This slide was left intentionally dark
The Standard Model
Contents

1) Motivation for dark matter

DM production: Weakly-Interacting Massive Particles (WIMPs)

2) DM (WIMP) detection

• Indirect searches
• direct searches
• collider searches

3) DM models
Dark Matter is a necessary (and abundant) ingredient in the Universe

Galaxies

- Rotation curves of spiral galaxies
- Gas temperature in elliptical galaxies

Clusters of galaxies

- Peculiar velocities and gas temperature
- Weak lensing
- Dynamics of cluster collision
- Filaments between galaxy clusters

Cosmological scales

Anisotropies in the Cosmic Microwave Background

\[ \Omega_{\text{CDM}} h^2 = 0.1196 \pm 0.003 \]

Planck 2013

It is one of the clearest hints of Physics Beyond the SM
Rotation curves of spiral galaxies become flat for large distances

From the luminous matter of the disc one would expect a decrease in the velocity that is not observed

\[ \frac{v_{\text{rot}}^2}{r} = \frac{G M(r)}{r^2} \rightarrow v_{\text{rot}} = \sqrt{\frac{G M(r)}{r}} \]

\[ M(r) = \text{cte} \rightarrow v_{\text{rot}} \propto \frac{1}{\sqrt{r}} \]

Galaxies contain vast amounts of non-luminous matter

\[ M \gg M_* \]

Faber, Gallagher '79
Bosma '78, '81
van Albada, Bahcall, Begeman, Sancisi '84

Rubin '75
Rotation curves of spiral galaxies become flat for large distances

From the luminous matter of the disc one would expect a decrease in the velocity that is not observed.

$$\frac{v_{\text{rot}}^2}{r} = \frac{G \, M(r)}{r^2} \rightarrow v_{\text{rot}} = \sqrt{\frac{G \, M(r)}{r}}$$

$$M(r) = cte \rightarrow v_{\text{rot}} \propto \frac{1}{\sqrt{r}}$$

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\[
\frac{v_{\text{rot}}^2}{r} = \frac{G M(r)}{r^2} \rightarrow v_{\text{rot}} = \sqrt{\frac{G M(r)}{r}}
\]

\[
M(r) = cte \rightarrow v_{\text{rot}} \propto \frac{1}{\sqrt{r}}
\]

Isothermal Spherical Cow Halo (a.k.a. Standard Halo Model)

Isotropic density distribution \( \rho(r) \propto r^{-2} \)

it has reached a steady state (Maxwell-Bolzmann distribution of velocities)
Rotation curves have also been measured for a large number of spiral galaxies. The mismatch in the shape cannot be compensated by modifying the contribution from luminous components (disk and bulge).

Faber, Gallagher ‘79
Bosma ‘78, ’81
van Albada, Bahcall, Begeman, Sancisi ‘84

Figure 2 Rotation curves of 25 galaxies of various morphological types from Bosma (1978).
The effect of DM has also been observed in the Milky Way...

- There is DM in the centre of our Galaxy

- Observations also show that there is need for DM in the solar neighbourhood

Bertone, Iocco, Pato 2015

Bovy, Tremaine 2012
There are substantial uncertainties in the description of our DM halo

- **local DM density**
  \[ \rho_{DM}(R_0) = 0.43(0.11)(0.10) \text{ GeV/cm}^3 \]
 \[ \rho_{DM}(R_0) = 0.32 \pm 0.07 \text{ GeV/cm}^3 \]
  \[ \rho_{DM}(R_0) = 1.3 \pm 0.3 \text{ GeV/cm}^3 \]

- **DM density profile**
  (DM density at the galactic centre)

  ![Density profile graph]

- **Velocity distribution of DM particles**
  Central and escape velocities
  Deviations from Maxwellian distribution

Nesti, Salucci 2012
Strigari, Trotta 2009
De Boer, Webber 2011
Sample Dark Matter halo from the Aquarius DM simulation

Andromeda (M31)
Galaxy clusters also contain large amounts of non-luminous matter

Peculiar motions of galaxies in the Coma cluster show that the total mass is much larger than the luminous one

Zwicky 1933, 1937

Weak lensing techniques also allow to “weigh” galaxy clusters by measuring the distortion (shear) of distant galaxies behind the cluster.

