Lattice inputs for flavour physics

Elvira Gámiz Sánchez

UNIVERSIDAD
DE GRANADA

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Introduction

Flavour-violating and CP-violating processes allow us to test high energy scales.

* **Unveiling New Physics effects**

* **Constraining NP models.**

* Tests not limited by available energy by available precision.

SM predictions for those observables depend on only a few parameters $\rightarrow$ can overconstrain the value of those parameters.

Interplay flavour physics with direct searches for new physics and electroweak precision studies

$\rightarrow$ `Which is the correct extension of the SM?`

Introduction: Lattice QCD

$$\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times \underbrace{(\text{matrix elements})}_{\text{lattice QCD}}$$

Parametrize MEs in terms of decay constants, form factors, bag parameters, ...

Lattice QCD: Numerical evaluation of QCD path integral (rely only on first principles) using Monte Carlo methods.

Phenomenology (in particular **flavour physics**) needs **precise** lattice QCD calculations $\rightarrow$

* Control and reliably estimate systematic errors.

* Not everything can be calculated **precisely** using lattice techniques, only some processes:

with estable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron

** This includes quark masses and α_s, hadron spectrum, weak decays (leptonic, semileptonic, mixing)...

Introduction

... and most CKM matrix elements

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ \pi \to \ell\nu & K \to \ell\nu & B \to \tau\nu \\ & K \to \pi\ell\nu & B \to \pi\tau\nu, B_s \to K\ell\nu \\ & & \Lambda_b \to p\ell\nu \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ D \to \ell\nu & D_s \to \ell\nu & B_{(s)} \to D_{(s)}, D^*_{(s)}\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ \langle B^0_d|\bar{B}^0_d\rangle & \langle B^0_s|\bar{B}^0_s\rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

$$(\rho, \eta) \quad \langle K^0|\bar{K}^0\rangle$$

Introduction: Lattice QCD

Development of new methods is allowing to increase the scope of LQCD calculations:

* Baryons
* Nonleoptonic decays ($K \to \pi\pi ...$)
* Resonances
* Scattering
* Long-distance effects
* QED effects ...

Introduction: Lattice QCD

[inspired by A.Kronfeld and A.El-Khadra]

$\langle \bar{B}^0_q | \mathcal{O}_i^{\Delta B=2} | B^0_q \rangle$

$\langle \bar{D}^0_q | \mathcal{O}_i^{\Delta C=2} | D^0_q \rangle$

$\hat{B}_K$, . . .

$f_{+,0}^{B\to D}(q^2)$, . . .

$f_+^{K\to\pi}$, $f_{+,0,T}^{B\to\pi}$, . . .

$f_{K^\pm}$ $f_{B_{(s)}}$, . . .

$\surd$

LQCD flagship results

$\langle (\pi\pi)_{I=2} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

First complete LQCD results, large errors

ΔM_K, ε_K

$\langle (\pi\pi)_{I=0} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

First results, phys. parameters incomplete system.

$B \to K^* \ell\ell \to K\pi\ell\ell$, . . .

$K^+ \to \ell^+ \nu(\gamma)$, . . .

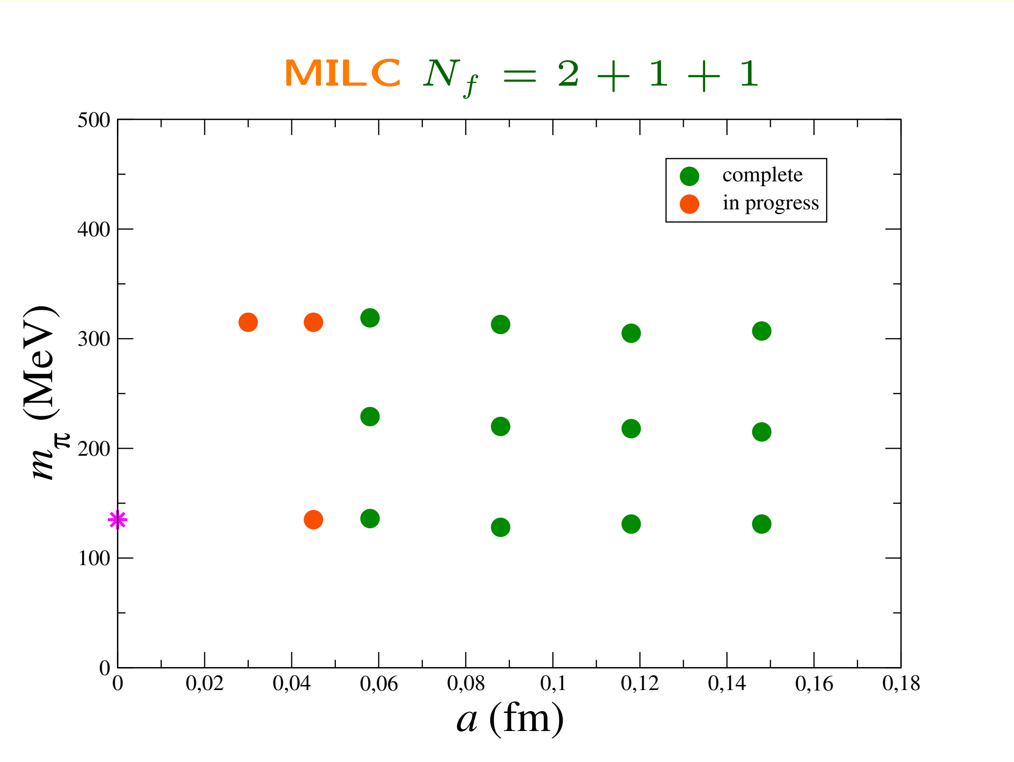
$K^+ \to \pi^+ \ell^+ \ell^-$, . . .

$K^+ \to \pi^+ \nu \bar{\nu}$, . . .

New methods, pilot projects

Introduction: Lattice QCD

Combined chiral-continuum extrapolation



Many lattice collaborations doing now simulations with **physical light-quark masses**; PACS-CS, BMW, MILC, RBC/UKQCD, ETM...

ChPT techniques still necessary to reduce errors and/or correct/estimate systematic effects: light and heavy quark discretization, finite volume, isospin-breaking, mass mistunings, ...

\# Next generation of gauge configurations: isospin-breaking ($N_f = 1 + 1 + 1 + 1$) and QED+QCD BMW, QCDSF, RBC, MILC ...

Kaons: leptonic, semileptonic and mixing

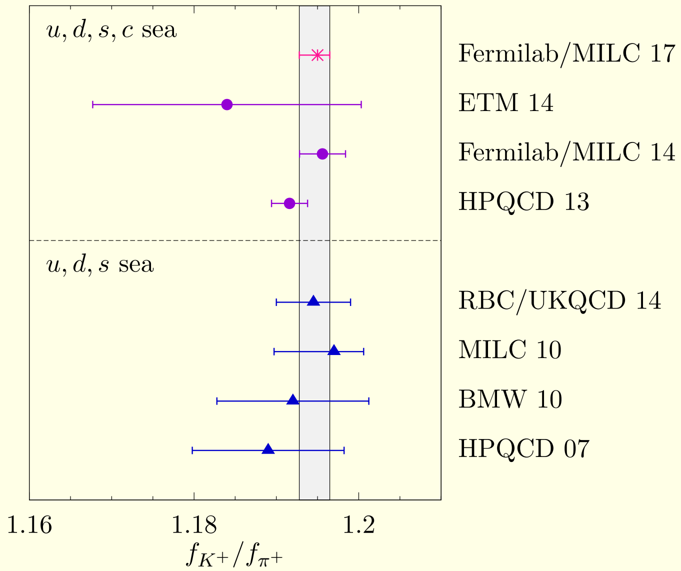
Leptonic Kaon decays: $K \to \ell\nu$

Decay constants come from simple matrix element

$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = i f_P p_\mu \;\; \left((m_{q_1} + m_{q_2})\langle 0|\bar{q}_1\gamma_5 q_2|P(p=0)\rangle = f_P M_P^2 \right)$$

$\rightarrow$ precise calculations on the lattice (even higher precision for ratios due to cancellation of stats. and systs.)

Many $N_f = 2+1, 2+1+1$ calculations $\rightarrow$ good test of lattice QCD

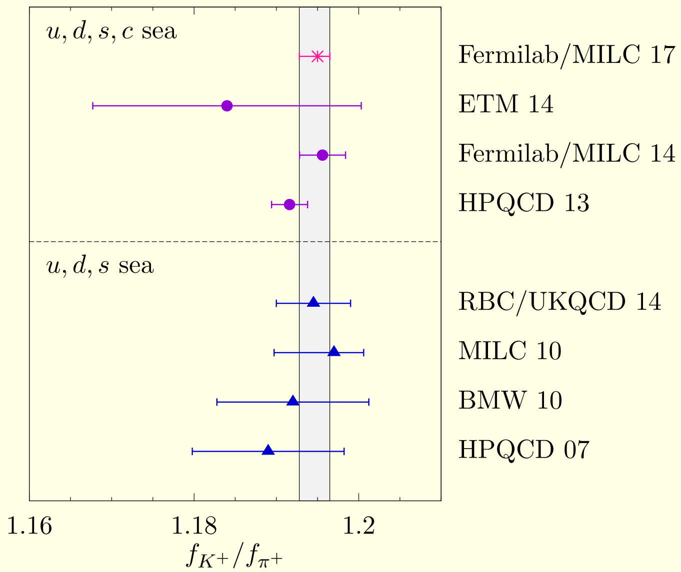


Most recent (and precise) **FNAL/MILC**, 1712.09262

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1950(14)_{\text{stat}}\left({}^{+4}_{-17}\right)_{\text{other syst}}(3)_{f_\pi, PDG}$$
$$= 1.1950\left({}^{+15}_{-22}\right)$$

(FLAG $N_f = 2+1+1$ average $= f_{K^+}/f_{\pi^+} = 1.193(3)$)

Leptonic Kaon decays: $K \to \ell\nu$



Most recent (and precise) **FNAL/MILC**, 1712.09262

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$$= 1.1950\left({}^{+15}_{-22}\right)$$

Using exp. data rates for K_{l2}/π_{l2} and radiative correction factors from **Rosner, Stone, Van de Water**, 1509.02220

$$\to \frac{|V_{us}|}{|V_{ud}|} = 0.2310(3)_{f_K/f_\pi}(2)_{expt}(2)_{EM}$$

$$\Gamma(K^+(\pi^+) \to l^+\nu_l(\gamma)) = (\text{known})\left(1 + \delta^l_{EM,K(\pi)}\right)|V_{us(ud)}|^2 f^2_{K^+(\pi^+)}$$

(δ^l_{EM} includes structure dependent EM corrections, currently estimated phenomenologically within ChPT **Cirigliano et al** 1107.6001)

Improvements underway, but will eventually require inclusion of QED and isospin-breaking in simulations

* RM123 approach: expand lattice path-integral in powers of $\alpha_{em}, (m_d - m_u)/\Lambda_{QCD}$

First results **Giusti et al**, 1711.06537 $\delta R^{lat}_{K\pi} = -1.22(16)\%$ agree with pheno estimates.

$$(\,\delta R^{lat}_{K\pi} = (\delta_{SU(2),K} + \delta_{EM,K} - \delta_{SU(2),\pi} - \delta_{EM,\pi})\,)$$

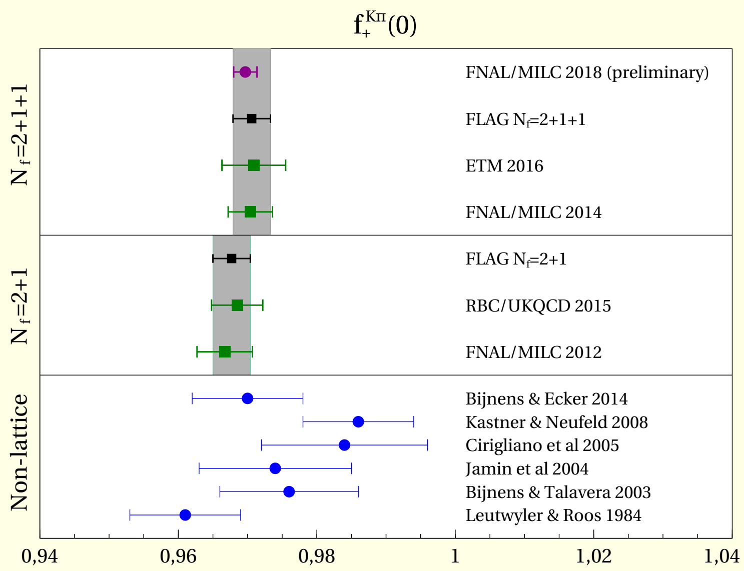
Semileptonic Kaon decays

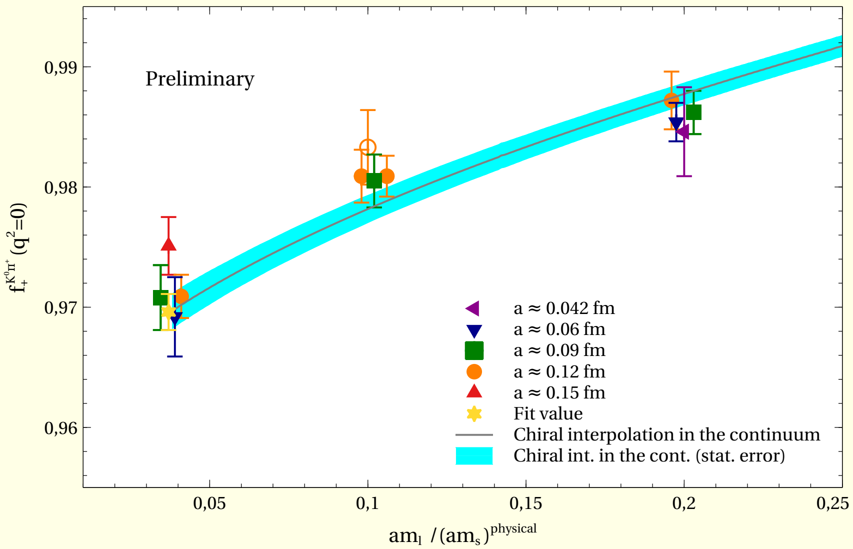
Direct determination of $|V_{us}|$ (no input needed for $|V_{ud}|$)

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Test different current (vector vs axial-vector) as with leptonic decays.

