CPAN
Ingenio 2010

II CPAN DAYS

from 29 November 2010 to 01 December 2010

Hotel Astoria Palace (Valencia)

Some highlights of theoretical nuclear Physics

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Departamento de Física Teórica

UAM
UNIVERSIDAD AUTONOMA DE MADRID

Nuclear Landscape

Ab Initio

Configuration Interaction

Density Functional Theory

Stable Nuclei

Known Nuclei

Terra Incognita

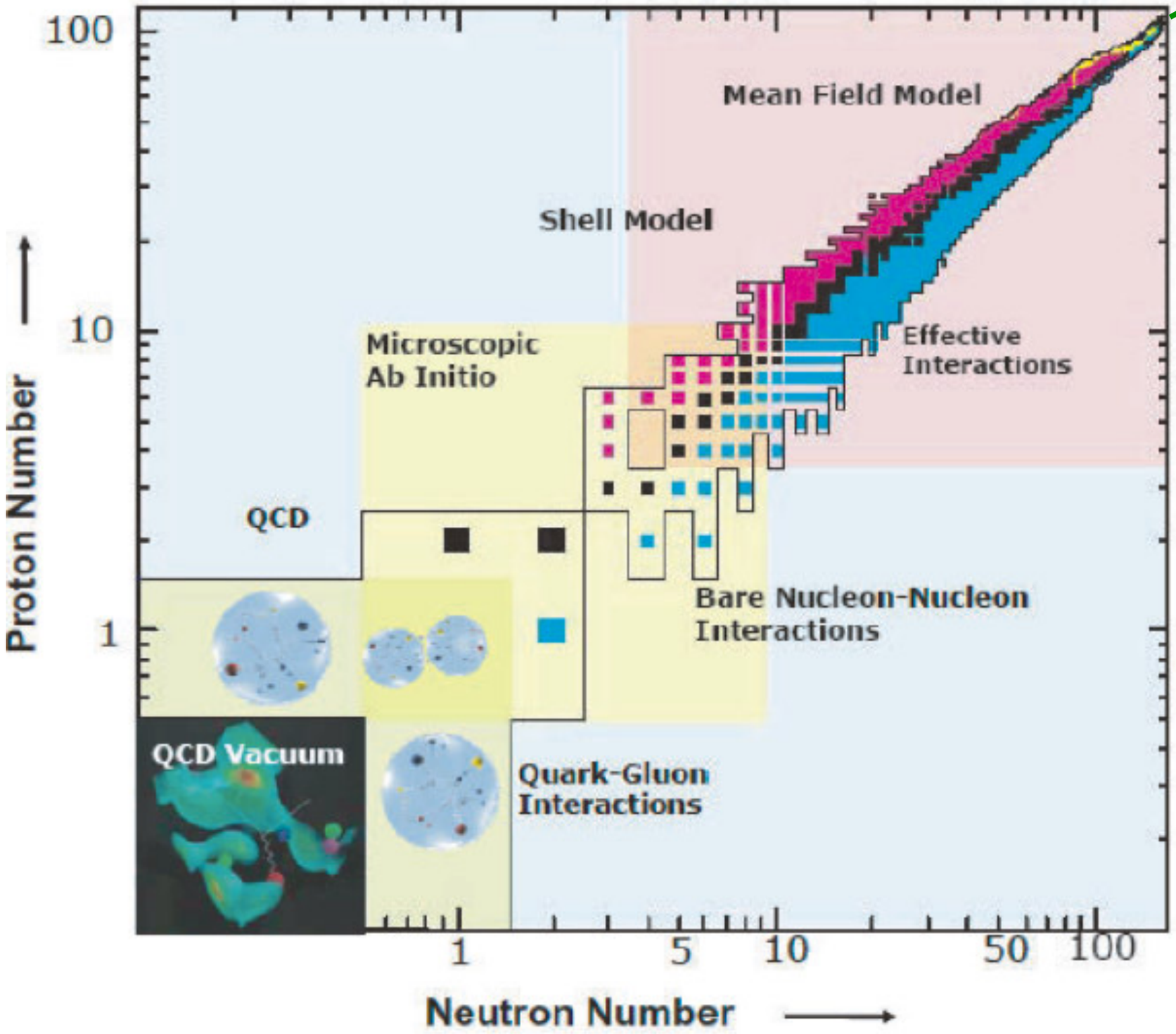
r-process

Protons

Neutrons

2 8 20 28 50 82 126

Nuclear Matter



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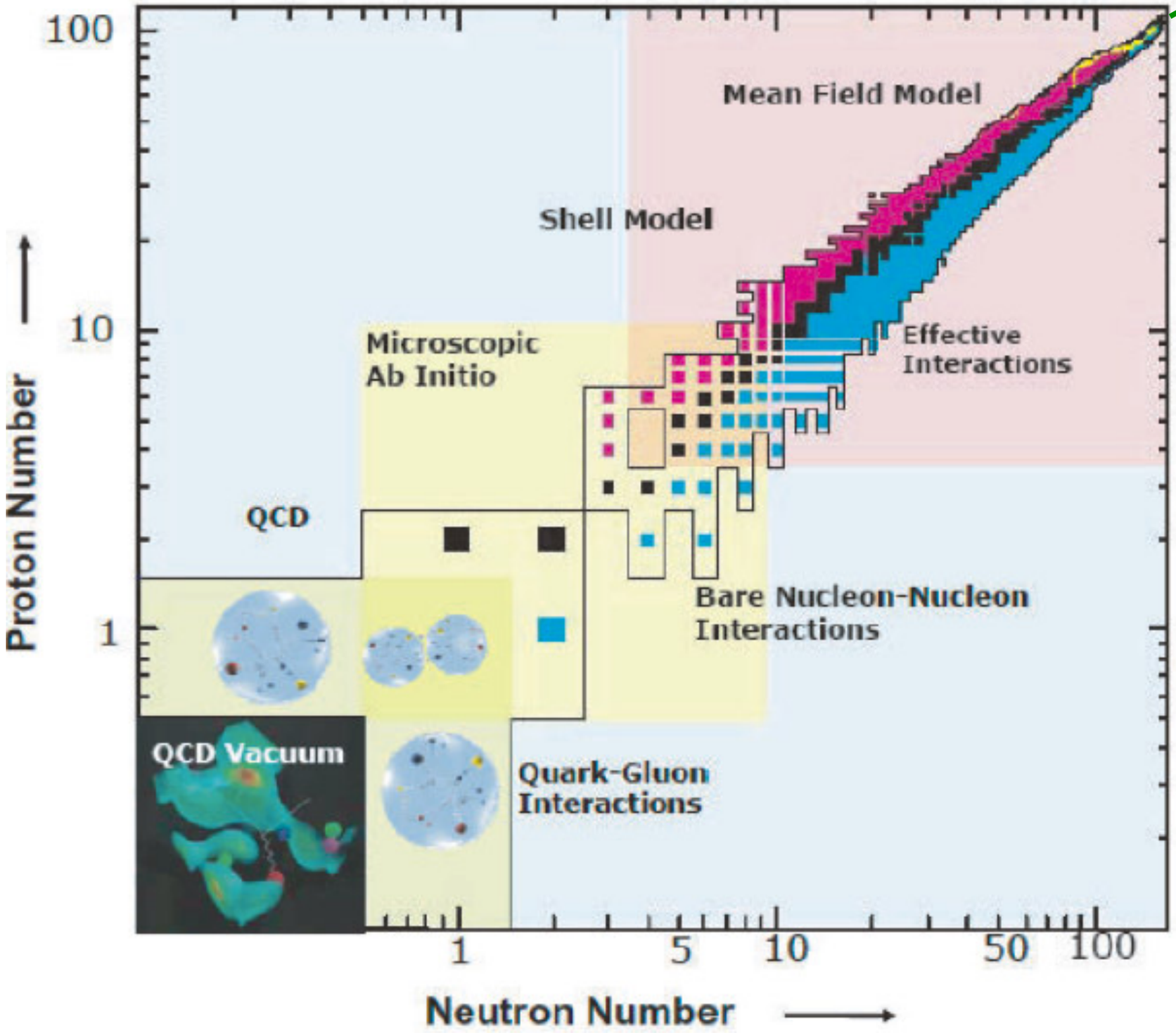

