The Lightest Higgs Boson Mass of the MSSM at Three-Loop Accuracy

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Outline

1. **Introduction**

2. **Three-Loop Corrections to M_h in MSSM**
   - Renormalization Scheme
   - Evaluation of Unrenormalized Topologies
   - Algebra of Numerators
   - Reduction to a Set of Scalar Integrals
   - Master Integrals
   - Partial Results

Introduction

- We are interested in determine the three-loop QCD corrections to the Light CP-Even Higgs boson (***h***) mass of the **r**MSSM.

With this project:

- Previous computations of these observables by P. Kant, R. V. Harlander, L. Mihaila, M, Steinhauser. '08 '10. We want to verify and provide an alternative computation of the results of P. Kant et al.

  - Renormalization procedure consistent with the others higher order corrections currently included in the public code FeynHiggs.

    Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '06 '10

    Then, our renormalization scheme also includes an on-shell renormalization of the squark sector.

- We have performed a calculation in a way that does not require expansions in particular mass hierarchies. So, we use an approach that does not depend on such expansions.

Renormalization Scheme

- Mass Matrix of the CP-Even Higgs Bosons (***h, H***)

$$\mathcal{M}_H^2 = \boxed{\mathcal{M}^2_{H,\text{tree}}} - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

$$\begin{pmatrix} \phi_1^0 & \phi_2^0 \end{pmatrix} \begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \frac{T_1}{\sqrt{2}v_1} & -\left(M_A^2 + M_Z^2\right) s_\beta c_\beta \\ -\left(M_A^2 + M_Z^2\right) s_\beta c_\beta & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \frac{T_2}{\sqrt{2}v_2} \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

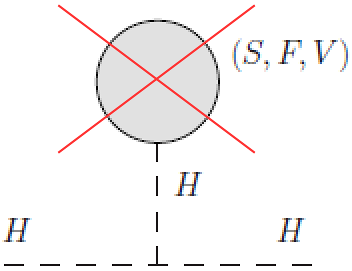
$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \quad ; \quad tan(2\alpha) = tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

Derived from the potential in the Higgs sector of the Lagrangian!

- VEV's are the minima of the full EP:

$$T_{1,2}^{tree} = 0 \quad ; \quad \delta^{(l)} T_{1,2} = -T_{1,2}^{(l)}$$

Self-energies corrections are gauge-dependent, we choose $\zeta=1$.



- Self-energies will be evaluated at $p=0$.
- Three-loop self-energies at $O(\alpha_t \alpha_s^2)$; EW gaugeless limit.

Renormalization Scheme

- Taking into account the above restrictions:

$$\hat{\sum}^{(3)}_{\phi_1} = \sum^{(3)}_{\phi_1} + \left(p^2 - M^2_{\phi_1}\right)\delta^{(3)}Z_{H_1} - \delta^{(2)}Z_{H_1}\delta^{(1)}M^2_{\phi_1} - \delta^{(1)}Z_{H_1}\delta^{(2)}M^2_{\phi_1} - \delta^{(3)}M^2_{\phi_1}$$

$$\hat{\sum}^{(3)}_{\phi_2} = \sum^{(3)}_{\phi_2} + \left(p^2 - M^2_{\phi_2}\right)\delta^{(3)}Z_{H_2} - \delta^{(2)}Z_{H_2}\delta^{(1)}M^2_{\phi_2} - \delta^{(1)}Z_{H_2}\delta^{(2)}M^2_{\phi_2} - \delta^{(3)}M^2_{\phi_2}$$

$$\hat{\sum}^{(3)}_{\phi_1\phi_2} = \sum^{(3)}_{\phi_1\phi_2} - \delta^{(3)}\Phi_{\phi_1\phi_2} - \delta^{(3)}M^2_{\phi_1\phi_2}$$

- Mass CT are obtained from the CP-Even Higgs bosons mass matrix.
- Corrections are in the physical basis (*h*, *H*).

$$C^h_{\alpha\beta} = c_\alpha C^2_{\alpha\beta} - s_\alpha C^1_{\alpha\beta}$$

Evaluated at p = 0.