Preliminary: $N_f = 2+1+1$ FNAL/MILC 2018



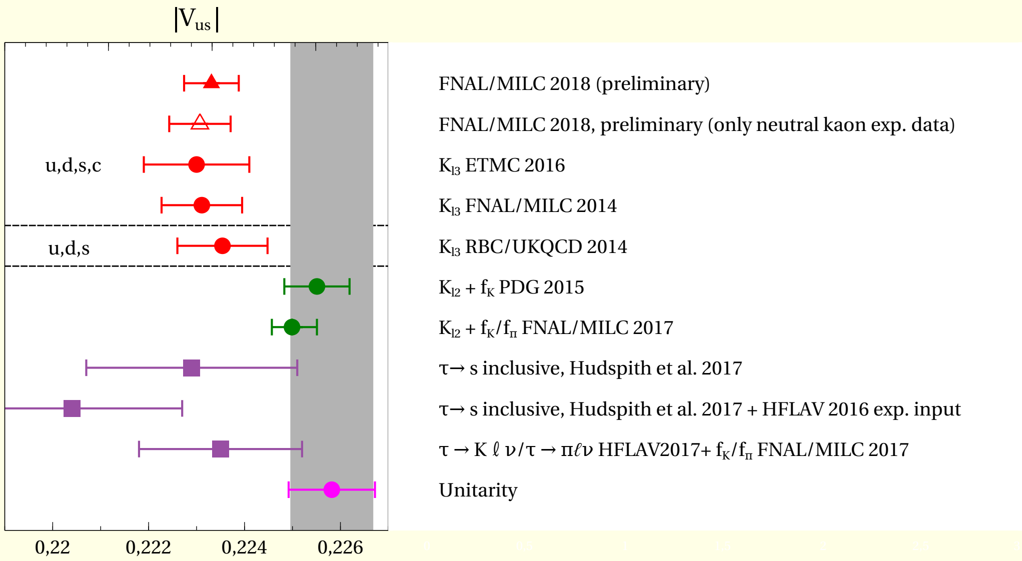


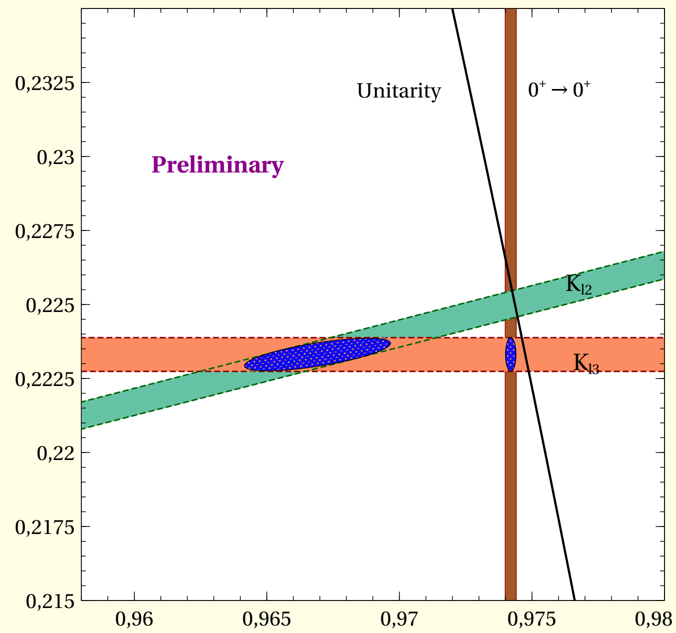
Grey bands: FLAG 2016 averages

* Includes FV corrections at one-loop, and strong isospin-breaking at two-loops ($f_+^{K^0\pi^-}$)

Using exp. average from M. Moulson 1704.04104 $\rightarrow$ $|V_{us}| = 0.22332(43)_{f_+(0)}(42)_{\rm exp}$ **Prelim.**

First row CKM unitarity





* With $|V_{us}|$ from the preliminary K_{l3} FNAL/MILC analysis and $|V_{ud}| = 0.97420(21)$ from superallowed β decays Hardy & Towner:

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00110(27)_{V_{us}}(41)_{V_{ud}} \quad 2.3\sigma \text{ tension}$$

* With $|V_{us}|$ and $|V_{us}|/|V_{ud}|$ from K decays with lattice inputs from FNAL/MILC preliminary K_{l3} analysis and FNAL/MILC K_{l2} analysis in 1712.09262

$$\Delta_u = -0.0151(38)_{f_+(0)}(35)_{f_{K^\pm}/f_{\pi^\pm}}(36)_{\rm exp}(27)_{\rm EM} \quad 2.2\sigma \text{ tension}$$

$\rightarrow$ Correlated analysis of f_{K^+}/f_{π^+} and $f_+^{K^0\pi^-}$ to reduce errors and test leptonic-semileptonic tension.

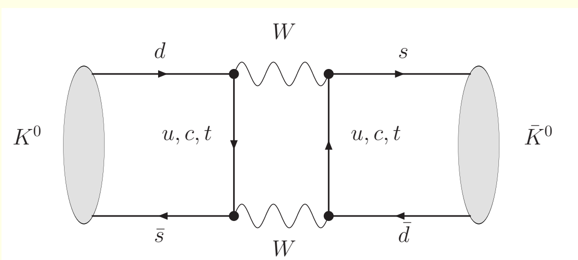
Indirect CP violation: $K^0 - \bar{K}^0$ mixing

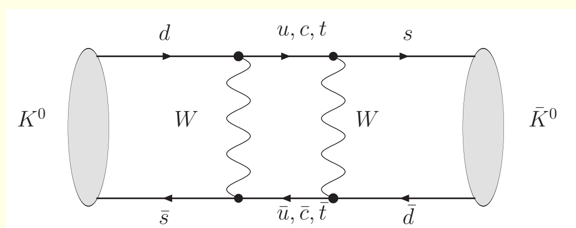
Parametrized via $\varepsilon_K \equiv \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$

That in the SM can be expressed as $(A_{0(2)} = A(K \to (\pi\pi)_{I=0(2)}))$

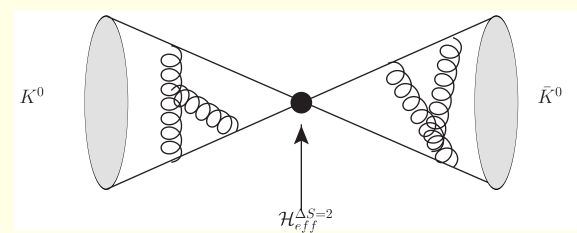
$$\varepsilon_K \approx e^{i\phi_\varepsilon} sin\phi_\varepsilon \left[\underbrace{C_\varepsilon \hat{B}_K X_{\rm SD}}_{\rm short-distance} + \underbrace{\xi_0 + \xi_{LD}}_{\rm long-distance} \right]$$

with $\xi_0 = \frac{Im A_0}{Re A_0}$ and X_{SD} the short distance contribution from





$\Longrightarrow$



$$X_{SD} = |V_{cb}|^2 \lambda^2 \eta \left(|V_{cb}|^2 (1 - \bar{\rho}) + \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \right)$$

and $\hat{B}_K \propto \langle \bar{K}^0 | \mathcal{O}^{\Delta S=2} | K^0 \rangle$.

* The dispersive, ξ_{LD}, and absorptive, ξ_0, contributions are $\sim 2, 5\%$ respectively.

Neutral meson mixing

Theoretical prediction following **Y. Jang, W. Lee et al.**, 1710.06614, talk at Lattice2017

Short-distance contribution

* Lattice QCD techniques have reduced $\hat{B}_K$ errors to $\sim 1.3\%$:

$$\hat{B}_K|_{\mathbf{FLAG,N_f=2+1}} = 0.7625(97)$$

$\rightarrow \hat{B}_K$ no longer the dominant uncertainty in $K - \bar{K}$ mixing, but $|V_{cb}|$

Long-distance contribution

* Absorptive part: $\xi_0 = \frac{Im A_0}{Re A_0}$, related to ε'_K through $Re\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) \propto (\xi_2 - \xi_0)$ with $\xi_2 = \frac{Im A_2}{Re A_2}$

** Direct LQCD calculation: $\xi_0 = -0.57(49) \cdot 10^{-4}$ **RBC/UKQCD** 1505.07863. Not very precise yet and $(\pi\pi)$ phase shift $\delta_0^{\mathrm{RBC/UKQCD15}}$ is 3σ away from experiment

** Indirect method: LQCD calculation of $\xi_2 = \checkmark$ + experimental $\frac{\varepsilon'_K}{\varepsilon_K}$
$\rightarrow \xi_0 = -1.63(19) \cdot 10^{-4}$ **T. Blum et al** 1502.00263

* Dispersive part, only rough LQCD estimate: $\xi_{LD} = (0 \pm 1.6)\%$

Christ, Izubuchi, Sachrajda, Soni & Yu 1212.5931

Neutral meson mixing

Theoretical prediction following **Y. Jang, W. Lee et al.**, 1710.06614, talk at Lattice2017

Wolfenstein parameters from an angle-only fit **UTfit** (not using ε_K, $\hat{B}_K$ and $|V_{cb}|$)

$$|\varepsilon_K| = (1.58 \pm 0.16) \cdot 10^{-3} \qquad \text{with } |V_{cb}|_{\text{excl.}}$$

$$|\varepsilon_K| = (2.05 \pm 0.1) \cdot 10^{-3} \qquad \text{with } |V_{cb}|_{\text{incl.}}$$

Error dominated by $|V_{cb}|$ (30%), followed by $\bar{\eta}$ (26%) and η_{ct} (21%).

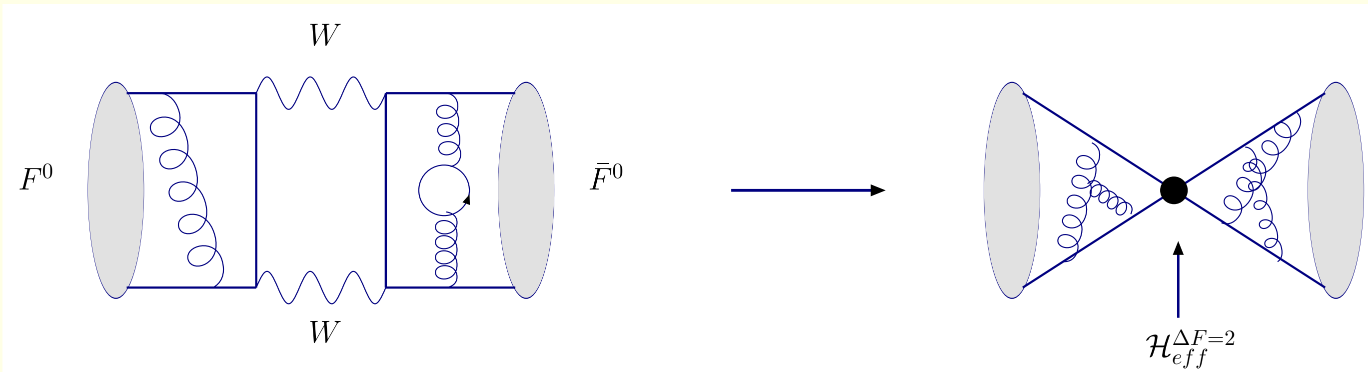
Long distance effects yield 2.5% (ξ_{LD}) and 1.3% (ξ_0) of the error.

Experimentally: $\varepsilon_K = 2.228(11) \cdot 10^{-3} e^{i\phi_\varepsilon}$ and $\phi_\varepsilon = 43.52(5)^o$

Need to clarify $|V_{cb}|_{\text{excl}} - |V_{cb}|_{\text{incl}}$ tension

Neutral meson mixing BSM

In the Standard Model and beyond, short-distance contributions to the mixing can be described via a $\mathcal{H}_{eff}^{\Delta F=2}$.