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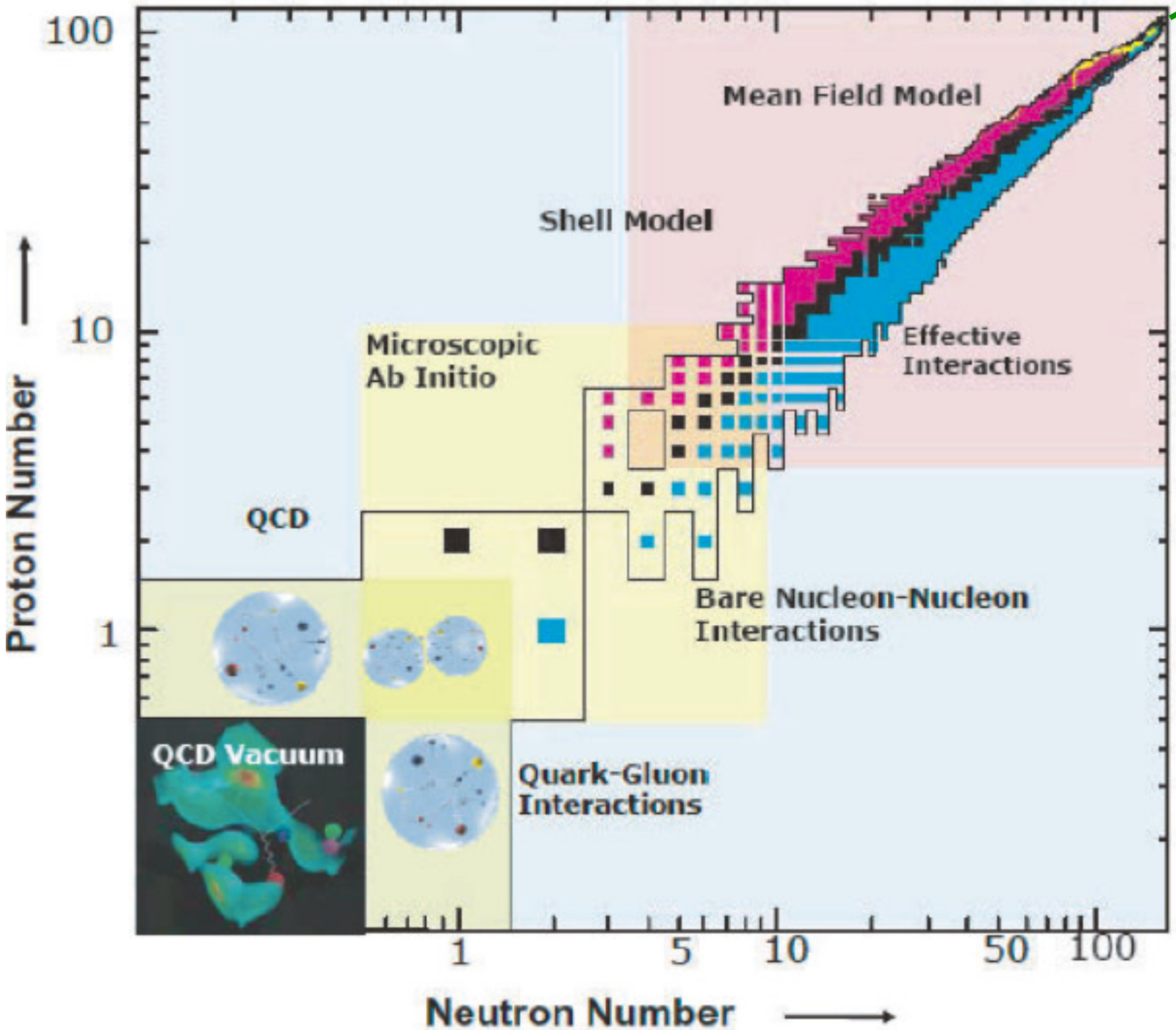
US IEM UHU

Nuclear Matter



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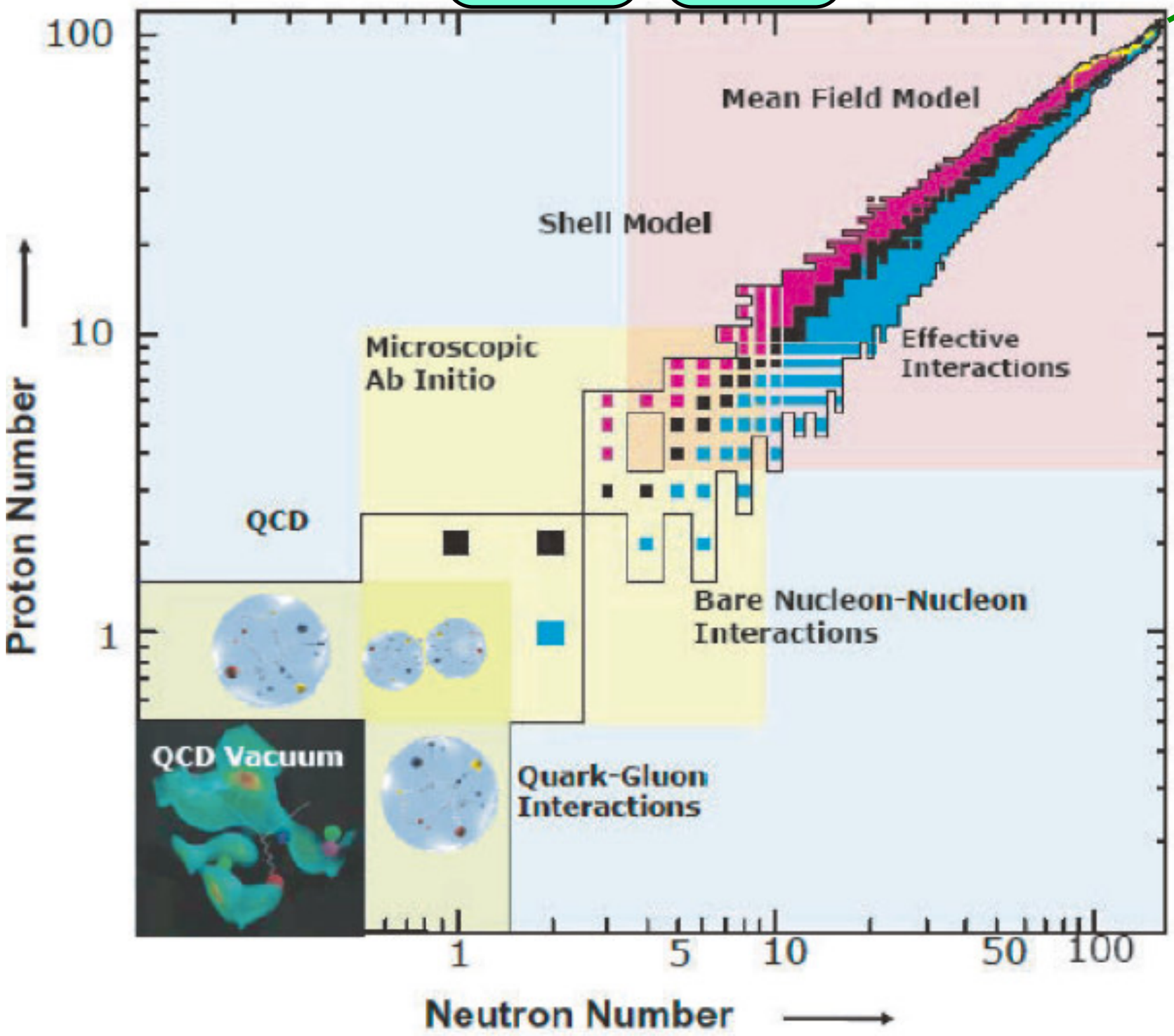

