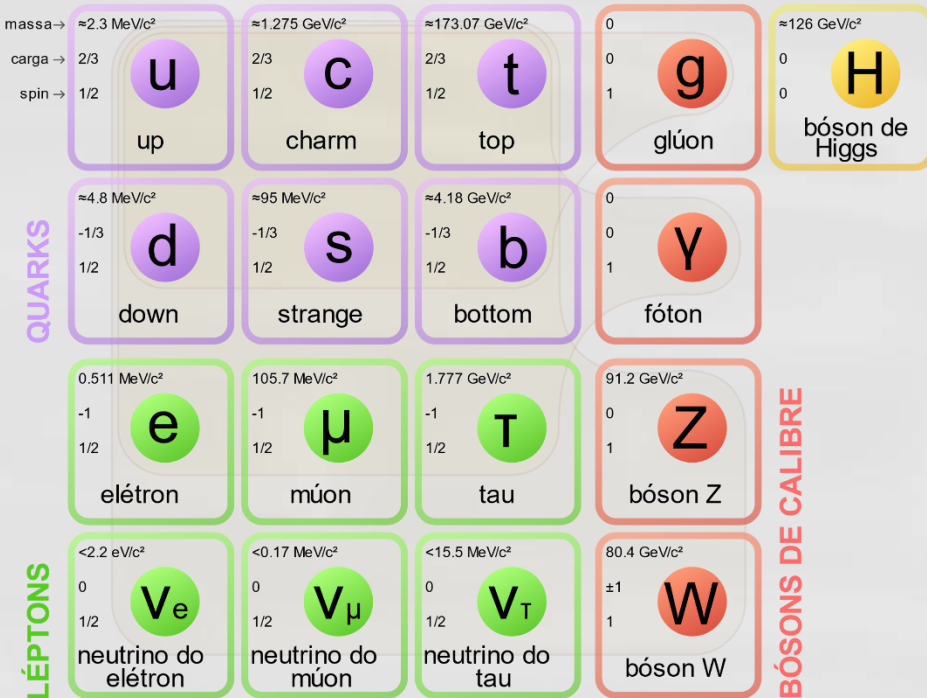


The precision frontier: higher order corrections to observables and predictions of the 2HDM; development of new regularization techniques.

Adriano Lana Cherchiglia - UFABC

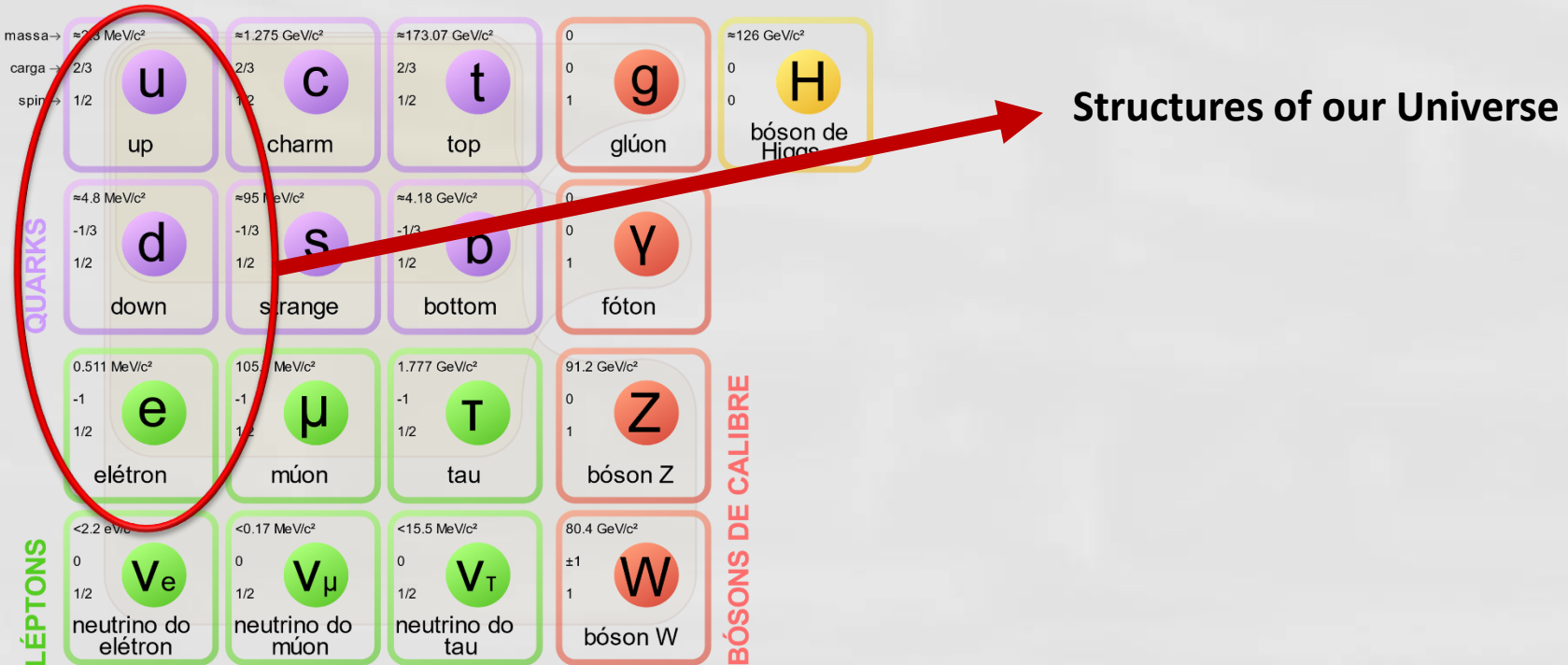
Valencia, 1st March 2018

Standard Model



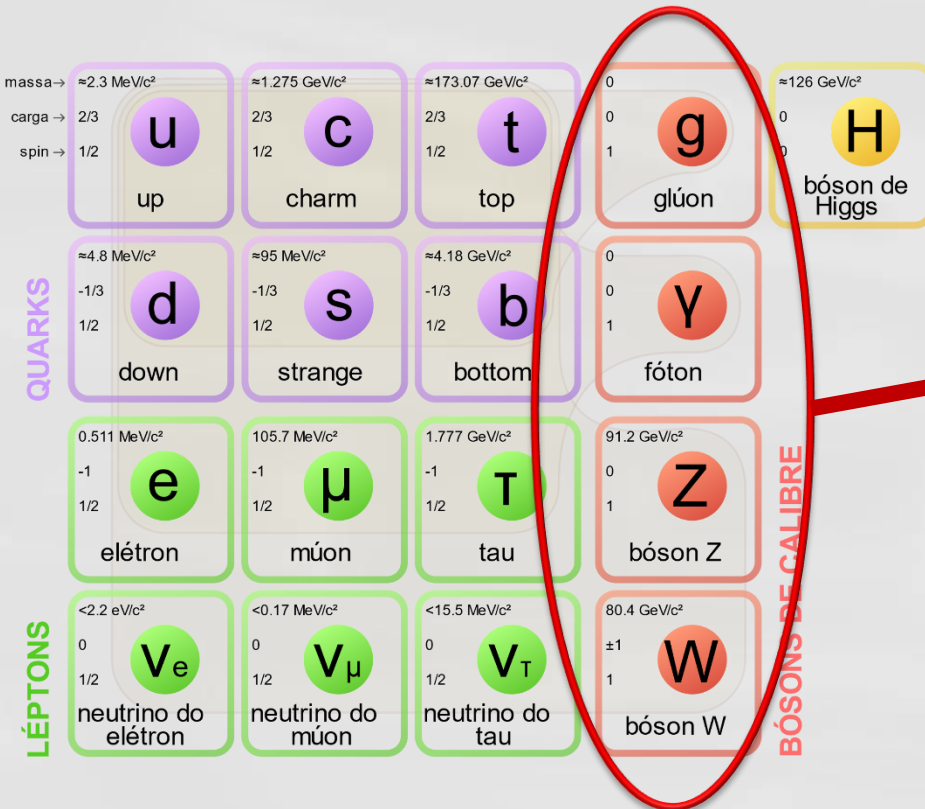
<https://commons.wikimedia.org/w/index.php?curid=49632920>

