_\mu\gamma^5\right)\Psi \ ,$$

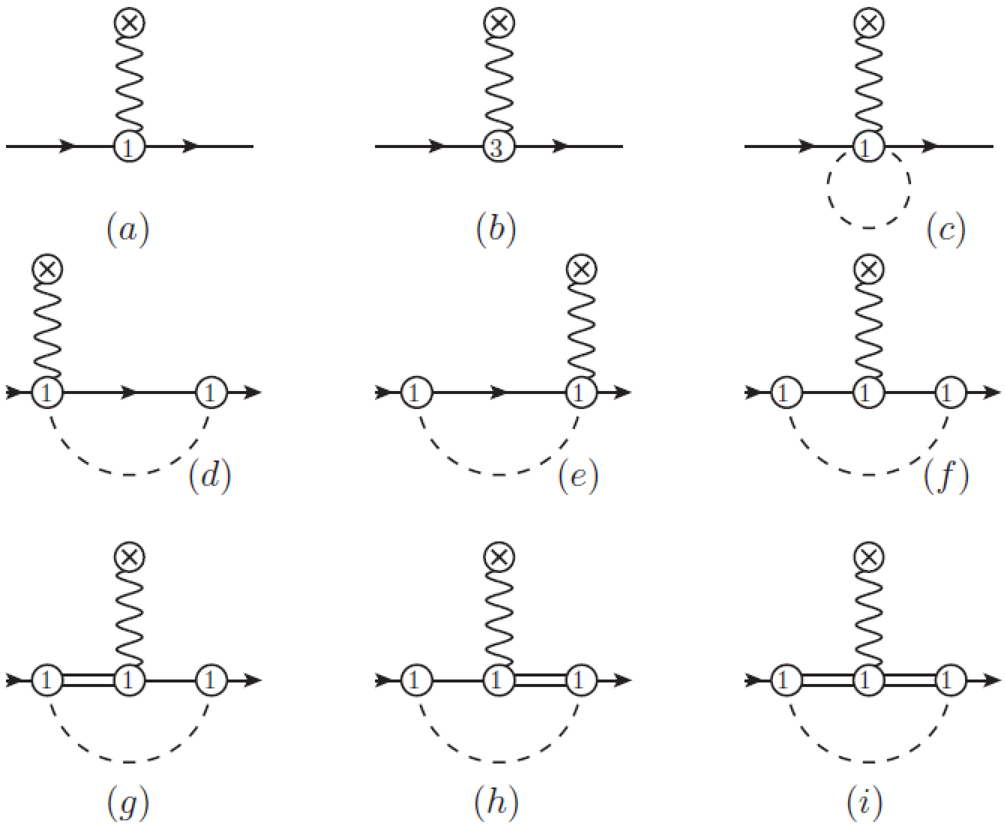


$$\mathcal{L}^{(3)}_{\pi N} \supset \bar{\Psi}\{\frac{d_{16}}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu + \frac{d_{22}}{2}\gamma^\mu\gamma_5[D_\nu, F^-_{\mu\nu}]\}\Psi \ ,$$

F_A in BChPT

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$

  Yao, LAR, Vicente Vacas, PRD 96 (2017)



$$F_A(Q^2, M_\pi^2) = g + 4d_{16}M_\pi^2 + d_{22}Q^2 + F_A^{(c)} + F_A^{(f)} + 2\,F_A^{(g)} + F_A^{(i)} + F_A^{(wf)}$$

F_A in BChPT

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- g, d_{16}, d_{22} are determined from a fit to LQCD data in both Q^2 and M_π

  Alexandrou et al., Phys. Rev. D 96 (2017)
  Capitani et al., arXiv:1705.06186
  Gupta, Phys.Rev. D96 (2017)

- Fit range:
  - M_π < ???
  - Q^2 < ???
- Explicit or implict $\Delta(1232)$?

F_A in BChPT

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  Alexandrou et al., Phys. Rev. D 96 (2017)
  Capitani et al., arXiv:1705.06186
  Gupta, Phys.Rev. D96 (2017)

- Fit range:
  - $M_\pi < 400$ MeV
    - reasonable
    - LQCD ensembles with $M_\pi > 400$ MeV increase χ^2
  - $Q^2 < ???$

- Explicit or implict $\Delta(1232)$?