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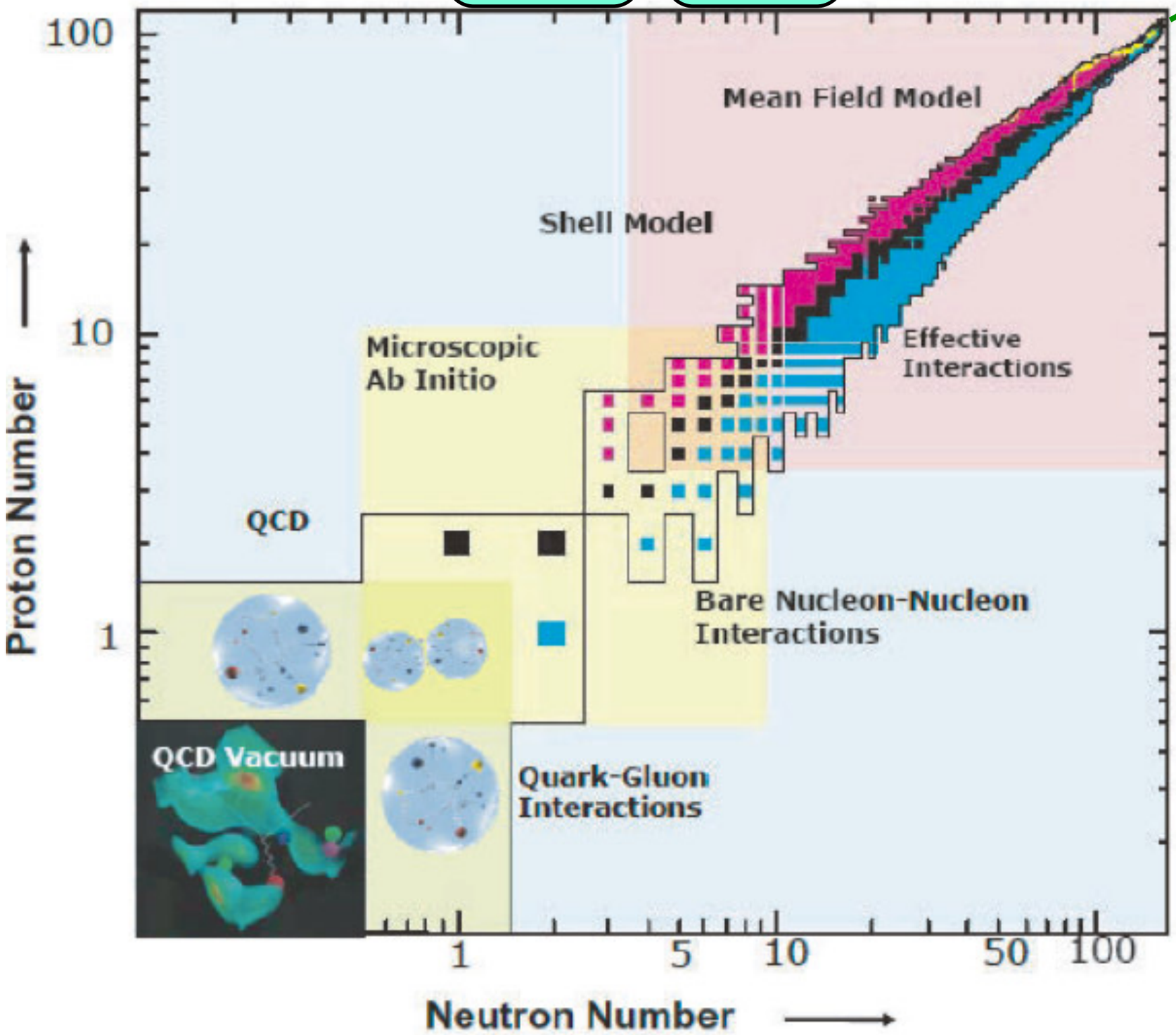

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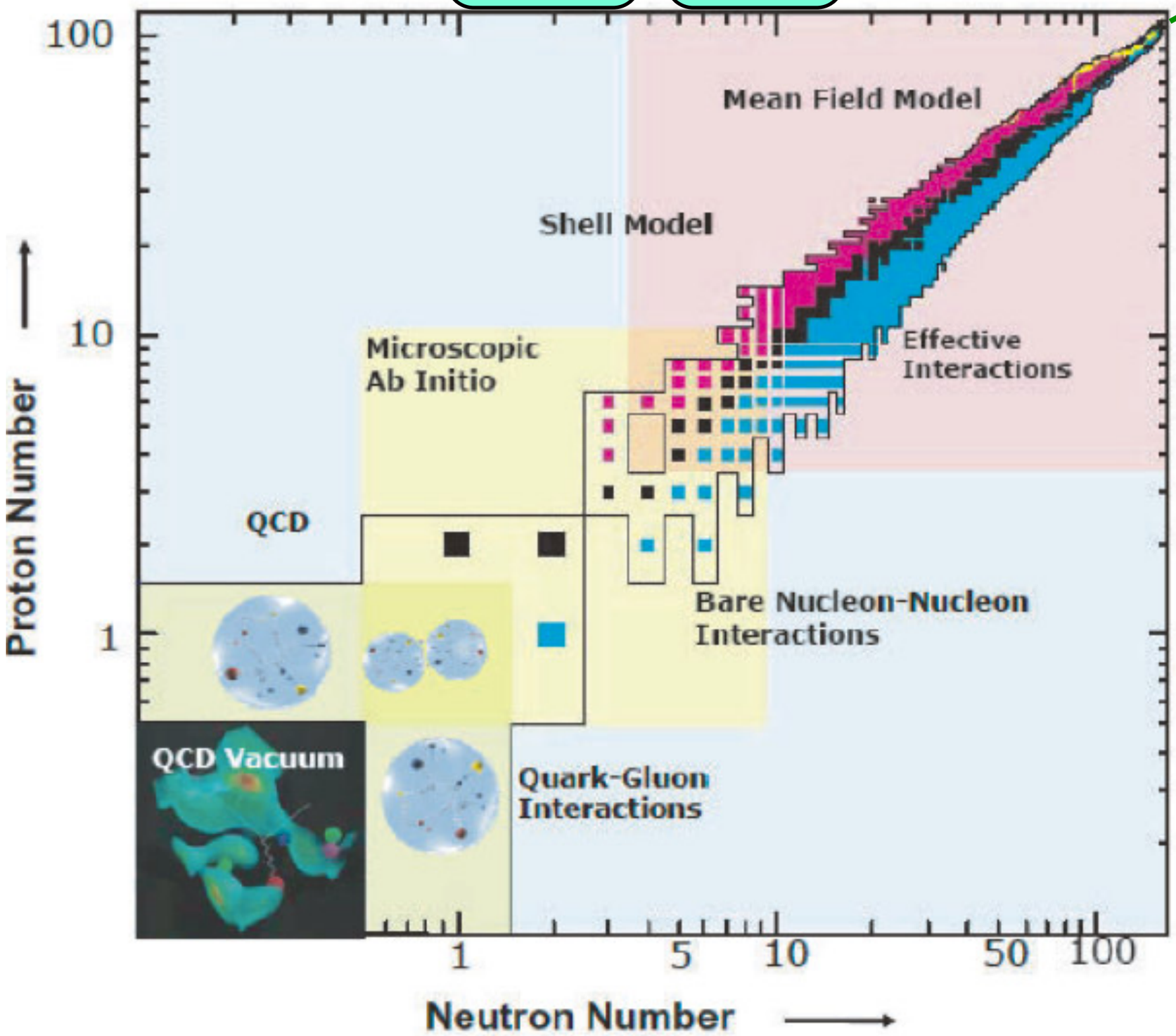