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Peculiar motions of galaxies in the Coma cluster show that the total mass is much larger than the luminous one

Zwicky 1933, 1937

Weak lensing techniques also allow to “weigh” galaxy clusters by measuring the distortion (shear) of distant galaxies behind the cluster.
- The mass of a galaxy (nebulae) cluster can be determined by different methods

  (Zwicky 1933)
  (Zwicky “On the masses of nebulae and of clusters of nebulae” 1937)

- Luminosity of galaxies (nebulae)
  This can only be used to determine the mass of the luminous component

- Peculiar motions of component galaxies (virial theorem)

  Isolated self-gravitating system

  \[ 2K + U = 0 \]

  \[ K = \frac{1}{2} M \langle v^2 \rangle \quad U = -\frac{\alpha GM^2}{R} \]
The DM in Galaxy clusters can also be observed through weak gravitational lensing.

Observe collective distortions in the shape of distant galaxies whose light has crossed a heavy object (such as a galaxy cluster).
The DM in Galaxy clusters can also be observed through weak gravitational lensing

Observe collective distortions in the shape of distant galaxies whose light has crossed a heavy object (such as a galaxy cluster)

E.g., reconstruction of the DM distribution using Hubble observations.
The bullet cluster (a.k.a. merging galaxy cluster 1E0657-56)

The observed displacement between the bulk of the baryons and the gravitational potential favours the dark matter hypothesis versus modifications of gravity.
Numerical simulations show the importance of DM for structure formation, showing a filamentary network.
... and in fact dark matter filaments might have been recently observed

Using weak-lensing techniques

Dietrich et al. 2012
Observations of the Cosmic microwave Background can be used to determine the components of our Universe

WMAP and Planck precision data of the CMB anisotropies allow the determination of cosmological parameters

COBE, WMAP, Planck

The dark matter abundance is measured accurately

$$\Omega_\Lambda h^2 = 0.3116 \pm 0.009$$

$$\Omega_c h^2 = 0.1196 \pm 0.003$$

$$\Omega_b h^2 = 0.02207 \pm 0.00033$$

Planck 2013
Challenges for **DARK MATTER** in the 80’s

The main questions concerning dark matter are whether it is really present in the first place and, if so, how much is there, where is it and what does it consist of.

**How much.** In general one wants to know the amount of dark matter relative to luminous matter. For cosmology the main issue is whether there is enough dark matter to close the universe. Is the density parameter $\Omega$ equal to 1?

**Where.** The problem of the distribution of dark matter with respect to luminous matter is fundamental for understanding its origin and composition. Is it associated with individual galaxies or is it spread out in intergalactic and intracluster space? If associated with galaxies how is it distributed with respect to the stars?

**What.** What is the nature of dark matter? Is it baryonic or non-baryonic or is it both?
Current challenges for **DARK MATTER**

- **Experimental detection:**
  Does DM feel other interactions apart from Gravity? Is the Electro-Weak scale related somehow related to DM? How is DM distributed?

- **Determination of the DM particle parameters:**
  Mass, interaction cross section, etc…

- **What is the theory for Physics beyond the SM:**
  DM as a window for new Physics Can we identify the DM candidate?
We don’t know yet what DM is... but we do know many of its properties

- Neutral
- Stable on cosmological scales
- Reproduce the correct relic abundance
- Not excluded by current searches
- No conflicts with BBN or stellar evolution

Many candidates in Particle Physics

- Axions
- Weakly Interacting Massive Particles (WIMPs)
- SuperWIMPs and Decaying DM
- WIMPzillas
- Asymmetric DM
- SIMPs, CHAMPs, SIDMs, ETCs...

... they have very different properties
The Standard Model does not contain any viable candidate for DM.

Neutrinos constitute a tiny part of (Hot) dark matter

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{91.5\text{eV}} \lesssim 0.003$$

Hot dark matter not consistent with observations on structure formation.