F_A in BChPT

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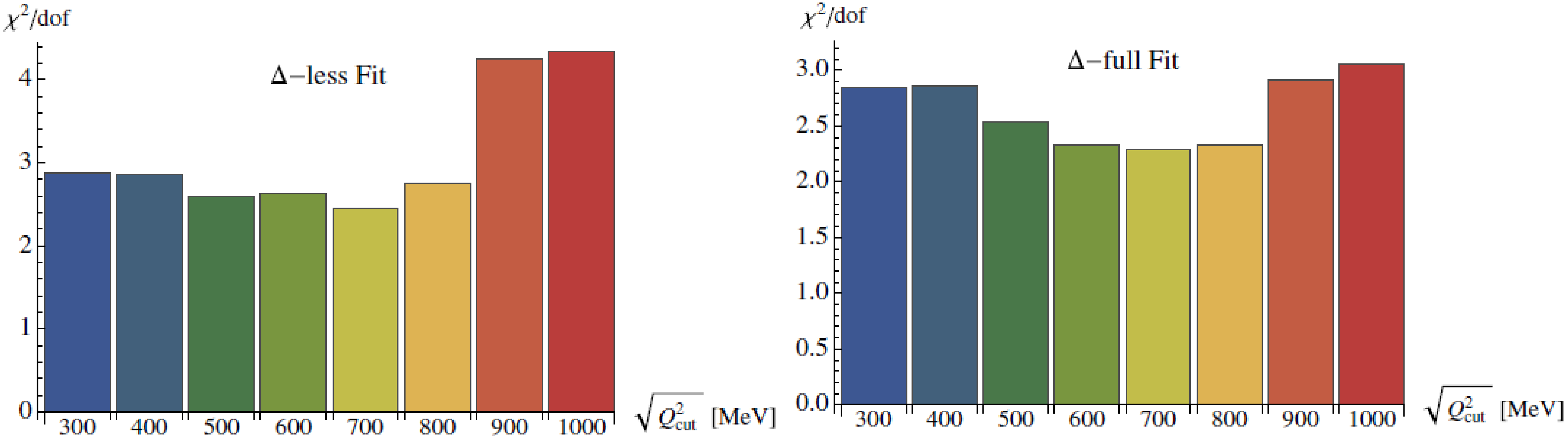