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Nuclear Theory Challenges

To describe the structure of nuclei near the particle drip lines and superheavy nuclei.

Some examples of the new physics:

1.- The magic numbers N=20 and N=28 fade away with the neutron number and new magic numbers N=14, N=16 and N=32 seem to appear.

2.- Unusual properties of nuclei far from stability: extrem radial extension of halo nuclei (radius of 11Li is the same as 208Pb).

3.- Relevance of weakly bound nucleons when the nucleonic binding gets small.

4.- Strong effects of 3-body forces and tensor forces in exotic nuclei.

Nuclear many-body methods

1. Shell-model and no-core shell-model calculations; Large-scale diagonalization. (UAM)

2. Perturbative many-body methods. (UG, USAL)

3. Coupled cluster theory. (UG)

4. Extension to weakly bound systems. Complex scaling and complex shell model. (US, IEM)

5. DFT, Relativistic and beyond mean field approaches (UAM, IEM, UCM,UG and UB)

6. Density matrix renormalization group. (IEM)

7. Variational, diffusion and path integral Monte Carlo methods. (IFIC, UG, UB)

8. Green's function theory, unitary operator method. (UB)

9. Algebraic models.(US, UHU, IEM)

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The exact solution of the many-body system

Let be h the Hamiltonian of a particle moving in a potential, of a Harmonic oscillator, for example. Its eigenfunctions

$$h\phi_k = \epsilon_k \phi_k$$

form a basis. In the same way, the N-body wave functions

$$\Phi_{p_1,...,p_N}(1,...,N) = \frac{1}{\sqrt{N!}} \sum_P (-1)^P P \phi_{p_1}(1)...\phi_{p_N}(N) = c^\dagger_{p_N}...c^\dagger_{p_1}|-\rangle$$

provide a basis for the N-body systems.

That means, the exact solution of the many body Hamiltonian can be formally expanded as

$$\Psi(1,...,N) = \sum_{p's} B_{p_1,...,p_N} \Phi_{p_1,...,p_N}(1,...,N)$$

The coefficients of the expansion being determined by the diagonalization of the Hamiltonian in the $\Phi_{p_1,...,p_N}(1,...,N)$ basis.

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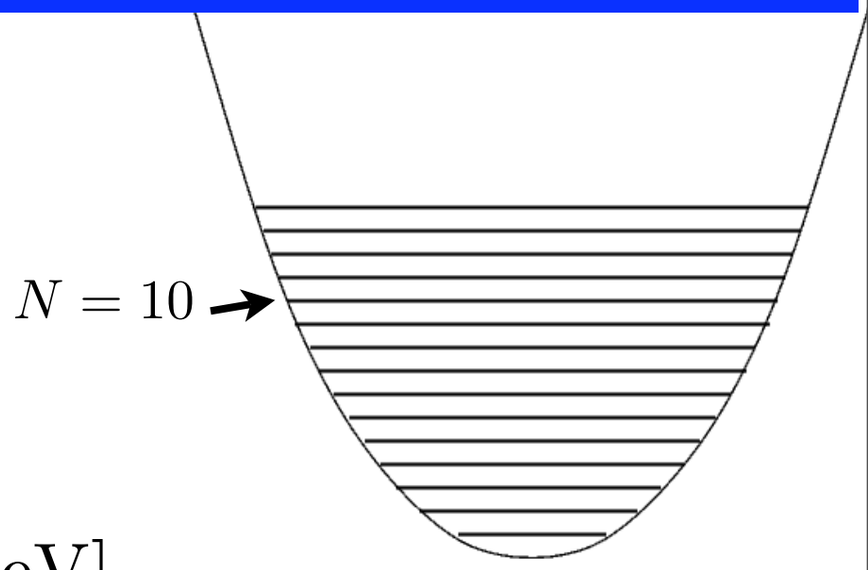
Size of the model space or How large is the basis do we need to calculate ^{16}O ?

For a H.O the energy of the shell N is given by

$$E_N = \hbar\omega(N + 3/2)$$



For the nuclear interaction, the energy cutoff Λ is about 600 MeV, i.e,

$$\Lambda = E_{N_{max}} = \hbar\omega\,(N_{max} + 3/2) = 600\,[\mathrm{MeV}]$$

since $\hbar\omega \approx (3/2)^{4/3}\dfrac{\hbar^2}{2m_N r_0^2}A^{-1/3} = 41 \times A^{-1/3}[\mathrm{MeV}] \Longrightarrow N_{max} = 35$

Taking into account the degeneracies of the H.O. the total number of single particle states is $M = 16872$

The number of posible ways to distribute 8 protons and 8 neutrons in 16872 positions is

$$\left(\begin{array}{c}16872\\8\end{array}\right) \times \left(\begin{array}{c}16872\\8\end{array}\right) \approx 10^{57}$$

This would be the size of the basis states to make the diagonalization !!

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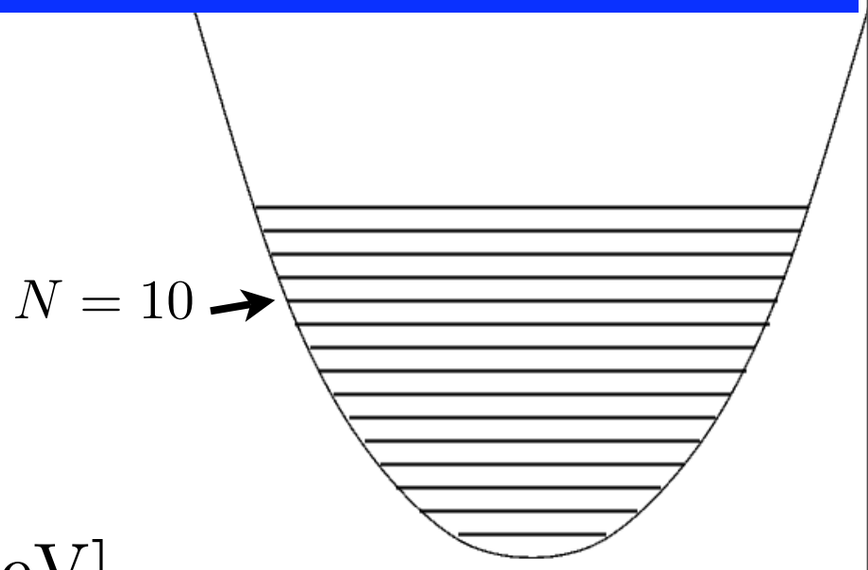

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... approximations are unavoidable.