Standard Model



<https://commons.wikimedia.org/w/index.php?curid=49632920>

Standard Model

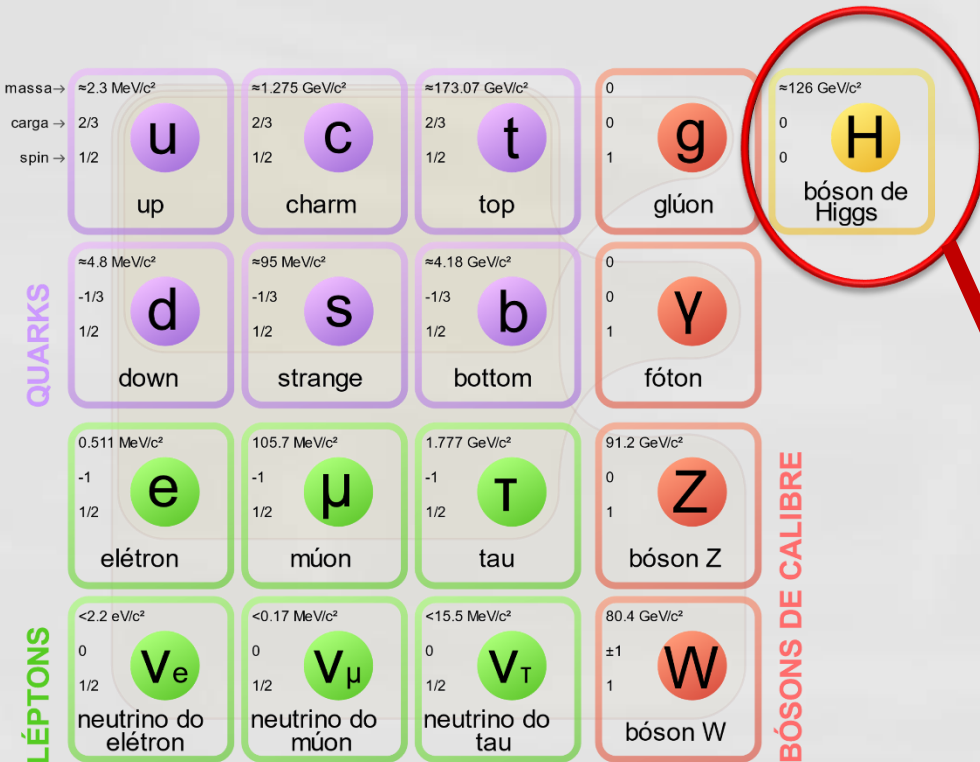


Structures of our Universe

Mediators of strong, weak, and electromagnetic forces

<https://commons.wikimedia.org/w/index.php?curid=49632920>

Standard Model

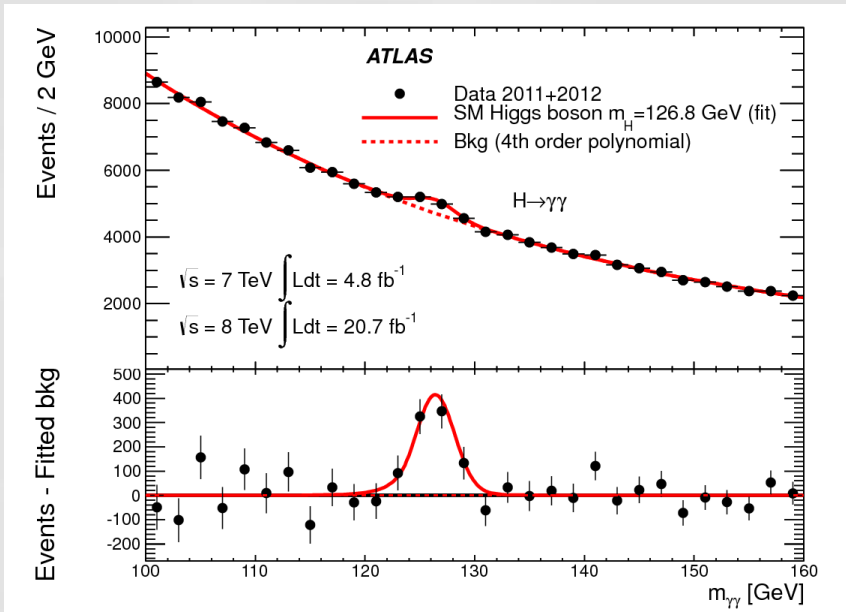


Structures of our Universe

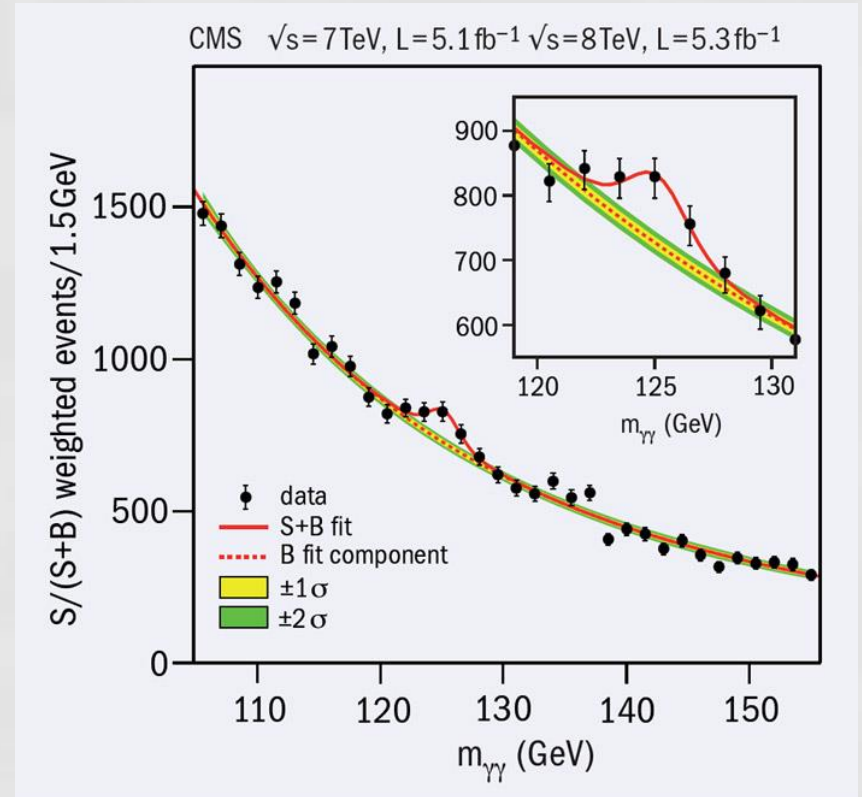
Mediators of strong, weak, and electromagnetic forces

Gives masses to all particles

Standard Model (2012)



Phys.Lett. B716 (2012) 1-29

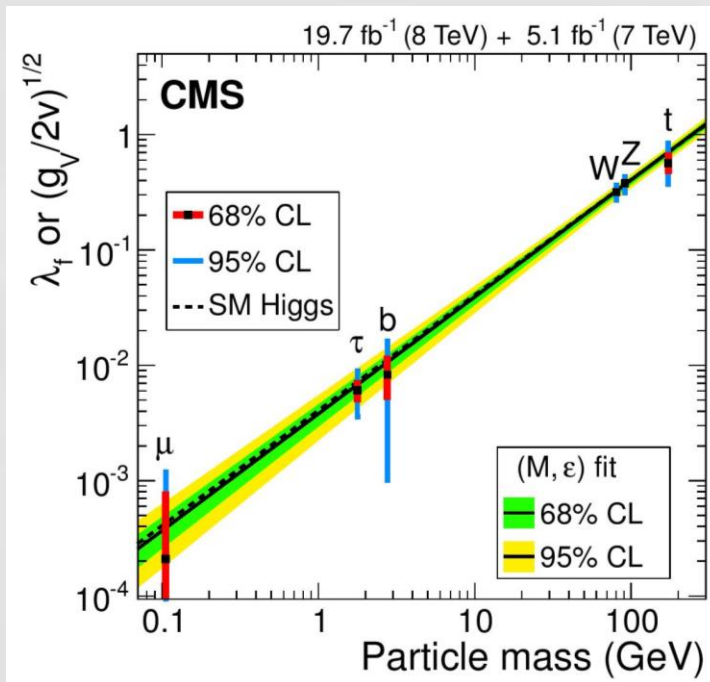


Phys.Lett. B716 (2012) 30-61

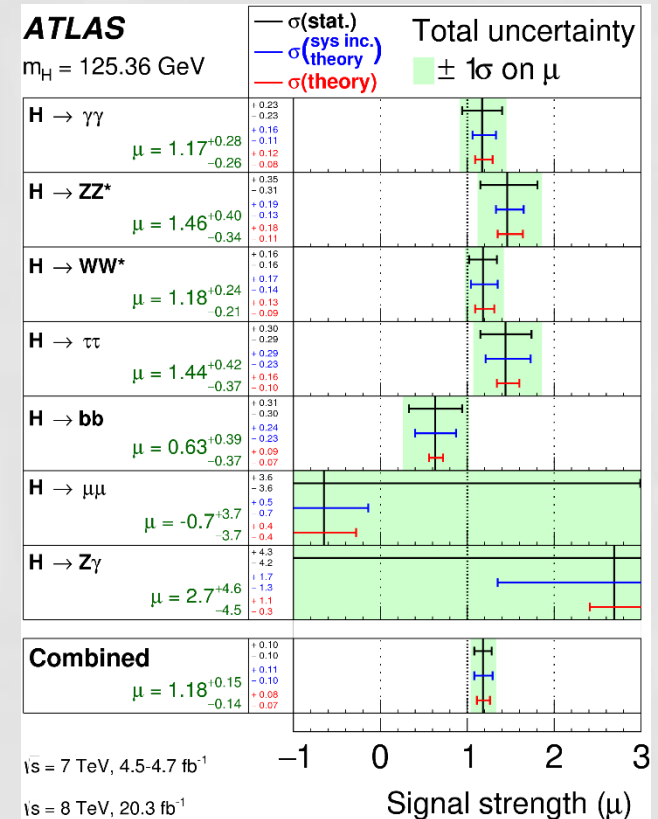
Higgs boson discovered!

Standard Model (pos-2012)

- Scalar sector scrutinized
 - Is the scalar found the one of the SM?
 - Are there other scalars?



Eur. Phys. J. C 75 (2015) 212



Eur. Phys. J. C 76 (2016) 6

Standard Model (pos-2012)

- High Energy Physics (colliders)
 - Small deviations

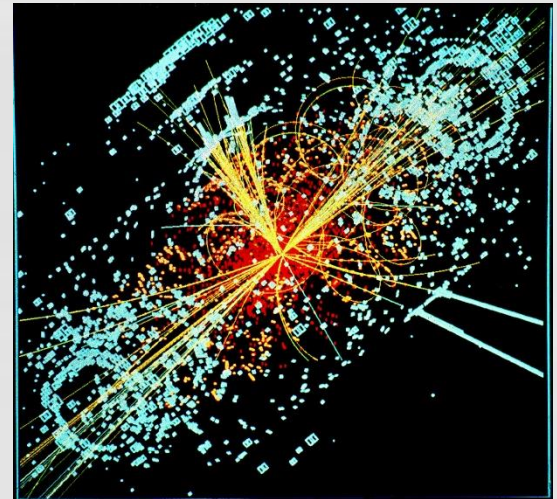


BSM predictions

Increase precision



- Open questions:
 - No gravitational interactions included;
 - No candidates for Dark Matter;
 - Matter-antimatter asymmetry;
 -



<http://cds.cern.ch/record/628469>

Beyond Standard Model

- Two-Higgs-Doublet-Model (2HDM)

- Minimal extension to scalar sector
- Four more scalars

Some variants (inert model) have dark matter candidate

- Supersymmetry (SUSY)

- Correlates bosons and fermions;
- Has a non-minimal scalar sector (2HDM);
- Predicts a partner to each particle of SM.



- Dark matter candidate;
- Solves naturalness problem.

