

Threshold and jet radius joint resummation for single-inclusive jet production

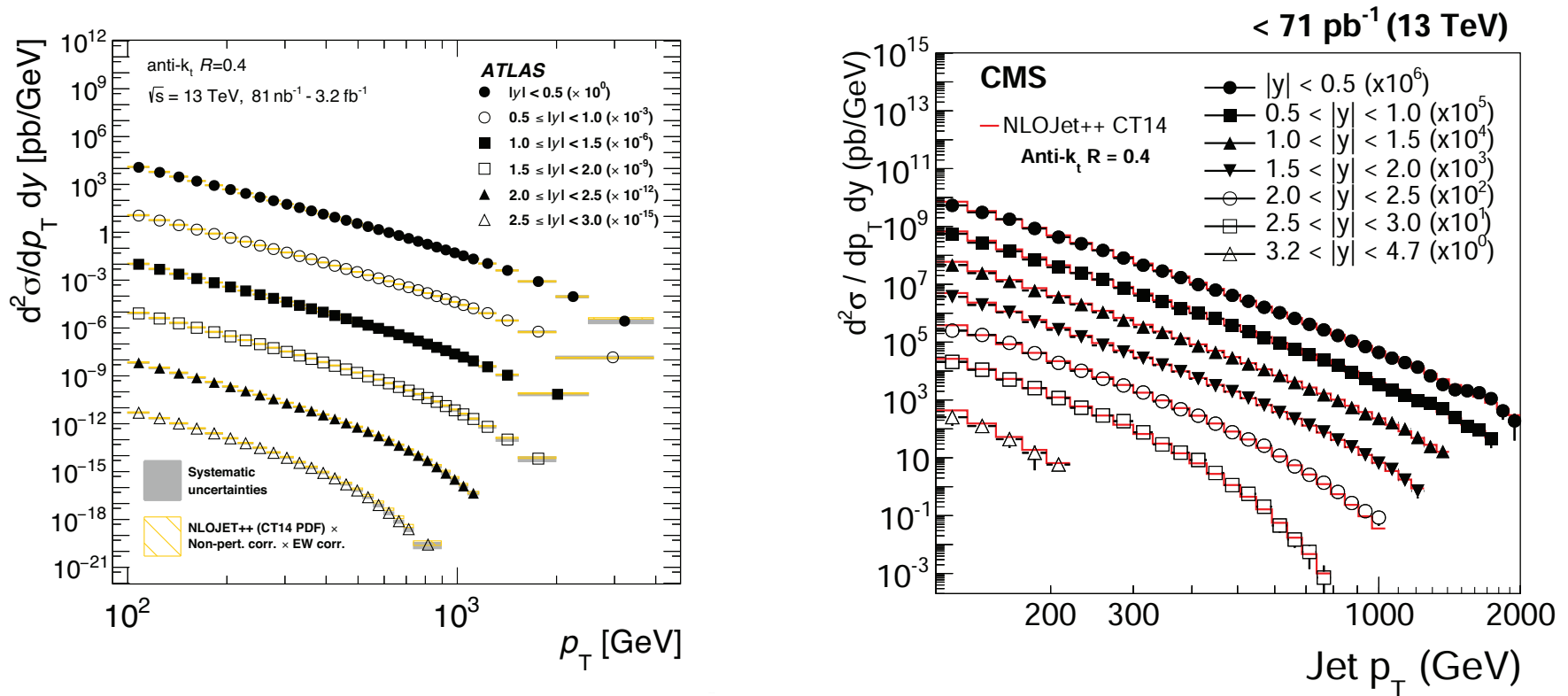
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Based on work done in collaboration with:

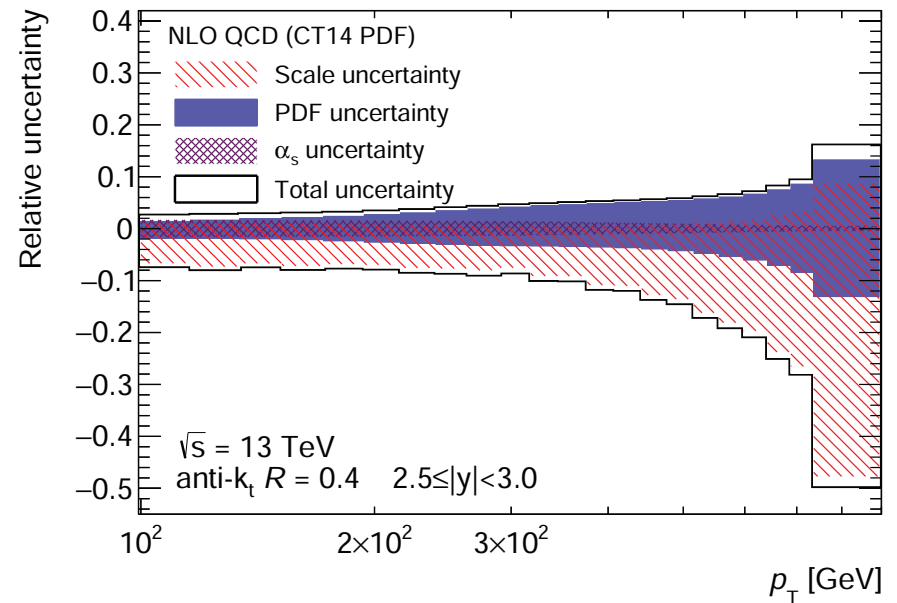
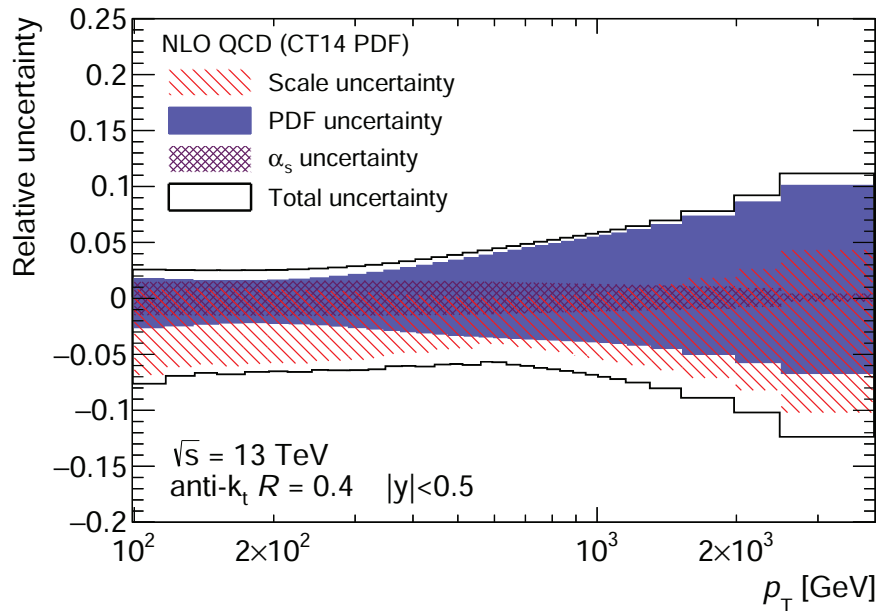
- *Threshold and jet radius joint resummation for single-inclusive jet production*
Xiaohui Liu, S. M. and Felix Ringer *Phys.Rev.Lett.* 119 (2017) no.21, 212001
[arXiv:1708.04641](#)
- *Phenomenology of single-inclusive jet production with jet radius and threshold resummation*
Xiaohui Liu, S. M. and Felix Ringer [arXiv:1801.07284](#)

