Dimensional regularization and γ_5

Dominik Stöckinger

TU Dresden

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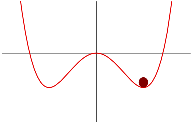
Why new physics?

Big questions. . . point to (TeV scale) new physics

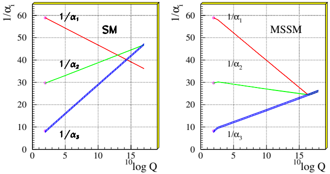


EWSB, Higgs, scalar particle?

hierarchy $M_{\rm Pl}/M_W$? Naturalness?

Dark Matter?

Baryon Asymmetry?



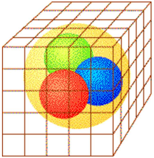
Grand Unification?
Flavor Structure?

Neutrinos?

Supersymmetry? Extended Higgs sector? Extended Flavour sector?
Need complementary experiments to discover and scrutinize new physics

Motivation

Regularization necessary to define QFT at the quantum level



cutoff-scale Λ

$\int_{|p|<\Lambda} d^4p$

DREG

$\mu^{4-D} \int d^Dp$

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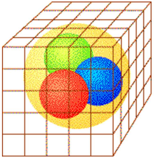

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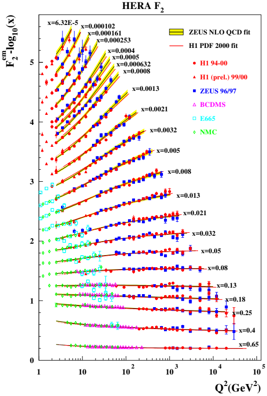

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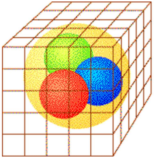

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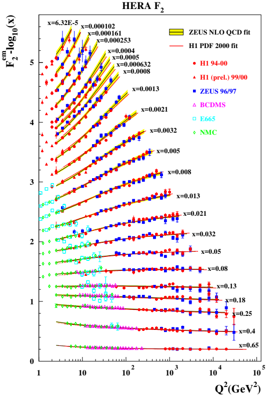
- e.g. anomaly/breaking of scale invariance



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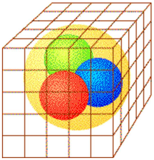

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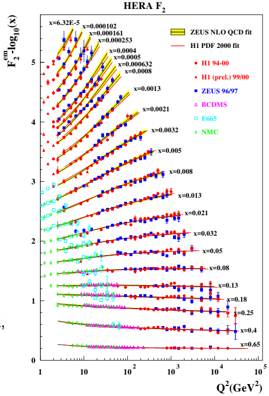
- regularization must be consistent with unitarity/causality (proven for DREG: [Speer '74][Breitenlohner, Maison '77], equivalence of DRED, no inconsistency: [Jack, Jones, Roberts '94][DS '05])



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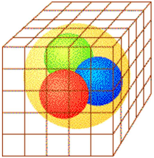

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Entertaining history: puzzles, problems

- DREG breaks SUSY
- "DRED is mathematically inconsistent [Siegel '80]"
- "DRED has IR factorization problem [van Neerven, Smith, et al '88 and '05][Zerwas et al]"
- "No DRED IR factorization problem found [Kunszt, Signer, Trocsanyi '94; Catani et al '97]"
- "DRED violates unitarity ['t Hooft, van Damme '84]"
- "Some published results therefore wrong [Harlander, Kant, Mihaila, Steinhauser '06; Kilgore '11]"

To d, or not to d:

Recent developments and comparisons of regularization schemes

C. Gnendiger[a,1], A. Signer[a,b], D. Stöckinger[c],
A. Broggio[d], A. L. Cherchiglia[e], F. Driencourt-Mangin[f],

computations. Are there more efficient dimensional schemes? Or is it ultimately advantageous to work completely in four dimension?

That is the question.

First reminder of necessary vs desirable properties, proven vs unproven statements

- Issue 1: Unitarity, causality etc
- Issue 2: Mathematical consistency
- Issue 3: (regularized) Quantum action principle
- Issue 4: Symmetries

$\Rightarrow$ discuss γ_5

Issue 1: Unitarity, Causality, Equivalence

- suppose, theory has been defined up to n-loop level
- then, at $(n+1)$-loop level: unitarity determines imaginary terms, causality determines nonlocal terms uniquely ("causal perturbation theory" [Bogoliubov et al, Epstein, Glaser])

Basic requirement:

any correct regularization must satisfy at the $(n+1)$-loop level:

- it may differ from BPHZ only by real, local terms
- any two correct regularizations may differ only by real, local terms

Dimensional regularization [Speer '74,Breitenlohner, Maison '77]

mathematical equivalence to analytic reg., then to BPHZ,

Dimensional reduction [Jack, Jones, Roberts '93]

equivalence to DREG making use of indep. ϵ-scalar renormalization],

other schemes?

