Integrand reduction for two-loop five-point amplitudes in QCD

BALATON2018 - Feynman Memorial Meeting

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in collaboration with

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some desired processes at NNLO_{QCD}

$pp \to H + 2$ jets
$pp \to \gamma\gamma +$ jet
$pp \to 3$ jets

Les Houches 2017: Physics at TeV Colliders Standard Model Working Group Report some desired processes at NNLO_{QCD}

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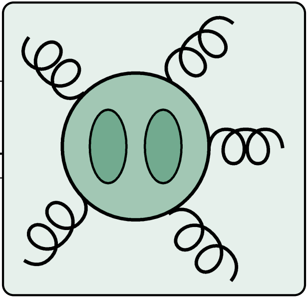
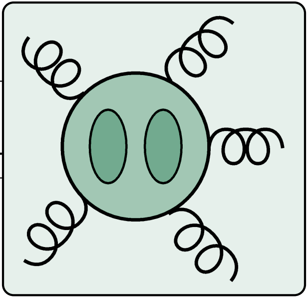
Les Houches 2017: Physics at TeV Colliders Standard M



some desired processes at NNLO$_{QCD}$

$pp \to H + 2$ jets
$pp \to \gamma\gamma$ + jet
$pp \to 3$ jets

Les Houches 2017: Physics at TeV Colliders Standard M



measurement of strong coupling from jet ratio $\frac{pp\to 3j}{pp \to 2j}$

$$\alpha_S(m_Z) = 0.1148 \pm 0.0014(\text{exp.}) \pm 0.0018(\text{PDF}) \pm 0.0050(\text{theory})$$

CMS Collaboration @ 7 TeV: arXiv:1304.7498 all-plus sector including non-planar contribution long known

- *A Two-Loop Five-Gluon Helicity Amplitude in QCD*
  Badger, Frellesvig, and Zhang 2013
- *A Complete Two-Loop, Five-Gluon Helicity Amplitude in Yang-Mills Theory*
  Badger, Mogull, Ochirov, and O'Connell 2015
- Analytic form of the two-loop planar five-gluon all-plus-helicity amplitude in QCD
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several recent results

- *A first look at two-loop five-gluon amplitudes in QCD*
  Badger, CBH, Hartanto, and Peraro 2017
- *Planar two-loop five-gluon amplitudes from numerical unitarity*
  Abreu, Febres-Cordero, Ita, Page, and Zeng 2017
- *Two-loop five-point massless QCD amplitudes within the IBP approach*
  Chawdhry, Lim, and Mitov 2018

leading colour gluon contribution

$$\mathcal{A}^{(2)}(1,2,3,4,5) = \sum_{\sigma \in S_5/Z_5} \mathrm{tr}\left(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(5)}}\right) \times A^{(2)}\left(\sigma(1), \sigma(2), \sigma(3), \sigma(4), \sigma(5)\right) \quad (1)$$

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colour-ordered amplitude

$$A^{(2)}\left(1,2,3,4,5\right) = \int\int \sum_{T} \frac{\Delta_T(\{k\},\{p\})}{\prod_{\alpha \in T} D_\alpha} \tag{2}$$

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sum over planar topologies

irreducible numerator

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sum over planar topologies

integrand reduction

Ossola, Papadopoulos, Pittau, Mastrolia, Badger, Frellesvig, Zhang, Peraro, Mirabella, . . . (2005-)