Single-inclusive jet production



- Double differential cross section for $pp \rightarrow \text{jet} + X$ at $\sqrt{s} = 13 \text{ TeV}$
 - transverse momentum p_T and rapidity y of signal-jet
 - [ATLAS arXiv:1711.02692](#) (left), [CMS arXiv:1605.04436](#) (right)
- Comparison with NLO perturbative QCD predictions (NLOJET++ [Nagy](#))
 - impressive agreement over several orders of magnitude

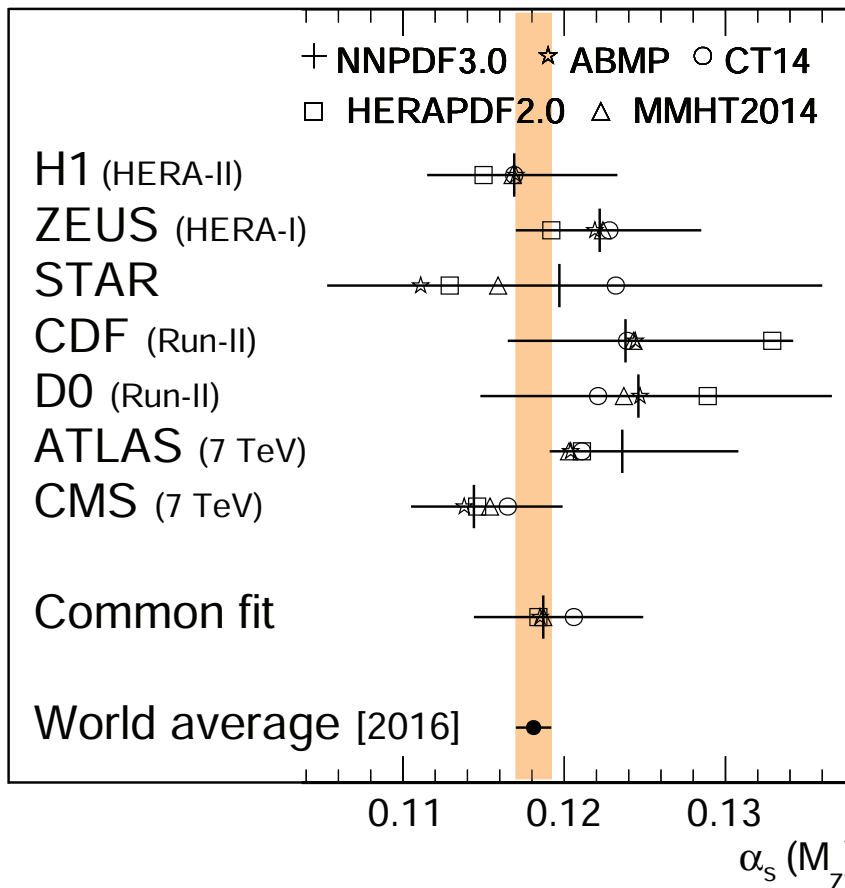
Uncertainties



- Relative QCD uncertainties in single-inclusive jet cross-sections at NLO
 - first and last $|y|$ bins for inclusive jet measurement
 - uncertainties due to renormalization and factorization scales, μ_R , μ_F , the strong coupling α_s with $\Delta\alpha_s = 0.0015$ and PDFs CT14
- Sizable uncertainties
 - $\mathcal{O}(10\%)$ for central rapidities $|y| < 0.5$
 - $\mathcal{O}(30 - 40\%)$ forward $2.5 \leq |y| < 3.0$ at large p_T

Uses of inclusive jet data

- Determination of $\alpha_s(M_Z)$ and PDFs (gluon at medium to large x)
 - partonic cross sections $\hat{\sigma}_{ij \rightarrow \text{jet}} \propto \alpha_s^2(\mu)$
 - jet cross section $d\sigma_{pp \rightarrow \text{jet}} = \alpha_s^2(\mu) \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes [\dots]$