Dark Matter is one of the clearest hints of Physics Beyond the SM
Some basics on Dark Matter Production

Dark matter was present in the Early Universe and it is present now, however, there are many different mechanisms to account for its correct abundance

- Thermal production (freeze-out)
- Out of equilibrium production (freeze-in)
- Late decays of unstable exotics
- Asymmetry
Cosmology 101

Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a homogeneous and isotropic universe that is expanding (or contracting)

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) = g_{\mu\nu}dx^\mu dx^\nu \]

\( k = \) curvature

\[ g_{00} = 1 \]
\[ g_{11} = \frac{-a(t)^2}{1 - kr^2} \]
\[ g_{22} = -r^2 a(t)^2 \]
\[ g_{33} = -r^2 \sin^2 \theta a(t)^2 \]

\( a(t) \) is the scale parameter
WIMP dilution

\[ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]

k=0 for a flat Universe

\[ H = \frac{\dot{a}(t)}{a(t)} = 1.66 \, g_*^{1/2} \frac{T^2}{M_P} \]
<table>
<thead>
<tr>
<th>Event</th>
<th>time $t$</th>
<th>redshift $z$</th>
<th>temperature $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>$10^{-34}$ s (?)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Baryogenesis</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>EW phase transition</td>
<td>20 ps</td>
<td>$10^{15}$</td>
<td>100 GeV</td>
</tr>
<tr>
<td>QCD phase transition</td>
<td>20 µs</td>
<td>$10^{12}$</td>
<td>150 MeV</td>
</tr>
<tr>
<td>Dark matter freeze-out</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Neutrino decoupling</td>
<td>1 s</td>
<td>$6 \times 10^9$</td>
<td>1 MeV</td>
</tr>
<tr>
<td>Electron-positron annihilation</td>
<td>6 s</td>
<td>$2 \times 10^9$</td>
<td>500 keV</td>
</tr>
<tr>
<td>Big Bang nucleosynthesis</td>
<td>3 min</td>
<td>$4 \times 10^8$</td>
<td>100 keV</td>
</tr>
<tr>
<td>Matter-radiation equality</td>
<td>60 kyr</td>
<td>3400</td>
<td>0.75 eV</td>
</tr>
<tr>
<td>Recombination</td>
<td>260–380 kyr</td>
<td>1100–1400</td>
<td>0.26–0.33 eV</td>
</tr>
<tr>
<td>Photon decoupling</td>
<td>380 kyr</td>
<td>1000–1200</td>
<td>0.23–0.28 eV</td>
</tr>
<tr>
<td>Reionization</td>
<td>100–400 Myr</td>
<td>11–30</td>
<td>2.6–7.0 meV</td>
</tr>
<tr>
<td>Dark energy-matter equality</td>
<td>9 Gyr</td>
<td>0.4</td>
<td>0.33 meV</td>
</tr>
<tr>
<td>Present</td>
<td>13.8 Gyr</td>
<td>0</td>
<td>0.24 meV</td>
</tr>
</tbody>
</table>
A system of particles in kinetic equilibrium has a phase space occupancy \( f \) given by the Bose-Einstein or Fermi-Dirac distributions at temperature \( T \):

\[
f(p) = \frac{1}{e^\frac{E-\mu}{T} \pm 1}
\]

The phase space distribution allows one to compute the associated number density \( n \), energy density \( \rho \) and pressure \( p \) for a dilute and weakly-interacting gas of particles with \( g \) internal degrees of freedom:

\[
n = g \int \frac{d^3p}{(2\pi)^3} f(p),
\]

\[
\rho = g \int \frac{d^3p}{(2\pi)^3} E(p) f(p),
\]

\[
p = g \int \frac{d^3p}{(2\pi)^3} \frac{|p|^2}{3E(p)} f(p).
\]

**Relativistic particles**

\( T \gg m \), \( E \sim |p| \)

\[
n_b = \frac{g}{\pi^2} \zeta(3) T^3,
\]

\[
\rho_b = \frac{\pi^2}{30} g T^4,
\]

\[
n_f = \frac{3}{4} \frac{g}{\pi^2} \zeta(3) T^3,
\]

\[
\rho_f = \frac{7}{8} \frac{\pi^2}{30} g T^4.
\]