$$\delta^{(3)}M^2_{hh} = Re\left[\sum^{(3)}_{AA}\right](s_\alpha s_\beta + c_\alpha c_\beta)^2 + \left(\frac{e}{2s_W M_W}\right)\left(\frac{T^{(3)}_h}{\sqrt{2}}C^h_{\alpha\beta} + \frac{T^{(3)}_H}{\sqrt{2}}C^H_{\alpha\beta}\right)$$

- $tan\beta = \frac{v_2}{v_1}$, $D(\alpha) = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}$.

$$C^H_{\alpha\beta} = c_\alpha C^1_{\alpha\beta} + s_\alpha C^2_{\alpha\beta}$$

Evaluation of Unrenormalized Topologies

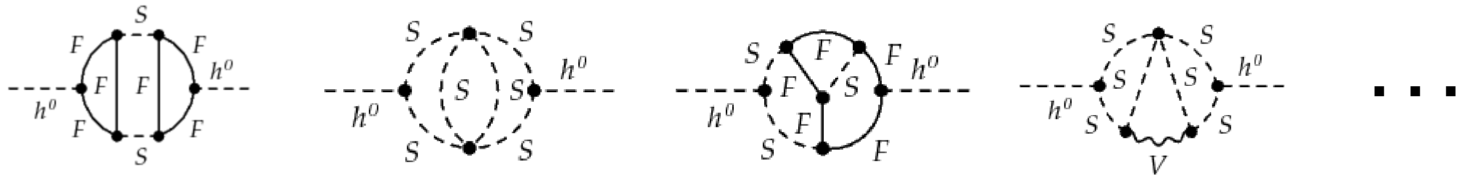
- We need to compute the unrenormalized contributions (*hh, HH, AA, h, H*):
  - Generation of diagrams, selection and generation of amplitudes at O($\alpha_t \alpha_s^2$)

T. Hahn ‘13, V. Shtabovenko et al. ‘16

```
$LoadFeynArts = True;
Get["~/FeynCalc901/FeynCalc/FeynCalc.m"];

Topology = CreateTopologies[ 3, 1 → 1, Options ];
DiagramSelect[ InsertFields[ Topology, h → h, Model → "MSSMCT",
Options  ], SelectionRules];

SE = CreateFeynAmp[%];
```

Generated integrals are in 4-dimensions (they are not regularized)

Topologies to be evaluated:



- 80 three-loop 1PI self-energy topologies. At O($\alpha_t \alpha_s^2$) 40 self-energy topologies make contributions. In total there are 3869 x 3 Amplitudes to evaluate.
- 15 3L 1PI tadpole Topologies. At O($\alpha_t \alpha_s^2$) 11 tadpole topologies make contributions. In total there are 3590 x 2 Amplitudes to evaluate.

DRED Regularization Scheme

- Amplitudes are regularized in DRED D. Stockinger et al. ‘05 ‘11.
- Two options:
  - i) Extended Lorentz algebra, or
  - ii) Dirac algebra in Q4S = QDS + QεS (no split).

$$\gamma^\mu = \begin{pmatrix} \hat{\gamma}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \tilde{\gamma}^\mu \end{pmatrix}; \quad g^{\mu\nu} = \begin{pmatrix} \hat{g}^{\mu\nu} & 0 \\ 0 & \tilde{g}^{\mu\nu} \end{pmatrix}; \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

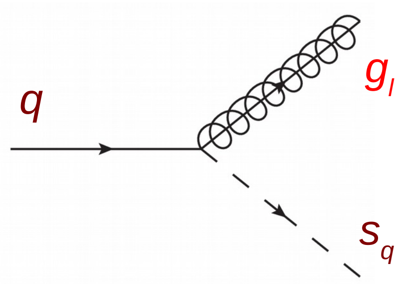
- Loop momenta have been continued from 4 to D dimensions (D = 4 - 2ε),
- while gamma matrices and gauge fields remain in 4 – dim.