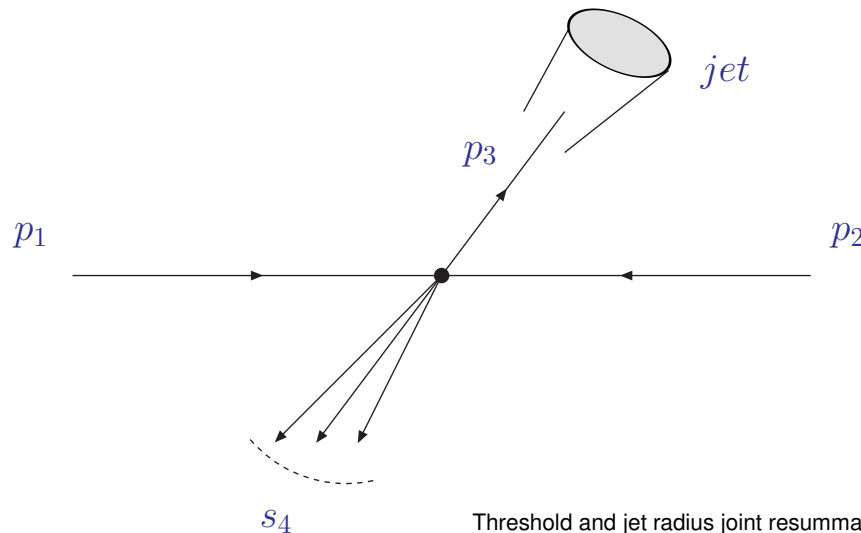
- $\alpha_s(M_Z)$ at NLO in QCD from inclusive jet cross section data
Britzger, Rabbertz, Savoiu, Sieber '17
- correlations between PDFs and $\alpha_s(M_Z)$ are important

QCD factorization

- Double differential cross section for $pp \rightarrow \text{jet} + X$
 - transverse momentum p_T and rapidity η of signal-jet

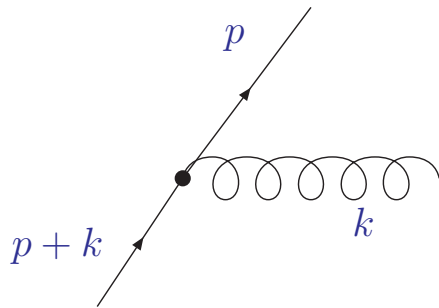
$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} dz \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} dv x_1^2 f_{i_1}(x_1) x_2^2 f_{i_2}(x_2) \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz}(v, z, p_T, R)$$

- PDFs f_i and variables $V = 1 - p_T e^{-\eta} / \sqrt{S}$, $VW = p_T e^{\eta} / \sqrt{S}$
- Partonic cross sections $\hat{\sigma}_{i_1 i_2}$ dependent on partonic kinematic variables on $s = x_1 x_2 S$, $v = u/(u+t)$ and partonic threshold $z = s_4/s \rightarrow 0$
- Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ with kinematics constraint $s + t + u = s_4$



Threshold logarithms

- Soft and collinear regions of phase space
 - double logarithms from singular regions in Feynman diagrams
 - propagator vanishes for: $E_g = 0$, soft $\theta_{qg} = 0$ collinear

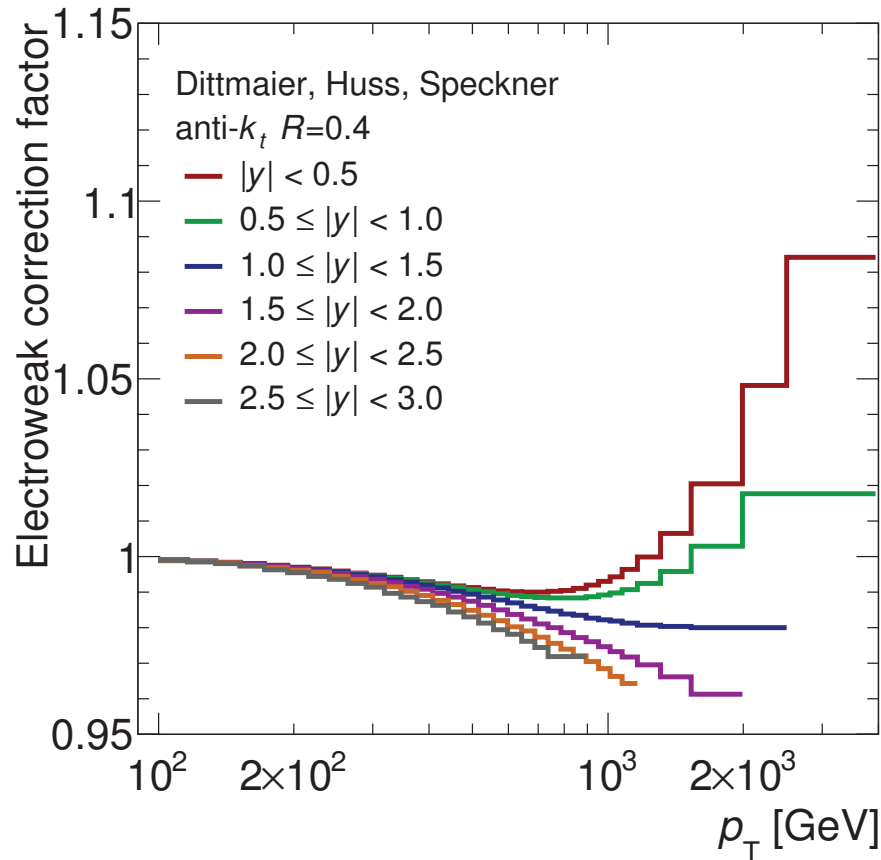


$$\begin{aligned}
 \alpha_s \int d^4 k \frac{1}{(p+k)^2} & \longrightarrow \alpha_s \int dE_g d\sin\theta_{qg} \frac{1}{2E_q E_g (1 - \cos\theta_{qg})} \\
 & \longrightarrow \alpha_s \ln^2(\dots)
 \end{aligned}$$

$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

- Large double-logarithmic corrections $\ln(\dots) \gg 1$ near threshold
- Single-inclusive jet production with threshold logarithms
 $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ for $z = s_4/s \rightarrow 0$
 - positive corrections enhance partonic cross sections $\hat{\sigma}_{i_1 i_2}$
 - long history of resummation [Sterman '87](#); [Catani, Trentadue '88](#); ...
- Same for weak radiative corrections: positive corrections for large $p_T \gg M_W$ [Dittmaier, Huss, Speckner '12](#)