Issue 2: Mathematical consistency

Basic requirement:

Mathematical consistency is required for any scheme

- Math. inconsistency means: possible to derive e.g. $0 = 1$
- In other words: one initial expression leads to different results, depending on the order of calculational steps

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Counter examples:

- Siegel's inconsistency of DRED (next slide)
- inconsistent γ_5, e.g. in

$$\mathrm{Tr}(\gamma^\mu \gamma_5 \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_\mu) \propto \begin{cases} \text{either} & D\epsilon^{\alpha\beta\gamma\delta} \\ \text{or} & (8 - D)\epsilon^{\alpha\beta\gamma\delta} \end{cases}$$

How does Q4S avoid Siegel's inconsistency?

Siegel: "With

$$\epsilon^{(4)}_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{(4)}_{\nu_1\nu_2\nu_3\nu_4} \propto \det((g^{(4)}_{\mu_i\nu_j}))$$

calculate

$$\epsilon^{(D)\mu\nu\rho\sigma}\ \epsilon^{(\epsilon)}{}_{\alpha\beta\gamma\delta}\ \epsilon^{(D)}{}_{\mu\nu\rho\sigma}\ \epsilon^{(\epsilon)\alpha\beta\gamma\delta}$$

in two different ways

$$\Rightarrow 0 = D(D-1)^2(D-2)^2(D-3)^2(D-4)$$

different calculational steps lead to different results,

mathematical inconsistency!!!" [Siegel'80]

Solution: Don't allow explicit index counting (step one) any more, because $g^{(4)}{}_{\mu\nu}$ $\in$quasi-4-dim space!

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Consistent versions of
DREG [Wilson '73, Collins-book]
DRED [DS '05]:
are ok, since all D-dim. quantities are mathematically defined.

Issue 3: Regularized quantum action principle

Basic statement:

Naive result from symmetry variation in path integral:

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\textstyle\int\delta\mathcal{L})\rangle$$

- property of regularization scheme, does not necessarily hold (no fundamental QFT requirement)
- if it holds, it is very useful!

$\rightarrow$ e.g. in QCD: can directly prove that DReg is gauge invariant, i.e. all STIs are automatically fulfilled at all orders!

(Important for finiteness, unitarity of S-matrix on physical states with positive norm)

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- if desired, must be proven for each regularization
- valid in BPHZ: [Lowenstein et al '71], and consistent versions of DREG: [Breitenlohner, Maison '77], DRED: [DS '05]

Symmetry transformations of Green functions — formally

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant) $$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

(1st order in δ) $$= \int \mathcal{D}\phi \; (1 + i\textstyle\int \delta\mathcal{L} + J\delta\phi)e^{i\int \mathcal{L}+J\phi}$$

result: $$0 = \int \mathcal{D}\phi \; (i\textstyle\int \delta\mathcal{L} + J\delta\phi)e^{i\int \mathcal{L}+J\phi}$$

formal “derivation” shows

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\textstyle\int\delta\mathcal{L})\rangle$$

Issue 4: symmetries: $S(\Gamma^{\rm reg} + \Gamma^{\rm ct}) = 0$

Case 1:	$S(\Gamma^{\rm reg}) = 0$	
Case 2a:	$S(\Gamma^{\rm reg}) = \Delta,$	$S(\Gamma^{\rm reg} + \Gamma^{\rm ct}) = 0$
Case 2b:	$S(\Gamma^{\rm reg}) = \Delta,$	$S(\Gamma^{\rm reg} + \Gamma^{\rm ct}) \neq 0$

In practice, life is easier with a symmetry-preserving regularization!

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Basic question:

for a given symmetry (gauge inv., SUSY,...):

$$S(\Gamma^{\text{reg}}) = 0$$

already on regularized level?

Example: DREG is "gauge invariant" (in QCD)

QCD and gauge invariance

$$\Delta \equiv \delta_{\rm BRS} \mathcal{L}_{\rm QCD}^{\rm DREG} = 0 \text{ vanishes!!}$$

Quantum action principle: $$i\,\delta_{\rm SUSY}\langle T\phi_1 \ldots \phi_n\rangle = \langle T\phi_1 \ldots \phi_n \Delta\rangle = 0$$

$\Rightarrow$ We know for decades that DREG preserves QCD gauge invariance at all orders!

