1) An experiment is searching for quarks of charge $2/3$, which are expected to produce $4/9$ the ionisation $I_0$ of unit charged particles. In an exposure in which $10^5$ cosmic ray tracks are observed, 1 track has its ionisation measured as $0.44I_0$. The detector is such that ionisation measurements are Gaussian distributed about their true values with standard deviation $\sigma$. Calculate the probability that this could be a statistical fluctuation on the ionisation of a unit charged particle for the following different assumptions:
   a) $\sigma = 0.07I_0$ for all $10^5$ track,
   b) For 99% of the tracks $\sigma = 0.07I_0$, while for the remainder it is $0.14I_0$.

2) An experiment is determining the decay rate $\lambda$ for a new particle $X$, whose probability density for decay at time $t$ is proportional to $\exp(-\lambda t)$. A total of nine decays are observed at decay times 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 picoseconds. Calculate the likelihood function $L(\lambda)$ at suitable values of $\lambda$ (most easily done by a simple computer program), and draw a graph of the results. Find the best estimate of $\lambda$ from the maximum of the likelihood curve, and a “±$\sigma$” range for $\lambda$ by finding the values of $\lambda$ where the logarithm to the base $e$ of the likelihood function decreases by 0.5 units from its maximum value.

3) i) A tracker has detector 6 elements at $x = -11, -10, -9, +9, +10$ and $+11$ cms, which each measure a track’s $y$-coordinate to an accuracy of $±1$ cm. A straight line $y = a + bx$ is fitted (for example by chi-squared) to the data from the 3 elements at positive $x$ (L1); a second (L2) for the data at negative $x$; and a third (L3) to all 6 detector elements. The inverse covariance matrix for $a$ and $b$ has elements $M_{aa} = \Sigma 1/\sigma^2$ $M_{bb} = \Sigma x^2/\sigma^2$ $M_{ab} = \Sigma x/\sigma^2$, where the measurements are $y_i ± \sigma_i$ at $x_i$. Evaluate the copvariance matrix for $a$ and $b$ for each of the 3 fits. How do the uncertainties and correlations compare with what you expect?
   ii) When two measurements for a pair of quantities are combined optimally, the uncertainties on the combined parameters are such that $M_c = M_1 + M_2$, where $M_c$ is the inverse covariance matrix for the combination, and $M_1$ and $M_2$ are those for the separate measurements. Determine the covariance matrix for the combination of the parameters of L1 and L2. Explain why the uncertainties for the combination are considerably smaller than those for L1 and L2 separately.

4) The coverage $C(\mu)$ is a property of a statistical technique for estimating a range for a parameter $\mu$ at a confidence level $\alpha$ (e.g. 68%, 90% or whatever). It is the fraction of times that, in repetitions of the procedure with different data each with its own statistical fluctuations, the estimated range contains the true value $\mu$.
   In a Poisson counting experiment with $n$ observed events, one method of estimating a range for the Poisson parameter $\mu$ uses the estimate $n ± \sqrt{n}$ i.e. from $n - \sqrt{n}$ to $n + \sqrt{n}$. This is
supposed to have 68% coverage. Determine the actual coverage $C(\mu)$ at $\mu = 3.41$ and 3.42 as follows:
Determine for which measured values of $n$ the nominal range from the "$n+\text{sqrt}(n)$" procedure includes the specified true value, and then add up the Poisson probabilities for obtaining these measured values, again assuming the specified value of the Poisson parameter.
Explain why plots of the coverage $C(\mu)$ as a function of the Poisson parameter value $\mu$ have discontinuities.

5) (a) Explain briefly the Bayesian and Frequentists approaches to ‘probability’.
(b) Describe how Bayesians and how Frequentists would obtain 90% upper limits on the Poisson parameter $\mu$ for a counting experiment in which $N$ events are observed. Provide enough detail, so that someone could use it as instructions for writing a computer programme to calculate 95% confidence (or credible) upper limits for a specified $N$.

6) An experiment is searching for the SM Higgs. With no Higgs production, 100 events are expected; if the Higgs is produced, 110 events are expected. The experiment observes 130 events, which is $3\sigma$ above the ‘No Higgs’ prediction, so the $p$-value for the null hypothesis is 0.1%. The Lab Publicity Officer announces that we now are 99.9% certain that the Higgs has been discovered.
Comment.