Two-loop five-point massless QCD amplitudes within the IBP approach

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BALATON2018 – Feynman Memorial Meeting
17th September 2018

Based on hep-ph/1805.09182 with M. A. Lim and A. Mitov

Introduction

- Precise calculations of scattering amplitudes in gauge theories (such as QCD) require the evaluation of multi-loop diagrams
- Multi-loop integrals are often evaluated using integration-by-parts (IBP) identities
- We introduce a new strategy for solving IBP identities
- As an example, we use it to reduce all planar integrals contributing to the QCD scattering amplitude $q\bar{q} \to q'\bar{q}'g$ at 2 loops

Outline

1. Introduction
   - Why 5-point QCD amplitudes?
   - Why IBPs?
2. Our approach to solving IBPs
   - Integration-by-parts (IBP) identities
   - Our strategy
3. Application to $2 \to 3$ at 2-loops
   - Results
   - Checks
4. Conclusion and future work

Why 5-point QCD amplitudes?

- NNLO phenomenology at the LHC has cleared $2 \to 2$ processes
- However, $2 \to 3$ processes remain an open problem.
  - e.g. 3-jet production; $H + 2$ jets; $\gamma\gamma + 1$ jet
- 2-loop amplitudes are the biggest missing ingredient
- Many developments in recent years:
  - Badger, Frellesvig, Zhang (2013)
  - Ita (2015)
  - Badger, Mogull, Ochirov, O'Connell (2015)
  - Gehrmann, Henn, Presti (2015)
  - Dunbar, Perkins (2016)
  - Dunbar, Godwin, Jehu, Perkins (2017)
  - Badger, Brønnum-Hansen, Hartanto, Peraro (2017)
  - Abreu, Cordero, Ita, Page, Zeng (2017)
  - Böhm, Georgoudis, Larsen, Schönemann, Zhang (2018)
  - Kosower (2018)
  - Badger, Brønnum-Hansen, Gehrmann, Hartanto, Henn, Lo Presti, Peraro (2018)
  - Abreu, Cordero, Ita, Page, Zeng (2018)
  - Gehrmann, Henn, Lo Presti (2018)
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Why IBPs?

- They contain an incredible amount of information about the problem
- They have been widely successful in the computation of multi-loop QCD amplitudes
- Some problems have so far remained beyond the reach of current IBP-solving methods
  - e.g. $2 \to 3$ at 2 loops; $2 \to 2$ at 3 loops; massive $2 \to 2$ at 2 loops
- Highly desirable to improve methods of solving IBPs
- Note: KIRA v1.1 makes steps in a similar direction

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Integration-by-parts (IBP) identities

- Generic bare amplitude is a sum of many Feynman integrals

$$M = \sum_{i=1}^{N} f_i I_i$$

  - I_i are scalar Feynman integrals:
  - N is large: for our $2 \to 3$ problem, $N \sim \mathcal{O}(10^4 - 10^5)$
  - f_i are rational functions
- Integration-by-parts (IBP) identities:

$$\int d^d k_1 \ldots d^d k_L \frac{\partial}{\partial k_j^\mu} \left(\frac{1}{\Pi_1^{n_1} \ldots \Pi_P^{n_P}} \right) = 0$$

Hence, many linear relations between integrals.

Integration-by-parts (IBP) identities (continued)

- Solve this system of equations to express all integrals in terms of a small basis of master integrals.

$$I_i = \sum_{m=1}^{\hat{N}} c_{i,m} \hat{I}_m$$

  - for our $2 \to 3$ problem, $\hat{N} \sim \mathcal{O}(100)$
- Can hence write original amplitude in terms of master integrals

$$M = \sum_{m=1}^{\hat{N}} \hat{c}_m \hat{I}_m, \text{ with } \hat{c}_m = \sum_{i=1}^{N} c_{i,m} f_i$$

- In our work we focus on solving the IBPs. Evaluating the masters is a separate problem (but the IBP solutions help here too).

Outline

Our strategy

- Recall:
  - We want to write each integral in terms of master integrals
    $$I_i = \sum_{m=1}^{\hat{N}} c_{i,m} \hat{I}_m$$
  - We‘ll use the IBP equations. They are linear and homogeneous.

Our strategy:

- Identify the master integrals (straight-forward e.g. using Reduze)
- Generate the IBP equations
- Pick one master integral. Set all other masters to zero.
- Solve the (greatly simplified!) IBP equations
- This will give the projection of each integral onto the chosen master integral.
- Now repeat for each of the other masters; this way, we build up the full solution (i.e. we obtain each $c_{i,m}$).

Note: we use Laporta algorithm but this is not essential.

Benefits of this strategy

- Simplifies the problem
  - Many integrals only have projections onto a subset of all the possible masters. (A given integral only projects onto masters lying in one of its sub-sectors.)
- Parallelisation
  - The IBP equations are "solved" many times – once for each master integral. These runs are independent of one another, so they can run in parallel.
  - Run times for different masters vary by several orders of magnitude. The overall running time is limited by a handful of 'difficult' masters.
- Reduced memory requirements
  - RAM usage is reduced, since far fewer coefficients need to be kept in memory at one time.

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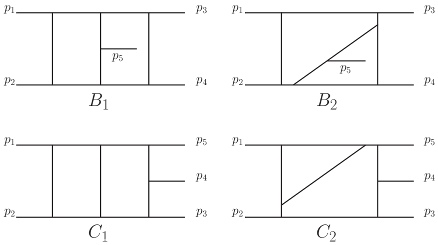
Application to 2 $\rightarrow$ 3: Setup

- We have implemented our strategy in a private C++ code and applied it to the QCD amplitude for $q\bar{q} \rightarrow q'\bar{q}'g$ at 2 loops
- The most complicated 2-loop 5-point topologies have 8 propagators:



Outline

Application to 2 $\to$ 3: Results

- Identified the masters:
  - 113 masters in B_1
  - 75 masters in B_2
  - 62 masters in C_1
  - 28 masters in C_2
- Full reduction for all planar (C_1 and C_2) integrals that contribute to $q\bar{q} \to q'\bar{q}'g$
- Some results for non-planar (B_1 and B_2) topologies:
  - Coefficients of the highest-weight masters, for all integrals with up to 6 numerator powers and up to 1 squared denominator
- Results available to download from: `www.precision.hep.phy.cam.ac.uk/results/amplitudes/`
  - Rational expressions are fully expanded
  - Files compressed (22GB)

Outline

Application to 2 $\to$ 3: Checks

- Computed 2 $\to$ 2 and cross-checked against `Reduze`
- Checked our results in the B_2 topology against those from `hep-th/1805.01873` (Böhm et al.)
- C_1 integrals with 5 numerator powers can be related to integrals with fewer numerator powers using `hep-th/1009.0472` (Gluza et al.) and `hep-th/1804.00131` (Kosower). We carried this out as a check and find full consistency with our solutions.

Outline

Conclusions and future work

- Proposed a new strategy for solving the IBP identities. Particularly useful for multi-scale problems.
- Derived analytic expressions for all integral coefficients needed to construct any planar 2-loop 5-point massless QCD amplitude with quarks and/or gluons. Results are now publicly available.
- Further reading – see our paper:

`hep-ph/1805.09182`

Next steps:

- We have begun working on the non-planar topologies
- Need *fast* numerical evaluation of masters and their coefficients, for use in collider phenomenology
- Ultimately, one would like a closed-form solution to the IBPs. This remains an open problem, although we hope our strategy could be of some help in this direction.

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