In general:

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i$$

SM:

$$\mathcal{O}_1 = (\bar{f}^\alpha \gamma_\mu L\, q^\alpha) \left(\bar{f}^\beta \gamma^\mu L\, q^\beta\right)$$

$$\mathcal{O}_2 = (\bar{f}^\alpha L\, q^\alpha) \left(\bar{f}^\beta L\, q^\beta\right)$$

$$\mathcal{O}_3 = \left(\bar{f}^\alpha L\, q^\beta\right) \left(\bar{f}^\beta L\, q^\alpha\right)$$

BSM:

$$\mathcal{O}_4 = (\bar{f}^\alpha L\, q^\alpha) \left(\bar{f}^\beta R\, q^\beta\right)$$

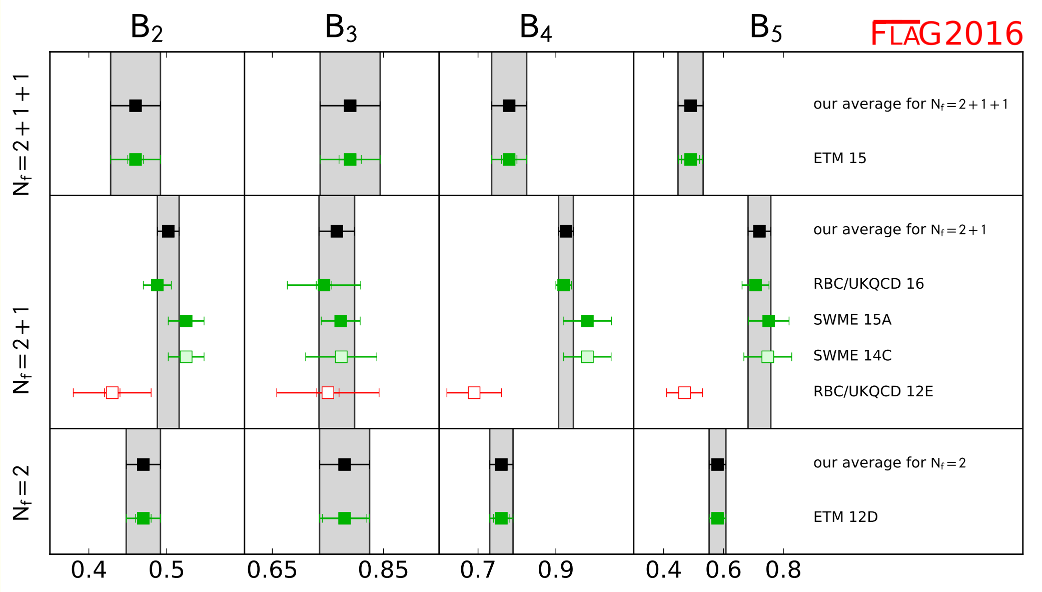
$$\mathcal{O}_5 = \left(\bar{f}^\alpha L\, q^\beta\right) \left(\bar{f}^\beta R\, q^\alpha\right)$$

$\widetilde{\mathcal{O}}_{1,2,3} = \mathcal{O}_{1,2,3}$ with the replacement $L(R) \to R(L)$

Recent and on-going calculations lattice calculations of K, D, and B mixing matrix elements for all five operators $\rightarrow$ constraints on BSM physics

Neutral meson mixing BSM

Several LQCD calculations of the complete basis available.



Apparent disagreements between $N_f = 2+1$ and $N_f = 2+1+1$ calculations of $\hat{B}_{4,5}$ due to the use of different intermediate renormalization schemes.

B and D mesons decay constants

Heavy quarks

Use improved relativistic actions to describe charm and bottom quarks

ETM, HPQCD, FNAL/MILC, RBC/UKQCD

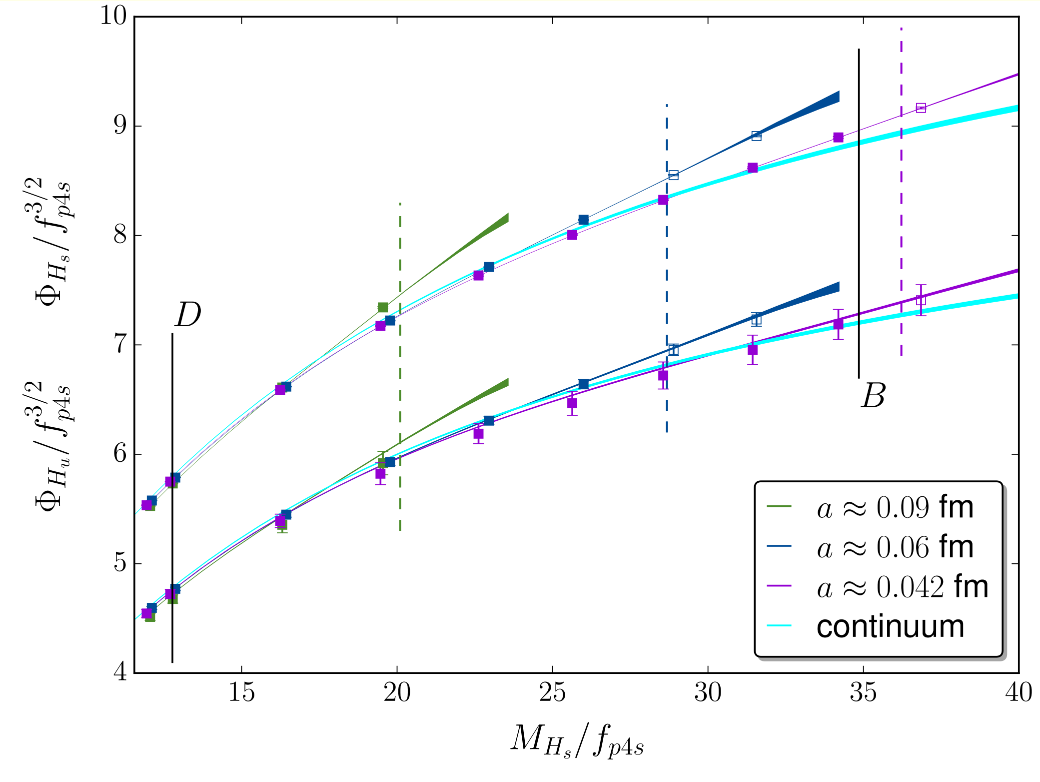
* Reduce errors. Potentially results as accurate as for light quarks
* Simplified renormalization or not renormalization at all
* Simpler tuning of masses.

For bottom

* Need lattice spacing small enough to safely simulate b ($am_b \leq 0.9$)
* Need HQ inspired parametrization to extrapolate (or interpolate) to the physical b

Heavy-Light mesons decay constants

Example: FNAL/MILC, 1712.09262



$\Phi_{H_x} \equiv f_{H_x}\sqrt{M_{H_x}}$ with H_x a heavy-light meson $\bar{h}x$.

f_{p4s} is a reference scale.

Dashed vertical lines indicate the cut $am_h = 0.9$.

Use several EFTs to construct the fit functions

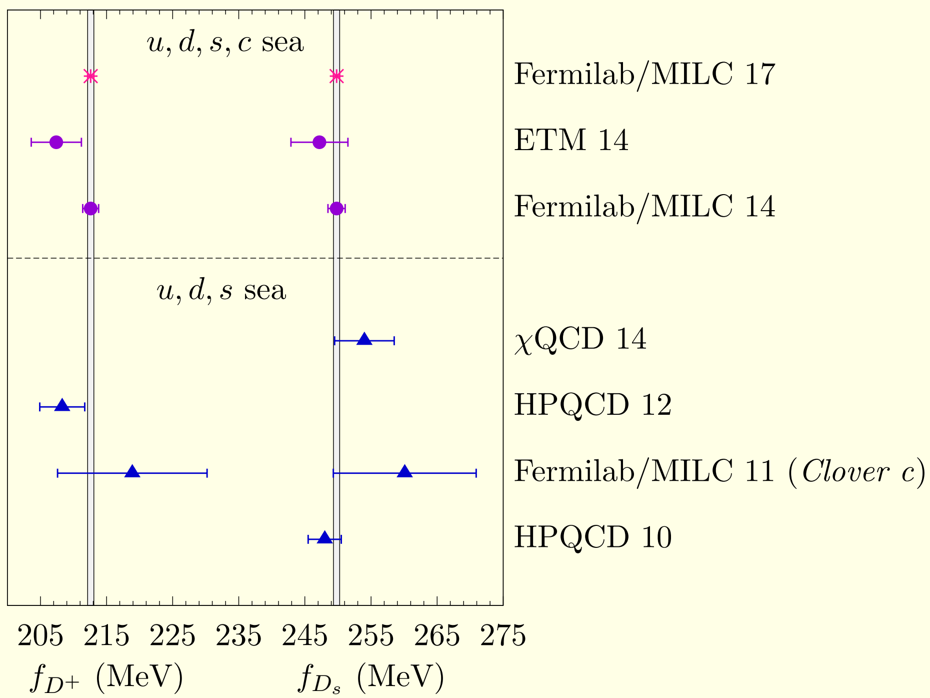
$$\Phi_{H_q} = (1+SET)(1+HQET)(1+HMrPQASChPT)\left(\frac{m_c'}{m_c}\right)^{3/27}\Phi_0$$

Symanzik Effective Theory (SET): $c_1\alpha_s\,(a\Lambda)^2 + \ldots + c_3\alpha_s(am_h)^2 + \ldots$

FNAL/MILC small errors due to: highly improved action, physical light quark masses, no renormalization, MILC ensembles with finer lattice spacings (down to 0.042 fm)

D and D_s decay constants

FNAL/MILC, 1712.09262: Errors ~ 2.5 smaller than previous calculations



$$f_{D^0} = 211.5(0.3)_{stat}(0.3)_{sys}(0.2)_{f_\pi,PDG}\text{MeV}$$

$$f_{D^+} = 212.6(0.3)_{stat}(0.3)_{sys}(0.2)_{f_\pi,PDG}\text{MeV}$$

$$f_{D_s} = 249.8(0.3)_{stat}(0.3)_{sys}(0.2)_{f_\pi,PDG}\text{MeV}$$

(largest syst. errors: FV and scale setting+tuned quark masses)

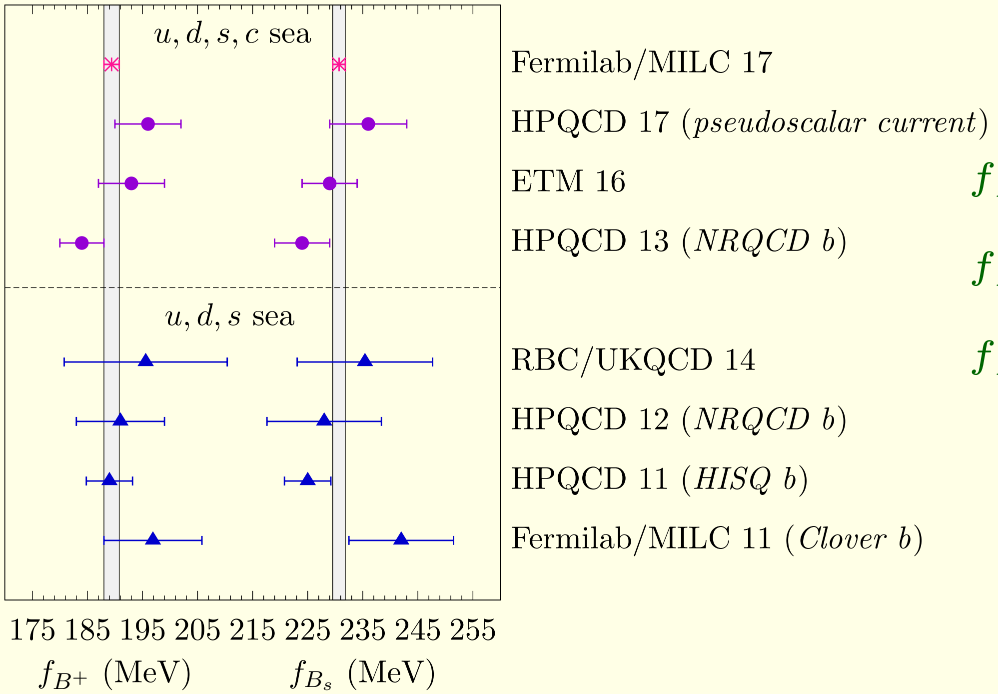
Using PDG16 experimental averages + known short- and long-distance EW corrections + uncertainty from unknown meson-structure-dependent EM corrections:

$$|V_{cs}|_{SM,f_{D_s}} = 0.997(2)_{f_{D_s}}(16)_{expt}(6)_{EM} \qquad |V_{cd}| = 0.2144(5)_{f_D}(49)_{expt}(13)_{EM}$$

EM error comes from unknown structure-dependent corrections and it is based on analogous corrections for pions and kaons: need a direct calculation.

B and B_s decay constants

FNAL/MILC, 1712.09262: Errors ~ 3 smaller than previous calculations



$$f_{B^+} = 189.4(0.8)_{stat}(1.1)_{sys}(0.3)_{f_\pi,PDG}\text{MeV}$$

$$f_{B^0} = 190.5(0.8)_{stat}(1.0)_{sys}(0.3)_{f_\pi,PDG}\text{MeV}$$

$$f_{B_s} = 230.7(0.8)_{stat}(0.8)_{sys}(0.2)_{f_\pi,PDG}\text{MeV}$$

(largest syst. errors: correlator fits)

Using f_{B^+} above and the exp. average **Rosner, Stone, van de Water**, 1509.02220

$$|V_{ub}| = 4.07(3)_{f_{B^+}}(37)_{expt} \cdot 10^{-3}$$

with the large uncertainty agrees with both inclusive and exclusive determinations.

Given the current and projected experimental uncertainties on the D and B meson leptonic decay rates, better lattice-QCD calculations of the decay constants are not needed in the near future.