**Non-Relativistic particles**

\( T \ll m \), \( E = (|p|^2 + m^2)^{1/2} = m \left( 1 + \frac{|p|^2}{m^2} \right)^{1/2} \approx m + \frac{|p|^2}{2m} \)

\[
n \approx g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}
\]
It is customary to define the Yield (equivalent to the number density but in a comoving volume) in terms of the entropy density (which scales as $a^3(t)$)

\[
Y = \frac{n}{s} \quad s = \frac{2\pi^2}{45} g_* s T^3
\]

For relativistic particles, we have

\[
n = \frac{g_{eff}}{\pi^2} \zeta(3) T^3 \quad \rightarrow \quad Y_{eq} = \frac{45}{2\pi^4} \zeta(3) \frac{g_{eff}}{g_* s}
\]

For non-relativistic particles, we have

\[
n = g_{eff} \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \quad \rightarrow \quad Y_{eq} = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g_{eff}}{g_* s} \left( \frac{m}{T} \right)^{3/2} e^{-m/T}
\]
We shall now apply the thermodynamics discussed in the previous section to the evolution of the early universe.

The primordial soup initially consists of all the different species of elementary particles. Their masses range from the heaviest known elementary particle, the top quark ($m \sim 175$ GeV) down to the lightest particles, the electron ($m = 511$ keV), the neutrinos ($m = ?$) and the photon ($m = 0$). In addition to the particles of the standard model, there may be other, so far undiscovered, species.

As the temperature falls, the various particle species become non-relativistic and annihilate at different times.

### Table 1: The particles in the standard model

**Quarks**

<table>
<thead>
<tr>
<th>Quark</th>
<th>Mass (GeV)</th>
<th>Spin</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$174.2 \pm 3.3$</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>$4.20 \pm 0.07$</td>
<td>3 colors</td>
<td></td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>$1.25 \pm 0.09$</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>$95 \pm 25$ MeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>$3\text{–}7$ MeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>$1.5\text{–}3.0$ MeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Gluons**

8 massless bosons

**Leptons**

<table>
<thead>
<tr>
<th>Lepton</th>
<th>Mass (GeV)</th>
<th>Spin</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^-$</td>
<td>$1777.0 \pm 0.3$ MeV</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\tau^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>$105.658$ MeV</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$\mu^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^-$</td>
<td>$510.999$ keV</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$e^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>$&lt; 18.2$ MeV</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\bar{\nu}_\tau$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>$&lt; 190$ keV</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\bar{\nu}_\mu$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>$&lt; 2$ eV</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Electroweak gauge bosons**

<table>
<thead>
<tr>
<th>Boson</th>
<th>Mass (GeV)</th>
<th>Spin</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+$</td>
<td>$80.403 \pm 0.029$</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$W^-$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z^0$</td>
<td>$91.1876 \pm 0.0021$ GeV</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$&lt; 6 \times 10^{-17}$ eV</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Higgs boson (SM)**

$H^0$ $125.5$ GeV

Number of relativistic degrees of freedom in the Standard Model

- Quarks: $72$ degrees of freedom
- Gluons: $16$ degrees of freedom
- Leptons: $12$ degrees of freedom
- Electroweak gauge bosons: $11$ degrees of freedom
- Higgs boson (SM): $1$ degree of freedom

Total: $g_f = 72 + 12 + 6 = 90$

$g_b = 16 + 11 + 1 = 28$
Number of relativistic degrees of freedom in the Standard Model

\[ g_*(T) = \sum_{\text{bos}} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{\text{fer}} g_i \left( \frac{T_i}{T} \right)^4 \]

- \( T \sim 200 \text{ GeV} \): all present 106.75
- \( T \sim 100 \text{ GeV} \): EW transition (no effect)
- \( T < 170 \text{ GeV} \): top annihilation 96.25
- \( T < 80 \text{ GeV} \): \( W^\pm, Z^0, H^0 \) 86.25
- \( T < 4 \text{ GeV} \): bottom 75.75
- \( T < 1 \text{ GeV} \): charm, \( \tau^- \) 61.75
- \( T \sim 150 \text{ MeV} \): QCD transition 17.25 (\( u, d, g \rightarrow \pi^{\pm,0}, \pi^0, \mu^0 \)) 37 \( \rightarrow 3 \)
- \( T < 100 \text{ MeV} \): \( \pi^\pm, \pi^0, \mu^- \) 10.75 \( e^\pm, \nu, \bar{\nu}, \gamma \) left
- \( T < 500 \text{ keV} \): \( e^- \) annihilation (7.25) \( 2 + 5.25(4/11)^{4/3} = 3.36 \)
The temperature and thus the quark energies have fallen so that the quarks lose their asymptotic freedom.