$$\gamma_\mu \gamma^\mu = 4; \quad g_{\mu\nu} g^{\mu\nu} = 4; \quad g^{\nu\rho} Tr\left\{ \left(\gamma^\mu \hat{q}_{1\mu}\right) \left(\gamma^\nu\right) \left(\gamma^\sigma \hat{q}_{2\sigma}\right) \left(\gamma^\rho\right) \right\} = -8 \left(\hat{q}_1 . \hat{q}_2\right)$$

However, $\boxed{\begin{pmatrix} \hat{q}_1^\mu & 0 \end{pmatrix} g_{\mu\nu} \begin{pmatrix} \hat{q}_2^\nu \\ 0 \end{pmatrix} \equiv \hat{q}_1 . \hat{q}_2}$

Q4S is an infinite-dim space with 4-dim properties !

- We must be careful with Q4S algebra for diagrams with the cubic vertex:

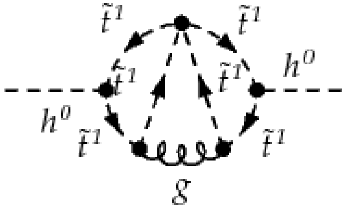


$$\boxed{\begin{array}{c} \{\gamma_5, \gamma^\mu\} = 0 \\ \mathrm{Tr}(\gamma_5(\text{arbitrary number of } \gamma^{\mu_i})) = 0 \end{array}}$$

Non-vanishing traces with a single γ_5 don’t occur!

Algebra of Numerators

- Traces are evaluated with the help of the function DiracTrace[ ] of FeynCalc.

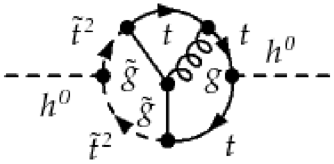


Diagrams without traces are contracted when its necessary with the help of the function Contract[ ] of FeynCalc.

Evaluation of sums over color indices:

Get["SUNSimplify.m"];

SetOptions[SUNSimplify, SUNTrace $\rightarrow$ True];
Amp = SUNSimplify[Amp]; $\longrightarrow$ $f_{abc} Tr(T_c T_b T_a)$

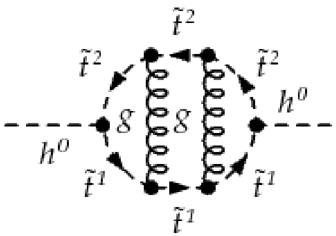


Options[SUNTrace];
{Explicit $\rightarrow$ False}
Amp = Calc[Amp /. False $\rightarrow$ True]; $\longrightarrow$ $-\frac{1}{2}\, iC_A{}^2 C_F$

Amplitudes are expressed in terms of the Casimir operator eigenvalues $C_A = 3$ and $C_F = 4/3$ for SU(3).

Algebra of Denominators

After performing the Dirac and Color Algebra:



$$\rightarrow \left[A q_i.q_j + B\,(q_i.q_j)\,(q_k.q_l) + C\,(q_i.q_j)\,(q_k.q_l)\,(q_m.q_n)\right] \left\{ \frac{1}{(k_1^2 - m_1^2)^a} \;,\; \ldots \right\}$$

Amp[2, 130]

k1 = q_1, k2 = q_2, k3 = q_3 , k4 = q_1- q_2, k5 = q_1- q_3, k6 = q_2- q_3

- We perform partial fractioning over propagators that have the same loop momenta but different masses:

$$\left\{ \frac{1}{\mathbf{k1^2 - MSf(1,3,3)^2}}^{1}, \frac{1}{\mathbf{k1^2 - MSf(2,3,3)^2}}^{1}, \boxed{\frac{1}{\mathbf{k2^2 - MSf(1,3,3)^2}}^{1}, \frac{1}{\mathbf{k2^2 - MSf(2,3,3)^2}}^{1}}, \frac{1}{\mathbf{k3^2 - MSf(1,3,3)^2}}^{1}, \frac{1}{\mathbf{k3^2 - MSf(2,3,3)^2}}^{1}, \frac{1}{\mathbf{k4^2}}^{1}, \frac{1}{\mathbf{k5^2}}^{0}, \frac{1}{\mathbf{k6^2}}^{1} \right\}$$

```
Get[ "SectorScript.m" ];

Amp[ n__, j__ ] := Amp[[ n, j ]] /. FeynAmpDenominator[ ___ ] :> Pafr[ j ];
```

$$\text{Amp[2, 130]} \;\sim\; \frac{1}{\mathbf{MSf(1,3,3)^2 - MSf(2,3,3)^2}} \left\{ \ldots, \frac{1}{\mathbf{k2^2 - MSf(1,3,3)^2}} - \frac{1}{\mathbf{k2^2 - MSf(2,3,3)^2}}, \ldots \right\}$$