irreducible numerator

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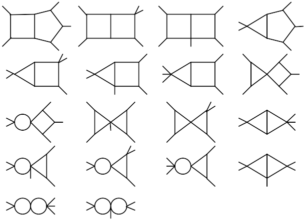
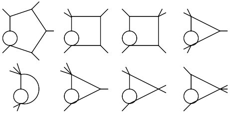
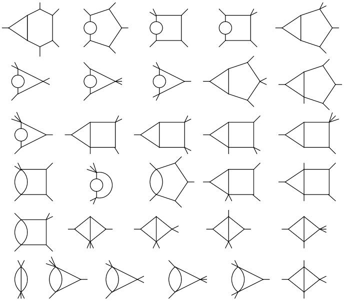
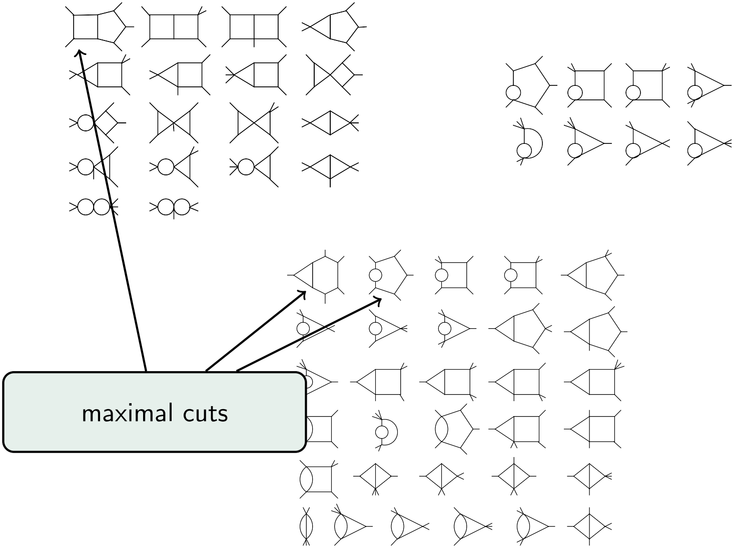
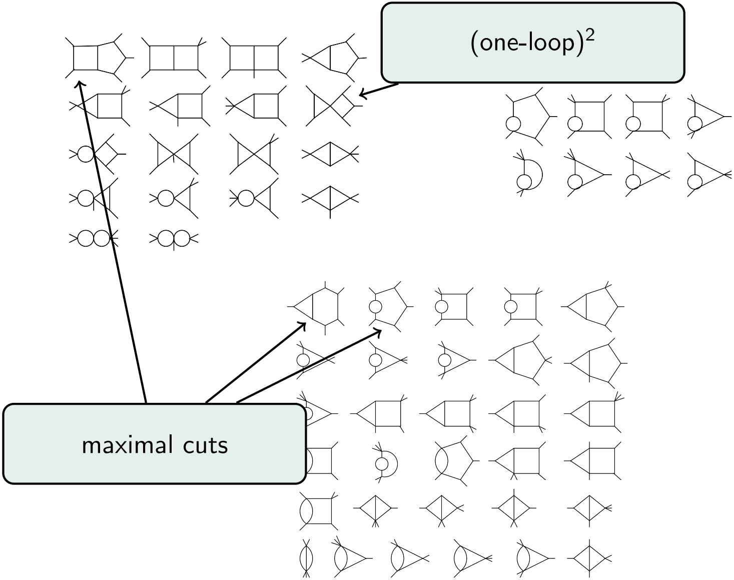







maximal cuts


(one-loop)^2
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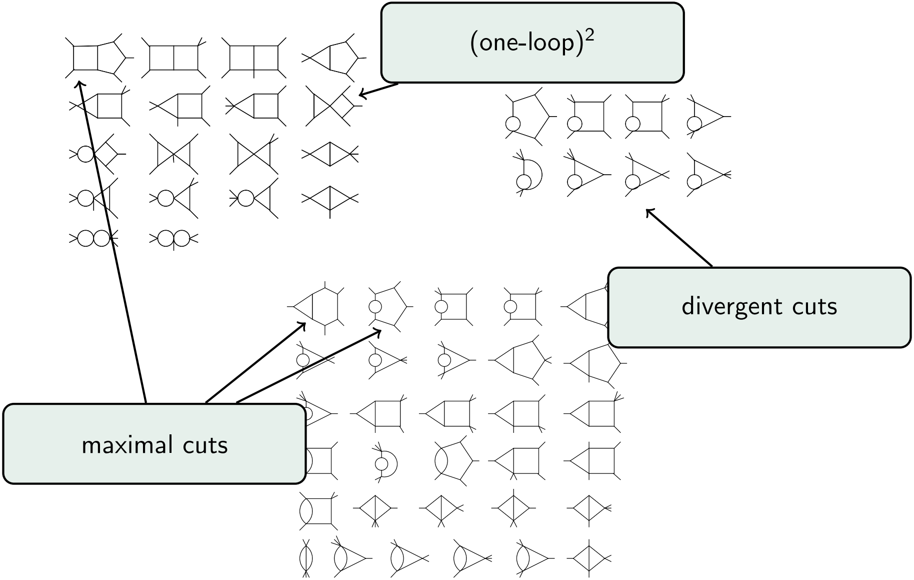
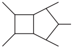
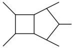
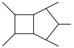
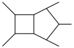