D mesons

Semileptonic D decays

$$\underbrace{\frac{d\Gamma(D \to P l \nu)}{dq^2}}_{experimental} = \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} |V_{cx}|^2$$

$$\left[\left(1 + \frac{m_l^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) \underbrace{|f_+(q^2)|^2}_{lattice\,QCD} + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 \underbrace{|f_0(q^2)|^2}_{lattice\,QCD} \right]$$

With vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ defined by

$$\langle P(p_P)|V_\mu|D(p_D)\rangle = \left(p_{P\mu} + p_{D\mu} - \frac{m_D^2 - m_P^2}{q^2} q_\mu\right) f_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_\mu \, f_0(q^2)$$

* At zero momentum transfer, $q^2 = 0$: $f_0(0) = f_+(0) \to$ extract $|V_{cd(cs)}|$

* At non-zero momentum transfer, $q^2 \neq 0$:

  Testing lattice QCD: shape of the form factors (lattice prediction **2004**)

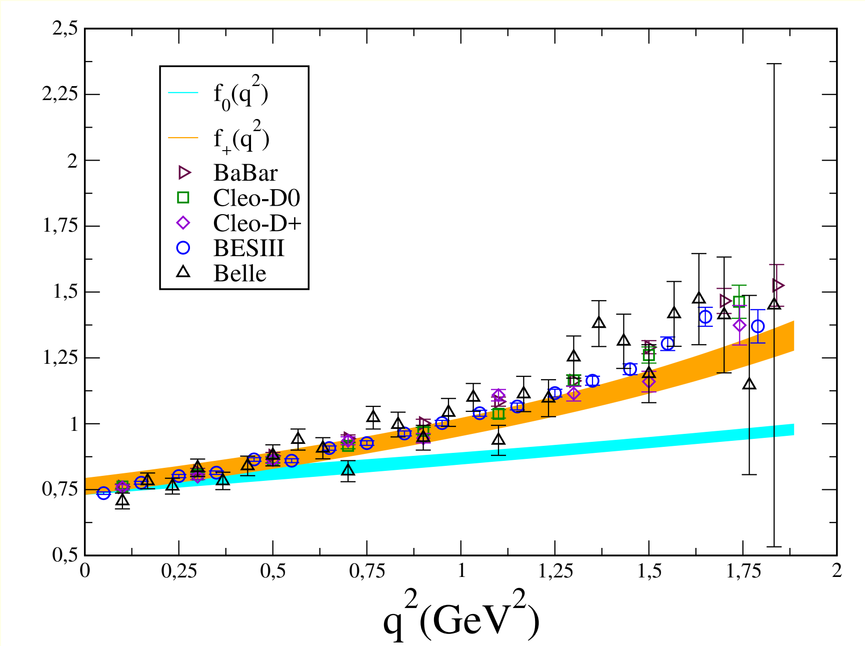
  $\to$ use same methodology for B semileptonic and rare decays

The errors on those studies are still dominated by errors in the calculation of the relevant form factors.

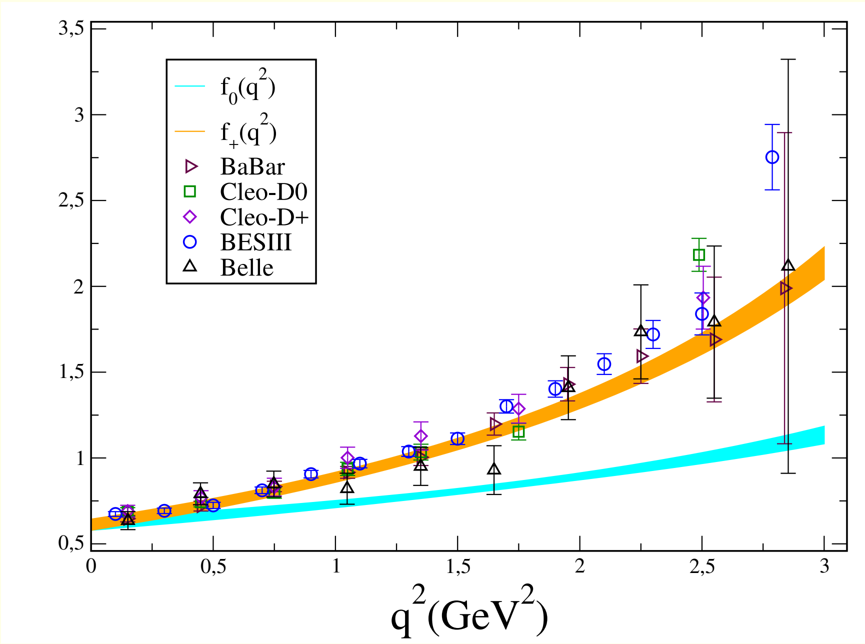
D semileptonic decays

$N_f = 2+1+1$ **ETMC17**, V.Lubicz et al, 1706.03017

$D \to K\ell\nu$



$D \to \pi\ell\nu$



* 3 values of a, different volumes, $m_\pi = 210 - 440\ MeV$

* Calculate f_+, f_0 over whole q^2 range

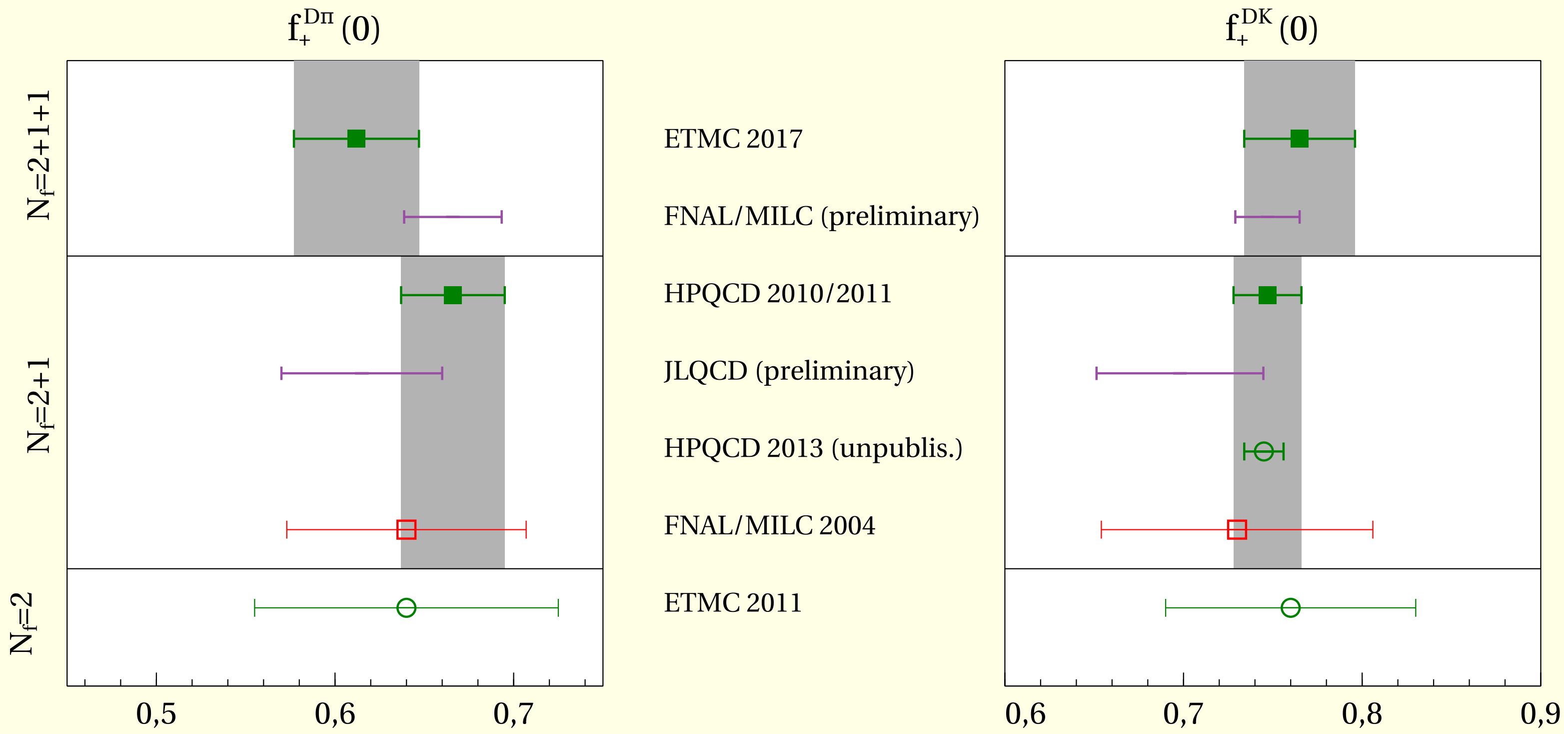
* modified z-expansion

* Correct for hypercubic discretization effects.

$$f_+^{D\pi}(0) = 0.612(35) \qquad f_+^{DK}(0) = 0.765(31)$$

In preparation: Combined experimental + lattice data fit to extract $|V_{cd(cs)}|$.

D semileptonic decays



Using the experimental averages from **Rosner, Stone, Van de Water**, 1509.02220 and **FLAG=HPQCD** form factors

$$|V_{cs}| = 0.975(25)_{lat}(7)_{exp} \qquad |V_{cd}| = 0.2140(93)_{lat}(29)_{exp}$$

* Non competitive with leptonic decays.

* Error dominated by LQCD calculation.

Need better LQCD form factor calculations

Another determinations of $|V_{cd(cs)}|$

HPQCD, 1311.6669

$D_s \to \phi l \nu$

* More challenging: more form factors (vector meson), unstable meson ...

* q^2 and angular distributions agree with **BaBar** data.

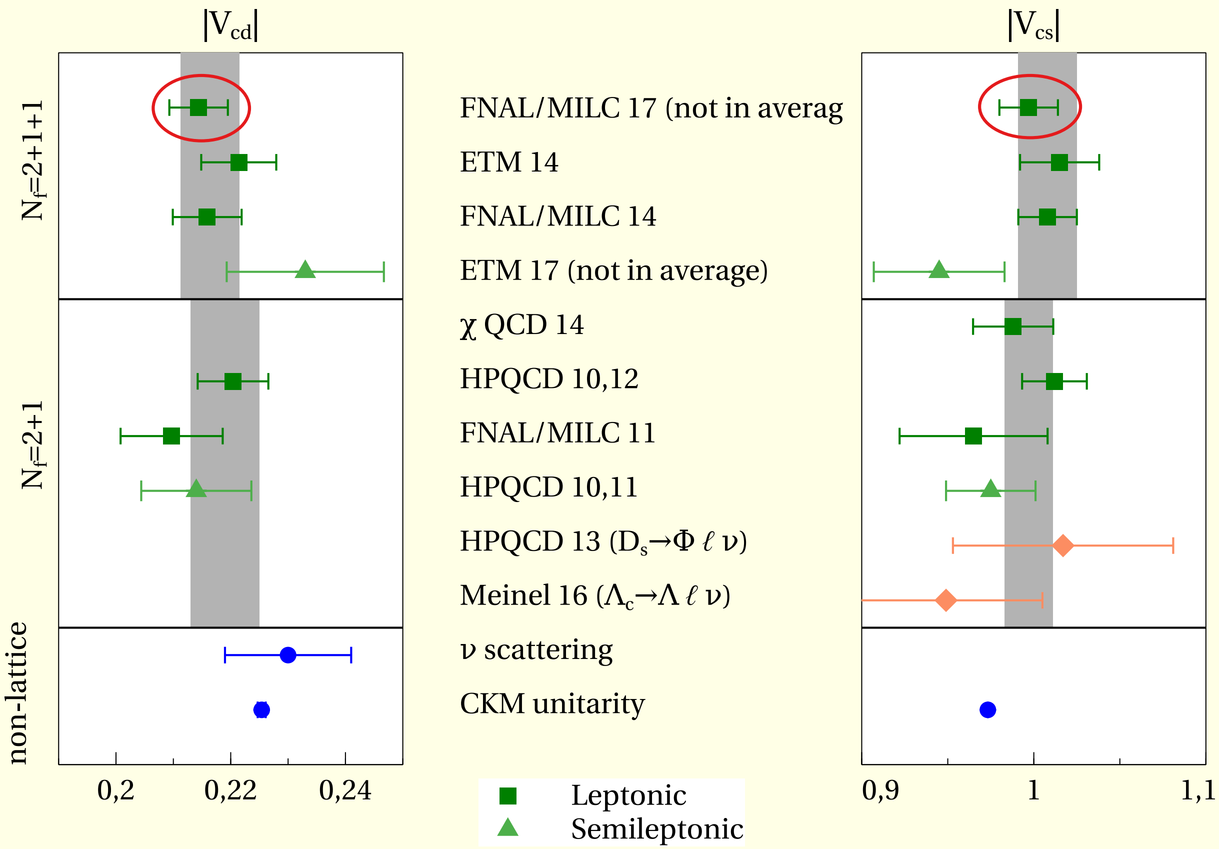
$$|V_{cs}| = 1.017(44)_{lat}(35)_{exp}(30)_{K\bar{K}}$$

$\Lambda_c \to \Lambda \ell \nu$

First lattice calculation of form factors, **S. Meinel** 1611.09696

$$|V_{cs}| = 0.949(24)_{lat}(14)_{\tau_{\Lambda_c}}(49)_{exp}$$

Second row CKM unitarity



Grey bands: **FLAG2016** averages including semileptonic and leptonic up to 2016

With $|V_{cd,cs}|$ from **FNAL/MILC**, 1712.09262 and $|V_{cb}|_{incl+excl} = 41.40(77) \cdot 10^{-3}$

$$\Delta_c \equiv |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.042(2)_{|V_{cd}|}(32)_{|V_{cs}|}(0)_{|V_{cb}|}$$

compatible with three-generation CKM unitarity within 1.3σ

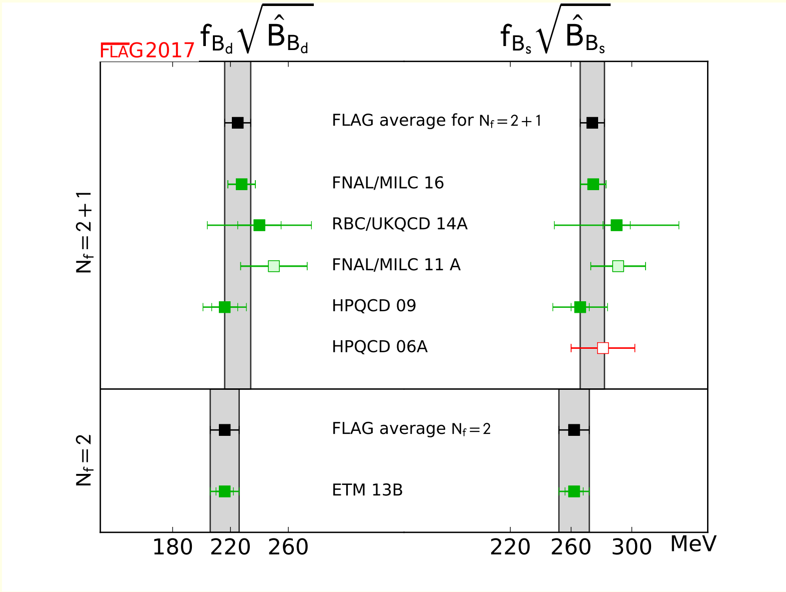
* Precision limited by exp. error on decay widths for $D_s \to \mu(\tau)\nu_{\mu(\tau)}$