In the search for New Physics

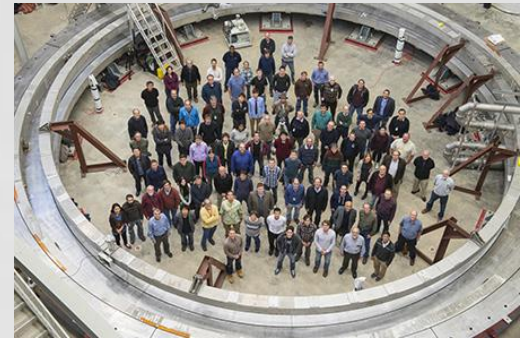
- High Energy Physics (colliders)
 - Still no sign of New Physics.



- Low-energy observables
 - Muon magnetic moment.



<https://cds.cern.ch/record/1295244>



<http://muon-g-2.fnal.gov/>

$(g - 2)_\mu$

- Basic idea

$$H_B = -\vec{\mu} \cdot \vec{B}$$

$$\vec{\mu}_s = g \left(\frac{q}{2m} \right) \vec{s}$$

- Dirac: $g = 2$
- Quantum Field Theory: $g = 2 + \dots$

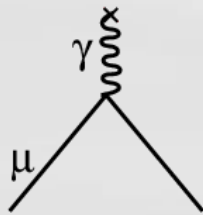


<http://muon-g-2.fnal.gov/>

→
$$a_\mu = \frac{(g - 2)}{2}$$

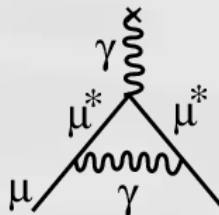
$(g - 2)_\mu$

- Theory (Standard Model)



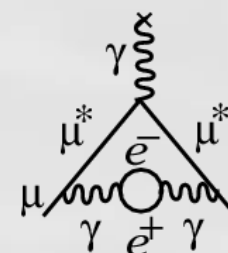
Dirac
(a)

$$a_\mu = 0$$



Schwinger
(b)

$$a_\mu = \frac{\alpha}{2\pi} \approx 0.0011623\dots$$



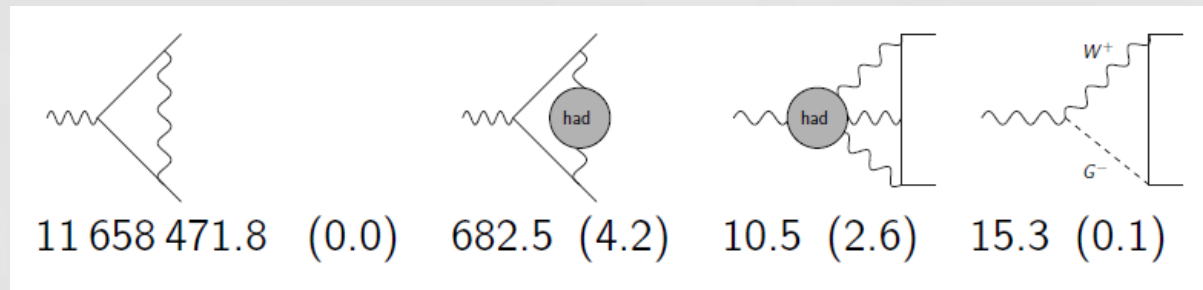
(c)

$$a_\mu = c_2 \left(\frac{\alpha}{\pi} \right)^2 \approx -0.0000176\dots$$

Rept.Prog.Phys.70:795,2007

$(g - 2)_\mu$

- Theory (Standard Model)



Thanks to D. Stöckinger

$$a_\mu^{th} = (11\,659\,180.8 \pm 3.6) \times 10^{-10}$$

“Muon $g-2$ Theory Initiative”

$(g - 2)_\mu$

- Experiment

- E821 BNL
- Final result in 2004



<http://www.g-2.bnl.gov/>

$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

Phys. Rev. Lett. 92, 161802 (2004)

$$(g - 2)_\mu$$

- Theory X Experiment


$$a_\mu^{th} = (11\,659\,180.8 \pm 3.6) \times 10^{-10}$$

$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

$(g - 2)_\mu$

- Theory X Experiment

$$a_\mu^{exp} - a_\mu^{th} = (28.1 \pm 7.3) \times 10^{-10}$$

3.8 σ ! 

$(g - 2)_\mu$

- Theory X Experiment

$$a_\mu^{exp} - a_\mu^{th} = (28.1 \pm 7.3) \times 10^{-10}$$



BSM



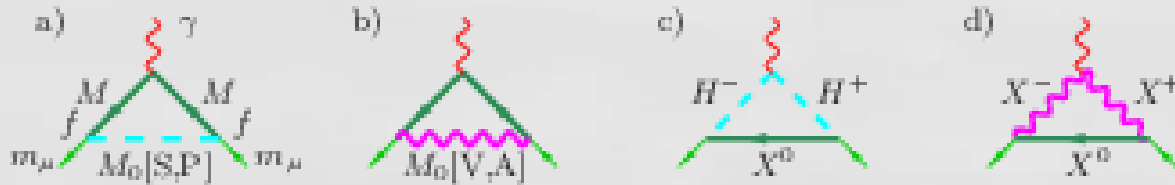
Increase precision



<http://muon-g-2.fnal.gov/>

$$(g - 2)_\mu$$

- Beyond Standard Model:
 - Radiative corrections

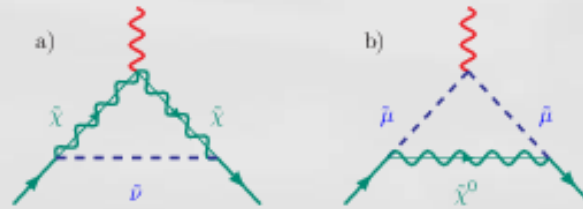


Phys.Rept. 477 (2009) 1-110

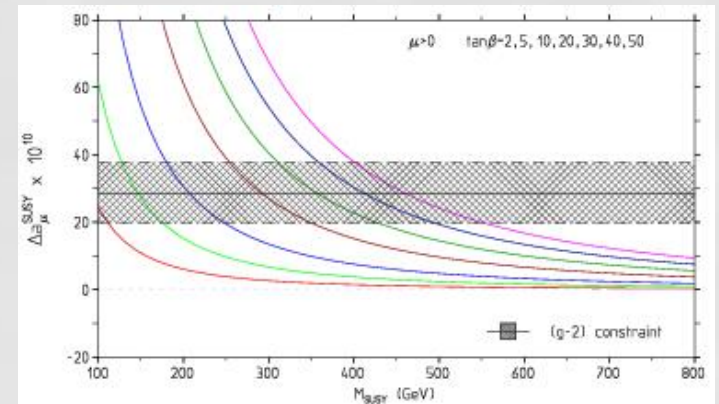
- Complementary information to direct searches (LHC)

$(g - 2)_\mu$

- Beyond Standard Model:
 - SUSY

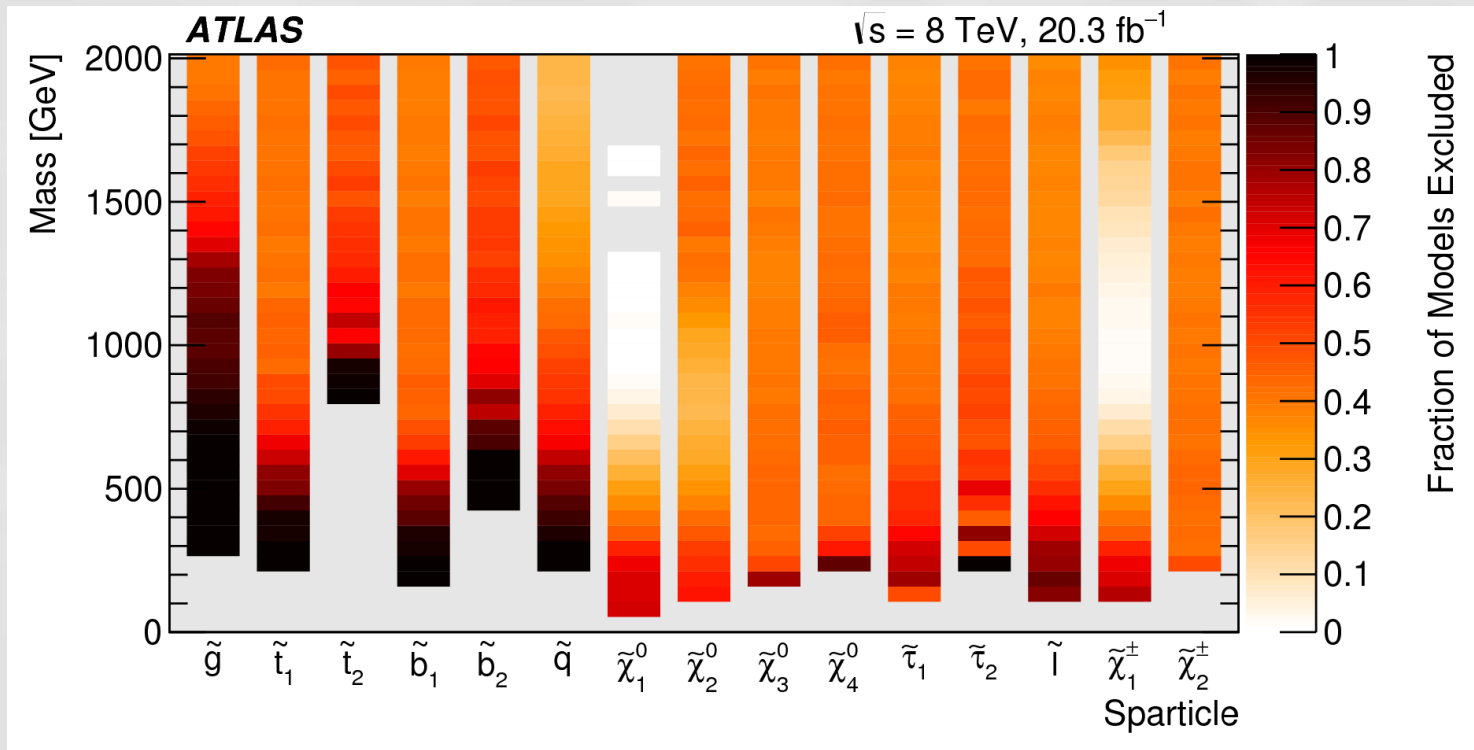


$$a_\mu(SUSY) \cong 123 \times 10^{-11} \left(\frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 \tan \beta$$



$$(g - 2)_\mu$$

- Beyond Standard Model:
 - SUSY



$$(g - 2)_\mu$$

- Beyond Standard Model:

- SUSY



Other scenarios



Ex: extensions to the scalar sector

$$\mathcal{L}_S = (D_\mu \phi_1)^\dagger (D^\mu \phi_1) + (D_\mu \phi_2)^\dagger (D^\mu \phi_2) - V(\phi_1, \phi_2)$$

2HDM



Invariant under CP



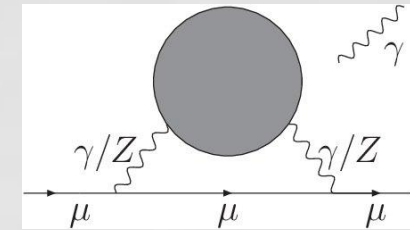
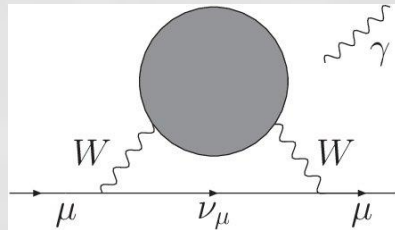
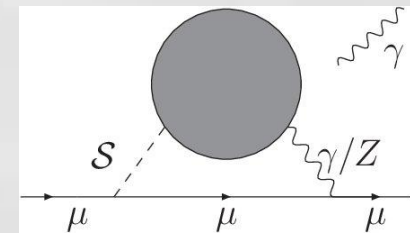
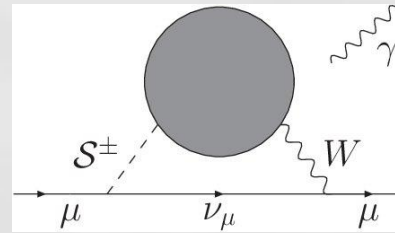
$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM

- More loops



Higher Precision



Complete two-loop prediction



Leading order (one-loop suppressed by m_μ^2/m_H^2)

$(g - 2)_\mu$

- Beyond Standard Model : 2HDM

S: h, H, A, H^\pm

$$\tan\beta = \frac{v_2}{v_1}$$

Flavour-aligned: $\varepsilon_l, \varepsilon_u, \varepsilon_d$

$$Y_f^h = s_{\beta\alpha} + c_{\beta\alpha}\varepsilon_f$$

$$Y_f^H = c_{\beta\alpha} - s_{\beta\alpha}\varepsilon_f$$

$$Y_{d,l}^A = -\varepsilon_{d,l} \quad Y_u^A = \varepsilon_u$$

$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM

S: h, H, A, H^\pm

$$\tan\beta = \frac{v_2}{v_1}$$

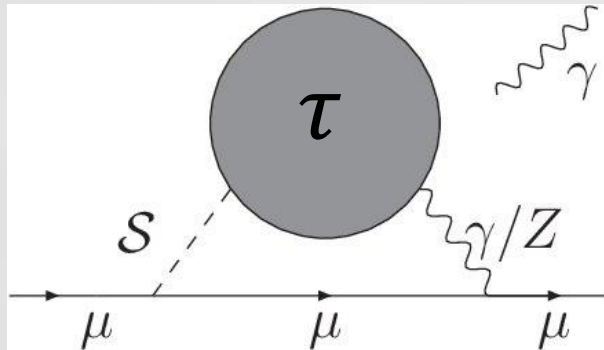
Flavour-aligned: $\varepsilon_l, \varepsilon_u, \varepsilon_d$

Constraints

- B-physics;
- Tau decay;
- $Z \rightarrow \tau\tau$;
- Collider;
- Theoretical;
- S, T, U parameters.

$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM
 - Tau loop



Constraints

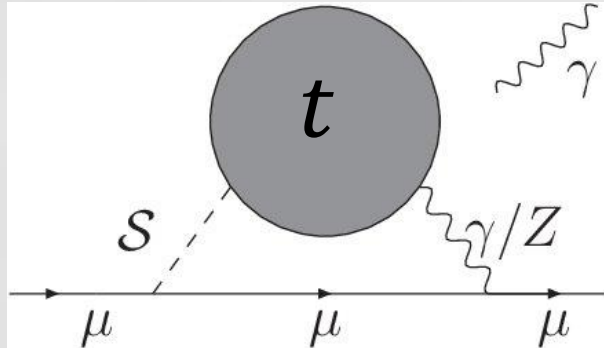
- B-physics;
- Tau decay;
- $Z \rightarrow \tau\tau$;
- Collider;
- Theoretical;
- S, T, U parameters.

$S: h, H, A, H^\pm$

Flavour-aligned: $\epsilon_l, \epsilon_u, \epsilon_d$

$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM
 - Top loop



Constraints

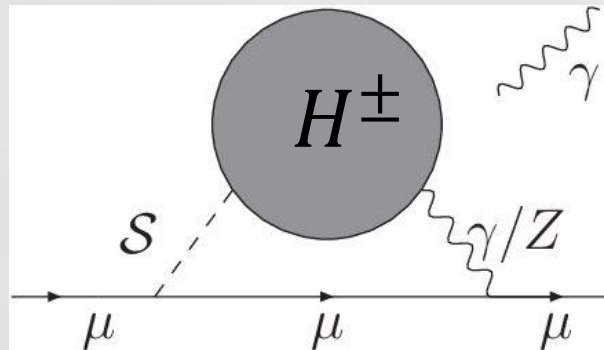
- B-physics;
- Tau decay;
- $Z \rightarrow \tau\tau$;
- Collider;
- Theoretical;
- S, T, U parameters.

S: h, H, A, H^\pm

Flavour-aligned: $\epsilon_l, \epsilon_u, \epsilon_d$

$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM
 - Bosonic



Constraints

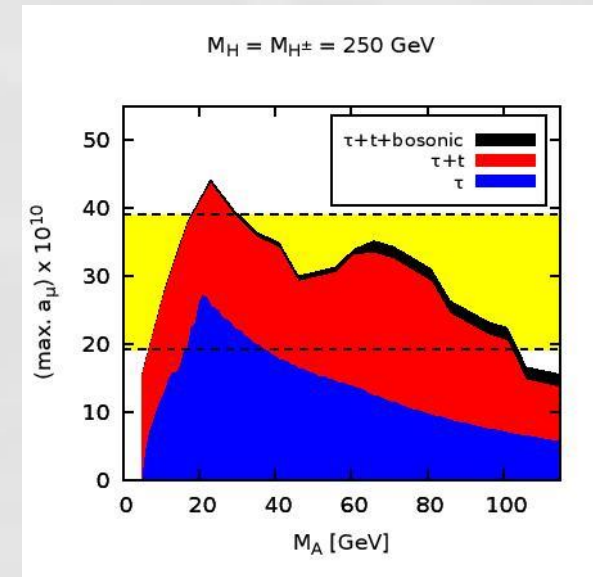
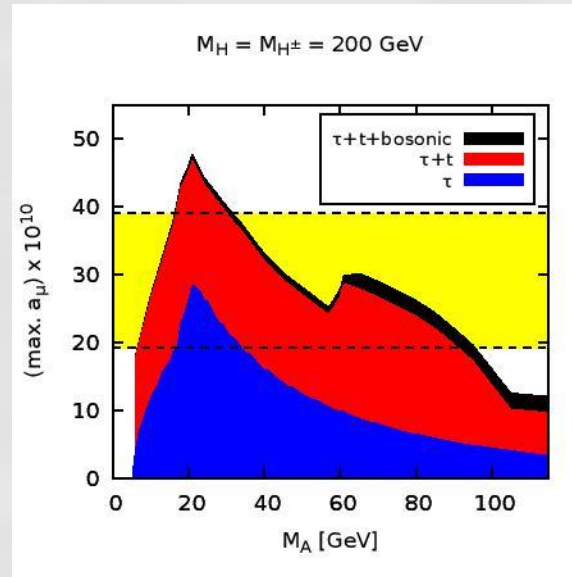
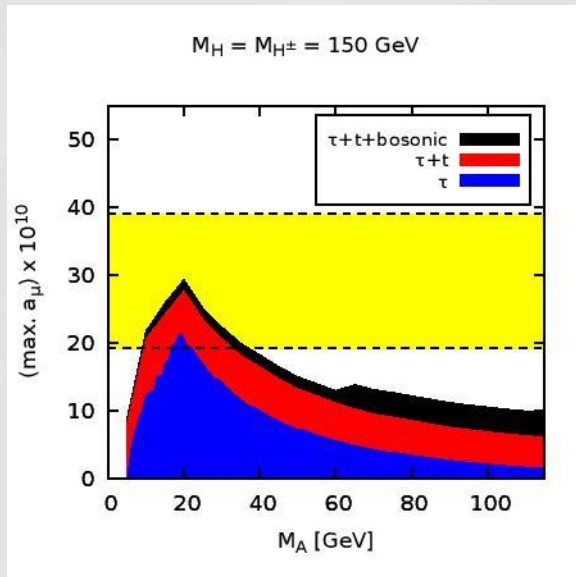
- B-physics;
- Tau decay;
- $Z \rightarrow \tau\tau$;
- Collider;
- Theoretical;
- S, T, U parameters.