[Breitenlohner, Maison '77]

- basic necessary properties: unitarity, consistency
- useful properties: manifest symmetries, quantum action principle
- how to know symmetries? q.a.p.!

γ_5 and DReg

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \tag{1}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \tag{2}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1)\,. \tag{3}$$

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- Best: HVBM (unitary, consistent, but most complicated and breaks gauge inv.)
- Many alternatives, see below

Alternative γ_5 schemes in DReg

$$\{\gamma_5, \gamma^\mu\} = 0, \tag{4}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \tag{5}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1)\,. \tag{6}$$

- Fully Anticommuting (and cyclic): should set $\mathrm{Tr}(\ldots) = 0$
  $\rightsquigarrow$ not unitary (imaginary parts=0!)

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- Improved "naive": reading point, give up cyclicity Kreimer, Körner,Schilcher
  $\leadsto$ quantum action principle and symmetries unclear, proofs not checked

Alternative: non-dimensional schemes

Implicit regularization (Cherchiglia, Sampaio, Nemes . . .), FDR (Pittau) face essentially the same problems in a different guise (e.g. cyclicity lost because of UV subtraction [Bruque,Cherchiglia,Perez-Victoria '18])

Big difference between QCD/QED and electroweak!

- QCD/QED:
  - vector current $\bar{\psi}\gamma^\mu\psi$ gauged, must be conserved
  - axial current can be anomalous. Recipes like naive, reading point etc easily keep gauge invariance manifest
- Electroweak:
  - L-current $\bar{\psi}\gamma^\mu\frac{1-\gamma_5}{2}\psi$ gauged, must be conserved
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$\Rightarrow$ Big open questions:

- Can such simplified schemes be applied at 3-loop?
- Consistent, consistent with unitarity?
- Can the quantum action principle be established?
- Do STIs hold manifestly?

HVBM scheme — the ultimate choice

consistent, unitary, q.a.p. holds; breaks symmetries, complicated

- "*D*-dim space": $\mu = 0, 1, 2, 3, 4, 5, \ldots$, but $g^{\mu}{}_{\mu} = D$

$$
\begin{aligned}
X^\mu &= \bar{X}^\mu + \hat{X}^\mu \\
\gamma_5 &= i\gamma_0\gamma_1\gamma_2\gamma_3 \\
\{\gamma_5, \bar{\gamma}^\mu\} &= 0 \\
[\gamma_5, \hat{\gamma}^\mu] &= 0
\end{aligned}
$$

Larin-scheme

Larin=HVBM applied to certain currents within QCD:

- in axial current: $\gamma^\mu\gamma_5 \to \epsilon^{\mu\nu\rho\sigma}\bar{\gamma}_\nu\bar{\gamma}_\rho\bar{\gamma}_\sigma$
  (actually this is the same as $\bar{\gamma}^\mu\gamma_5$ but can drop the “bar”)
- add certain non-$\overline{\text{MS}}$ counterterm

The current can of course be written in this way. But the counterterm is not general.

Finally: γ_5 and electroweak SM in HVBM

First question: what is $\mathcal{L}$ in D-dim?

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} \gamma^\mu P_L A_\mu \psi + \ldots$$

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- in all cases: gauge invariance broken.
- Choice $\gamma^\mu P_L$ not fully chirally projected
- Choice $P_R \gamma^\mu P_L = \bar{\gamma}^\mu P_L$ breaks even QED gauge invariance

Finally: γ_5 and electroweak SM in HVBM

Second question: How is gauge invariance broken, how is it restored?

- Option 1: calculate Green functions, check STI, adjust CTs until fulfilled
- Option 2: calculate breaking of STI using q.a.p. C.P.Martin '99, page 45

$\Rightarrow$ More complicated numerator algebra, no gauge invariance, more complicated, non-multiplicative counterterms

Outlook

Renormalization and regularization for EWSM: very interesting QFT problem

- super-naive schemes can violate unitarity, are inconsistent
- “Naive” scheme can work: make sure to preserve gauge invariance, unitarity $\leadsto$ how to control at 3-loop?
  - e.g. compare with HVBM Heinemeyer, DS, Weiglein ’04 for g-2
  - it may be that the “problems” of naive scheme cancel simultaneously with the chiral gauge anomaly (proof?)
- HVBM:
  - how to restore gauge invariance at 2-loop, 3-loop?
  - use direct STIs or compute breaking via q.a.p.?
  - algebra and momentum integrals with $\bar{p}^\mu$ manageable?