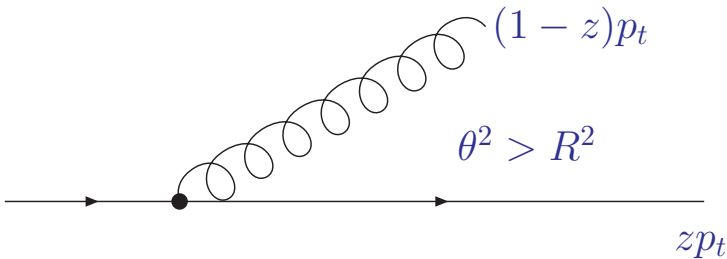
Electroweak corrections at NLO



- Electroweak correction factors for inclusive jet cross-section as function of jet p_T for all $|y|$ bins Dittmaier, Huss, Speckner '12
 - negative corrections $\mathcal{O}(1 - 3\%)$ for p_T in range $\mathcal{O}(\text{few } 100) \text{ GeV}$
 - sizeable (positive) effect $\mathcal{O}(10\%)$ at large $p_T \gg M_W$

Jet radius logarithms (I)

- Collinear singularity when the jet becomes very narrow
 - partons radiated outside of jet (not recombined with jet by chosen jet algorithm) become more and more collinear to emitter
- Example
 - loss of transverse momentum for leading jet
 - quasi-collinear branching of quark with transverse momentum p_T



$$\delta p_T = (1-z)p_T - p_T = -z p_T \text{ for } (1-z) > z$$

$$\delta p_T = (1-z)p_T - p_T = -z p_T \text{ for } z > (1-z)$$

- Perturbative radiation loss for average $\langle \delta p_T \rangle_q$

$$\langle \delta p_T \rangle_q = p_T \frac{\alpha_s}{2\pi} \int_{R^2}^1 \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) P_{qq}(z)$$

Jet radius logarithms (II)

- Leading order result for quark and gluon jets Dasgupta, Magnea, Salam '07

$$\langle \delta p_T \rangle_q = C_F \frac{\alpha_s}{\pi} p_T \ln(R) \left(2 \ln 2 - \frac{3}{8} \right) = 0.43 \alpha_s \ln(R)$$

$$\langle \delta p_T \rangle_g = \frac{\alpha_s}{\pi} p_T \ln(R) \left[C_A \left(2 \ln 2 - \frac{43}{96} \right) + T_f n_f \frac{7}{48} \right] = 1.02 \alpha_s \ln(R)$$

- Large single-logarithmic corrections $\ln(1/R) \gg 1$ for small R
 - negative corrections decrease partonic cross sections $\hat{\sigma}_{i_1 i_2}$
 - resummation ...

Joint resummation

SCET factorization

- Factorization in small- R and $z \rightarrow 0$ threshold limit
 - assume anti- k_t jet algorithm, $z \sim R$, and small finite mass of jet

$$\begin{aligned} \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz} &= s \int ds_X ds_c ds_G \delta(zs - s_X - s_G - s_c) \\ &\quad \times \text{Tr} [\mathbf{H}_{i_1 i_2}(v, p_T, \mu_h, \mu) \mathbf{S}_G(s_G, \mu_{sG}, \mu)] J_X(s_X, \mu_X, \mu) \\ &\quad \times \sum_m \text{Tr} [J_m(p_T R, \mu_J, \mu) \otimes_{\Omega} S_{c,m}(s_c R, \mu_{sc}, \mu)] \end{aligned}$$

- Specific functions for individual kinematic regions
 - hard functions for $2 \rightarrow 2$ scattering $\mathbf{H}_{i_1 i_2}$ (known to 2-loops Broggio, Ferroglia, Pecjak, Zhang '14)
 - inclusive jet function $J_X(s_X)$ dependent on invariant mass s_X of the recoiling collimated radiation (known to order α_s^2 Becher, Neubert '06, Becher, Bell '10)
 - global soft function \mathbf{S}_G accounts for wide-angle soft radiation which cannot resolve the small radius R (known to NLO Liu, S.M., Ringer '17)

Joint resummation

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- Specific functions for individual kinematic regions
 - signal-jet function $J(p_T R)$ accounts for energetic radiation inside jet
Becher, Neubert, Rothen, Shao '15
 - soft-collinear (“coft”) function $S_c(s_c R)$ captures soft radiation near jet boundary
Becher, Neubert, Rothen, Shao '15, Chien, Hornig, Lee '15
- Sum runs over all collinear splittings and traces taken in color space
- ‘ \otimes_{Ω} ’ denotes associated angular integrals Becher, Neubert, Rothen, Shao '15