- We rewrite the scalar products on the numerator as:

$$q_i.q_j = \frac{1}{2}\left[P^{-1}_{i+j+1} - P^{-1}_j - P^{-1}_i + f(m_i, m_j)\right]; \quad P_j = \left(k_j^2 - m_j^2\right)^{-1}$$

Reduction to a Set of Scalar Integrals

- Amplitudes can be expressed as a superposition of a set of scalar integrals:

$$\mathrm{Amp}[\,\mathrm{n}\,,\ \mathrm{m}\,] = \sum_j c_j\, \mathrm{INT}[\{\nu_{j_1},\ m_{j_1}\},\{\nu_{j_2},\ m_{j_2}\},\{\nu_{j_3},\ m_{j_3}\},\{\nu_{j_4},\ m_{j_4}\},\{\nu_{j_5},\ m_{j_5}\},\{\nu_{j_6},\ m_{j_6}\}]$$

where

$$\mathrm{INT}[\{a,\ m_1\},\{b,\ m_2\},\{c,\ m_3\},\{d,\ m_4\},\{e,\ m_5\},\{f,\ m_6\}] =$$

$$\left\langle \frac{1}{(k_1^2-m_1^2)^a}\frac{1}{(k_2^2-m_2^2)^b}\frac{1}{(k_3^2-m_3^2)^c}\frac{1}{(k_4^2-m_4^2)^d}\frac{1}{(k_5^2-m_5^2)^e}\frac{1}{(k_6^2-m_6^2)^f}\right\rangle_{3l}$$

$$\langle\ \ldots\ \rangle_{3l} = \prod_{j=1}^{3}\int d^D q_j$$

with a, b, c … positive, negative or zero integer numbers

- Classification of scalar integrals:

```
Amps = Flatten[ Table[ Amp[ n, m ], {n, 1, 40 or 11 }, {m, 1, Length[ Amp[ n, m ] ]}]];
ScalarInt = Union @ Cases[ Amps, INT[ ___ ] , Infinity ];
```

Length[ScalarInt] =

- Self-energies: In total there are 3525 scalar integrals.
- Tadpoles: There are 788 scalar integrals.

Master Integrals

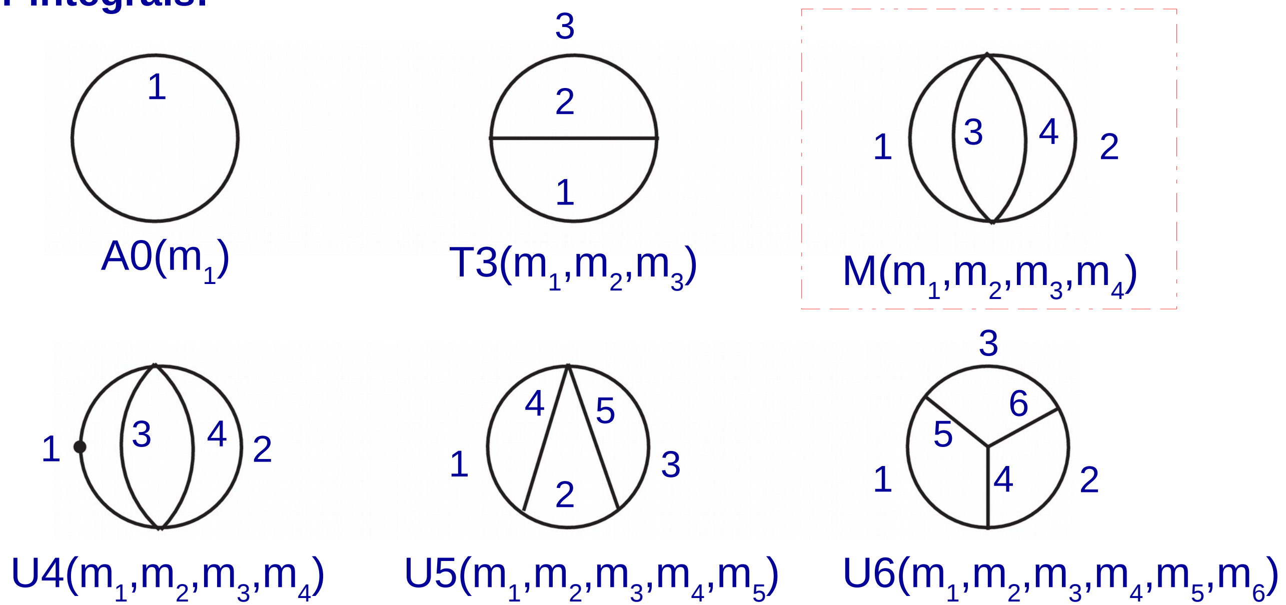
- The set of scalar integrals are not independent of each other but related by the IBP identities and the LI identities.