(one-loop)^2
divergent cuts
maximal cuts

determine coefficients from *d*-dimensional unitarity cuts

$\Delta\Big($  $\Big) = \mathrm{Cut}\Big($  $\Big)$ (3)

determine coefficients from d-dimensional unitarity cuts

Δ () = Cut () (3)

generalised unitarity cuts

Bern, Rozowsky, Yan, Dixon, Kosower, de Freitas, Wong, ... 1997-

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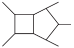
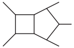
$\Delta\Big($  $\Big) = \mathrm{Cut}\Big($  $\Big)$ (3)

finite field reconstruction

Peraro 2016

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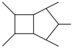
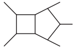
$\Delta \Big($  $\Big) = \mathrm{Cut} \Big($  $\Big)$ (3)

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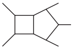
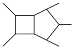
$\Delta\Big($  $\Big) = \mathrm{Cut}\Big($  $\Big)$ (3)

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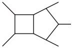
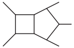
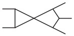
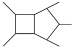
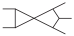
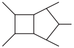
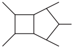
$\Delta\Big($  $\Big) = \mathrm{Cut}\Big($  $\Big)$ (3)

simultaneously determine coefficients for simpler cut solutions

$\Delta\Big($  $\Big) + \dfrac{\Delta\Big(\text{ }$  $\Big)}{D_{\text{off-shell}}} = \mathrm{Cut}\Big($  $\Big)$ (4)

determine coefficients from d-dimensional unitarity cuts

Δ () $=$ Cut () (3)

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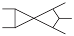
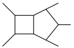
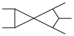
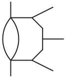
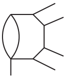
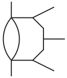
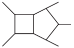
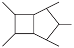
Δ () $+$ Δ () / $D_{\text{off-shell}}$ $=$ Cut () (4)

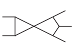
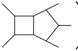
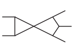
simultaneously determine coefficients to avoid divergent cuts

Δ () $+$ Δ () / $D_{\text{off-shell}}$ $=$ Cut () $-$ subtractions (5)

determine coefficients from d-dimensional unitarity cuts

$\Delta\Big($  $\Big) = \mathrm{Cut}\Big($  $\Big)$ (3)

simultaneously determine coefficients for simpler cut solutions

$\Delta\Big($  $\Big) + \dfrac{\Delta\Big($  $\Big)}{D_{\text{off-shell}}} = \mathrm{Cut}\Big($  $\Big)$ (4)

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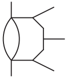
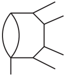
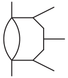
$\Delta\Big($  $\Big) + \dfrac{\Delta\Big($  $\Big)}{D_{\text{off-shell}}} = \mathrm{Cut}\Big($  $\Big) - \text{subtractions}$ (5)

see also Abreu, Febres Cordero, Ita, Jaquier, and Page 2017

split loop momentum in parallel and perpendicular components

$$k_i = k_{\parallel,i} + k_{\perp,i} \tag{6}$$

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relation to extra-dimensional ISPs

$$\begin{aligned} \mu_{ij} &= -k_{\perp,i}^{[-2\epsilon]} \cdot k_{\perp,j}^{[-2\epsilon]} \\ &= k_i \cdot k_j - k_{\parallel,i} \cdot k_{\parallel,j} - k_{\perp,i}^{[4]} \cdot k_{\perp,j}^{[4]} \end{aligned} \tag{7}$$

construction of a simple "no-μ" basis

$$\Delta_T = \sum_i c_i \prod_{m_j \in S} m_j^{\alpha_{ij}} \tag{8}$$

construction of a simple “no-μ” basis

rational coefficient
in external kinematics

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$$S = \{k \cdot p\} \cup \{k \cdot \omega\}$$

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construction of "μ" containing basis

i) take an over-complete set of monomials in $k_i \cdot p_j$, $k_i \cdot \omega_j$, and μ_{ij}

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construction of "μ" containing basis

i) take an over-complete set of monomials in $k_i \cdot p_j$, $k_i \cdot \omega_j$, and μ_{ij}
ii) choose a set of criteria to order the set of monomials

construction of a simple "no-μ" basis

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ii) choose a set of criteria to order the set of monomials
iii) map each monomial containing μ_{ij} from the set of step i) onto a linear combination of monomials of the simple basis
iv) solve for the independent monomials

new algorithm

- no polynomial division

new algorithm

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- integrand basis in preferred variables

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- integrand basis in preferred variables
- contains spurious monomials