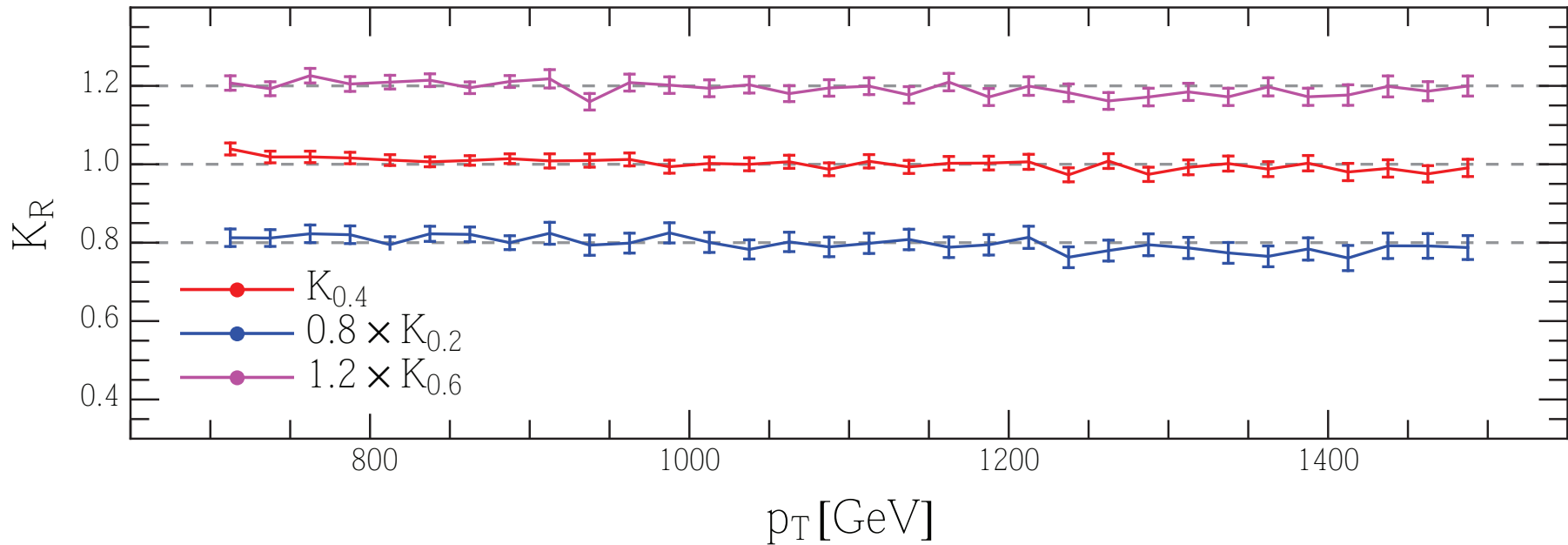
Phenomenology

- Evaluation of cross section in SCET (resummation) with renormalization group equations
 - evolution of all functions from their natural scales μ_i to common hard scale $\mu = p_T^{\max}$
- Matching of NLL resummed results with full NLO calculation (need to avoid double counting)

$$d\sigma = d\sigma_{\text{NLL}} - d\sigma_{\text{NLO}_{\text{sin}}} + d\sigma_{\text{NLO}}$$

- resummed cross section $d\sigma_{\text{NLL}}$
- fixed order NLO result in singular limit $d\sigma_{\text{NLO}_{\text{sin}}}$
- complete fixed order NLO result $d\sigma_{\text{NLO}}$

Validation



- Comparison of fixed order NLO result in singular limit $d\sigma_{\text{NLO}_{\text{sin}}}$ with complete fixed order NLO result $d\sigma_{\text{NLO}}$
- Ratio $K_R = \frac{d\sigma_{\text{NLO}_{\text{sin}}}}{d\sigma_{\text{NLO}}}$ as function of jet p_T for $1.5 < |\eta| < 2$ at $\sqrt{S} = 13 \text{ TeV}$ (error bars for numerical uncertainty)

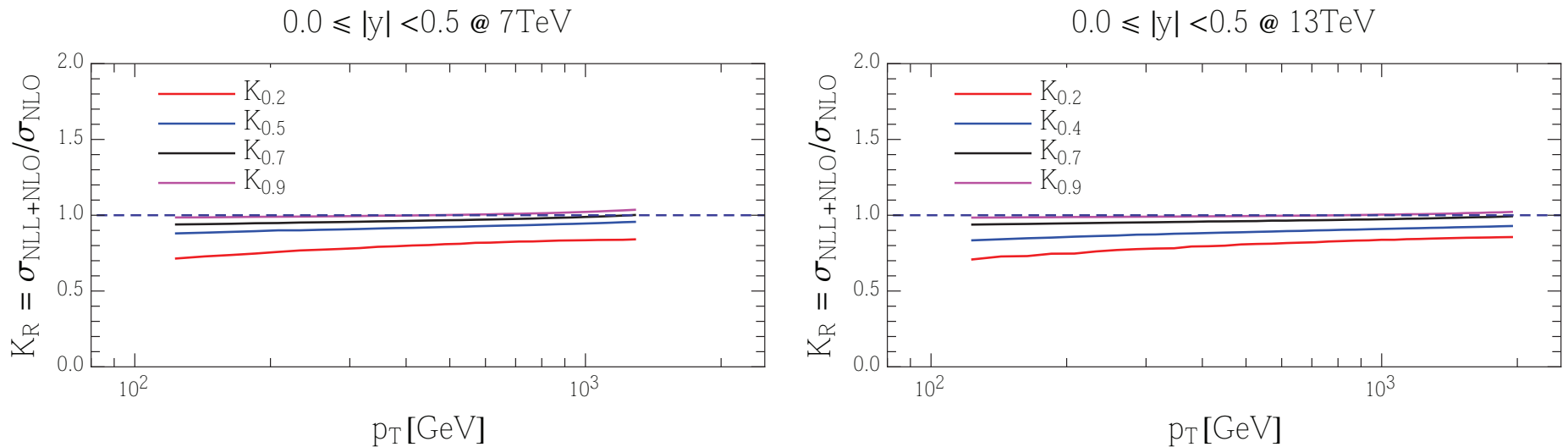
Jet radius dependence (I)

Resummation for various R

- Ratio K_R of NLO + NLL and NLO cross sections for different jet radii

$$K_R = \frac{\sigma_{\text{NLL+NLO}}(R)}{\sigma_{\text{NLO}}(R)}$$

- LHC at $\sqrt{S} = 7 \text{ TeV}$ (left) and 13 TeV (right) with NLO PDF set of **MMHT**

