Feynman integral evaluation at supercomputers

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There's Plenty of Room at the Bottom

Talk by Richard Feynman on December 29, 1959

I don't know how to do this on a small scale in a practical way, but I do know that computing machines are very large; they fill rooms. Why can't we make them very small, make them of little wires, little elements – and by little, I mean little. For instance, the wires should be 10 or 100 atoms in diameter, and the circuits should be a few thousand angstroms across. Everybody who has analyzed the logical theory of computers has come to the conclusion that the possibilities of computers are very interesting – if they could be made to be more complicated by several orders of magnitude...

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...If they had millions of times as many elements, they could make judgments. They would have time to calculate what is the best way to make the calculation that they are about to make. They could select the method of analysis which, from their experience, is better than the one that we would give to them. And in many other ways, they would have new qualitative features.
...But there is plenty of room to make them smaller. There is nothing that I can see in the physical laws that says the computer elements cannot be made enormously smaller than they are now.

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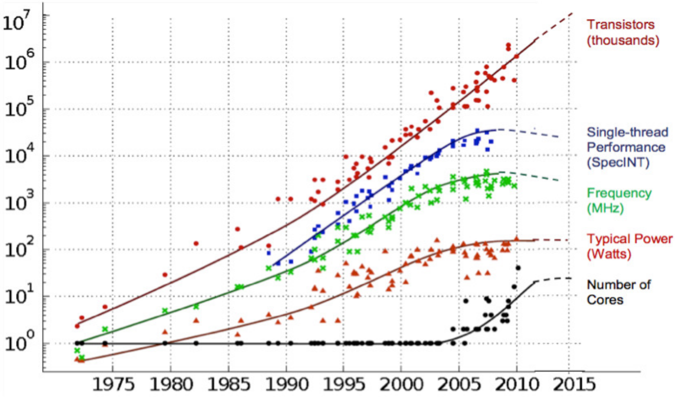
- More or less valid till 2005. And till 2010 counting CPU cores. What now?


10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
1975 1980 1985 1990 1995 2000 2005 2010 2015
Transistors (thousands)
Single-thread Performance (SpecINT)
Frequency (MHz)
Typical Power (Watts)
Number of Cores

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- Four-loop results J. M. Henn, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'16 J. Henn, A. V. Smirnov, V. A. Smirnov, M. Steinhauser and R. N. Lee'17 R. N. Lee, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'17 A. von Manteuffel and R. M. Schabinger'17

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- MANY nodes!!!!
- Time limits for jobs and high chance of failure due to hardware
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- One needs a special code structure and special resource for parallelization.

Evaluation of Feynman integrals

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- evaluation of master integrals.

α-representation

Feynman parametric representation:

$$\mathcal{F}(a_1 \ldots, a_L; d) = \frac{\mathrm{i}^{a+h(1-d/2)} \pi^{hd/2}}{\prod_l \Gamma(a_l)}$$
$$\times \int_0^\infty \ldots \int_0^\infty \prod_l \alpha_l^{a_l-1} U^{-d/2} \mathrm{e}^{\mathrm{i}F/U - \mathrm{i}\sum m_l^2 \alpha_l} \mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_L \ .$$

where U и F are polynomials α that can be algorithmically determined by the initial diagram.

Sector decomposition

$$\int_0^1 \int_0^1 \frac{1}{(x+y)^{2-\varepsilon}} dydx$$

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$$2 \int_0^1 \int_0^1 \frac{x}{(x+xz)^{2-\varepsilon}} dzdx = 2 \int_0^1 \int_0^1 x^{-1+\varepsilon} \frac{1}{(1+z)^{2-\varepsilon}} dzdx$$

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- *Example*:
  $\mathtt{SDEvaluate[UF[\{k\},\{-k^2,-(k+p_1)^2,-(k+p_1+p_2)^2,}$
  $\mathtt{-(k+p_1+p_2+p_4)^2\}, \{p_1^2 \to 0, p_2^2 \to 0, p_4^2 \to 0,}$
  $\mathtt{p_1\ p_2 \to -S/2, p_2\ p_4 \to -T/2, p_1\ p_4 \to (S+T)/2,}$
  $\mathtt{S \to 3, T \to 1\}], \{1,1,1,1\},0]}$

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- *Example*: SDEvaluate[UF[{k},{-k^2,-(k+p_1)^2,-(k+p_1+p_2)^2, -(k+p_1+p_2+p_4)^2}, {p_1^2 $\rightarrow$0,p_2^2 $\rightarrow$0,p_4^2 $\rightarrow$0, p_1 p_2 $\rightarrow$-S/2,p_2 p_4 $\rightarrow$-T/2,p_1 p_4 $\rightarrow$(S+T)/2, S$\rightarrow$3,T$\rightarrow$ 1}], {1,1,1,1},0]
- *Answer*: -4.38658 + 1.3333/ep^2 - 0.732466/ep + 0.001 pm9

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- 2) The number of sectors in the sector-decomposition approach.

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- Analyze the results with Mathematica

Some results obtained on supercomputers evaluating master integrals with FIESTA.

- Corrections to the muon anomalous magnetic moment at four-loop order A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'15
- Quark Mass Relations to Four-Loop Order in Perturbative QCD P. Marquard, A. V. Smirnov, V. A. Smirnov and M. Steinhauser'15 P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser and D. Wellmann'17

Multiple programs for Feynman integral reduction

- AIR
- FIRE
- Reduze
- LiteRed
- Kira
- different private implementations
- more public algorithms going to appear?

Parallel approach to reduction

Reduction is solving a huge sparse matrix with polynomial coefficients
Current diagrams need (A LOT OF RAM) and (A LOT OF TIME)!

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- Parallel reduction in sectors of same level
- Multiple fermat workers (GCD application)
- Prime field approach (Manteuffel, Panzer, Schabinger)
- Separate evaluation of coefficients at different masters (Chawdhry, Lim, Mitov)

Prime field approach

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- Take different large prime numbers, move from Z to Zp
- Run MANY reductions that are much more simple than the original one
- Reconstruct the coefficients

100 values of d, 100 values of x, 20 prime numbers -> 200000 reductions, each of those takes time and use threads -> fits for a super computer.

Rational reconstruction

- An integer is unequely reconstructed by enough of its projections to Zp
- When reconstructing a rational number, we look for smallest possible numerator and denominator
- A few extra prime numbers are for checks

Polynomial reconstruction

- Newton approach

$f(x) = c0 + (x - x_0)(c_1 + (x - x_1)(c_2 + \ldots) \ldots)$
coefficients c_i are algorithmically evaluated from the values $f(x_i)$.

Rational reconstruction

- Thiele approach

$f(x) = c0 + (x - x_0)/(c_1 + (x - x_1)/(c_2 + \ldots) \ldots)$
coefficients c_i are algorithmically evaluated from the values $f(x_i)$.

Rational reconstruction (multiple variables)

Combine two approaches (when coefficients are again functions)

- Newton-Newton (for polynomials)
- Newton-Thiele (when polynomial in one variable)
- Thiele-Newton (something in between)
- Thiele-Thiele (universal but too complex)

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- Then the results are multiplied by the worst denominator and Newton-Thiele is used.
- Can we have such a basis with proper coefficients? We beleive that YES!