Two main streams :

I.- Large Shell model approximations (Configuration Interaction): The exact diagonalization is performed in a small configuration space (few subshells). The interaction is fine-tuned for each configuration space.

PROS: Very accurate description of most observables.
CONS: Limited predictability.

II.- Density functional Theory (Mean Field and Beyond): Large Configuration space (~15 H.O. shells) are used at the mean field level. Final diagonalization performed in a highly correlated space. Only one interaction (density dependent) for all nuclei.

PROS: High predictability of gross properties.
CONS: Not very high precision for some observables.

The mean field: Hartree-Fock (HF) approach

The HF approach uses the variational principle to extract from a two body potential the optimal one body Hamiltonian

$$H = \sum_i T(i) + \sum_{i<j} V(i,j) = \sum_i h_{HF}(i) + V_{res}$$

The one body HF wave functions are given by

$$h_{HF}\varphi_k = \epsilon_k \varphi_k$$

and the many-body one by

$$\Phi^{HF}_{p_1,...,p_N}(1,...,N) = \frac{1}{\sqrt{N!}} \sum_P (-1)^P P \varphi_{p_1}(1)...\varphi_{p_N}(N) = a^\dagger_{p_N}...a^\dagger_{p_1}|-\rangle$$

since the one body wave functions form a complete basis is obvious that one can expand the HF one body w.f. in terms of the Harmonic oscillator ones:

H.F → $a^\dagger_p = \sum_q U_{pq} c^\dagger_q$ ← H.O.

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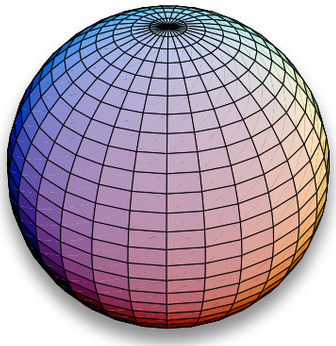
HF and the spontaneous symmetry breaking mechanism

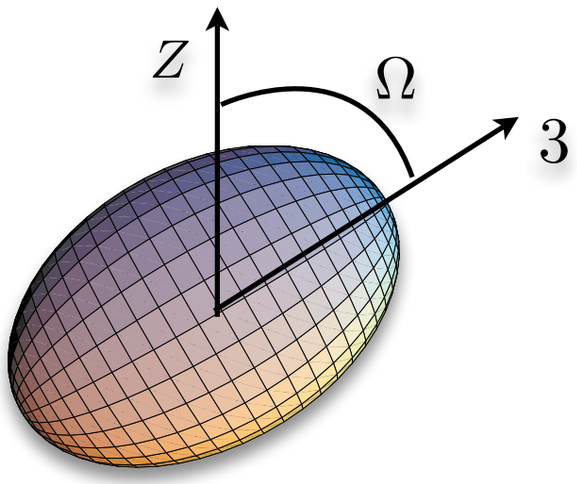
The quantum numbers of the Harmonic Oscillator are nlj. The p quantum numbers label the eigenstates of the HF Hamiltonian and in principle one would also expect conservation of symmetries, i.e.,



$$a^{\dagger}_{(lj)n'} = \sum_{n} U^{(lj)}_{n'n} c^{\dagger}_{(lj)n}$$

In this case only states with different n's are mixed, for large basis about 20 states.

Interestingly, in many cases the HF Hamiltonian is not rotational invariant, a large mixing takes place (~4000 states) and a collaps of the wave function along a direction occurs.

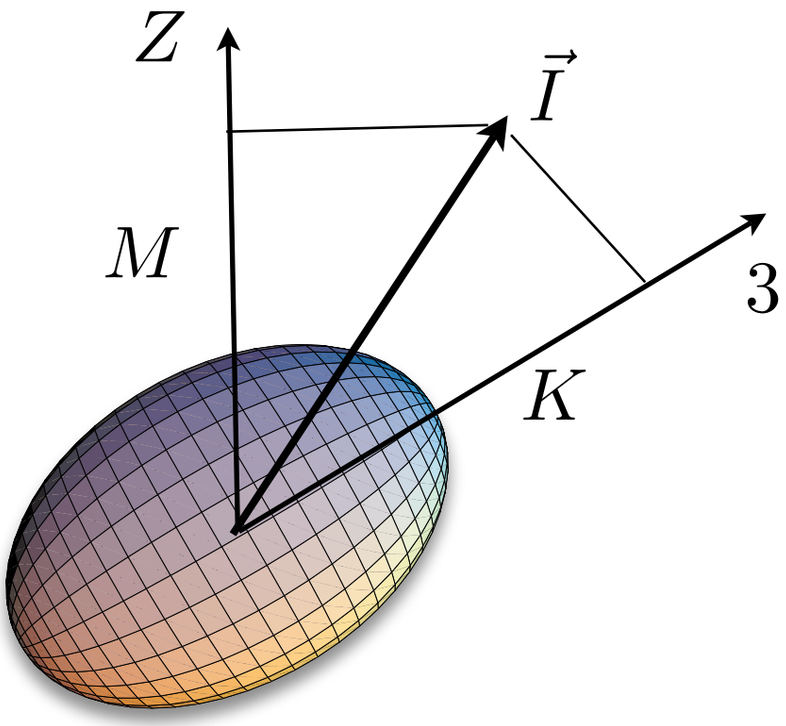


$$a^{\dagger}_{p} = \sum_{nlj} U^{lj}_{pn} c^{\dagger}_{nlj}$$

HF and the spontaneous symmetry breaking mechanism

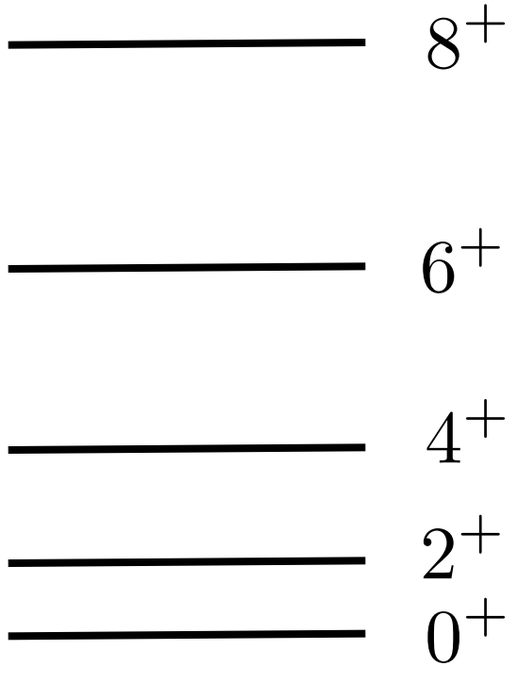
Since all orientations are degenerated we can lower the energy by a convenient linear combination and at the same time recover the rotational invariance.

$$|\Psi^{IM}\rangle = \frac{2I+1}{8\pi^2}\sum_K g_K^I \int \mathrm{d}\Omega \mathcal{D}^{I*}_{MK}(\Omega)\hat{R}(\Omega)|\Phi^{HF}\rangle = P^I_M|\Phi^{HF}\rangle$$



where Ω are the Euler angles, $\hat{R}(\Omega)$ the rotation operator and $\mathcal{D}^{I*}_{MK}(\Omega)$ the Wigner matrix

For different I values we obtain a rotational spectrum.



By means of the symmetry breaking mechanism we can build highly correlated HF wave functions of collective states!!