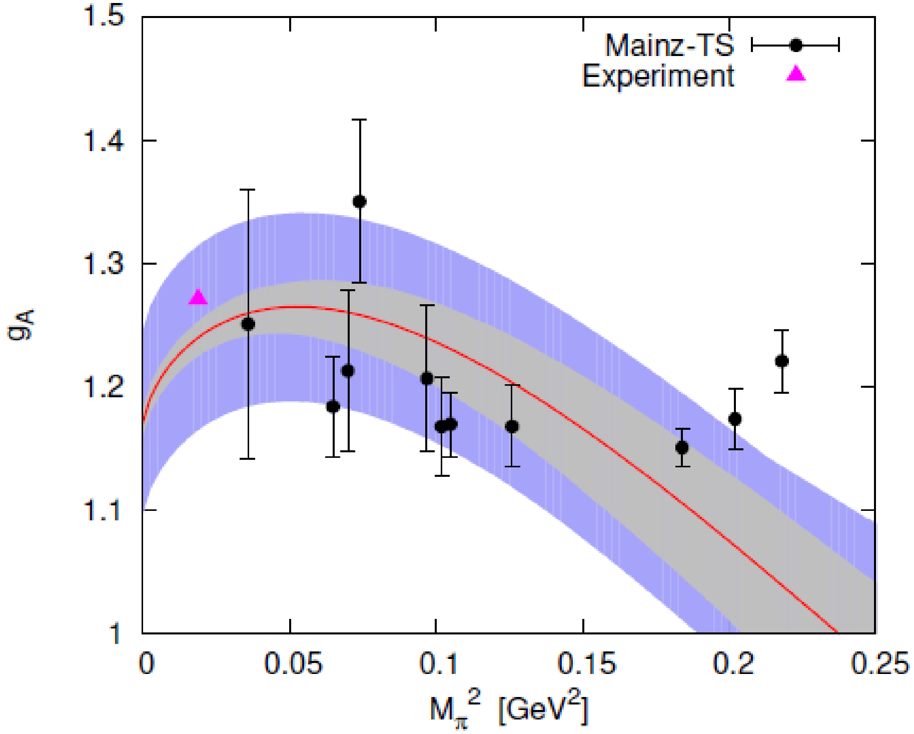
  - $Q^2 < 0.25$ (0.36) GeV^2

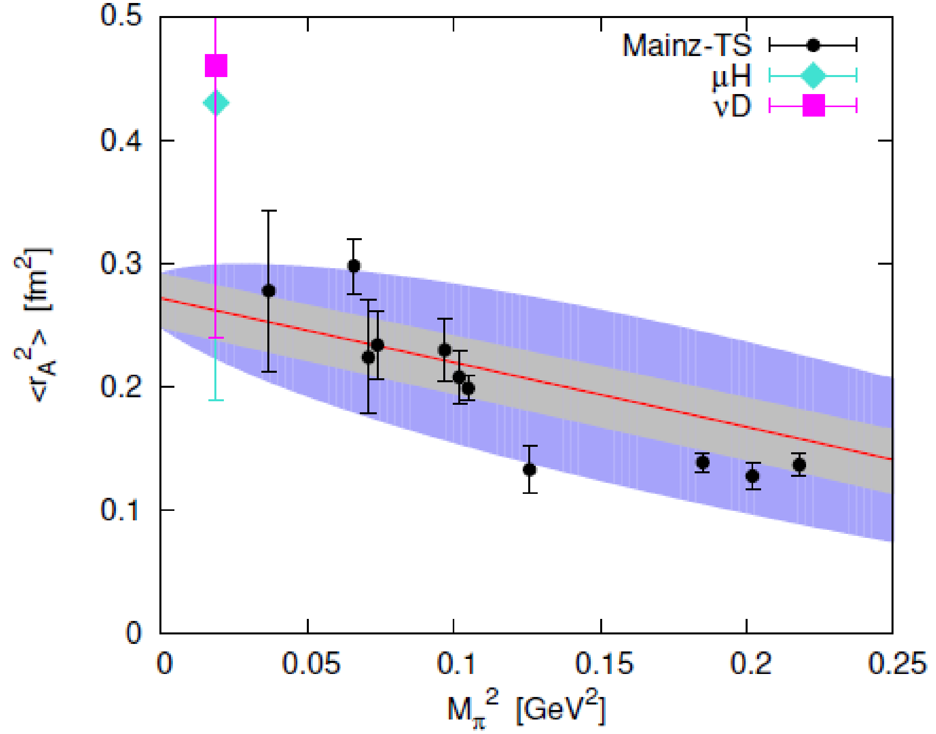
- Explicit $\Delta(1232)$
  - better χ^2/dof

F_A & LQCD

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$

Yao, LAR, Vicente Vacas, PRD 96 (2017)