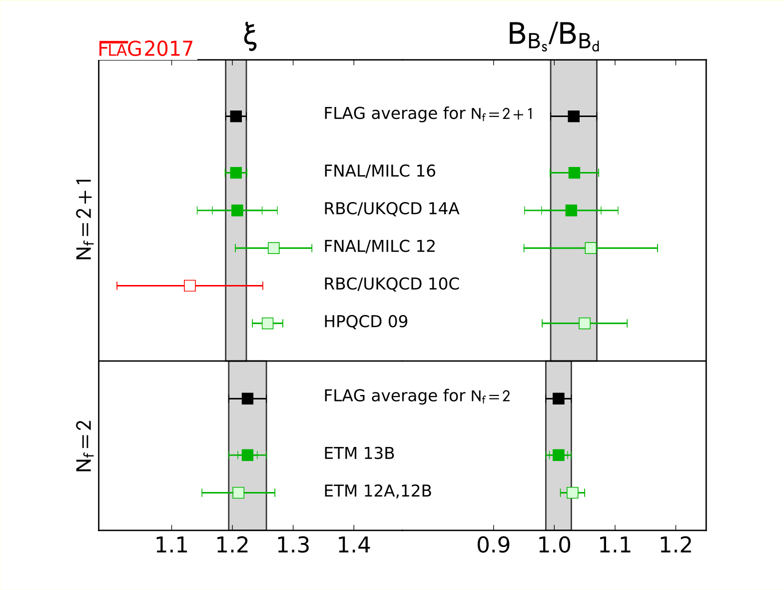
B mesons

Neutral B-meson mixing

In the Standard Model, it is dominated by the short-distance top contribution (for $q = d, s$):

$$\Delta M_q \propto \left|V_{tq}^* V_{tb}\right|^2 f_{B_q}^2 \hat{B}_{B_q}^{(1)}, \text{ where } \tfrac{8}{3} f_{B_q}^2 B_{B_q}^{(1)}(\mu) M_{B_q}^2 = \langle \bar{B}^0 | \mathcal{O}_1^q | B^0 \rangle (\mu)$$





With ξ the $SU(3)$-breaking ratio $\xi = \sqrt{\frac{f_{B_s}^2 \hat{B}_{B_s}^{(1)}}{f_{B_d}^2 \hat{B}_{B_d}^{(1)}}} \propto \frac{\Delta M_s}{\Delta M_d}$

(Results provided for $\hat{B}_i$ with $i = 1, \ldots 5$ by FNAL/MILC and ETMC)

Neutral B-meson mixing

In the SM, using tree-level inputs for the CKM matrix elements **CKMfitter** and **Fermilab-MILC** 1602.03560 results

$$f_{B_d}\sqrt{\hat{B}^{(1)}_{B_d}} = 227.7(9.5)(2.3)\ \mathrm{MeV}\,,\ f_{B_s}\sqrt{\hat{B}^{(1)}_{B_s}} = 274.6(8.4)(2.7)\ \mathrm{MeV} \quad ,$$
$$\xi = 1.206(18)(6)$$

we get

$$\Delta M_d^{SM} = 0.630(53)(42)(5)(13)\,ps^{-1} \qquad \Delta M_d^{expt,HFLAV16} = 0.5064(19)\,ps^{-1}$$
$$\Delta M_s^{SM} = 19.6(1.2)(1.0)(0.2)(0.4)\,ps^{-1} \qquad \Delta M_s^{expt,HFLAV16} = 17.757(21)\,ps^{-1}$$
$$(\Delta M_d/\Delta M_s)^{SM} = 0.0321(10)(15)(0)(3)\,ps^{-1}$$

(where the errors are from lattice, CKM matrix elements, other inputs in SM expression, omission of charm quark on the sea, respectively)

* Agreement at the 1.8σ, 1.1σ and 2.0σ level, respectively.

Rare decays $\mathcal{B}(B_{s(d)} \to \mu^+\mu^-)$

Bag parameters describing $B-$**meson** mixing in the SM can be used for an indirect theoretical prediction of $\overline{\mathcal{B}}(B \to \mu^+\mu^-)$ **Buras**, hep-ph/0303060

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Using $\hat{B}_{B_s} = 1.42(9)$, $\hat{B}_{B^0} = 1.43(12)$ (from **FNAL/MILC**, 1602.03560 matrix elements and **FNAL/MILC** 1712.09262 decay constants)

$$10^9 \cdot \overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.27(21)_{\hat{B}_s}(0)_{\Delta M_s}(6)_{other}$$

$$10^{11} \cdot \overline{\mathcal{B}}(B^0 \to \mu^+\mu^-) = 0.88(8)_{\hat{B}^0}(0)_{\Delta M_d}(2)_{other}$$

Error dominated by uncertainty in the bag parameter

* On-going: FNAL/MILC (including decay constants-MEs correlations), ETMC ($N_f = 2+1+1$), RBC/UKQCD ...

Rare decays $\mathcal{B}(B_{s(d)} \to \mu^+\mu^-)$

* Indirect determination (assuming SM)

$$10^9 \cdot \overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.27(21)_{\hat{B}_s}(0)_{\Delta M_s}(6)_{other} = 3.27(22)$$

$$10^{11} \cdot \overline{\mathcal{B}}(B^0 \to \mu^+\mu^-) = 0.88(8)_{\hat{B}^0}(0)_{\Delta M_d}(2)_{other} = 0.88(8)$$

* Direct determination using the most recent (and precise) decay const.

$f_{B^0} = 190.5(1.3)\ MeV$ and $f_{B_s} = 230.7(1.2)\ MeV$ **FNAL/MILC**, 1712.09262

$$10^9 \cdot \overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{SM} = 3.64(4)_{f_{B_s}}(8)_{CKM}(7)_{other} = 3.64(11)$$

$$10^{11} \cdot \overline{\mathcal{B}}(B \to \mu^+\mu^-)_{SM} = 1.00(1)_{f_{B^0}}(2)_{CKM}(2)_{other} = 1.00(3)$$

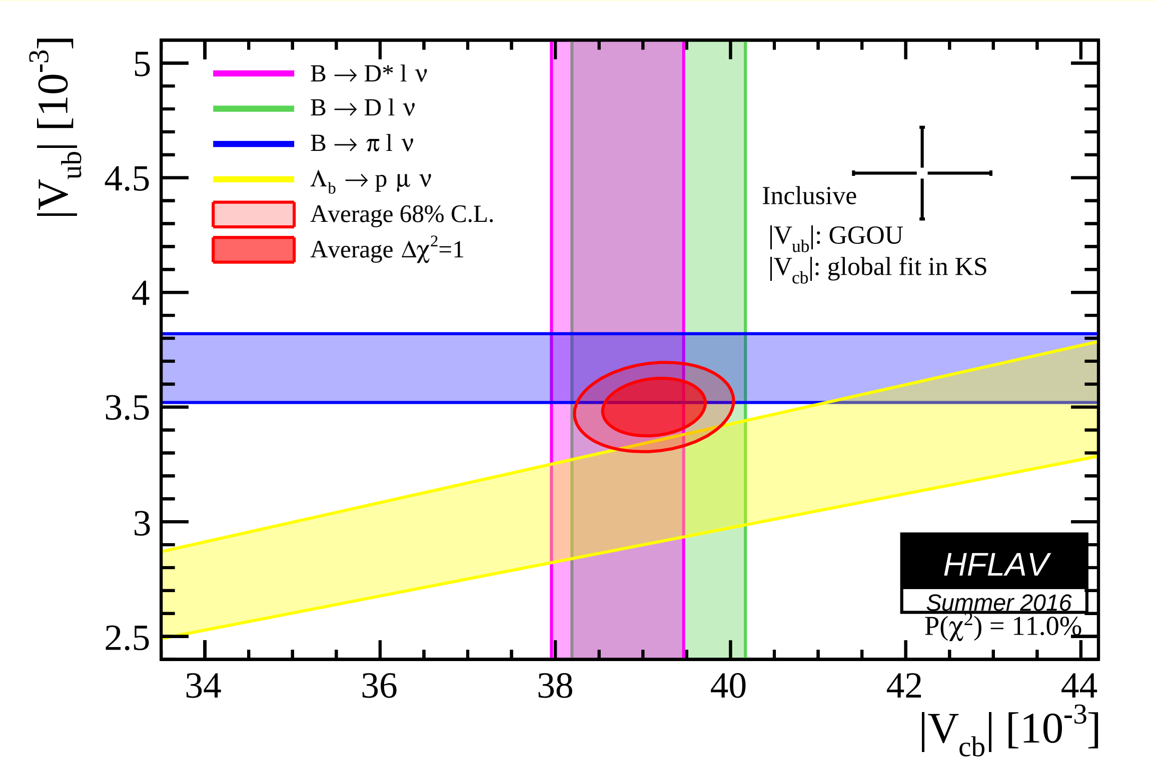
* To be compared with the most recent experimental measurements

$$10^9 \cdot \overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\mathbf{HFLAV\,17}} = 3.1(7)\,, \quad 10^{11} \cdot \overline{\mathcal{B}}(B \to \mu^+\mu^-)_{\mathbf{LHCb,CMS}} = 39\left(^{+16}_{-14}\right)$$

SM predictions consistent with exp. measurements but ample room for NP

Exclusive vs inclusive $|V_{ub}|$ and $|V_{cb}|$

Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ at the $\sim 3\sigma$ level.



LQCD inputs

* $B \to \pi \ell \nu$: $f_+(q^2)$ $(f_0(q^2))$
* $\Lambda_b \to p \mu \nu / \Lambda_c \to \Lambda_d \mu \nu$: Six form factors for each channel
* $B \to D^* \ell \nu$: $\mathcal{F}(\omega = 1)$
* $B \to D \ell \nu$: $\mathcal{G}(\omega)$ (related to $f_+(q^2)$)

$|V_{cb}|$ from exclusive B decays ($w = v_B \cdot v_D$ velocity transfer to the leptonic pair)

$$\frac{d\Gamma(B \to D^* l \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$

$$\frac{d\Gamma(B \to D l \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

Exclusive determination of $|V_{cb}|$

$|V_{cb}|$ extracted from exclusive B decays ($w = v_B \cdot v_D$ is the velocity transfer to the leptonic pair)

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$
$$\frac{d\Gamma(B \to D l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

* How to parametrize the ω dependence?

Model-independent `z-expansion:` map ω onto a complex variable z via the conformal transformation $z = (\sqrt{\omega + 1} - \sqrt{2})/(\sqrt{\omega + 1} + \sqrt{2})$

   * Coefficients in the z-expansion are subject to unitarity bounds based on analyticity.

BCL (Bourrely-Caprini-Lellouch), 0807.2722

BGL (Boyd-Grinstein-Lebed), hep-ph/9412324

CLN (Caprini-Lellouch-Neubert), hep-ph/9712417: + model-dependent NLO HQET constraints to reduce the error: heavy-quark corrections underestimated?

Bigi&Gambino, 1703.06124 **Grinstein&Kobach**, 1703.08170

$B \to D\ell\nu$: Exclusive determination of $|V_{cb}|$

$B \to D\ell\nu$ channel:

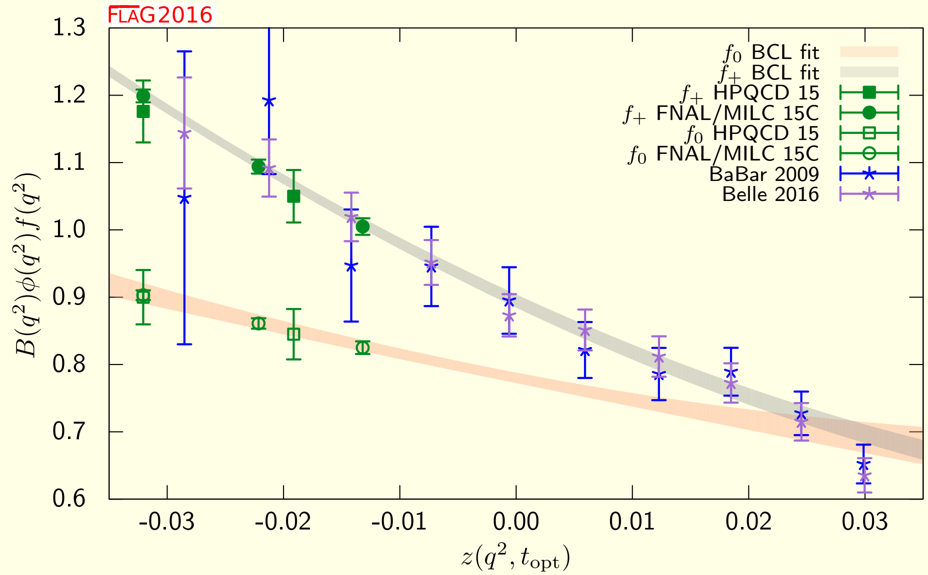
Combined BGL fit to experimental and lattice data on different q^2 regions

Bigi&Gambino,1606.08030

HPQCD, 1505.03925 **FNAL/MILC**, 1503.07237

$$|V_{cb}| = 40.49(97) \cdot 10^{-3}$$

* LQCD form factor error smaller than experiment.