$$\int d^D q_j \frac{\partial}{\partial q_j^\mu}\left[k^\mu \mathbf{I}'\left(p_1,\ldots,p_m,q_1,\ldots,q_l\right)\right]=0 \quad ; \quad \sum_{n=1}^{m}\left(p_n^\nu \frac{\partial}{\partial p_{n\mu}} - p_n^\mu \frac{\partial}{\partial p_{n\nu}}\right)\mathbf{I}\left(p_1,\ldots,p_m\right)=0$$

- Homogeneous system of linear equations can be reduced to a small set of the so called Master Integrals. There are thousands of equations.

- **Reduze 2.1**: C++ program, Laporta algorithm. Simplification of the prefactors with GiNaC, Fermat, etc. C. Studerus et al. ‘10 ‘12

- **Basis of master integrals:**



Master Integrals

- A0[__] and T3[__] are analytically well known. We need:

$$A0[\_\_] = A_{div}[\_\_]\,\varepsilon^{-1} + A_{fin}[\_\_] + \varepsilon\, A_{eps}[\_\_] + \varepsilon^2 A_{eps2}[\_\_]$$
$$T3[\_\_] = T_{div2}[\_\_]\,\varepsilon^{-2} + T_{div1}[\_\_]\,\varepsilon^{-1} + T_{fin}[\_\_] + \varepsilon T_{eps}[\_\_]$$

- For X = U4, U5 and U6 :
  There are up to four different mass scales,

$$X[\_\_] = X_{fin}[\_\_] + X_{div}[\_\_]\,\varepsilon^{-1} + X_{div2}[\_\_]\,\varepsilon^{-2} + X_{div3}[\_\_]\,\varepsilon^{-3}$$

Master Integrals

- A0[__] and T3[__] are analytically well known. We need:

A0[__] = Adiv[__] ε^{-1} + Afin[__] + ε Aeps[__] + ε^2Aeps2[__]
T3[__] = Tdiv2[__] ε^{-2} + Tdiv1[__] ε^{-1} + Tfin[__] + εTeps[__]

- For X = U4, U5 and U6 :
There are up to four different mass scales,

X[__] = Xfin[__] + Xdiv[__] ε^{-1} + Xdiv2[__] ε^{-2} + Xdiv3[__] ε^{-3}

- Analytic results known for m1, m2 ≠ 0 in some specific cases.
- Integrals with three and four scales must be numerically evaluated. There are two equivalent possibilities:

i) TVID (A. Freitas et al. '17) based on the dispersion method.

$$\int_{s_0}^{\infty} ds \, \frac{f(s)}{s - s' \pm i\varepsilon}$$

Gauss-Kronrod algorithm
QUADPACK library

ii) 3VIL (S. P. Martin et al. '16) based on differential equation method.