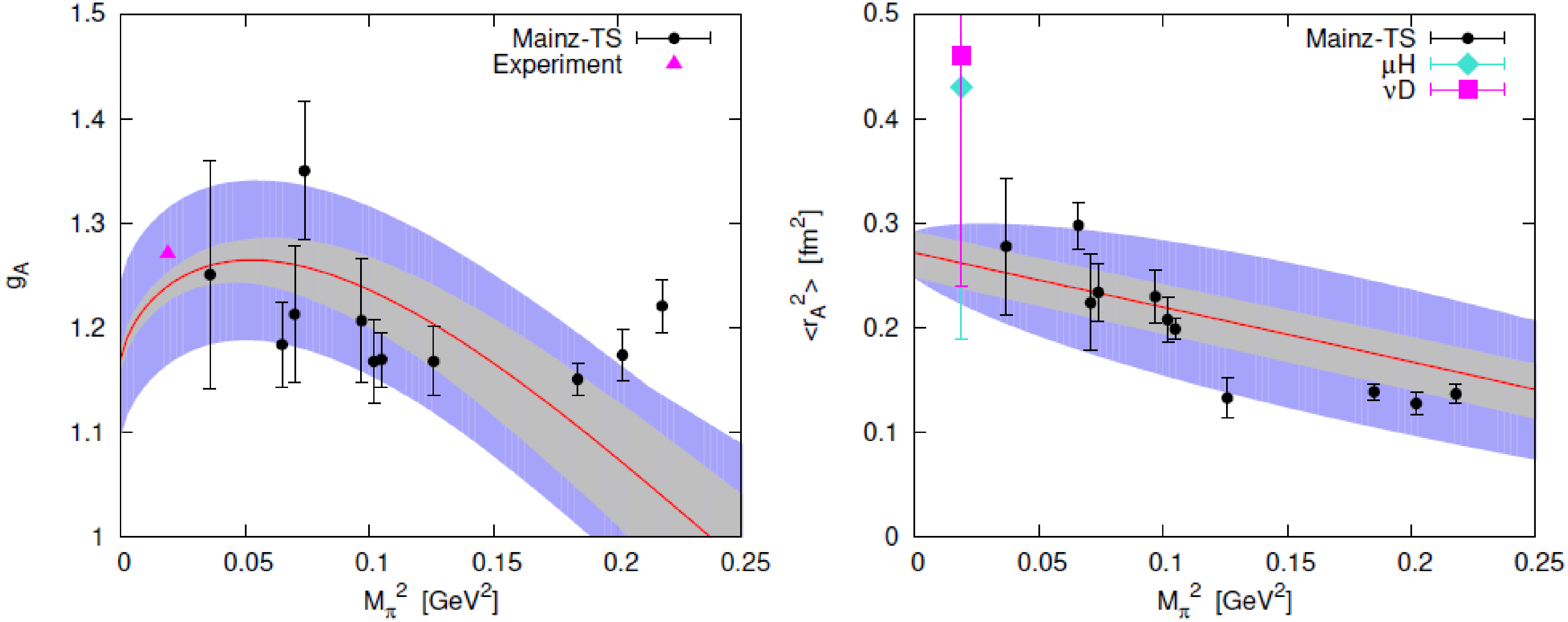


- At the physical point: g_A = 1.237(74) , $<r_A^2>$ = 0.263(38) fm^2
- Mainz-TS points: z-expansion results from Capitani et al., arXiv:1705.06186, **not used in the fit**.

F_A & LQCD

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$

  Yao, LAR, Vicente Vacas, PRD 96 (2017)