Nice agreement with BCL FLAG global fit ($|V_{cb}| = 40.1(1.0) \cdot 10^{-3}$)

In acceptable agreement with either $B \to D^*$ exclusive and inclusive determinations.

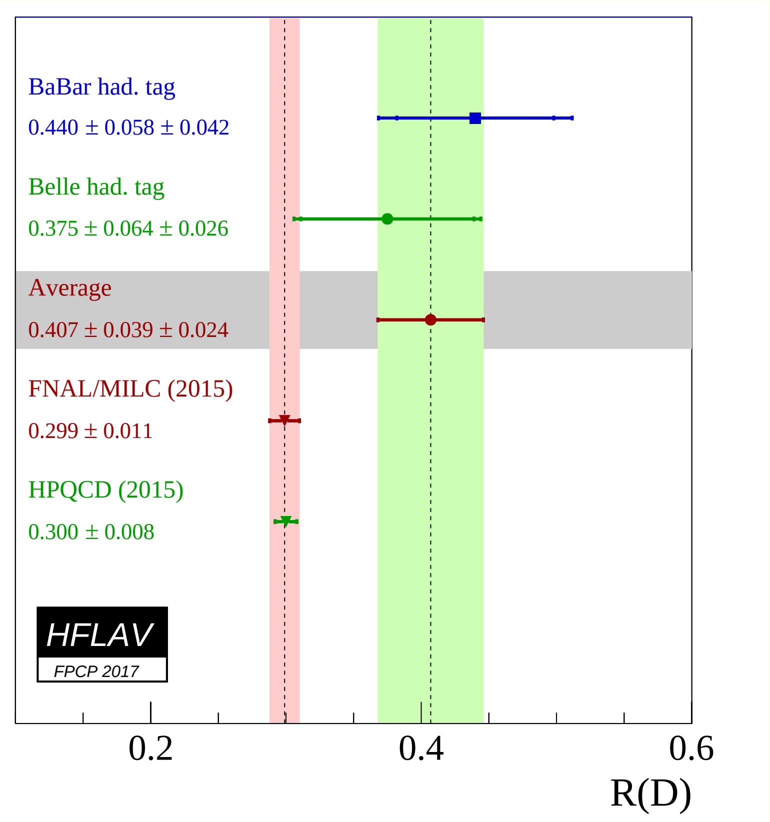
* HFLAV quotes $|V_{cb}| = 39.18(1.04) \cdot 10^{-3}$ when using only the data at zero recoil from a CLN fit.

$B \to D\ell\nu$: LFV ratios

* Lattice $f_+(q^2)$ and $f_0(q^2)$ form factors can be used to calculate:

$$R(D) \equiv \frac{\mathcal{B}(B\to D\tau\nu_\tau)}{\mathcal{B}(B\to D\ell\nu)}_{FLAG2016} = 0.300(8)$$

This result is $\sim 2.3\sigma$ lower than the experimental average



Critical to control reliability of form factor parametrization at low q^2.

* Global fit experiment+lattice data with BGL parametrization: $R(D) = 0.299(3)$
Bigi & Gambino 1606.08030

In progress: **RBC/UKQCD, FNAL/MILC,HPQCD, LANL/SNU...**

$B \to D^*\ell\nu$: LFV ratios

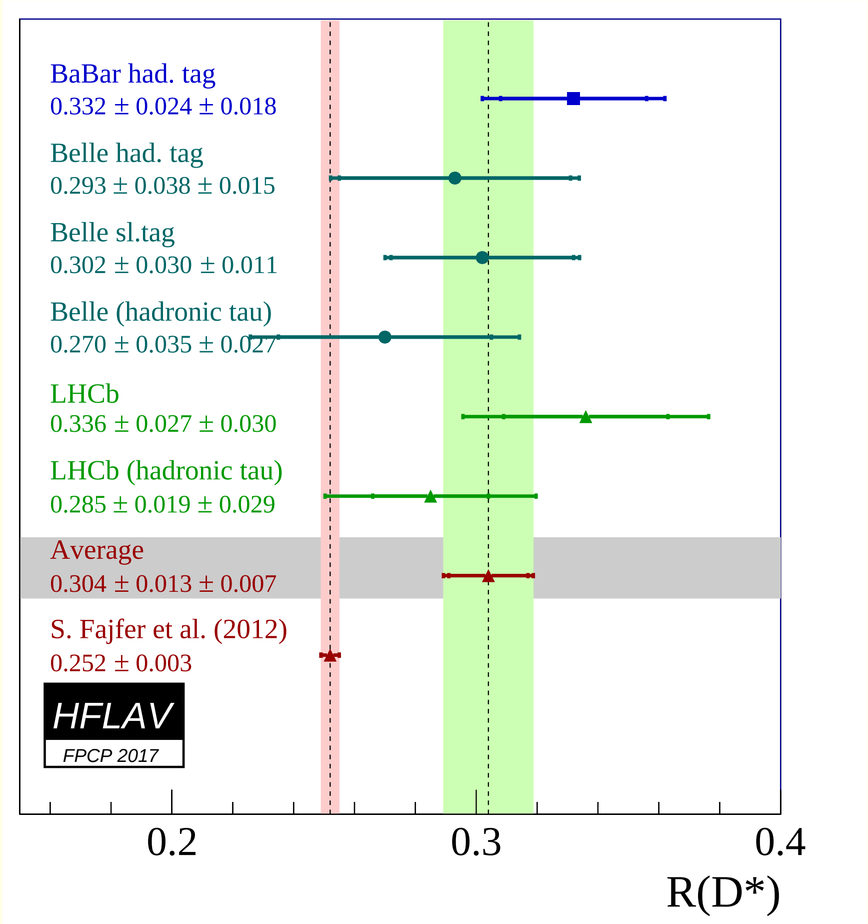
* SM calculation in **Fajfer et al** 1203.2654 based on exp. $B \to D^*$ measurements which use CLN parametrization.

$$R(D^*) \equiv \frac{\mathcal{B}(B\to D^*\tau\nu_\tau)}{\mathcal{B}(B\to D^*\ell\nu)}_{FLAG2016} = 0.252(3)$$

This result is $\sim 3.4\sigma$ lower than the experimental average

Needs reliable LQCD calculation of all form factors involved.

In progress: **RBC/UKQCD, FNAL/MILC...**



$B \to D^*\ell\nu$: Exclusive determination of $|V_{cb}|$

$B \to D^*\ell\nu$ channel: $\eta_{EW}|V_{cb}|\mathcal{F}(\omega=1)$

* Until now, lattice data for $\mathcal{F}(\omega=1)$ only available at zero recoil $\omega=1$.

FNAL/MILC 1403.0635, **HPQCD** 1711.11013

* Experimental data extrapolated to $\omega=1$ using CLN parametrization.

$$|V_{cb}|_{FLAG2016} = 39.27(56)(49)\cdot 10^{-3} \qquad 2.7\sigma \text{ from inclus.}$$

For the first time, **Belle** 1702.01521 provided unfolded fully-differential decay rate and associated covariance matrix.

Bigi, Gambino & Schacht 1703.06124,1707.09509 **Grinstein & Kobach** 1703.08170

$|V_{cb}|_{CLN} = 38.2(1.5)\cdot 10^{-3} \to |V_{cb}|_{BGL} = 41.7(2.0)\cdot 10^{-3}$, agree with incl.

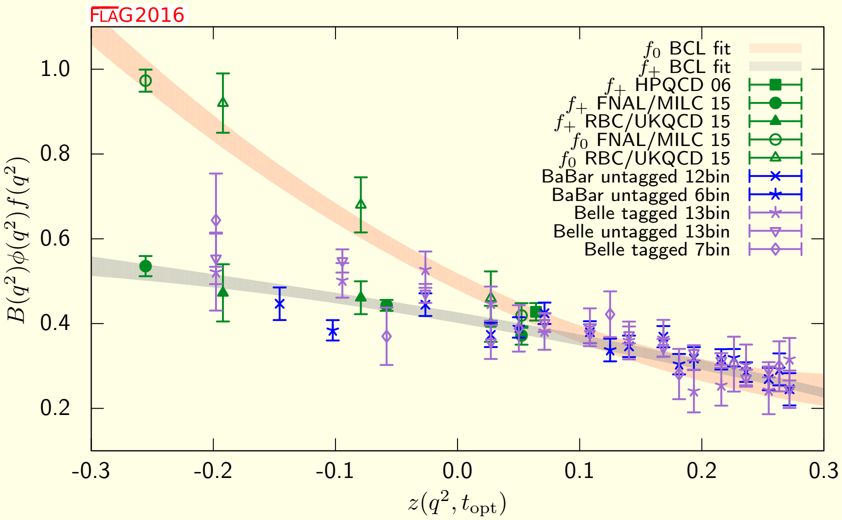
Bigi, Gambino & Schacht 1707.09509 $R(D^*) = 0.260(8)$ $(3.4 \to 2.6\sigma)$

* Large difference not necessarily true for other exp. data.

Need LQCD calculation of form factors at non-zero recoil

$B \to \pi \ell \nu$ Exclusive determination of $|V_{ub}|$

$|V_{ub}|$ extracted from exclusive mode $B \to \pi l \nu$



Combined BCL fit to experimental and lattice data on different q^2 regions using z-expansion.

RBC/UKQCD, 1501.05373

FNAL/MILC, 1503.07839

HPQCD, hep-lat/0601021

$$|V_{ub}|^{FLAG2016} = 3.73(14) \cdot 10^{-3}$$

Good consistency between lattice and experimental shapes and commensurate errors.

$$|V_{ub}|^{\rm inclusive, HFLAV} = 4.62(20)(29) \cdot 10^{-3} \qquad \sim 2.5\sigma \text{ disagreement.}$$

* **In progress: FNAL/MILC, RBC, ALPHA**

* **In progress: Belle-II** is expected to reduce the exp. error for both inclus. and exclus.

Other exclusive determinations of $|V_{ub}|$

Extraction of $\frac{|V_{ub}|}{|V_{cb}|}$ from $\frac{\Gamma(\Lambda_b \to p\mu\nu)}{\Gamma(\Lambda_b \to \Lambda_c\mu\nu)}$ measured at the **LHCb** 1504.01568 and LQCD form factors from **Detmold, Lehner and Meinel**, 1503.01421

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.080(4)_{\mathrm{expt}}(4)_{\mathrm{theor}}$$

* Combining it with $|V_{cb}|$, it could give $|V_{ub}|$ with an error competitive with $B \to \pi\ell\nu$

Using inclusive value of $|V_{cb}|$: $|V_{ub}| = 3.38(25) \cdot 10^{-3}$ 2.9σ tension with $|V_{ub}|_{inclus.}$

Alternative way of getting $|V_{ub}|$: $B_s \to K\ell\nu$.

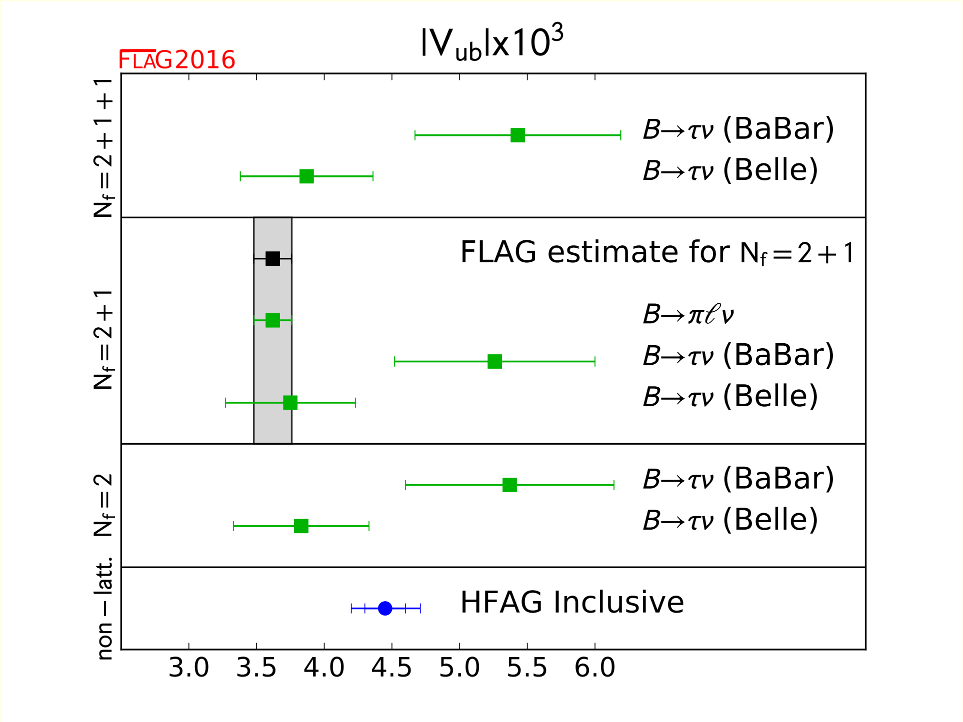
* Two LQCD calculations of the relevant form factors:
  HPQCD 1406.2279, **RBC/UKQCD** 1501.05373

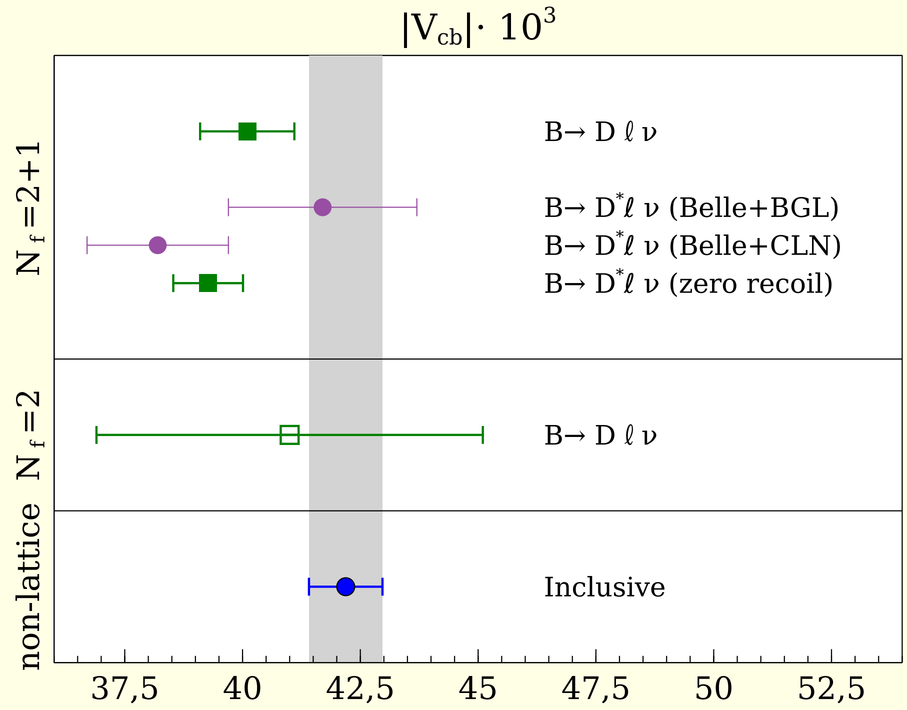
  On-going: **FNAL/MILC, ALPHA**

* LQCD error smaller than for $B \to \pi$ form factors

* Experimentally: expected to be measured at **Belle-II**

Inclusive vs Exclusive determinations of $|V_{cb(ub)}|$





Tension persists

* Alternative calculations $B_s \to K$, $\Lambda_b \to p(\Lambda_c)$, $B \to \tau\nu$ ...