Master Integrals

- A0[__] and T3[__] are analytically well known. We need:

A0[__] = Adiv[__] ε $^{-1}$ + Afin[__] + ε Aeps[__] + ε^{2}Aeps2[__]

T3[__] = Tdiv2[__] ε $^{-2}$ + Tdiv1[__] ε $^{-1}$ + Tfin[__] + εTeps[__]

- For X = U4, U5 and U6 :
  There are up to four different mass scales,

X[__] = Xfin[__] + Xdiv[__] ε $^{-1}$ + Xdiv2[__] ε $^{-2}$ + Xdiv3[__] ε $^{-3}$

- U4 contains:

  ε $^{-1}$F(Afin[__], Aeps[__]),
  ε $^{-2}$F(Afin[__], m_i);

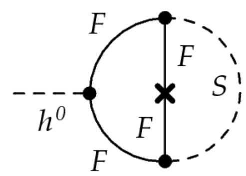
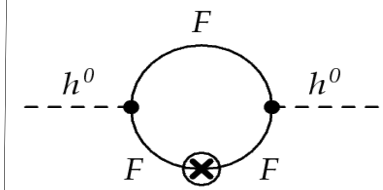
$$\frac{1}{\epsilon^3}\sum_{i=2}^{4}\frac{m_i^2}{3}+\frac{1}{\epsilon^2}\left[-\frac{m_1^2}{6}+\sum_{i=2}^{4}m_i^2\left(\frac{5}{6}-\frac{\log m_i^2}{2}-\frac{\log m_1^2}{2}\right)\right]$$
$$+\frac{1}{\epsilon}\left[m_1^2\left(-1+\frac{\log m_1^2}{2}\right)+\sum_{i=2}^{4}m_i^2\left(\frac{4}{3}+\frac{\pi^2}{12}-\log m_i^2-\frac{3}{2}\log m_1^2\right.\right.$$
$$\left.\left.+\frac{1}{4}\log^2 m_i^2+\frac{1}{4}\log^2 m_1^2+\log m_i^2\log m_1^2\right)\right]$$

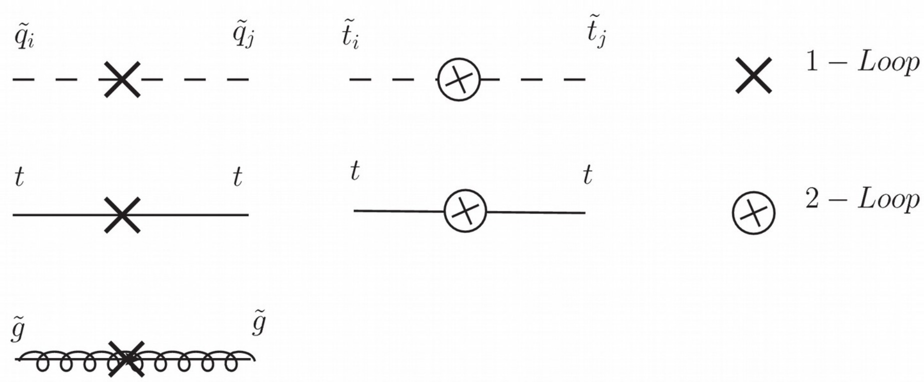
- U5 contains:
  ε $^{-1}$F(Tfin[__], Afin[__], Aeps[__], m_i), ε $^{-2}$F(Afin[__], m_i).

- U6 contains:
  ε $^{-1}$(2ζ(3))

$$U_6(m_1^2,m_2^2,m_3^2,m_4^2,m_5^2,m_6^2)=\frac{1}{\epsilon}2\zeta(3)+U_{6,\mathrm{fin}}(m_1^2,m_2^2,m_3^2,m_4^2,m_5^2,m_6^2)$$

Sub - Renormalization Procedure

Diagrams	Tadpoles 523 x 2 diagrams	Self-energies 3491 x 4 diagrams
• 2 and 1 -loop diagrams with 1 and 2 -loop counter-term insertions respectively .		