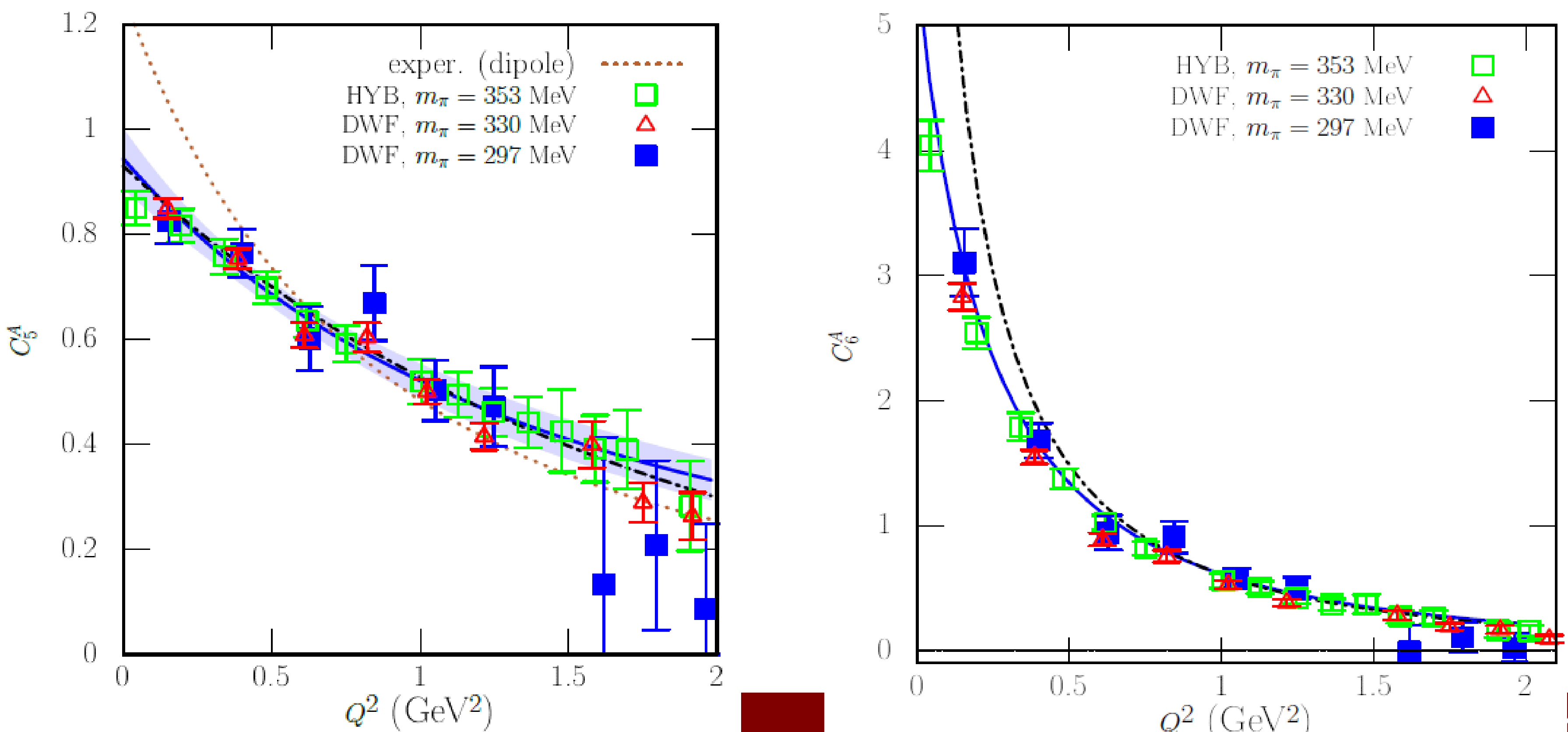


- At the physical point: g_A = 1.237(74) , $<r_A^2>$ = 0.263(38) fm^2

- $O(p^5)$ might be needed to improve the M_π dependence of $<r_A^2>$

Axial N-Resonance transitions

- Goldberger-Treiman relations (PCAC) can be derived for leading couplings
- No information about Q^2 dependence
- Calculations assume dipole shapes with M_A = 1 GeV
- Few LQCD results
  - N-Δ axial form factors: Alexandrou et al., PRD83 (2011)

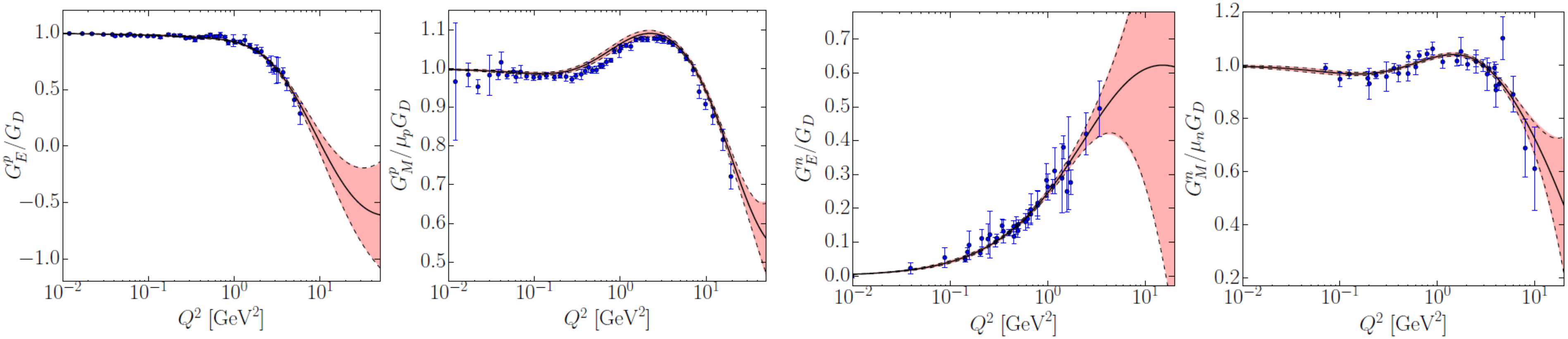


Conclusions

- Good understanding and realistic modeling of neutrino interactions with matter are crucial for current and future oscillation experiments
- Neutrino-nucleon cross sections are insuficiently constrained: direct or indirect measurements are required
- Complementary information from LQCD on nucleon axial form factors is also valuable

Dipole nucleon form factors?