Superfluid nuclei: The Hartree-Fock-Bogoliubov theory

The symmetry breaking mechanism is also known from BCS theory and its generalizations as the HFB. In this approach

$$|\Phi^{HFB}\rangle = \alpha_M \dots \alpha_1 |-\rangle,$$

with the quasiparticles α given by the most general linear transformation:

$$\alpha_\mu = \sum_i U^*_{i\mu} c_i + \sum_i V^*_{i\mu} c^\dagger_i,$$

The matrices U and V are determined by the variational principle:

$$\delta \langle \Phi^{HFB} | \hat{H} - \lambda \hat{N} | \Phi^{HFB} \rangle = 0,$$

the parameter λ being determined by the constraint

$$\langle \Phi^{HFB} | \hat{N} | \Phi^{HFB} \rangle = N,$$

with N the number of particles of our system.

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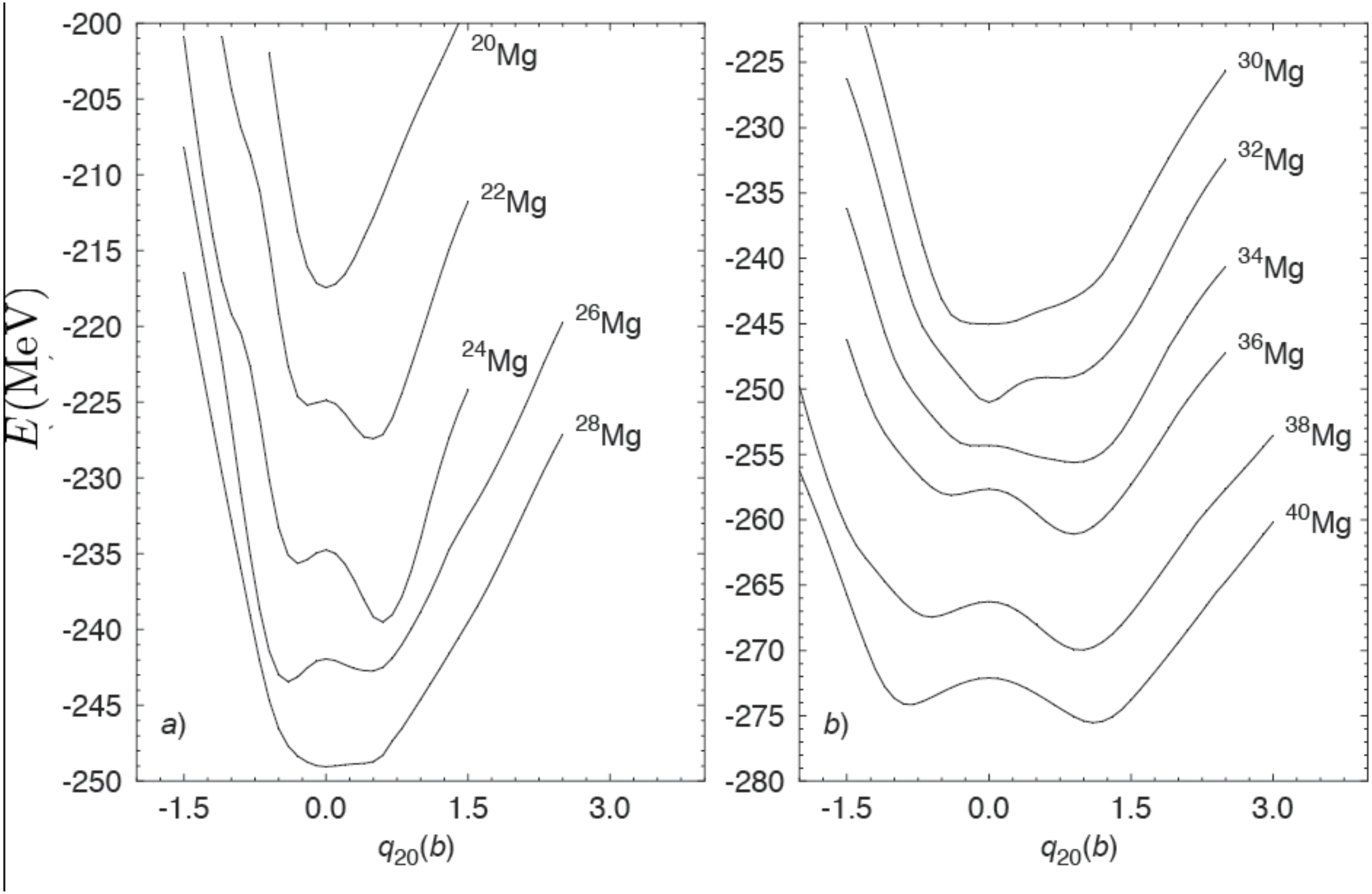
Nuclear shapes along an isotopic chain



The Projected Hartree-Fock-Bogoliubov theory

In nuclear many body approaches one breaks several symmetries as : Translational and rotational invariance, spatial parity and particle number, isospin, etc.

Of course nuclear system do have good quantum numbers and we have to recover them by means of projectors

$$|\Psi^{N,Z,\pi,...IM}\rangle = P^N P^Z P^\pi ... P^I_M |\Phi^{HFB}\rangle$$

Notice that the variational parameters are still the matrices U and V which determine the HFB transformation.

Modern Beyond mean field Theories

The idea is to construct a highly correlated Hilbert space. For this one chooses the relevant degrees of freedom $q \equiv \{q_1, q_2, ..., q_n\}$ and generates the wave functions $\Phi(q_1, q_2, ..., q_n) \equiv \Phi(q)$. The weights of

$$\left|\Psi^{N,Z}_{\sigma I}\right\rangle = \int dq\; f^{N,Z}_{\sigma I}(q)\; \hat{P}^{I}_{M} ... \hat{P}^{N}\, \hat{P}^{Z}\, |\Phi(q)\rangle\,,$$

are determined by diagonalization of the Hamiltonian:

$$\int dq'\; \mathcal{H}^{N,Z}_{I}(q,q')\; f^{N,Z}_{\sigma I}(q') = E^{N,Z}_{\sigma I} \int dq'\; \mathcal{N}^{N,Z}_{I}(q,q')\; f^{N,Z}_{\sigma I}(q'),$$

this is the Hill-Wheeler equation.

$$\mathcal{N}^{N,Z}_{I}(q,q') = \langle\Phi(q)|\, \hat{P}^{I}_{M} ... \hat{P}^{N}\, \hat{P}^{Z}\, |\Phi(q')\rangle\,,$$