* Wait for better experimental data.

* Improvements from LQCD in progress.

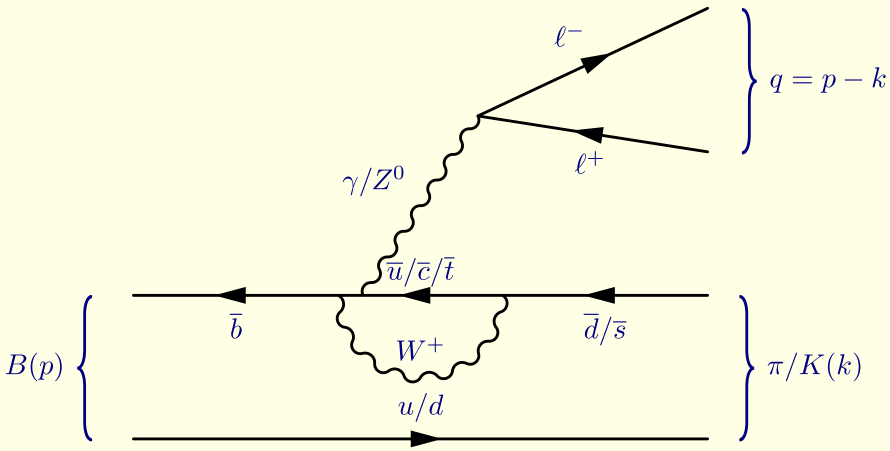
Further study of parametrization dependence

* Crucial: LQCD form factors at non-zero recoil (in progress)

* Also relevant for $R(D^*)$.

Form factors for $B \to \pi(K) l^+ l^-$

Flavor-changing neutral currents $b \to q$ transitions are potentially sensitive to NP effects $B \to K^* \gamma$, $B \to K^{(*)} \ell^+ \ell^-$, $B \to \pi \ell^+ \ell^-$



Most important contributions to all this type of decays are expected to come from matrix elements of current (vector, axial and tensor) operators

Need vector, f_+, scalar, f_0 and tensor, f_T form factors from LQCD

D. Du et al 1510.02349

$$\frac{d\Gamma}{dq^2} = (\text{known}) |\vec{k}| |V_{tb} V_{tq}^*|^2 \left[\frac{2}{3} \beta_l^2 |\vec{k}|^2 \left| C_{10}^{\text{eff}} f_0(q^2) \right|^2 + \frac{m_l^2 (M_B^2 - M_P^2)^2}{q^2 M_B^2} \left| C_{10}^{\text{eff}} f_+(q^2) \right|^2 \right.$$
$$\left. + \left(1 - \frac{1}{3}\beta_l^2\right) |\vec{k}|^2 \left| C_9^{eff} f_+(q^2) + 2 C_7^{eff} \frac{m_b + m_q}{M_B + M_P} f_T(q^2) \right|^2 \right]$$

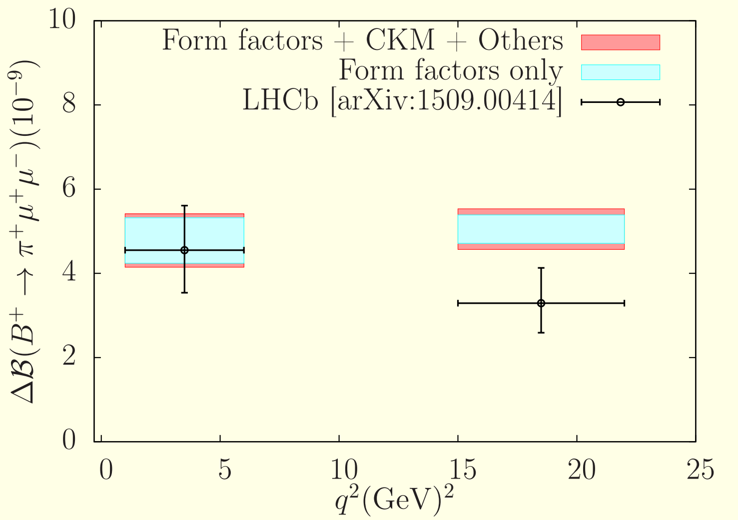
* Determine CKM matrix elements or constrain Wilson coeff. C_9 and C_{10}.

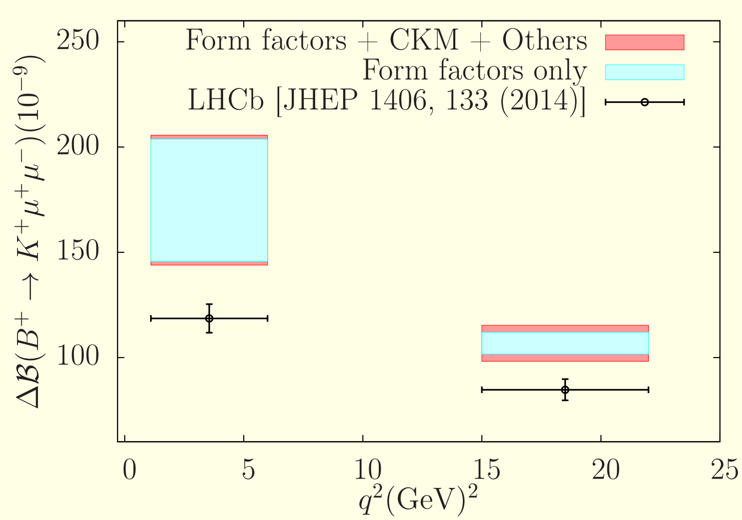
Form factors for $B \to \pi(K)l^+l^-$

$B \to K\ell^+\ell^-$: **HPQCD** 1306.0434, 1306.2384, **FNAL/MILC**, 1509.06235

$B \to \pi\ell^+\ell^-$: **FNAL/MILC**, 1507.01618, 1510.02349. Take f_+ and f_0 from combined fit of lattice + experimental data for $B \to \pi\ell\nu$ (assume not significant NP effects at tree level).

From **D. Du et al** 1510.02349 using **FNAL/MILC** 1507.01618,1509.06235, focus on large bins above and below charmoninum resonances

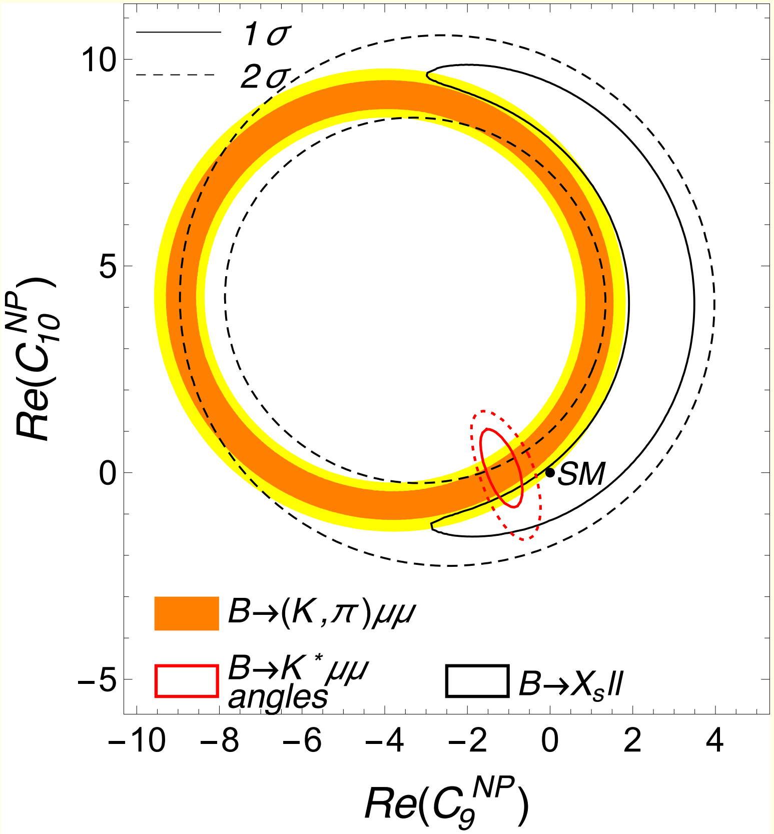


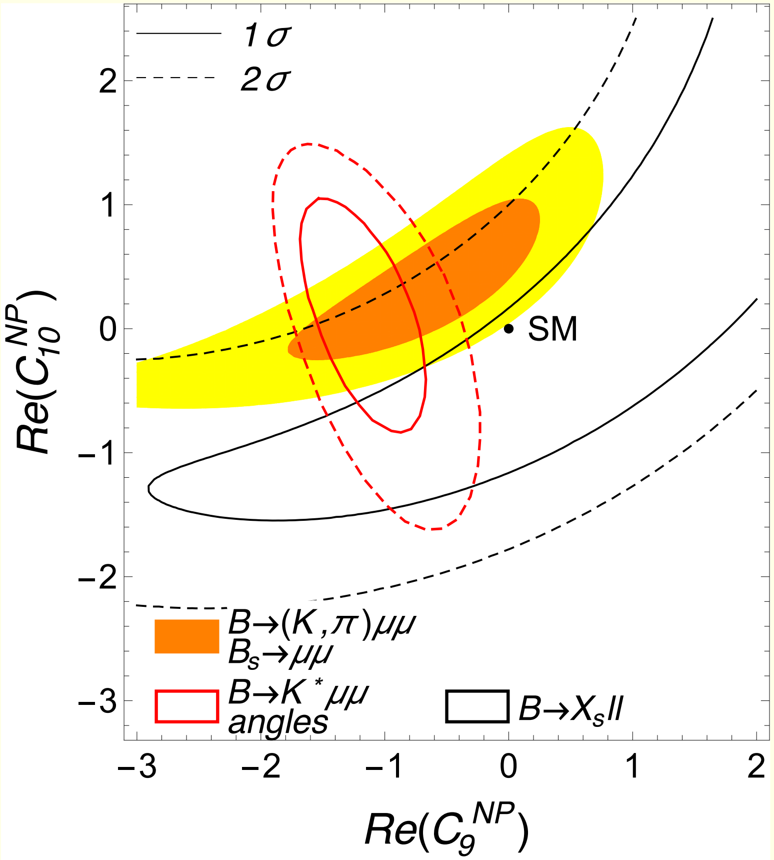


$1-2\sigma$ experiment-SM tensions

On-going: **FNAL/MILC** with physical quark masses.

Phenomenology for $B \to \pi(K)l^+l^-$



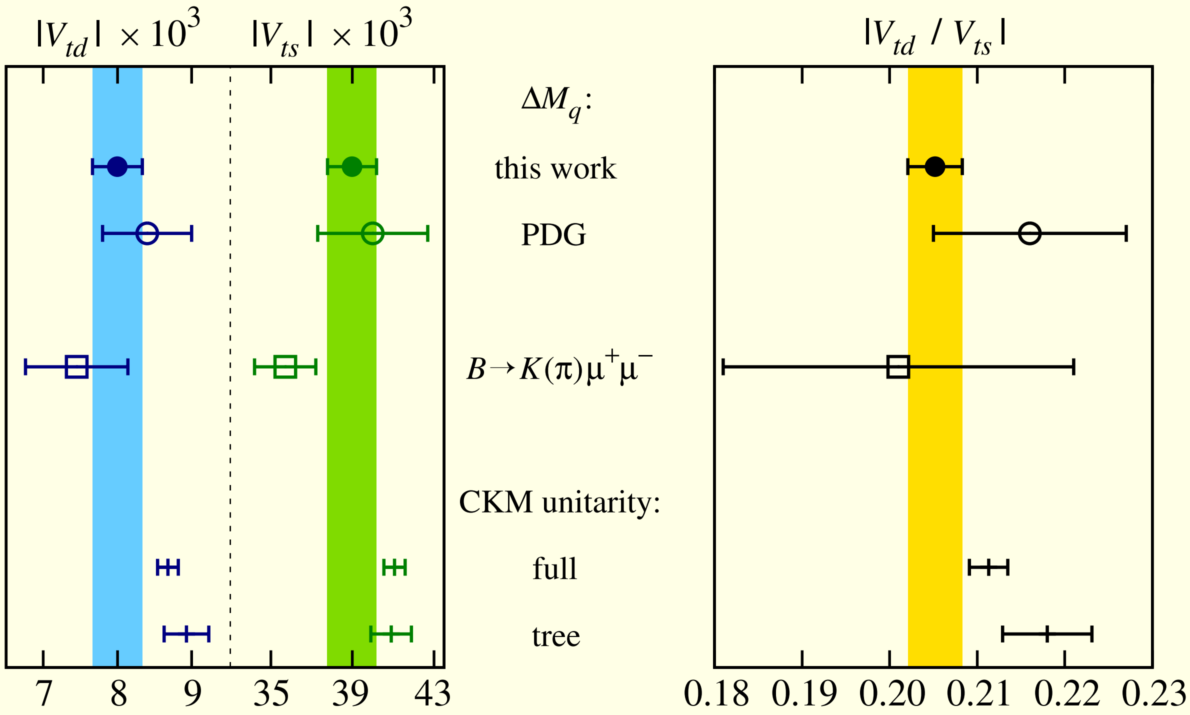


* 2σ tension with the SM.