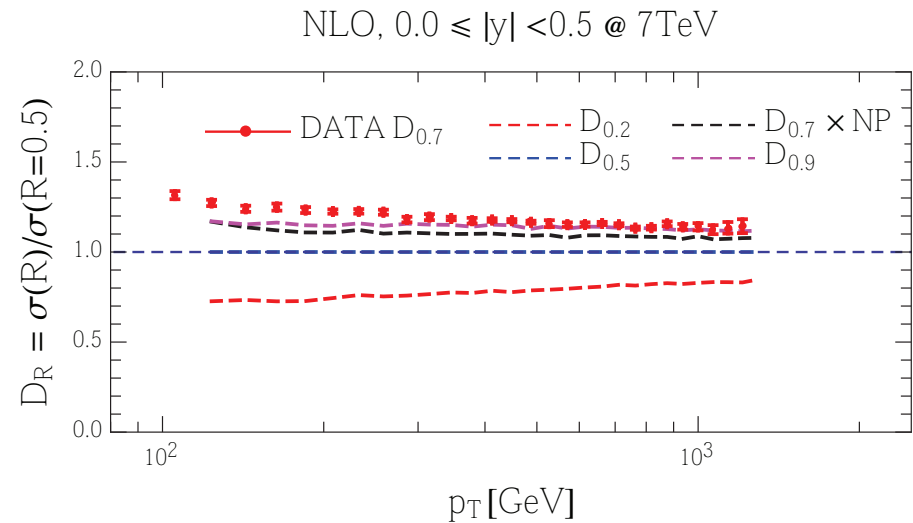
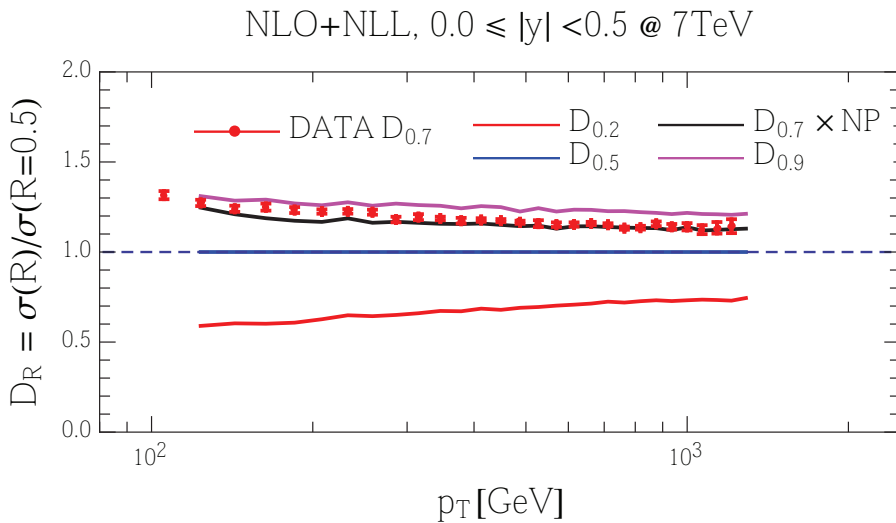


- Significant effect for small jet radii; reduction of $\mathcal{O}(20\%)$ for $R = 0.2$ in entire range of p_T

Jet radius dependence (II)

Impact of joint resummation

- Cross section ratio $D_R = \frac{\sigma(R)}{\sigma(R_{\text{fixed}})}$
- D_R for NLO + NLL (left) and NLO (right) cross sections for $R_{\text{fixed}} = 0.5$ at $\sqrt{S} = 7 \text{ TeV}$ with NLO PDF set of **MMHT14**
- Single-inclusive jet data from collected at $\sqrt{S} = 7 \text{ TeV}$ with $R = 0.7$ by **CMS arXiv:1406.0324** (with NP correction factors)



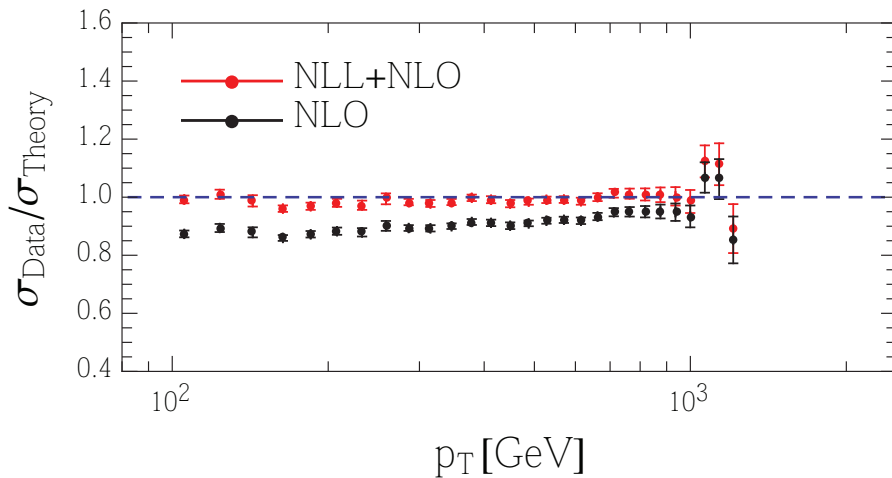
- NLO result overshoots; joint resummation agrees with data for $D_{R=0.7}$

Data vs. theory (I)

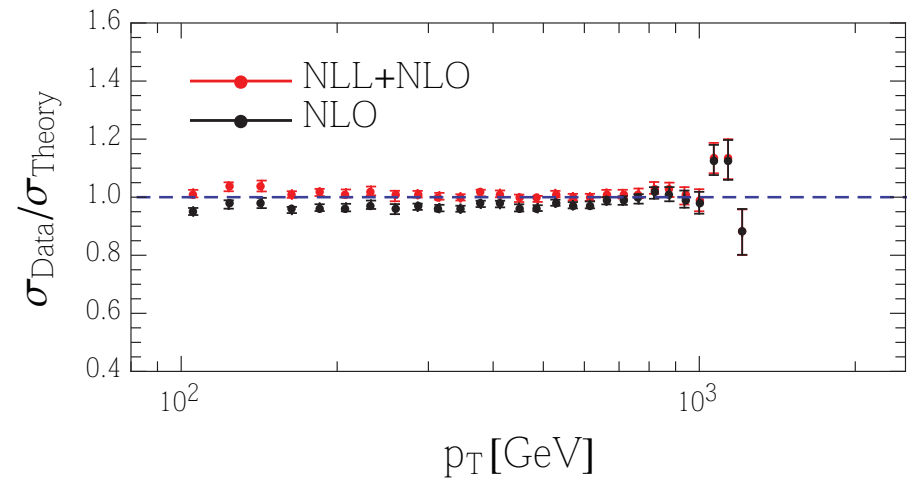
LHC data at $\sqrt{S} = 7$ TeV

- Ratio $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$ with $R = 0.5$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
 - NLO PDF set of [MMHT14](#)
 - data at $\sqrt{S} = 7$ TeV by [CMS arXiv:1406.0324](#)