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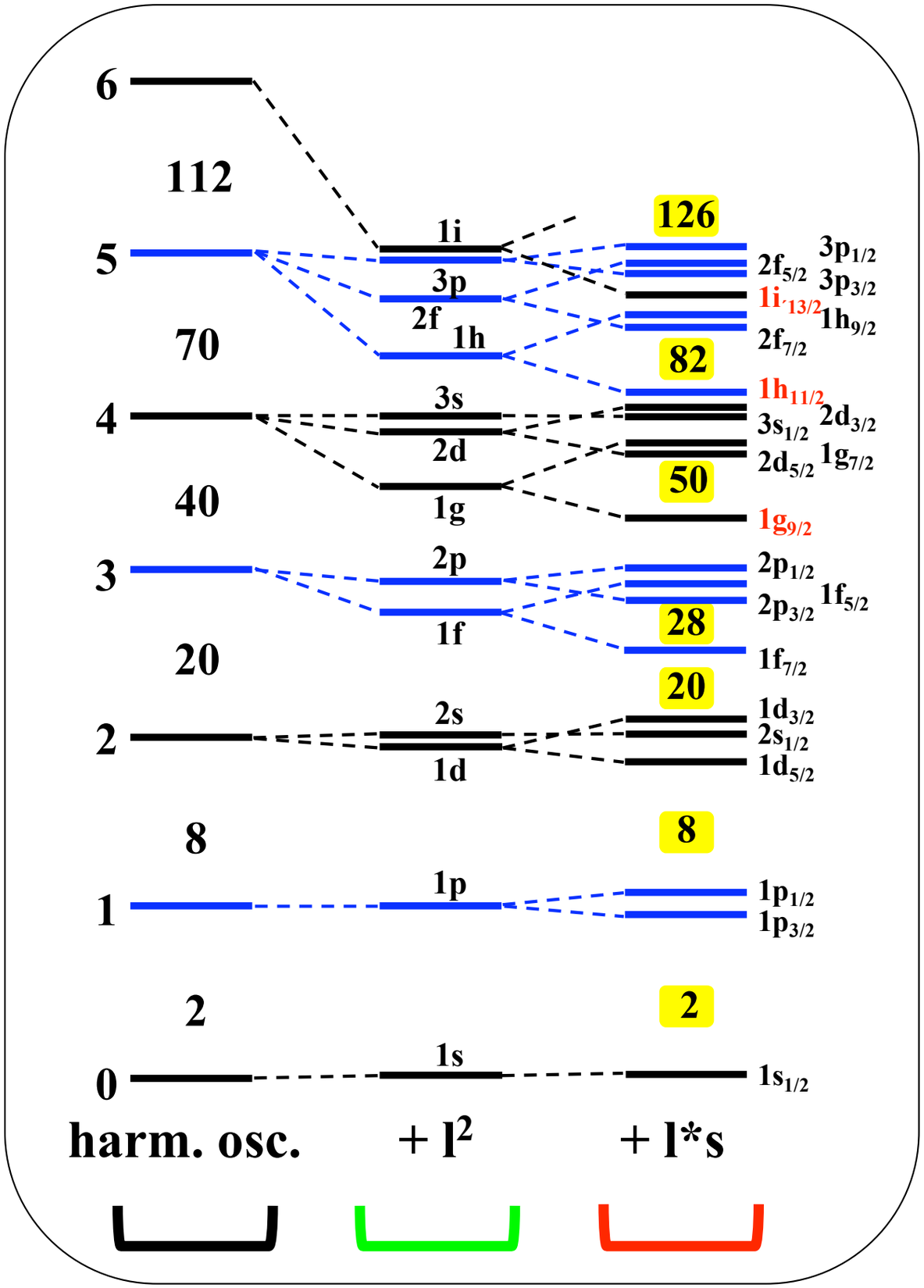
Applications

Single particle energies



Hamiltonian

$$h = t + \frac{m}{2}\,\omega_0^2\,(r^2 - R^2) + C\,l^2 + D\;\mathbf{ls}$$

Eigenvalues

$$\epsilon_{nlj} = \hbar\omega_0\,(N + \frac{3}{2}) \quad - \quad V_0 + Cl(l+1)$$

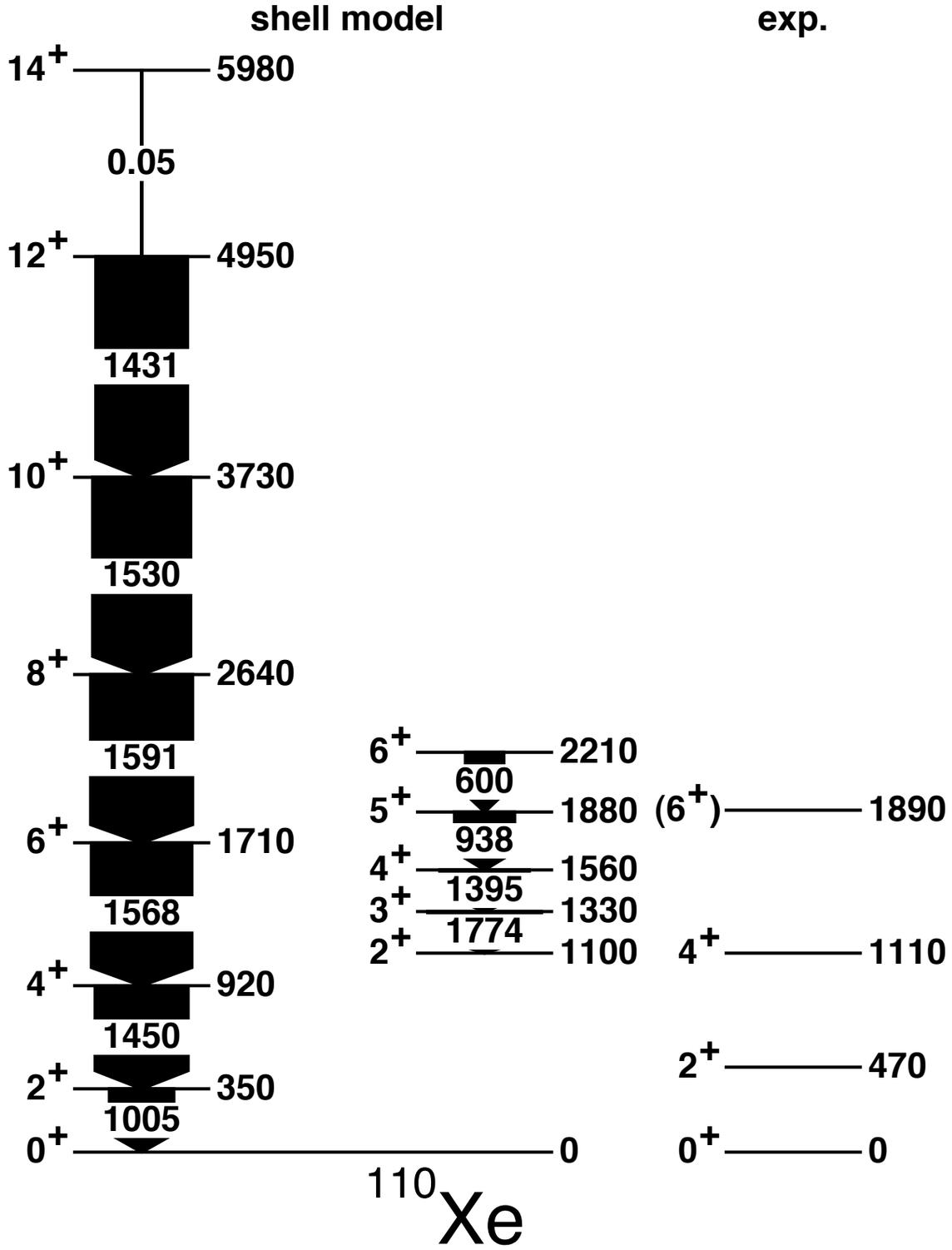
$$+ \quad \frac{D}{2}\begin{cases} l & \text{if } j = l + \frac{1}{2} \\ -(l+1) & \text{if } j = l - \frac{1}{2} \end{cases}$$

$$N = 2(n-1) + l$$

Collectivity in the light Xenon isotopes

Configuration space: $^{100}_{50}\mathrm{Sn}_{50}$ core + $\pi, \nu : 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

Interaction BonnC+monopole fit (GCN50:82)