- DR Scheme

  L. Mihaila 2013

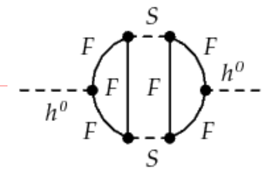
- OS Scheme: OS renormalization of squark sector.

Renormalized SE

Local UV divergences

$$\hat{\sum}^{(3)}_{\phi_i \phi_j} = \boxed{\sum^{(3)}_{\phi_i \phi_j}} - \delta^{(3)} M^2_{\phi_i \phi_j}$$

Sub - divergences



INT[{2,m1},{2,0},{0,m1},{1,m2},{-1,0},{1,m2}]

= (A0[m1]^3/m1^2 – U4[__]) ε $^{-1}$

≈ U4fin[__] / ε + ...

Partial Results

- There are two non-trivial checks:

  - Three-loop correction to M_h is free of IR and UV divergences.

  - Supersymmetric limit: $M_t = M_{st}$ and the gluino and the other squark masses are equal to zero:

    The three – loop quantum corrections to the Higgs boson mass vanish, as required by supersymmetry.

    The individual diagrams are different from zero, the calculation imposes a strong check on our setup.

  - Numerical analysis is coming . . .

Partial Results

SLHA = SUSY Les Houches Accord

P. Skands et al., JHEP 0407 (2004) 036

```
Install[ "MfeynHiggs" ];

Get[ "inparam.m" ];

   MHiggs[ i_, looplevel_ ] :=
   Block[ { ____ },
          mssmpart = 4;   fieldren = 0;   tanbren = 0;   higgsmix = 2;
          p2approx = 0;   runningMT = 1;   botResum = 1;  tlCplxApprox = 0;

          FHSetFlags[ mssmpart, fieldren, tanbren, higgsmix, p2approx,
                      looplevel, runningMT, botResum, tlCplxApprox ];

    FHSetSLHA["SLHA/SLHA.out"];         (* SOFTSUSY *)
```

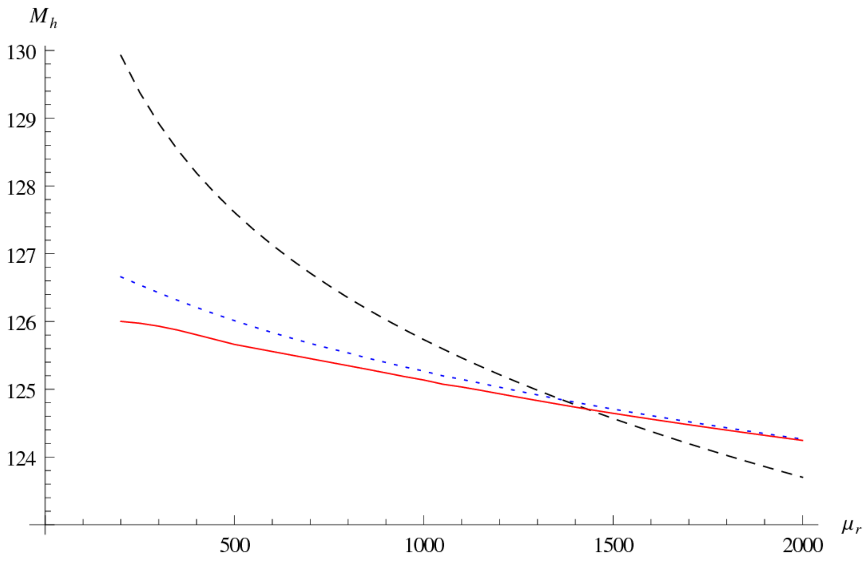
B. C. Allanach (2002).

```
    Return[ ( MHiggs /. FHHiggsCorr[  ] )[ [ i ] ] ];

          ]

 SetOptions[ H3GetSLHA, Model -> Inparam[ 100., 20., 1500.] ];
 H3GetSLHA[  ];

 Print["Mh = "<>ToString[MHiggs[1, 1]]<>" GeV" ]
 Print["Mh = "<>ToString[MHiggs[1, 2]]<>" GeV" ]


 Quit[  ];
```

```
Inparam[ muRen_, TB_, At_] := {

"MODSEL" -> {
EV -> muRen, EC -> " renormalisation
scale" …}
…
 "MINPAR" -> {
TB, EC -> " tan(beta)"}
…
"SMINPUTS" → {
127.934, -> " alpha^(-1)",
0.0000116637, -> " G_Fermi",
0.1184, EC -> " alpha_s(Mz)",
91.199, EC -> " Mz(pole)",
173.3, EC -> " Mt SM pole"}
 …
"EXTPAR" -> {
 800., EC -> " M3",
 At, EC -> " At",
 EV -> 1000., EC -> " Msusy"} }
```