- EM form factors from (e,e') scattering

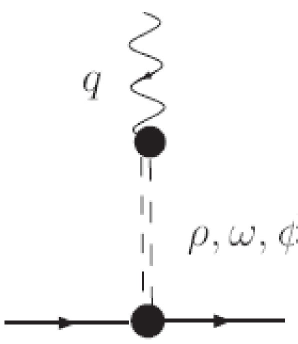


Ye et al., arXiv:1707.09063

- Dipole behavior for $Q^2 \lesssim 1$ GeV^2

$$G_D = \left(1 + \frac{Q^2}{0.71\,\mathrm{GeV}^2}\right)^{-2}$$

  - Exponential charge distributions (in the static limit)
  - In the VMD picture, a dipole might arise from two mesons with similar masses and opposite couplings



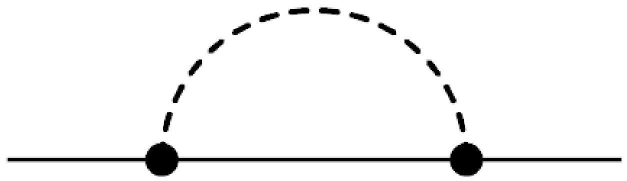
F_A in BChPT

- Example: nucleon mass in SU(2)

O(p) $\mathcal{L}_1 = -\bar{\psi} M_0 \psi + \ldots \qquad M = M_0$

O(p²) $\mathcal{L}_2 = 4 c_1 m_\pi^2 \bar{\psi}\psi + \ldots \qquad M = M_0 - 4 c_1 m_\pi^2$

O(p³) Loops $M = M_0 - 4 c_1 m_\pi^2 + \boxed{\frac{1}{16\pi^2}\left(\frac{g_A}{f_\pi}\right)^2 m_\pi^2 M_0} + \ldots$



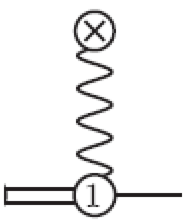
O(p²) !

EOMS: $c_1 \rightarrow c_1 + \frac{1}{64\pi^2}\left(\frac{g_A}{f_\pi}\right)^2 M_0$

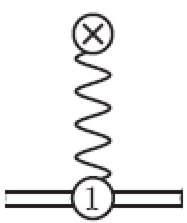
F_A in BChPT

$$A^a_\alpha = \bar{u}(p')\left[\gamma_\alpha\gamma_5 F_A + \frac{q_\alpha}{m_N}\gamma_5 F_P\right]\frac{\tau^a}{2}u(p)$$

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$
  Yao, LAR, Vicente Vacas, PRD 96 (2017)



$$\mathcal{L}^{(1)}_{\pi N\Delta} = h_A\,\bar{\Psi}^i_\mu \xi^{\frac{3}{2}}_{ij}\omega^{\mu,j}\Psi + h.c.$$