S : h, H, A, H^\pm

Flavour-aligned: $\epsilon_l, \epsilon_u, \epsilon_d$

$$(g - 2)_\mu$$

- Beyond Standard Model : 2HDM



Precision Frontier

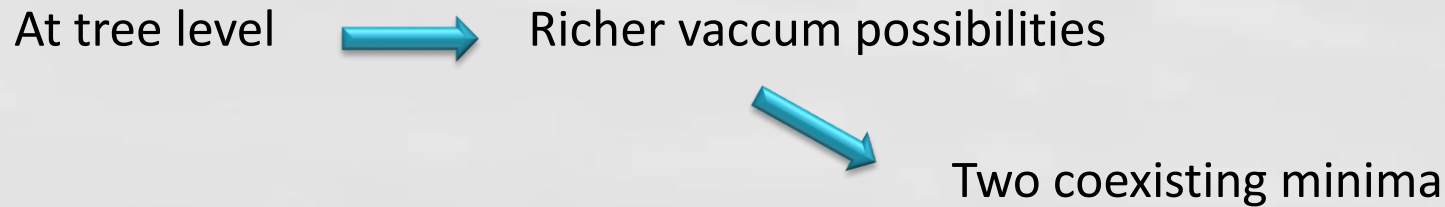
- One-loop (and Beyond?): example in the 2HDM

At tree level

$$V_0 = m_{11}^2 |\phi_1|^2 + m_{22}^2 |\phi_2|^2 - [m_{12}^2 \phi_1^\dagger \phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 |\phi_1|^4 + \frac{1}{2} \lambda_2 |\phi_2|^4 \\ + \lambda_3 |\phi_1|^2 |\phi_2|^2 + \lambda_4 |\phi_1^\dagger \phi_2|^2 + \left\{ \frac{1}{2} \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{h.c.} \right\}$$

Precision Frontier

- One-loop (and Beyond?): example in the 2HDM



$$\langle \varphi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle \varphi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

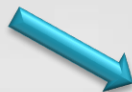
Precision Frontier

- One-loop (and Beyond?): example in the 2HDM

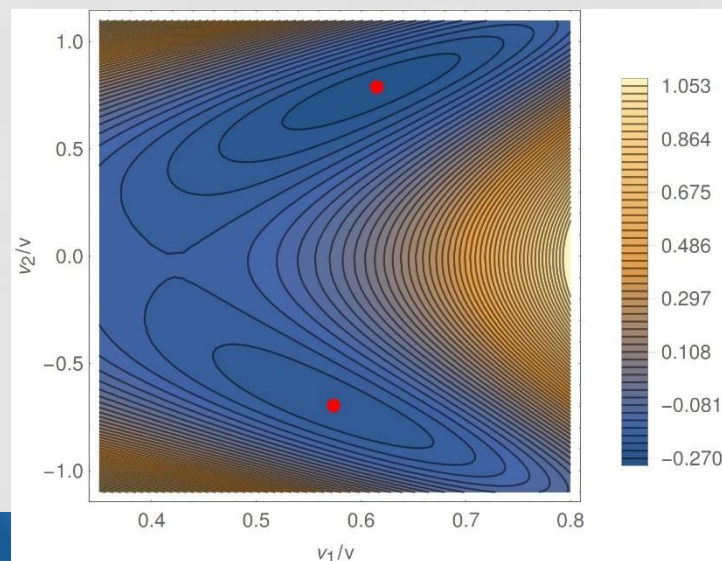
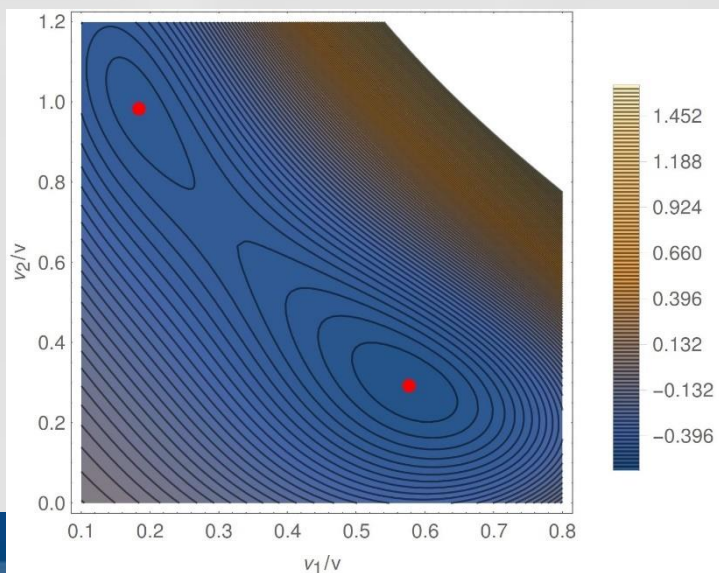
At tree level



Richer vacuum possibilities



Two coexisting minima



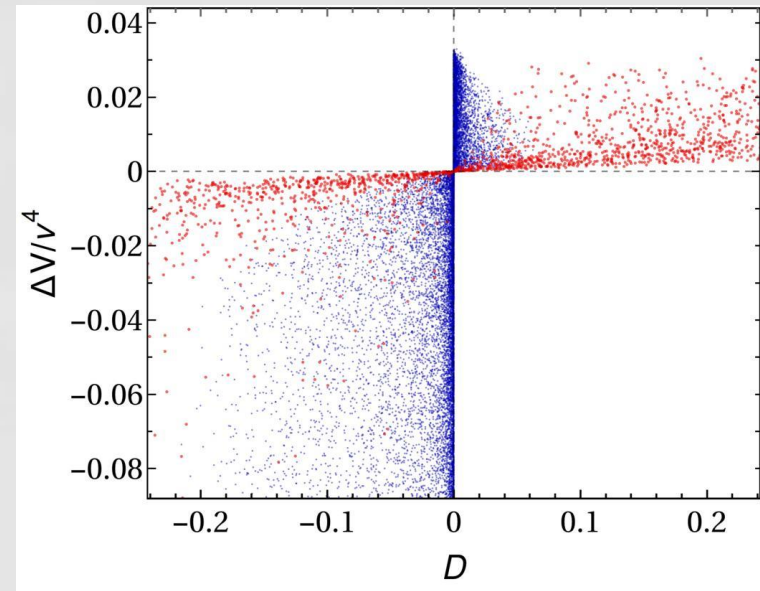
Precision Frontier

- One-loop (and Beyond?): example in the 2HDM

At tree level \longrightarrow Straightforward to find global minimum

$$D = \frac{1}{v^4} m_{12}^2 (m_{11}^2 - k^2 m_{22}^2) s_{2\beta} (t_\beta^2 - k^2)$$

$$k \equiv (\lambda_1/\lambda_2)^{1/4}$$



Precision Frontier

- One-loop (and Beyond?): example in the 2HDM

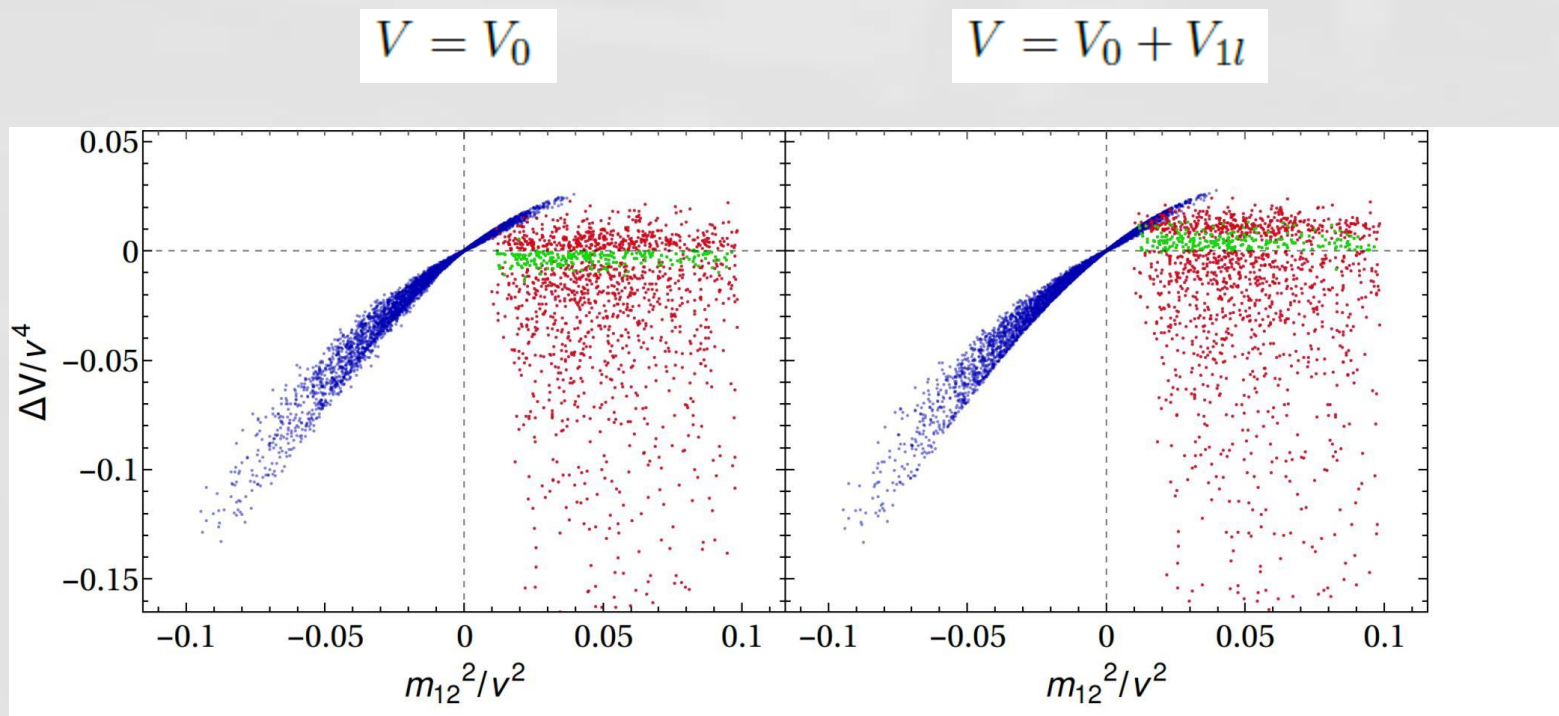
Adding one-loop

$$V = V_0 + V_{1l}$$

$$V_{1l} = \frac{1}{64\pi^2} \sum_k c_k M_k^4(\varphi_i) \left(\ln \frac{M_k^2(\varphi_i)}{\mu^2} - \frac{3}{2} \right)$$