Partial Results

a)



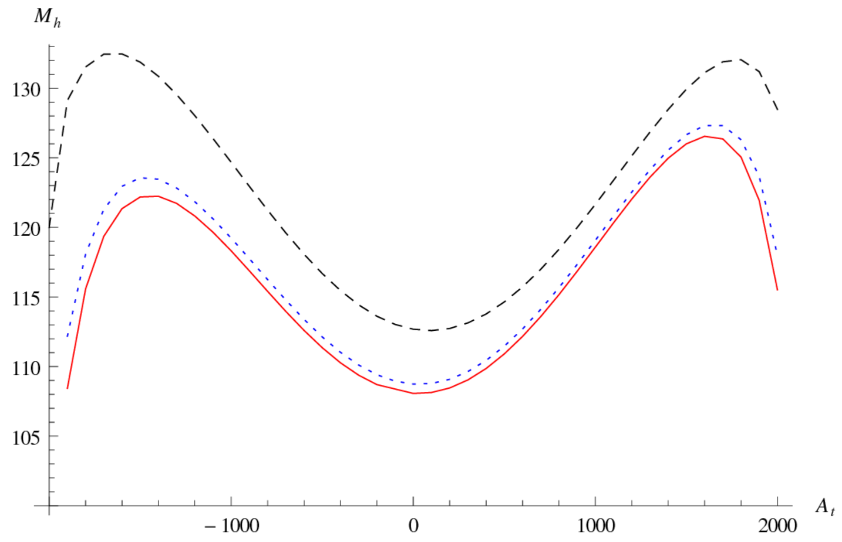
a) At = 1500. ; TB = 10.

b) muren = 200. ; TB = 10.

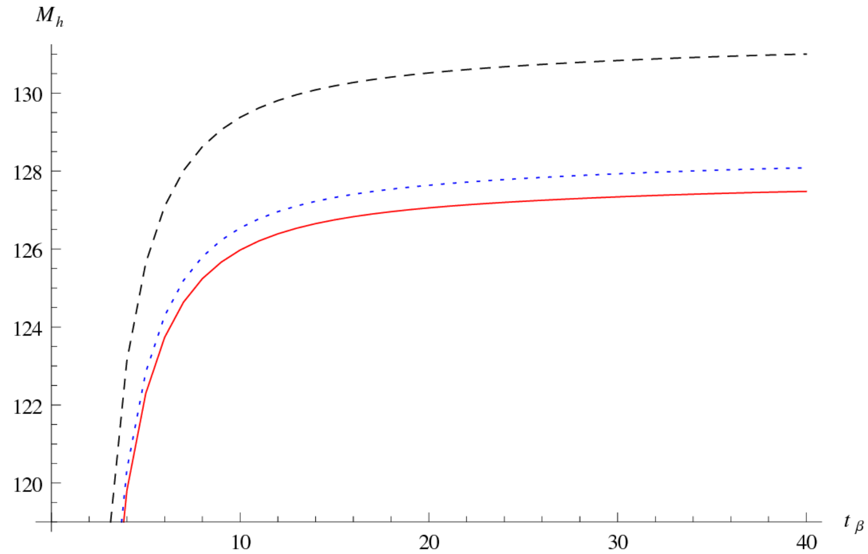
c) At = 1500. ; muren = 250.

The **black dashed**, **blue dotted** and **red solid line** corresponds to the **one-**, **two**- and **three**- loop prediction.

b)



c)



Thanks for your attention !