There are no more free quarks and gluons; the quark-gluon plasma has become a hadron gas.

The lightest baryons are the nucleons: the proton and the neutron. The lightest mesons are the pions.

All except pions are nonrelativistic below the QCD phase transition temperature.

Thus the only particle species left in large numbers are the pions \((g=3)\), muons \((4)\), electrons \((4)\), neutrinos \((2\times3)\), and the photons \((2)\).

\(g^* = 17.25\)
\[ Y_{eq} = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g_{eff}}{g_{*s}} \left( \frac{m}{T} \right)^{3/2} e^{-m/T} \]
EXAMPLE 1.1

It is easy to estimate the value of the Yield that we need in order to reproduce the correct DM relic abundance, \( \Omega h^2 \approx 0.1 \), since

\[
\Omega h^2 = \frac{\rho_x h^2}{\rho_c} = \frac{m_x n_x h^2}{\rho_c} = \frac{m_x Y_0 s_0 h^2}{\rho_c},
\]

where \( Y_0 \) corresponds to the DM Yield today and \( s_0 \) is today’s entropy density. We can assume that the Yield did not change since DM freeze-out and therefore

\[
\Omega h^2 = \frac{m_x Y_f s_0 h^2}{\rho_c}.
\]

Using the measured value \( s_0 = 2970 \text{ cm}^{-3} \) and the value of the critical density \( \rho_c = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3} \), as well as Planck’s result on the DM relic abundance we arrive at

\[
Y_f \approx 3.55 \times 10^{-10} \left( \frac{1 \text{ GeV}}{m_x} \right).
\]
For non-relativistic particles, the “magic number is $x \approx 20$.

\[ Y_{eq} = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g_{\text{eff}}}{g_{*s}} \left( \frac{m}{T} \right)^{3/2} e^{-m/T} \]

For DM masses in the range 1 GeV – 1 TeV.
The time evolution of the phase space distribution function is dictated by Liouville’s operator (which ensures conservation of density in the phase space) and the Collisional operator, which encodes number changing processes.

\[ \hat{L}[f] = C[f] \]

The Liouville operator can be written in a covariant way

\[ \hat{L} = \frac{d}{d\tau} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\sigma\rho} p^\sigma p^\rho \frac{\partial}{\partial p^\mu} \]

Where the affine connection is related to derivatives of the metric as follows

\[ \Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} (g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma}) \]

Notice that this terms incorporates gravity and the actual geometry of space-time.
If we apply this to the FRW metric, which only depends on $t$ and $E$

$$f(x^\mu, p^\mu) = f(t, E)$$

We find that Liouville operator can be greatly simplified

**Exercise 1**

$$\hat{L} = E \frac{\partial}{\partial t} - \Gamma^0_{\sigma \rho} p^\sigma p^\rho \frac{\partial}{\partial E}$$

$$= E \frac{\partial}{\partial t} - H|p|^2 \frac{\partial}{\partial E}$$

Ultimately, we are interested in the time evolution of the number density

$$n = \frac{g}{2\pi^3} \int f(p) d^3p$$
Thus, we integrate Liouville’s operator in the momentum space

$$\frac{g}{2\pi^3} \int \hat{L}[f] \, d^3p = \frac{g}{2\pi^3} \int C[f] \, d^3p.$$ 

Exercise 2

Prove the following relation

$$\frac{g}{(2\pi)^3} \int \frac{d^3\vec{p}}{E} \left[ E \frac{\partial f}{\partial t} - H|\vec{p}|^2 \frac{\partial f}{\partial E} \right] = \frac{dn}{dt} + 3Hn$$