Precision Frontier

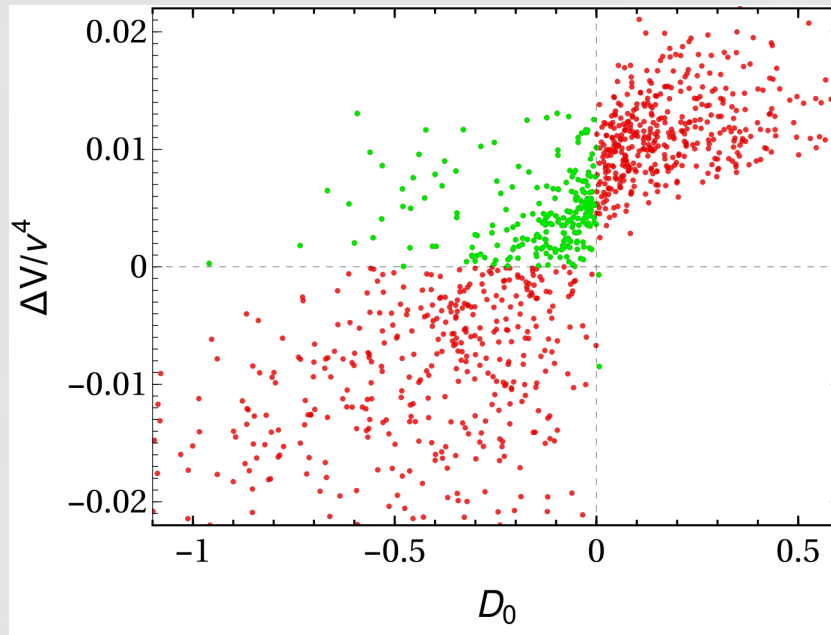
- One-loop (and Beyond?): example in the 2HDM



Precision Frontier

- One-loop (and Beyond?): example in the 2HDM

Adding one-loop  **NO** straightforward way to find global minimum



Precision Frontier

- High Energy Physics



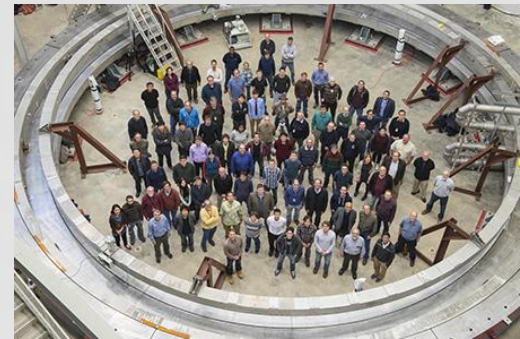
Precision calculations



- Low-energy observables



<https://cds.cern.ch/record/1295244>



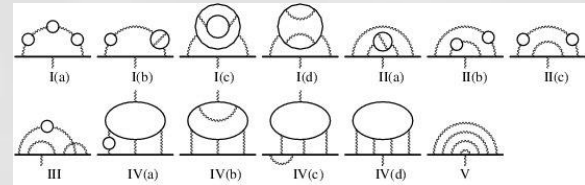
<http://muon-g-2.fnal.gov/>

Precision Frontier

- More loops



More efficient algorithms



Phys.Rev.Lett. 109 (2012) 111808



New regularization techniques

Precision Frontier

• New regularization techniques:

Eur. Phys. J. C (2014) 74:3197
DOI 10.1140/epjc/s10052-014-3197-4

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

On the four-dimensional formulation of dimensionally regulated amplitudes

A. R. Fazio^{1,a}, P. Mastrolia^{2,3,b}, E. Mirabella^{3,c}, W. J. Torres Bobadilla^{1,2,d}

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Four-dimensional unsubtraction with massive particles

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german.rodrigo@csic.es

 [Journal of High Energy Physics](#)
November 2012, 2012:151

A four-dimensional approach to quantum field theories

Authors [Authors and affiliations](#)

R. Pittau 

Article
First Online: [27 November 2012](#)
DOI: [10.1007/JHEP11\(2012\)151](https://doi.org/10.1007/JHEP11(2012)151)

Cite this article as:
Pittau, R. J. High Energ. Phys. (2012) 2012:
151. doi:10.1007/JHEP11(2012)151

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Precision Frontier

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article - Theoretical Physics

To d , or not to d : recent developments and comparisons of regularization schemes

C. Gnendiger^{1,a}, A. Signer^{1,2}, D. Stöckinger³, A. Broggio⁴, A. L. Cherchiglia⁵, F. Driencourt-Mangin⁶, A. R. Fazio⁷, B. Hiller⁸, P. Mastrolia^{9,10}, T. Peraro¹¹, R. Pittau¹², G. M. Pruna¹, G. Rodrigo⁶, M. Sampaio¹³, G. Sborlini^{6,14,15}, W. J. Torres Bobadilla^{6,9,10}, F. Tramontano^{16,17}, Y. Ulrich^{1,2}, A. Visconti^{1,2}

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¹¹ Higgs Centre for Theoretical Physics, The University of Edinburgh, Edinburgh EH9 3FD, UK

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¹⁶ Dipartimento di Fisica, Università di Napoli, 80126 Naples, Italy

¹⁷ INFN, Sezione di Napoli, 80126 Naples, Italy

Precision Frontier

- New regularization techniques:
 - Implicit Regularization

O. A. BATTISTEL, A. L. MOTA, and M. C. NEMES, *Mod. Phys. Lett. A* **13**, 1597 (1998). DOI: <http://dx.doi.org/10.1142/S0217732398001686>

CONSISTENCY CONDITIONS FOR 4-D REGULARIZATIONS

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M. C. NEMES

Departamento de Física, ICEX, Universidade Federal de Minas Gerais, Belo Horizonte, CEP 30161-970, C.P. 702, MG, Brazil

Received: 21 November 1997

From the study of well-known divergent amplitudes we deduce three consistency conditions which are necessary and sufficient to eliminate ambiguities and symmetry violations. The conditions relate divergent integrals of the same degree of divergence and are automatically satisfied within the context of dimensional regularization. We show how the deduced consistency conditions can be satisfied in four-dimensional regularizations. An important conclusion of the present study is the possibility of working with 4-D regularization schemes which preserve the virtues of dimensional regularization avoiding, however, its restrictions.



Sistematization of ambiguities

PHYSICAL REVIEW D **87**, 065011 (2013)

(Un)determined finite regularization-dependent quantum corrections: The Higgs boson decay into two photons and the two-photon scattering examples

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²Departamento de Física, Universidade Federal do Tocantins, P.O. Box 132, Araguaina, Tocantins 77804-970, Brazil

(Received 7 November 2012; published 15 March 2013)

We investigate the appearance of arbitrary, regularization-dependent parameters introduced by divergent integrals in two *a priori* finite but superficially divergent amplitudes: the Higgs decay into two photons and the two-photon scattering. We use a general parametrization of ultraviolet divergences which makes explicit such ambiguities. Thus we separate in a consistent way using implicit regularization the divergent, finite, and regularization-dependent parts of the amplitudes which in turn are written as surface terms. We find that, although finite, these amplitudes are ambiguous before the imposition of physical conditions, namely, momentum routing invariance in the loops of Feynman diagrams. In the examples we study, momentum routing invariance turns out to be equivalent to gauge invariance. We also discuss the results obtained by different regularizations and show how they can be reproduced within our framework allowing for a clear view on the origin of regularization ambiguities.

DOI: 10.1103/PhysRevD.87.065011

PACS numbers: 11.10.Gh, 13.40.Hg, 14.80.Bn

PHYSICAL REVIEW D **86**, 025016 (2012)

Momentum routing invariance in Feynman diagrams and quantum symmetry breakings

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We illustrate with examples that quantum symmetry breakings in perturbation theory are connected to breakdown of momentum routing invariance (MRI) in the loops of a Feynman diagram. We show that MRI is a necessary and sufficient condition to preserve Abelian gauge symmetry at arbitrary loop order. We adopt the implicit regularization framework in which surface terms that are directly connected to momentum routing can be constructed to arbitrary loop order. The interplay between momentum routing invariance, surface terms and anomalies is discussed. We also illustrate that MRI is important to preserve supersymmetry. For theories with poor symmetry content, such as scalar field theories, MRI is shown to be important in the calculation of renormalization group functions.

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Precision Frontier

- New regularization techniques:
 - Implicit Regularization

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**SYSTEMATIC IMPLEMENTATION OF
IMPLICIT REGULARIZATION FOR
MULTILOOP FEYNMAN DIAGRAMS**

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Implicit Regularization (IReg) is a candidate to become an invariant framework in momentum space to perform Feynman diagram calculations to arbitrary loop order. In this work we present a systematic implementation of our method that automatically displays the terms to be subtracted by Bogoliubov's recursion formula. Therefore, we achieve a twofold objective: we show that the IReg program respects unitarity, locality and Lorentz invariance and we show that our method is consistent since we are able to display the divergent content of a multiloop amplitude in a well-defined set of basic divergent integrals in one-loop momentum only which is the essence of IReg. Moreover, we conjecture that momentum routing invariance in the loops, which has been shown to be connected with gauge symmetry, is a fundamental symmetry of any Feynman diagram in a renormalizable quantum field theory.



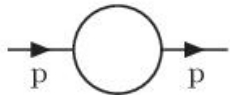
Satisfies Lorentz invariance,
locality and unitarity

Precision Frontier

- New regularization techniques :



Efficiency in the identification and removal of divergences.



$$\int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2} \frac{1}{(k-p)^2}$$



Divergent Part + Finite Part



$$\frac{1}{\varepsilon}$$



$$\int_k \frac{1}{(k^2 - \mu^2)^3}$$

Precision Frontier

- New regularization techniques :