$0.0 \leq |\eta| < 0.5$, $R = 0.5$ @ 7TeV



$0.0 \leq |y| < 0.5$, $R = 0.7$ @ 7TeV



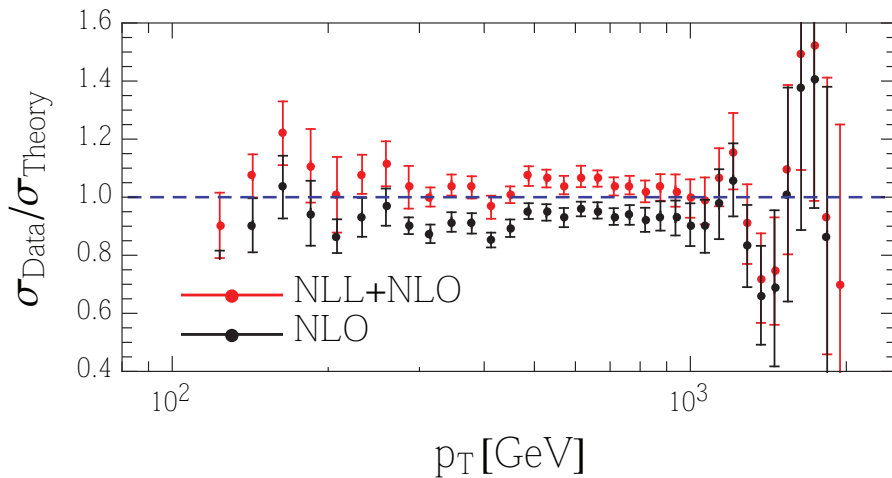
- Joint resummation agrees well with data; ratio with NLO predictions undershoots

Data vs. theory (II)

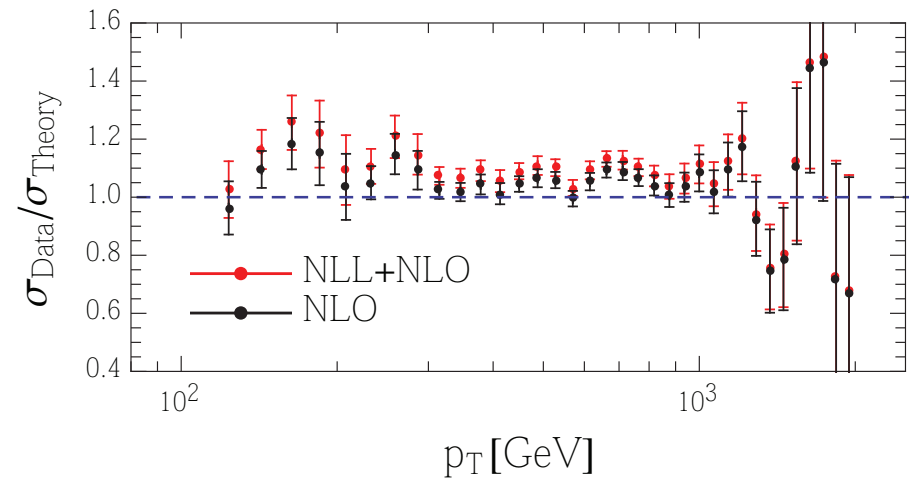
LHC data at $\sqrt{S} = 13$ TeV

- Ratio $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$ with $R = 0.4$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
 - NLO PDF set of **MMHT14**
 - data at $\sqrt{S} = 13$ TeV by **CMS arXiv:1605.04436**

$0.0 \leq |y| < 0.5$, $R = 0.4$ @ 13TeV



$0.0 \leq |y| < 0.5$, $R = 0.7$ @ 13TeV

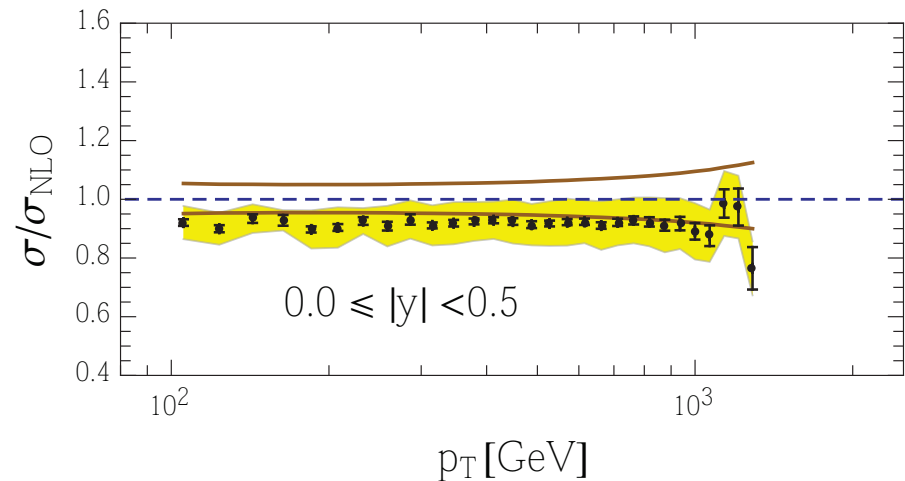
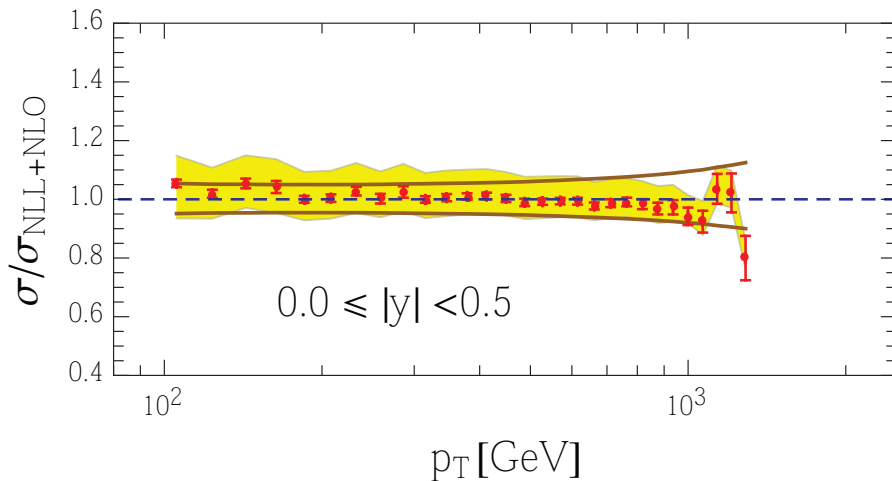