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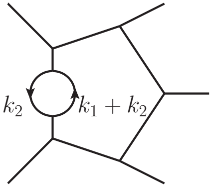

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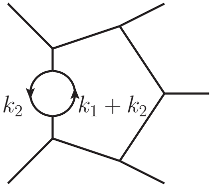

- $k_2 \to -k_1 - k_2$
- $\mu_{12} \to -\mu_{11} - \mu_{12}$

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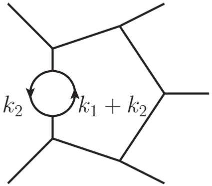

- $k_2 \to -k_1 - k_2$
- $\mu_{12} \to -\mu_{11} - \mu_{12}$
- $\mu_{11} + 2\mu_{12}$ is spurious

$$s_{12} = -1, \quad s_{23} = -\frac{37}{78}, \quad s_{34} = -\frac{2023381}{3194997}, \quad s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$$

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	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	27.7526	-23.7728	-168.1162	-175.2103
$P^{(2),[0]}_{--+++}$	12.5	27.7526	-23.7728	-168.1163	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8084	69.6695
$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

$$\widehat{A}^{(2),[i]}_{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \frac{A^{(2),[i]}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4},5^{\lambda_5})}{A^{\mathrm{LO}}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4},5^{\lambda_5})}$$

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$s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$

$(d_s - 2)^i$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
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universal pole structure

$P^{(2)} = \mathbf{I}^{(1)} A^{(1)} + \mathbf{I}^{(2)} A^{(0)}$

Catani 1996; Becher, Neubert 2009; Gnendiger, Signer, Stöckinger 2014

$$\widehat{A}^{(2),[i]}_{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \frac{A^{(2),[i]}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4},5^{\lambda_5})}{A^{\mathrm{LO}}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4},5^{\lambda_5})}$$

$s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$

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$P^{(2),[0]}_{--+++}$	12.5	27.7526	-23.7728	-168.1163	—
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$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

reduced to master integrals
Gehrmann, Henn, Lo Presti, Papadopoulos, Tommasini, Wever

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$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[1]}_{+++++}$	0	0	-2.5	-6.4324	-5.3107
$P^{(2),[1]}_{+++++}$	0	0	-2.5	-6.4324	—
$\widehat{A}^{(2),[1]}_{-++++}$	0	0	-2.5	-12.7492	-22.0981
$P^{(2),[1]}_{-++++}$	0	0	-2.5	-12.7492	—
$\widehat{A}^{(2),[1]}_{--+++}$	0	-0.625	-1.8175	-0.4869	3.1270
$P^{(2),[1]}_{--+++}$	0	-0.625	-1.8175	-0.4869	—
$\widehat{A}^{(2),[1]}_{-+-++}$	0	-0.625	-2.7759	-5.0018	0.1807
$P^{(2),[1]}_{-+-++}$	0	-0.625	-2.7759	-5.0018	—

	$\widehat{A}^{(2),[2]}_{+++++}$	$\widehat{A}^{(2),[2]}_{-++++}$	$\widehat{A}^{(2),[2]}_{--+++}$	$\widehat{A}^{(2),[2]}_{-+-++}$
ϵ^{0}	3.6255	-0.0664	0.2056	0.0269

$$s_{12} = \frac{113}{7}, \quad s_{23} = -\frac{152679950}{96934257}, \quad s_{34} = \frac{1023105842}{138882415}, \quad s_{45} = \frac{10392723}{3968069}, \quad s_{15} = -\frac{8362}{32585}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24014 - 221.41370 i	388.44694 - 167.45494 i
$P^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24013 - 221.41373 i	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80016 - 216.36601 i	342.75366 - 309.25531 i
$P^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80018 - 216.36604 i	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[1]}_{+++++}$	0	0	-2.5	0.60532 - 12.48936 i	35.03354 + 9.27449 i
$P^{(2),[1]}_{+++++}$	0	0	-2.5	0.60532 - 12.48936 i	—
$\widehat{A}^{(2),[1]}_{-++++}$	0	0	-2.5	-7.59409 - 2.99885 i	-0.44360 - 20.85875 i
$P^{(2),[1]}_{-++++}$	0	0	-2.5	-7.59408 - 2.99885 i	—
$\widehat{A}^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	-0.55509 - 6.22641 i
$P^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	—
$\widehat{A}^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	-0.62978 + 2.07080 i
$P^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	—