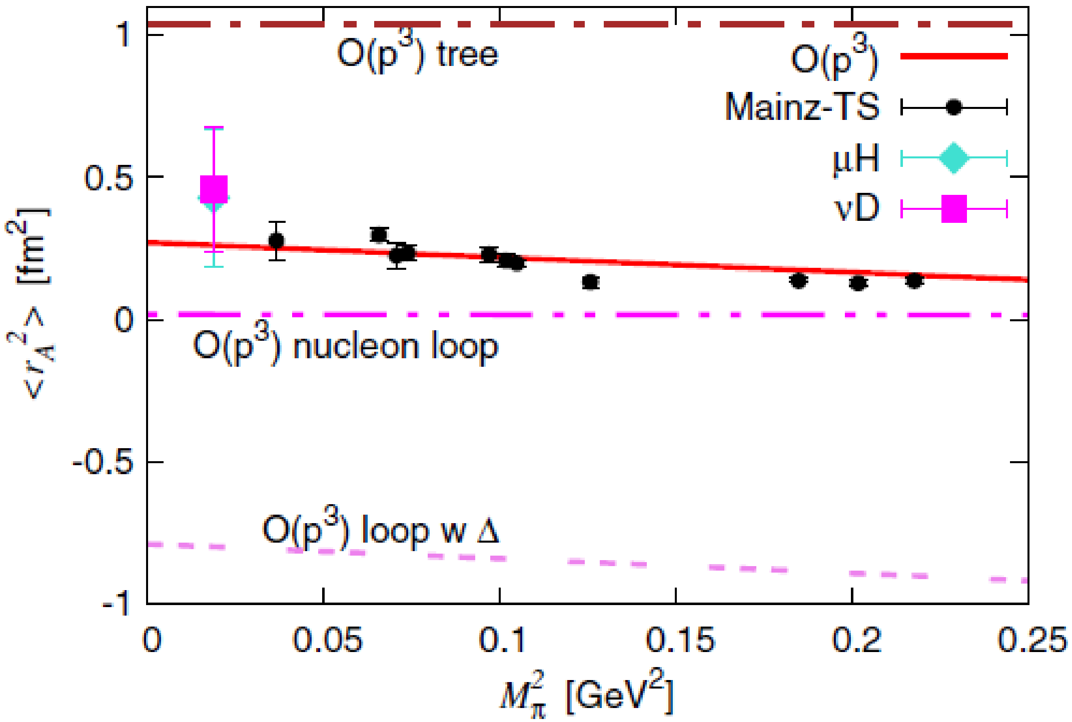
$$\mathcal{L}^{(1)}_{\pi\Delta} \supset -\bar{\Psi}^i_\mu \xi^{\frac{3}{2}}_{ij}\left\{\frac{g_1}{2}\not{u}^{jk}\gamma_5 g^{\mu\nu}\right\}\xi^{\frac{3}{2}}_{kl}\Psi^l_\nu$$

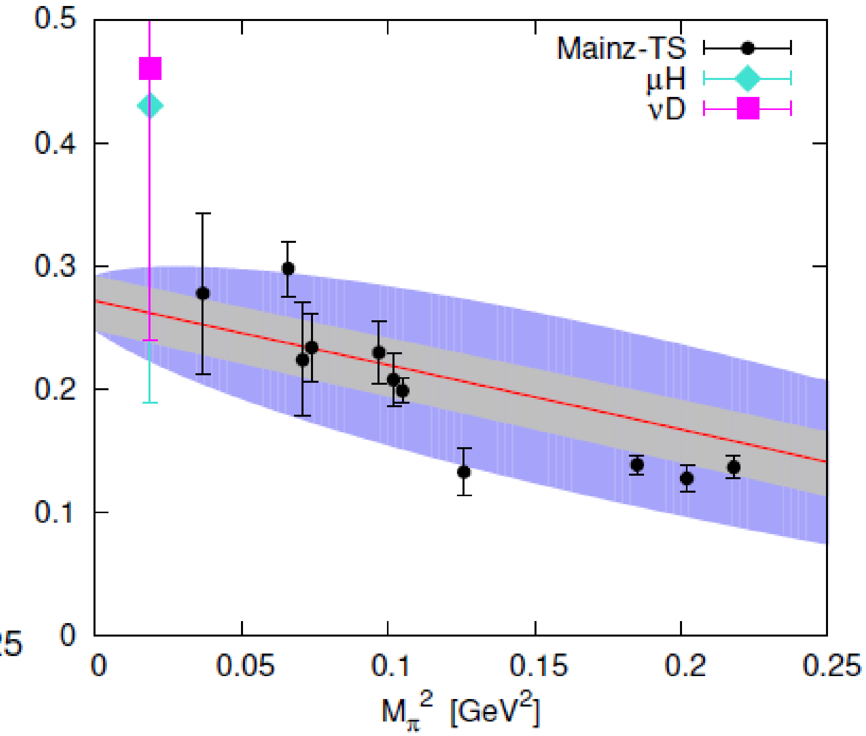
- h_A , g_1 fixed in πN scattering Yao et al., JHEP 05 (2016)

F_A & LQCD

- $F_A(Q^2, M_\pi)$ calculated in covariant ChPT up to $O(p^3)$ with explicit $\Delta(1232)$

  Yao, LAR, Vicente Vacas, PRD 96 (2017)





- At the physical point: g_A = 1.237(74) , $\langle r_A^2 \rangle$ = 0.263(38) fm^2
- Loops with $\Delta(1232)$ significantly improve $\langle r_A^2 \rangle$
- $O(p^5)$ might be needed to improve the M_π dependence of $\langle r_A^2 \rangle$