- Same trend as for data at $\sqrt{S} = 7$ TeV, but still large experimental uncertainties

Data vs. theory (III)

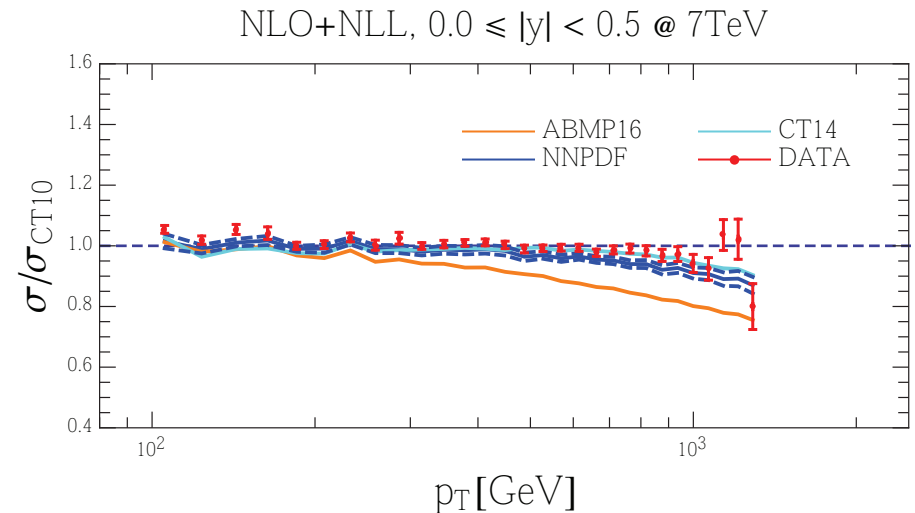
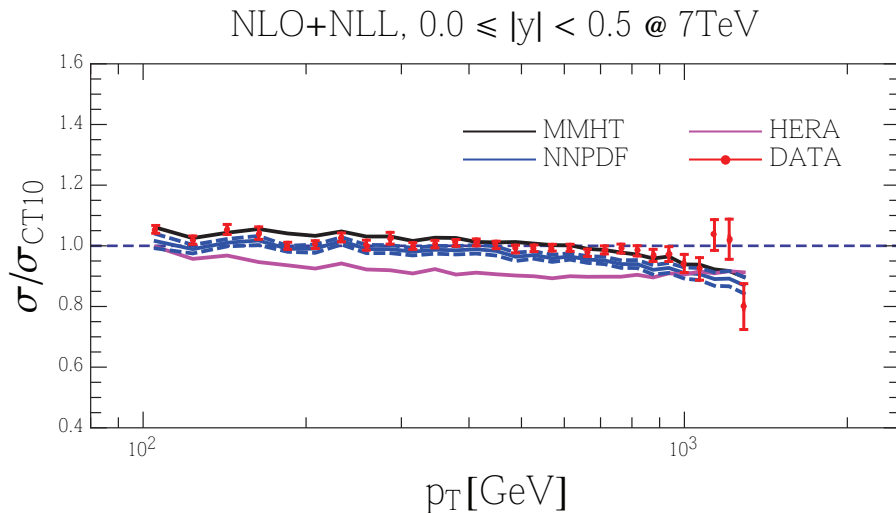
Experimental and theoretical uncertainties

- Ratio $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$ with $R = 0.5$ to theoretical results at NLO + NLL (left) and at NLO (right) accuracy at $\sqrt{S} = 7 \text{ TeV}$ using the NLO PDF set by CT10 for data collected by CMS [arXiv:1406.0324](#) (with NP correction factors)
 - error bars represent experimental statistical errors
 - solid (brown) lines represent experimental systematic ones
 - band (yellow) indicates theoretical scale uncertainties



PDF dependence

- Cross sections $\sigma_{\text{NLO+NLL}}$ at NLL + NLO accuracy with $R = 0.5$ at $\sqrt{S} = 7 \text{ TeV}$ normalized to the one with the NLO PDF set by CT10
 - central NLO PDF sets HERAPDF2.0, MMHT14 and NNPDF3.1 (left)
 - central NLO PDF sets ABMP16, CT14 and NNPDF3.1 (right)
 - data collected by CMS [arXiv:1406.0324](https://arxiv.org/abs/1406.0324) (with NP correction factors) is superimposed



Summary

Theory framework

- Joint resummation of threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ and small jet-radius logarithms $\alpha_s^n \ln^n(R)$ to all orders at NLL accuracy
- Resummation at NLO + NLL accuracy in good agreement with CMS data
- Residual theoretical uncertainties from scale variation still of similar size as experimental systematics

Uses of jet measurements

- Determinations of strong coupling constant $\alpha_s(M_Z)$ and gluon PDF at medium to large- x based on results fixed order perturbation theory likely to incur bias
- Precision of $\lesssim 1\%$ makes resummed theoretical predictions at NNLO + NNLL accuracy in QCD mandatory