	$\widehat{A}^{(2),[2]}_{+++++}$	$\widehat{A}^{(2),[2]}_{-++++}$	$\widehat{A}^{(2),[2]}_{--+++}$	$\widehat{A}^{(2),[2]}_{-+-++}$
ϵ^{0}	0.60217 - 0.01985 i	-0.10910 - 0.01807 i	-0.06306 - 0.01305 i	-0.03481 - 0.00699 i

$$s_{12} = \frac{113}{7}, \quad s_{23} = -\frac{152679950}{96934257}, \quad s_{34} = \frac{1023105842}{138882415}, \quad s_{45} = \frac{10392723}{3968069}, \quad s_{15} = -\frac{8362}{32585}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24014 - 221.41370 i	388.44694 - 167.45494 i
$P^{(2),[0]}_{--+++}$	12.5	-9.17716 + 47.12389 i	-107.40046 - 25.96698 i	17.24013 - 221.41373 i	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80016 - 216.36601 i	342.75366 - 309.25531 i
$P^{(2),[0]}_{-+-++}$	12.5	-9.17716 + 47.12389 i	-111.02853 - 12.85282 i	-39.80018 - 216.36604 i	—

Badger, CBH, Gehrmann, Hartanto, Henn, Lo Presti, Peraro
arXiv:1807.09709

	ϵ^{-4}	[illegible]	[illegible]	[illegible]	ϵ^{0}
$\widehat{A}^{(2),[1]}_{+++++}$	0	[illegible]	[illegible]	[illegible]	35.03354 + 9.27449 i
$P^{(2),[1]}_{+++++}$	0	[illegible]	[illegible]	[illegible]	—
$\widehat{A}^{(2),[1]}_{-++++}$	0	[illegible]	[illegible]	[illegible]	-0.44360 - 20.85875 i
$P^{(2),[1]}_{-++++}$	0	[illegible]	[illegible]	[illegible]	—
$\widehat{A}^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	-0.55509 - 6.22641 i
$P^{(2),[1]}_{--+++}$	0	-0.625	-0.65676 - 0.42849 i	-1.02853 + 0.30760 i	—
$\widehat{A}^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	-0.62978 + 2.07080 i
$P^{(2),[1]}_{-+-++}$	0	-0.625	-0.45984 - 0.97559 i	1.44962 + 0.53917 i	—

	$\widehat{A}^{(2),[2]}_{+++++}$	$\widehat{A}^{(2),[2]}_{-++++}$	$\widehat{A}^{(2),[2]}_{--+++}$	$\widehat{A}^{(2),[2]}_{-+-++}$
ϵ^{0}	0.60217 - 0.01985 i	-0.10910 - 0.01807 i	-0.06306 - 0.01305 i	-0.03481 - 0.00699 i

summary

- *d*-dimensional integrand reduction, generalised unitarity cuts, and IBP reduction
- finite field reconstruction
- pole structure check

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- analytic results
- improved integrand basis
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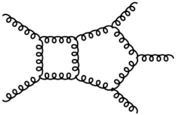
Gehrmann, Henn, Lo Presti
arXiv:1807.09812

thank you



planar gluon scattering

		$2 \to 2$		$2 \to 3$	
		$\mathcal{N}=4$	QCD	$\mathcal{N}=4$	QCD
one loop	integrand basis	1	65	5	175
	master integrals	1	2	1	2
two loops	integrand basis	2	15360	15	55580
	master integrals	1	7	3	61

helicity	flavour	non-zero coefficients	non-spurious coefficients	contributions @ $\mathcal{O}(\epsilon^0)$
	$(d_s-2)^0$	50	50	0
$+++++$	$(d_s-2)^1$	175	165	50
	$(d_s-2)^2$	320	90	60
	$(d_s-2)^0$	1153	761	405
$-++++$	$(d_s-2)^1$	8745	4020	3436
	$(d_s-2)^2$	1037	100	68
	$(d_s-2)^0$	2234	1267	976
$--+++$	$(d_s-2)^1$	11844	5342	4659
	$(d_s-2)^2$	1641	71	48
	$(d_s-2)^0$	3137	1732	1335
$-+-++$	$(d_s-2)^1$	15282	6654	5734
	$(d_s-2)^2$	3639	47	32

TABLE I. The number of non-zero coefficients found at the integrand level both before ('non-zero') and after ('non-spurious') removing monomials which integrate to zero. Last column ('contributions @ $\mathcal{O}(\epsilon^0)$') gives the number of coefficients contributing to the finite part. Each helicity amplitude is split into the components of $d_s - 2$.