* Favored region consistent with inclusive decays

* Competitive with constraints from $B \to K^*\ell\nu$

Extraction of CKM matrix elements



* $B \to K(\pi)\mu^+\mu^-$ results from **D. Du et al**, 1510.02349

* This work: B-meson mixing matrix elements **FNAL/MILC**, 1602.03560 + ΔM_q^{expt}

* Full/tree CKM unitarity results come from CKMfitter's fit 2015 using all inputs/only observable mediated at tree level of weak interactions.

2σ tensions between loop processes and CKM unitarity contraints from tree level processes

Phenomenology for $B \to \phi(K^*)l^+l^-$

$B \to K^* l^+ l^-$ (and $B_s \to \phi l^+ l^-$) have additional challenges for LQCD: K^* is unstable, more form factors ...

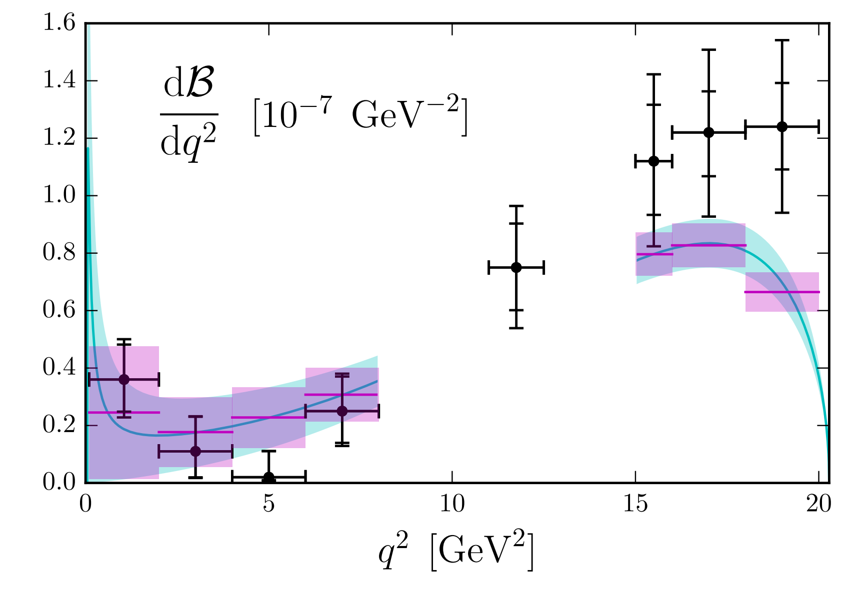
* First unquenched results **Horgan, Liu, Meinel and Wingate**, 1310.3722, 1310.3887: differential branching fractions $\sim 2\sigma$ larger than experiment

* Still a lot of space for technical and strategical improvements

* Finite-volume methods to include the width of an unstable final state hadron exist, but not implemented yet.

On-going work: **RBC/UKQCD** 1511.06622, ...

Phenomenology for $\Lambda_b \to \Lambda l^+ l^-$

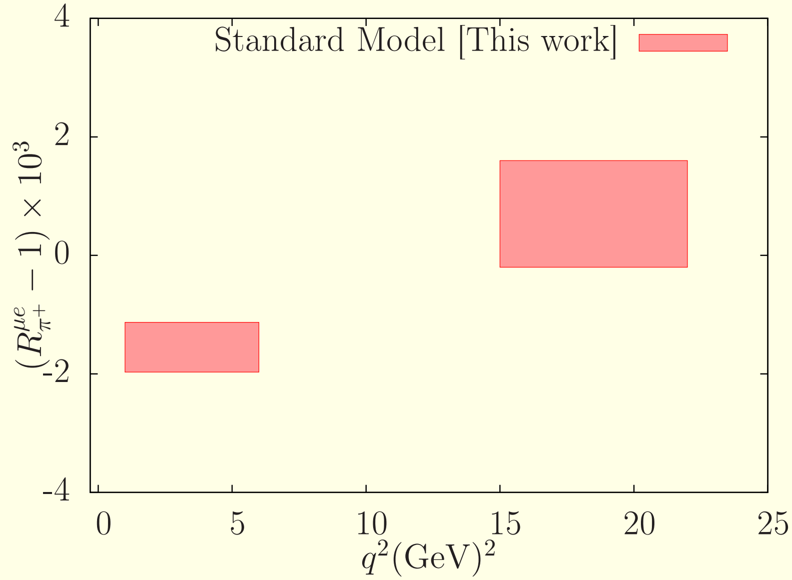


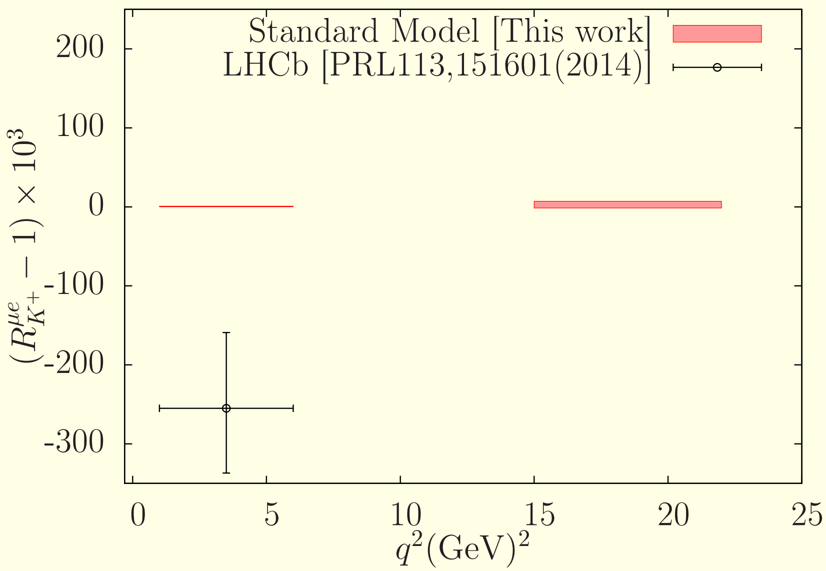
Tensions also observed in the channel $\Lambda_b \to \Lambda l^+ l^-$

* **LHCb** 1503.07138 data vs SM LQCD calculation **Detmold & Meinel** 1602.01399

Rare decays: Lepton Universality Tests

D. Du et al. 1510.02349 with FNAL/MILC form factors





2.6σ tension LHCb-SM

$$R_P^{\ell_1 \ell_2}(q^2_{min}, q^2_{max}) = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to P \ell_1^+ \ell_1^-)}{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to P \ell_2^+ \ell_2^-)}$$

D. Du et al. 1510.02349 SM prediction for $R_\pi = \frac{\mathcal{B}(B \to \pi \tau \nu_\tau)}{\mathcal{B}(B \to \pi \ell \nu)} = 0.641(17)$.

Expected to be measured at **Belle-II**

Rare decays: Lepton Universality Tests

$$R_P^{\ell_1 \ell_2}(q^2_{min}, q^2_{max}) = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to P \ell_1^+ \ell_1^-)}{\int_{q^2_{min}}^{q^2_{max}} dq^2 d\mathcal{B}(B \to P \ell_2^+ \ell_2^-)}$$

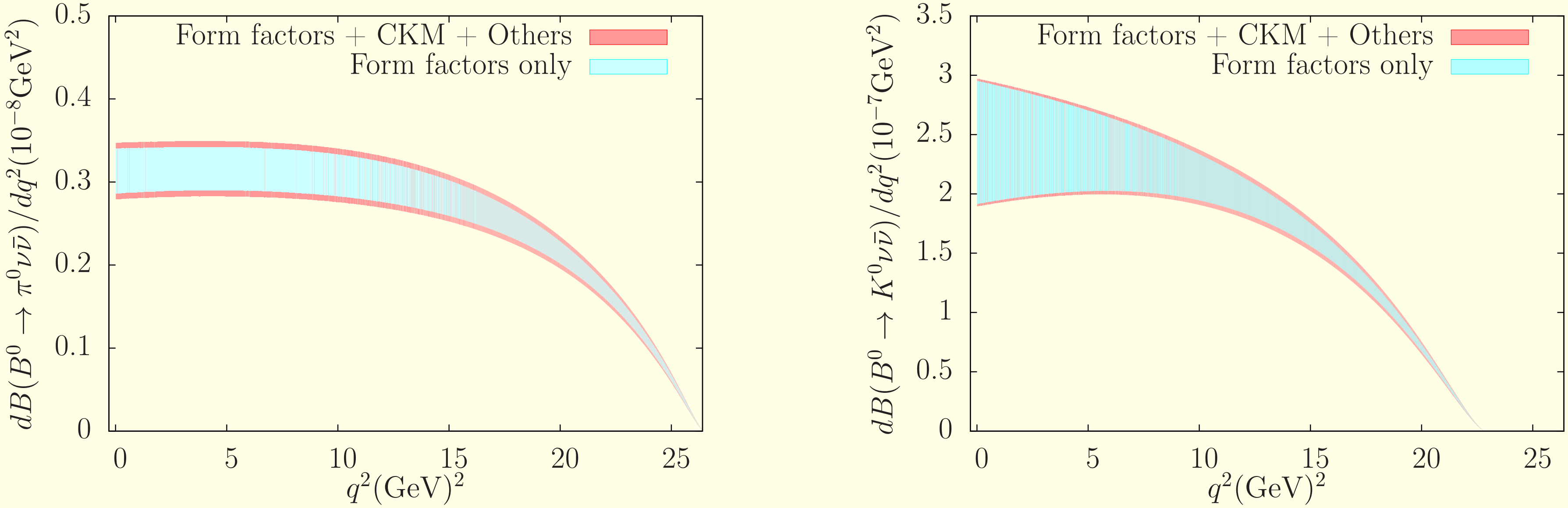
Renewed interest due to recent **LHCb** measurement of R_{K^*}, **R. Ajai et al** 1705.05802: $2 - 2.5\sigma$ tension with SM.

* Not LQCD inputs for this yet.

** Finite-volume methods to include the width of an unstable final state hadron exist, but not implemented yet.

Rare semileptonic B decays to $\nu\bar{\nu}$ states

D. Du et al. 1510.02349 with FNAL/MILC form factors



Predictions for both neutral and charged channels.

Theoretically clean (SD proportional to f_+)

Difficult to measure experimentally, maybe at Belle-II

$$\mathcal{B}(B^0 \to \pi^0 \nu\bar{\nu}) \cdot 10^7 = 0.668(41)(49)(16)$$
$$\mathcal{B}(B^0 \to K^0 \nu\bar{\nu}) \cdot 10^7 = 40.1(2.2)(4.3)(0.9)$$

$$\mathcal{B}(B^+ \to \pi^+ \nu\bar{\nu}) \cdot 10^6 = 9.62(1)(92);\ \mathcal{B}(B^+ \to K^+ \nu\bar{\nu}) \cdot 10^6 = 4.94(52)(6)$$

Conclusions and outlook

* Growing number of results on ensembles of configurations with physical quark masses

  ** Including charm on the sea $\surd$

* Relativistic description of c and b allow an important reduction of errors

* First complete calculations of weak $\Lambda_{b,c}$ decay form factors

* Theoretical framework for semileptonic B decays to vector meson final states exists **Briceño et al** 1406.5965, **Agadjanov et al** 1605.03386

  ** Pilot studies of form factors for $B_s \to K^*\ell\nu$, $B \to K^*\ell\nu$,... underway

* Need to include subdominant effects

  ** Strong isospin breaking: eventually, $N_f = 1 + 1 + 1 + 1$

  ** Lots of effort to include QED in lattice simulations **A. Patella** 1702.03857 (kaons, $(g-2)_\mu \ldots$), **Giusti et al** 1711.06537, **T. Blum et al.** 1801.07224

* Increasing number of quantities: $(g-2)_\mu$, $K \to \pi\pi$, nucleon matrix elements, resonances ... will become high precision. New quantities will be studied (inclusive decays ...)

Conclusions and outlook

Some tensions need theory (lattice) and experimental inprovements to clarify:

* $|V_{cb}|_{\rm incl.} - |V_{cb}|_{\rm excl.}$: $B \to D^* \ell\nu$ at non-zero recoil available soon.

  ** Also relevant for $R(D^*)$.

* $R(D)$, R_K, $R_{K^*}\ldots$

  ** Form factors for $B_{(s)} \to K^*\ell^+\ell^-$, $B \to K\ell^+\ell^-$, $B_{(s)} \to \pi\ell^+\ell^-$.

* First-row CKM unitarity.

  ** Leptonic vs semileptonic extraction of $|V_{us}|$

* Inclusive vs exclusive $|V_{ub}|$

* Consistency in tree-level and loop extractions of $|V_{td,ts}|$

…