Where we have divided by E for convenience
\[
d\Pi_i = \frac{g_i}{2\Pi^3} \frac{d^3 p_i}{2E_i}
\]

\[d\Pi = \frac{g}{2\pi^3} \frac{C[f]}{E} d^3 p = - \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta (p_A + p_B - p_1 - p_2) \]
\[
\left[ |\mathcal{M}_{12\rightarrow AB}|^2 f_1 f_2 - |\mathcal{M}_{AB\rightarrow 12}|^2 f_A f_B \right]
\]
\[
= -\langle \sigma v \rangle \left( n^2 - n_{eq}^2 \right)
\]

We have defined the thermally averaged annihilation cross section
\[
\langle \sigma v \rangle \equiv \frac{1}{n_{eq}^2} \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta (p_A + p_B - p_1 - p_2) |\mathcal{M}|^2 f_1^{eq} f_2^{eq}
\]

No CP violation in DM sector
\[
|\mathcal{M}_{12\rightarrow AB}|^2 = |\mathcal{M}_{AB\rightarrow 12}|^2
\]

Energy Conservation
\[
f_A f_B = f_A^{eq} f_B^{eq} = e^{-\frac{E_A+E_B}{T}} = e^{-\frac{E_1+E_2}{T}} = f_1^{eq} f_2^{eq}
\]

EXAMPLE 1.2

\[
L_{12} f_{12} = L_{eq} f_{eq}^{12}
\]
Non-relativistic species

\[
\frac{dn}{dt} + 3Hn - \langle \sigma v \rangle (n^2 - n_{eq}^2)
\]

- \[ \frac{dY}{dt} = \frac{d}{dt} \left( \frac{n}{s} \right) = \frac{d}{dt} \left( \frac{\alpha^3 n}{a^3 s} \right) = \frac{1}{a^3 s} \left( 3\alpha^2 \dot{a} n + a^3 \frac{dn}{dt} \right) = \frac{1}{s} \left( 3Hn + \frac{dn}{dt} \right) \]

- \[ x = \frac{m}{T} \]

\[
\frac{d}{dt} (\alpha^3 s) = 0 \rightarrow \frac{d}{dt} (aT) = 0 \rightarrow \frac{d}{dt} \left( \frac{a}{x} \right) = 0 \quad \rightarrow \quad \frac{dx}{dt} = H x
\]

\[
\frac{dY}{dt} = \frac{dY}{dx} \frac{dx}{dt} = \frac{dY}{dx} H x
\]

\[
\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} (Y^2 - Y_{eq}^2)
\]

\[
\lambda = \frac{2\pi^2}{45} \frac{M_P g_{*s}}{1.66 g_{*}^{1/2} m}
\]
Exercise 3

\[
\frac{dY}{dx} = -\frac{\lambda <\sigma v>}{x^2} (Y^2 - Y^2_{eq})
\]

\[
\Delta_Y \equiv Y - Y_{eq}
\]

\[
\Delta_Y = -\frac{dY_{eq}}{dx} \frac{x^2}{2\lambda <\sigma v>}, \quad 1 < x \ll x_f
\]

\[
\Delta_{Y\infty} = Y_{\infty} = \frac{x_f}{\lambda \left( a + \frac{b}{3x_f} \right)}, \quad x \gg x_f
\]

This leads to:

\[
\Omega h^2 \approx \frac{m_\chi Y_{\infty} s_0 h^2}{\rho_c} \frac{10^{-10} \text{ GeV}^{-2}}{(a + \frac{b}{60})} \frac{10^{-27} \text{ cm}^3 \text{ s}^{-1}}{(a + \frac{b}{60})}
\]

\[
\Omega \approx \frac{m_\chi Y_{\infty} s_0 h^2}{\rho_c} \frac{10^{-10} \text{ GeV}^{-2}}{(a + \frac{b}{60})} \frac{10^{-27} \text{ cm}^3 \text{ s}^{-1}}{(a + \frac{b}{60})}
\]
• Very different scales conjure up to lead to the electroweak scale

A typical electroweak scale cross section for a non-relativistic particle

\[ \sigma v \approx \alpha^2 \frac{m^2}{M_W} = G_F^2 m^2 \]

\[ G_F \approx 10^{-5} \text{ GeV}^{-2} \]