Efficiency in the identification and removal of divergences

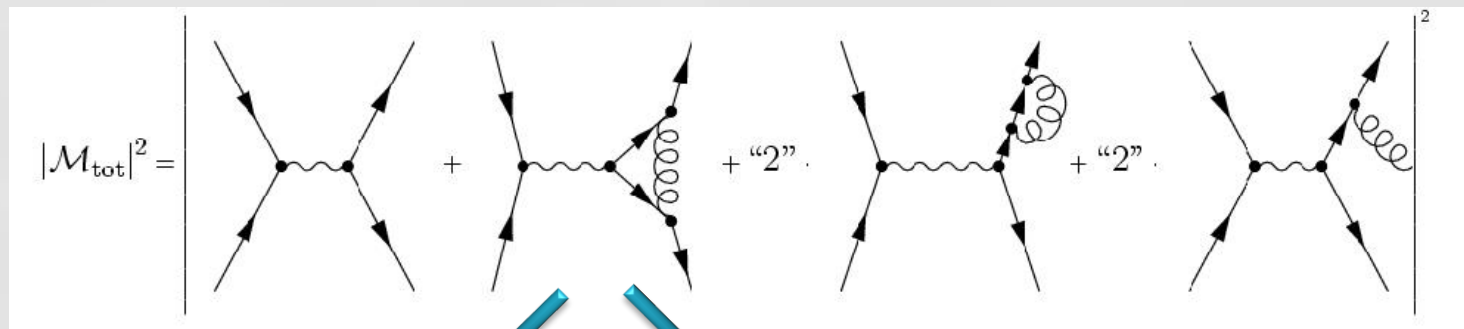


Allows numerical implementation

Precision Frontier

- Example of NLO observable with IReg

$$e^+e^- \rightarrow q\bar{q} (g)$$



Ultraviolet

Infrared

<https://www.ippp.dur.ac.uk>

Precision Frontier

- Example of NLO observable with IReg

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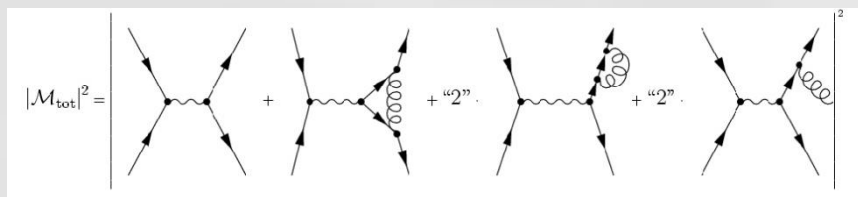
To d , or not to d : recent developments and comparisons of regularization schemes

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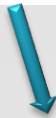
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Precision Frontier

- Example of NLO observable with IReg



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$$\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p_q) \gamma^\nu (\not{k} + \not{p}_q) \gamma_\mu (\not{k} - \not{p}_{\bar{q}}) \gamma_\nu u(p_{\bar{q}})}{k^2 (k + p_q)^2 (k - p_{\bar{q}})^2}$$

Precision Frontier

- Example of NLO observable with IReg

$$\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p_q)\gamma^\nu(\not{k} + \not{p}_q)\gamma_\mu(\not{k} - \not{p}_{\bar{q}})\gamma_\nu u(p_{\bar{q}})}{k^2(k + p_q)^2(k - p_{\bar{q}})^2}$$

- Perform Dirac algebra in 4D;
- Add μ^2 in propagators;
- Use

$$\frac{1}{(k - p)^2 - \mu^2} = \frac{1}{(k^2 - \mu^2)} + \frac{(-1)(p^2 - 2p \cdot k)}{(k^2 - \mu^2)[(k - p)^2 - \mu^2]}$$

Precision Frontier

- Example of NLO observable with IReg

$$\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p_q)\gamma^\nu(\not{k} + \not{p}_q)\gamma_\mu(\not{k} - \not{p}_{\bar{q}})\gamma_\nu u(p_{\bar{q}})}{k^2(k + p_q)^2(k - p_{\bar{q}})^2}$$

- Perform Dirac algebra in 4D;
- Add μ^2 in propagators;
- Isolate UV divergences as

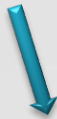
$$I_{\log}^{\nu_1 \dots \nu_{2N}}(\mu^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{k^{\nu_1} \dots k^{\nu_{2N}}}{(k^2 - \mu^2)^{N+2}}$$

- Infrared divergences appear, after integration, as logarithms in μ^2 .

Precision Frontier

- Example of NLO observable with IReg

$$\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p_q) \gamma^\nu (\not{k} + \not{p}_q) \gamma_\mu (\not{k} - \not{p}_{\bar{q}}) \gamma_\nu u(p_{\bar{q}})}{k^2 (k + p_q)^2 (k - p_{\bar{q}})^2}$$

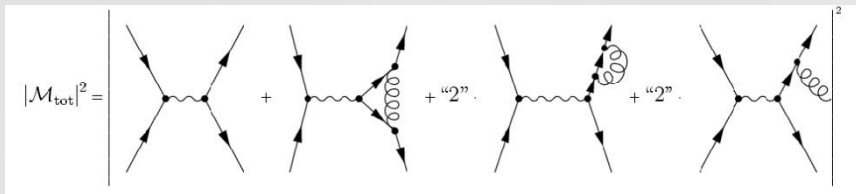


$$\sigma_{\text{IREG}}^{(v)} = \sigma^{(0)} \left(\frac{\alpha_s}{\pi} \right) C_F \left[-\frac{\ln^2(\mu_0)}{2} - \frac{3}{2} \ln(\mu_0) - \frac{7 - \pi^2}{2} + \mathcal{O}(\mu_0) \right],$$

$$\mu_0 \equiv \mu^2/s \quad s \equiv (p_q + p_{\bar{q}})^2 = 2p_q \cdot p_{\bar{q}}$$

Precision Frontier

- Example of NLO observable with IReg



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$$\begin{aligned} \sigma_{\text{IREG}}^{(r)} &= \frac{1}{2s} \int \frac{d^3 k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3 k_2}{(2\pi)^3 2\omega_2} \\ &\times \int \frac{d^3 k_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta^{(4)}(q - k_1 - k_2 - k_3) \\ &\times M_{\text{IREG}}^{(0)}(q\bar{q}g), \end{aligned}$$

Precision Frontier

- Example of NLO observable with IReg

$$\begin{aligned}\sigma_{\text{IREG}}^{(r)} &= \frac{1}{2s} \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3k_2}{(2\pi)^3 2\omega_2} \\ &\times \int \frac{d^3k_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta^{(4)}(q - k_1 - k_2 - k_3) \\ &\times M_{\text{IREG}}^{(0)}(q\bar{q}g),\end{aligned}$$

- Add mass μ to quark and gluon;
- Infrared divergences appear, after integration, as logarithms in μ^2 .

Precision Frontier

- Example of NLO observable with IReg

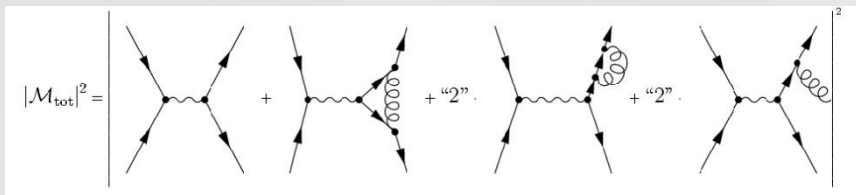
$$\begin{aligned}\sigma_{\text{IREG}}^{(r)} &= \frac{1}{2s} \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3k_2}{(2\pi)^3 2\omega_2} \\ &\times \int \frac{d^3k_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta^{(4)}(q - k_1 - k_2 - k_3) \\ &\times M_{\text{IREG}}^{(0)}(q\bar{q}g),\end{aligned}$$



$$\begin{aligned}\sigma_{\text{IREG}}^{(r)} &= \sigma^{(0)} \left(\frac{\alpha_s}{\pi} \right) C_F \\ &\times \left[\frac{\ln^2(\mu_0)}{2} + \frac{3}{2} \ln(\mu_0) + \frac{17}{4} - \frac{\pi^2}{2} + \mathcal{O}(\mu_0) \right]\end{aligned}$$

Precision Frontier

- Example of NLO observable with IReg



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$$\sigma^{(1)} = \sigma^{(0)} + \sigma_{\text{IREG}}^{(v)} + \sigma_{\text{IREG}}^{(r)}|_{\mu_0 \rightarrow 0} = \frac{Q_q^2 N_c}{3s} \left(\frac{e^4}{4\pi} \right) \times \left[1 + \left(\frac{\alpha_s}{4\pi} \right) 3C_F \right].$$

Conclusions

Standard Model

masa →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
carga →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u	c	t	g	H
	up	charm	top	gluon	bóson de Higgs
QUARKS					
	≈4.8 MeV/c ²	≈95 MeV/c ²	≈4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d	s	b	γ	
	down	strange	bottom	fóton	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e	μ	τ	Z	
	elétron	múon	tau	bóson Z	
LÉPTONS					BÓSONS DE CALIBRE
	≈2.2 eV/c ²	≈0.17 MeV/c ²	≈15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e	ν_μ	ν_τ	W	
	neutrino do elétron	neutrino do múon	neutrino do tau	bóson W	

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THEORY



<https://cds.cern.ch/record/1295244>

Small deviations

EXPERIMENT

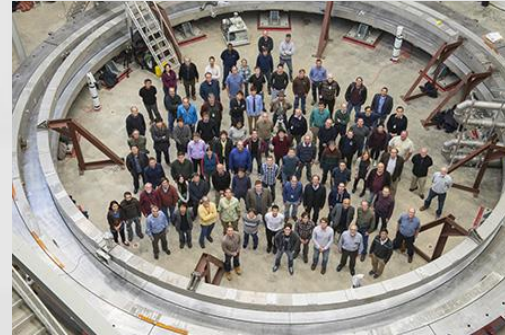
Conclusions

Standard Model

massa →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
carga →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g glúon	H bóson de Higgs
	↓	↓	↓	↓	↓
QUARKS	d down	s strange	b bottom	γ fóton	
	↓	↓	↓	↓	
	e elétron	μ múon	τ tau	Z bóson Z	
	↓	↓	↓	↓	
LÉPTONS	ν_e neutrino do elétron	ν_μ neutrino do múon	ν_τ neutrino do tau	W bóson W	BÓSONS DE CALIBRE