Notice that this implies

\[ \Omega h^2 \sim \frac{1}{\langle \sigma_A v \rangle} \sim \frac{1}{m^2} \]  

(non-relativistic particle)

Imposing

\[ \Omega \leq 1 \quad \rightarrow \quad m \leq 340 \text{ TeV} \]  

(Griest, Kamionkowski '90)
WIMPs can be thermally produced in the early universe in just the right amount

The freeze-out temperature (and hence the relic abundance) depends on the DM annihilation cross-section

\[ \frac{dn}{dt} + 3H n = - \langle \sigma v \rangle \left( n^2 - n_{eq}^2 \right) \]

\[ \Omega_\chi h^2 \simeq \text{const.} \cdot \frac{T_0^3}{M_{\text{Pl}}^3 \langle \sigma_A v \rangle} \simeq \frac{0.1 \text{ pb} \cdot c}{\langle \sigma_A v \rangle} \]

\[ T_o \approx 10^{-13} \text{ GeV} \]

\[ H_{100} = 100 \text{ km sec}^{-1} \text{ Mpc} \approx 10^{-42} \text{ GeV} \]

\[ M_{\text{Planck}} = 1/G_N^{1/2} = 10^{19} \text{ GeV} \]

A generic (electro)Weakly-Interacting Massive Particle can reproduce the observed relic density.
\[ Y_{eq} = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g_{eff}}{g_{*s}} \left( \frac{m}{T} \right)^{3/2} e^{-m/T} \]

\[ m_x = 100 \text{ GeV} \]
Special cases

- The low-temperature expansion for the annihilation cross section

\[ \sigma_A v = a + \frac{b}{x} \]

is not valid in some cases:

Resonant annihilation

Thresholds

\[(\text{Gondolo, Gelmini '91})\]

Coannihilations with other particles close in mass

\[(\text{Griest, Seckel '91})\]
Special cases

- Resonant annihilation:

\[(2 \, m_{\text{WIMP}}^2) = (m_X)^2\]

\[s = (2 \, m_{\text{WIMP}})^2\]

The resonant increase in the cross section implies a sharp decrease in the relic abundance.

General expression for thermal average of annihilation cross section

(Gondolo, Gelmini '91)
Special cases

- Resonant annihilation

The annihilation cross section is significantly increased in the pole of the propagator.

As a consequence, the relic density decreases rapidly.

Thermal motion allows resonant annihilation when

\[ m_W < \frac{m_A}{2} \]

This is not possible for

\[ m_W > \frac{m_A}{2} \]

(Griest, Seckel '91)
However....

- Not applicable to non-thermal candidates (for which the relic abundance is calculated differently, such as decays of heavier particles) (e.g., gravitinos and axinos)

- Non-standard Cosmology, e.g.,

  The presence of scalar fields in the Early Universes may induce a period of a much higher expansion rate. WIMPs decouple earlier and have a much larger relic abundance (by several orders of magnitude).

  (Catena, Fornengo, Masiero, Pietroni, Rosati, '04)

Changes in the cosmology also affect the calculation of relic abundance for non-thermal dark matter. E.g., quintessence-motivated kination models.

  (Gómez, Lola, Pallis, Rodríguez-Quintero '08)

Late entropy production which dilutes the DM density

  See, e.g., (Giudice, Kolb, Riotto '01)

  E.g., decay of heavy sterile neutrinos

  (Abazajian, Koushiappas '06)

  (Asaka, Shaposnikov, Kusenko '06)
Late (out of equilibrium) decay of semistable particles induce an injection of entropy.

The only constraint to be considered is not to spoil BBN predictions

\[ T_R > 5 \text{ MeV} \]

There are various possibilities depending on whether \( T_R \) is smaller or larger than the DM freeze out temperature and on whether the decay produces more DM particles.

\[ T_R > T_f \]
... probing **DIFFERENT** aspects of their interactions with ordinary matter

**Constraints** in one sector affect observations in the other two.

“**Redundant**” detection can be used to extract DM properties.
Dark matter **MUST BE** searched for in different ways...

- Direct DM detection
- Collider DM searches
- Astro/Cosmo probes
- Indirect DM detection