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THEORY



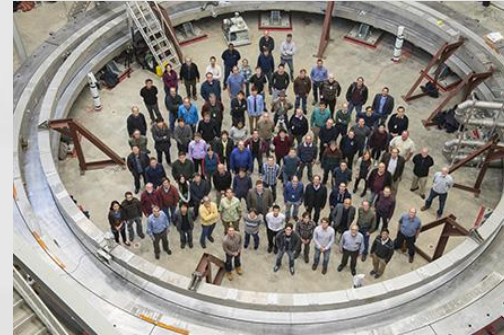
<http://muon-g-2.fnal.gov/>

3.8σ deviation

EXPERIMENT

Conclusions

Standard Model



<http://muon-g-2.fnal.gov/>

3.6 σ deviation

massa → carga → spin →	$\approx 2.3 \text{ MeV}/c^2$ 2/3 1/2 u up	$\approx 1.275 \text{ GeV}/c^2$ 2/3 1/2 c charm	$\approx 173.07 \text{ GeV}/c^2$ 2/3 1/2 t top	0 0 1 g gluon	$\approx 126 \text{ GeV}/c^2$ 0 0 H bóson de Higgs
	$\approx 4.8 \text{ MeV}/c^2$ -1/3 1/2 d down	$\approx 95 \text{ MeV}/c^2$ -1/3 1/2 s strange	$\approx 4.18 \text{ GeV}/c^2$ -1/3 1/2 b bottom	0 0 1 \gamma fóton	
QUARKS	$0.511 \text{ MeV}/c^2$ -1 1/2 e elétron	$105.7 \text{ MeV}/c^2$ -1 1/2 \mu múon	$1.777 \text{ GeV}/c^2$ -1 1/2 \tau tau	0 0 1 Z bóson Z	BÓSONS DE CALIBRE
	$\ll 2.2 \text{ eV}/c^2$ 0 1/2 \nu_e neutrino do elétron	$\ll 0.17 \text{ MeV}/c^2$ 0 1/2 \nu_\mu neutrino do múon	$\ll 15.5 \text{ MeV}/c^2$ 0 1/2 \nu_\tau neutrino do tau	$\approx 80.4 \text{ GeV}/c^2$ ± 1 1 W bóson W	
LÉPTONS					

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THEORY

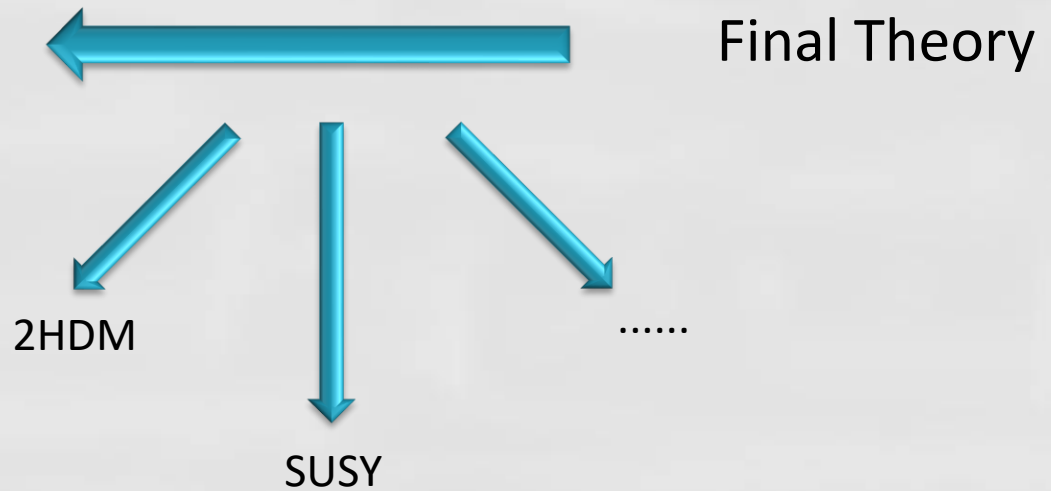
EXPERIMENT

Conclusions

Standard Model

massa →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
carga →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g glúon	H bóson de Higgs
	d down	s strange	b bottom	γ fóton	
QUARKS					
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e elétron	μ múon	τ tau	Z bóson Z	
	ν_e neutrino do elétron	ν_μ neutrino do múon	ν_τ neutrino do tau	W bóson W	
LÉPTONS					BÓSONS DE CALIBRE

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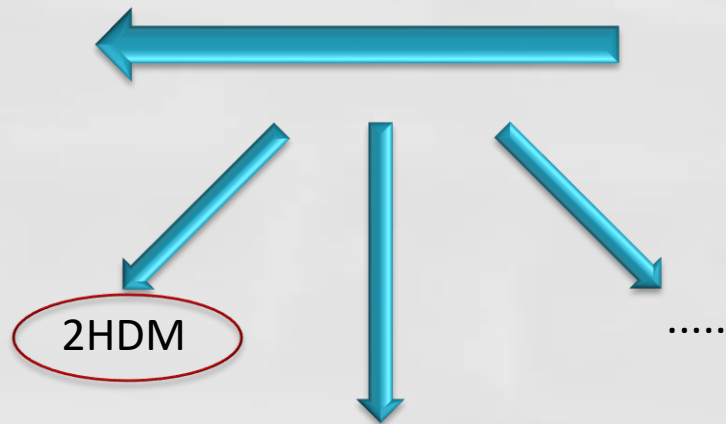


Conclusions

Standard Model

massa → carga → spin →	≈2.3 MeV/c ² 2/3 1/2 u up	≈1.275 GeV/c ² 2/3 1/2 c charm	≈173.07 GeV/c ² 2/3 1/2 t top	0 0 1 g glúon	≈126 GeV/c ² 0 0 H bóson de Higgs
QUARKS	≈4.8 MeV/c ² -1/3 1/2 d down	≈95 MeV/c ² -1/3 1/2 s strange	≈4.18 GeV/c ² -1/3 1/2 b bottom	0 0 1 γ fóton	
	0.511 MeV/c ² -1 1/2 e elétron	105.7 MeV/c ² -1 1/2 μ múon	1.777 GeV/c ² -1 1/2 τ tau	91.2 GeV/c ² 0 1 Z bóson Z	BÓSONS DE CALIBRE
LÉPTONS	<2.2 eV/c ² 0 1/2 ν_e neutrino do elétron	<0.17 MeV/c ² 0 1/2 ν_μ neutrino do múon	<15.5 MeV/c ² 0 1/2 ν_τ neutrino do tau	80.4 GeV/c ² ±1 1 W bóson W	

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Final Theory

Phenomenology
 $(g - 2)_\mu$

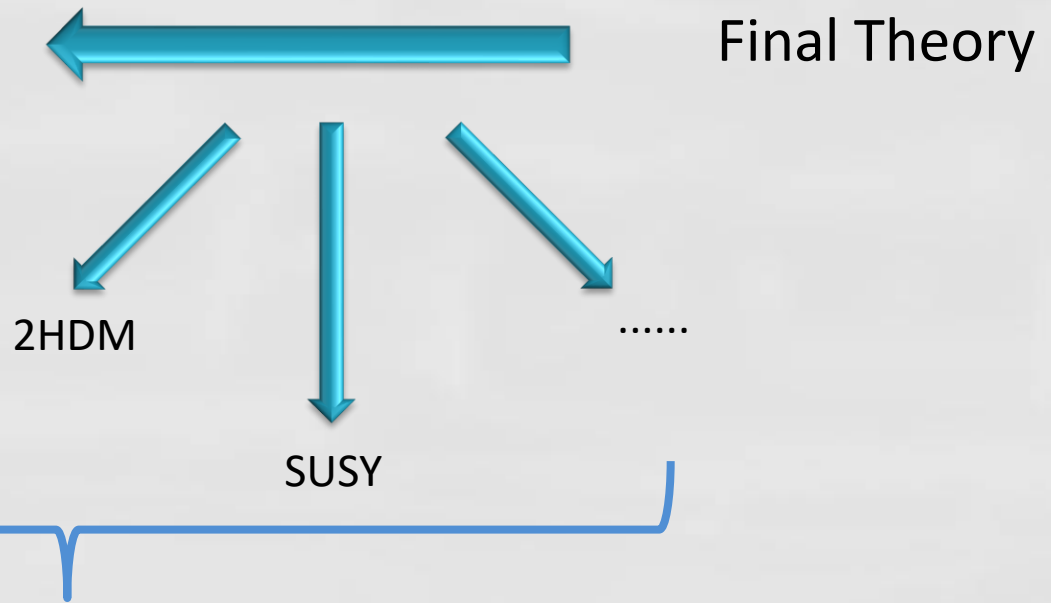
SUSY

Conclusions

Standard Model

massa →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
carga →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g glúon	H bóson de Higgs
	d down	s strange	b bottom	γ fóton	
QUARKS					
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e elétron	μ múon	τ tau	Z bóson Z	
	ν_e neutrino do elétron	ν_μ neutrino do múon	ν_τ neutrino do tau	W bóson W	
LÉPTONS					
					BÓSONS DE CALIBRE

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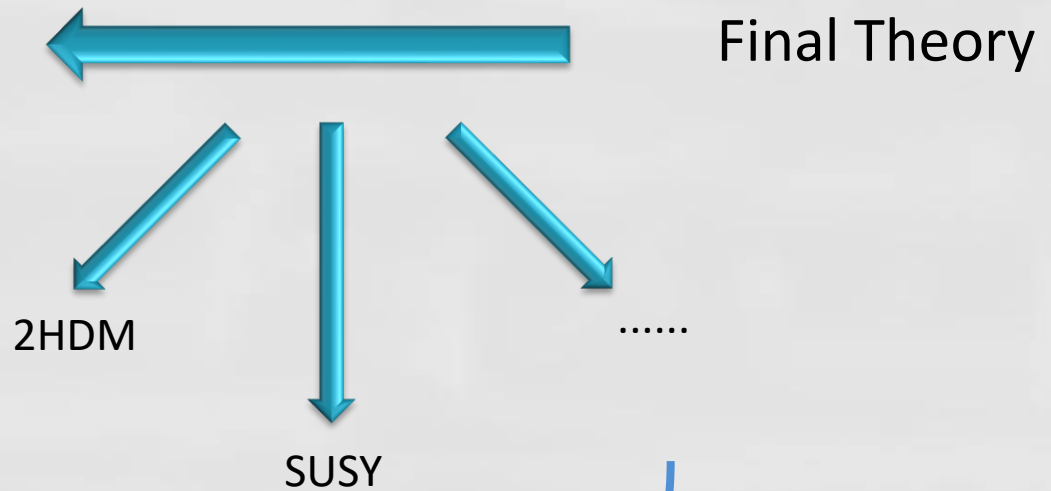
Precision calculations

Conclusions

Standard Model

masa →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
carga →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H bóson de Higgs
	d down	s strange	b bottom	γ fóton	
QUARKS	≈4.8 MeV/c ²	≈99 MeV/c ²	≈4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	e elétron	μ muón	τ tau	Z bóson Z	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	ν_e neutrino do elétron	ν_μ neutrino do muón	ν_τ neutrino do tau	W bóson W	
LÉPTONS	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
				BÓSONS DE CALIBRE	

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Precision calculations

Implicit Regularization

Thanks!