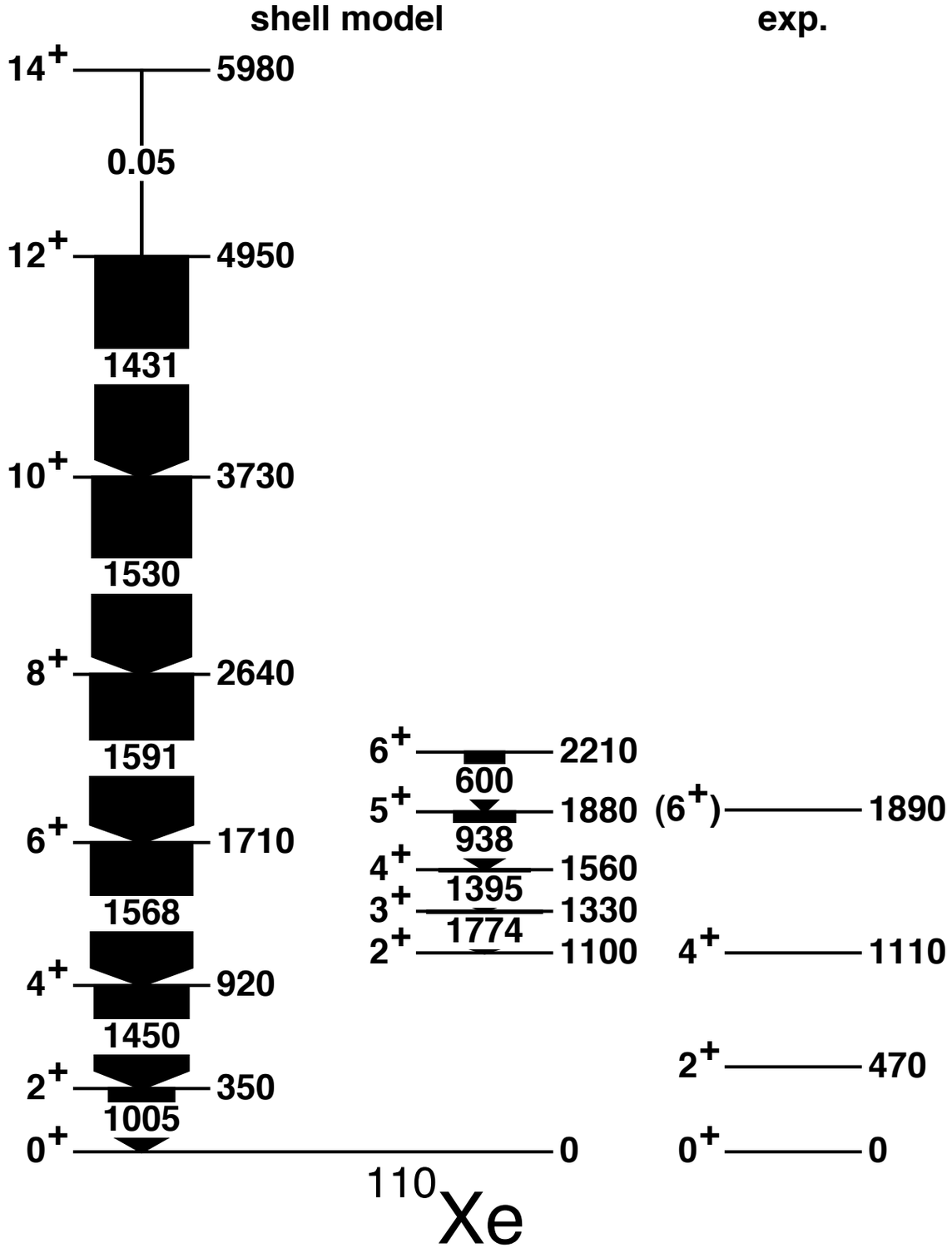


E. Caurier, F. Nowaki, A.Poves and K. Sieja,
arXiv:1009.3813, Phys. Rev. C in press

Collectivity in the light Xenon isotopes

Configuration space: $^{100}_{50}\mathrm{Sn}_{50}$ core + $\pi, \nu : 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

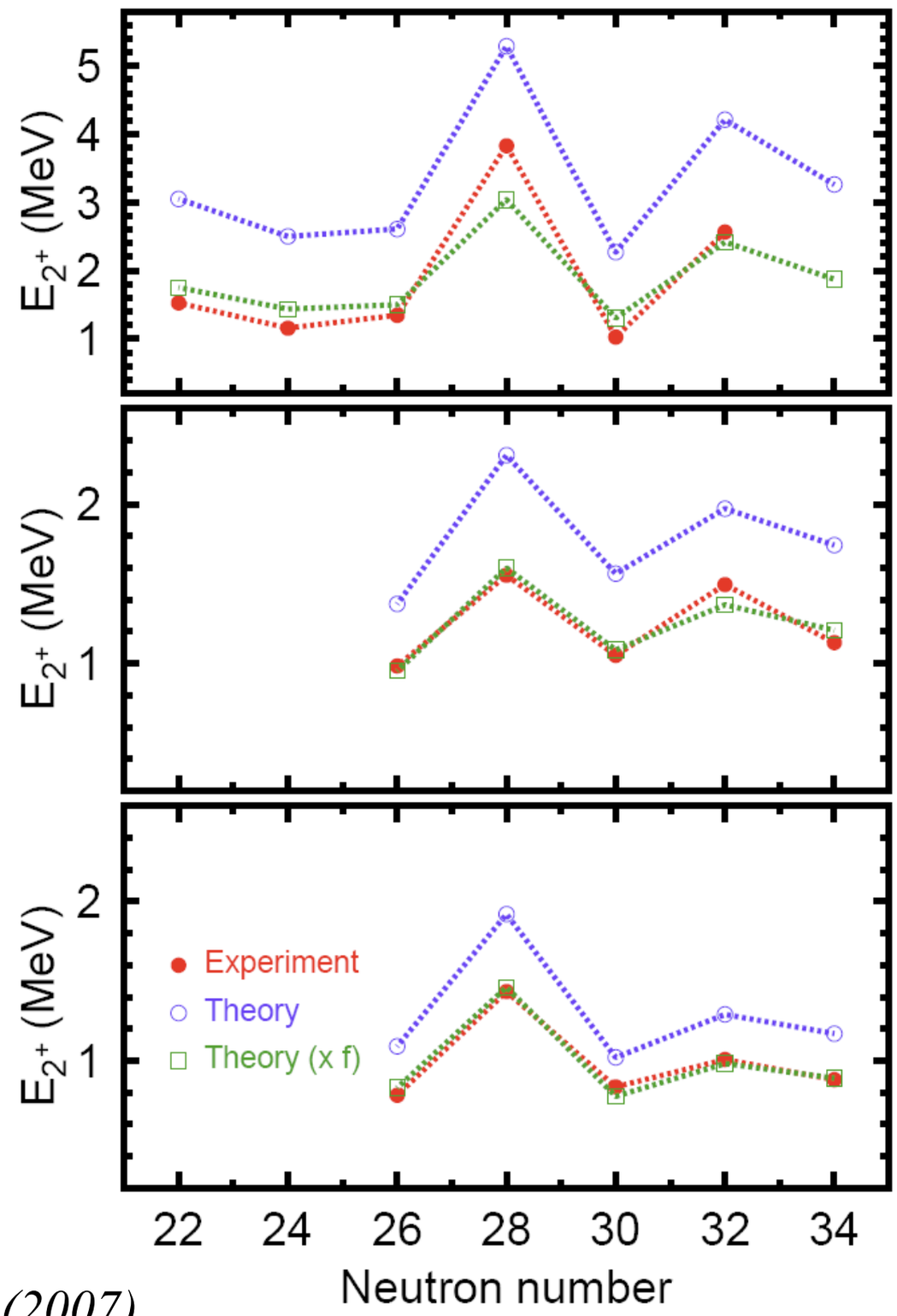
Interaction BonnC+monopole fit (GCN50:82)



10^{10} basis states !!

E. Caurier, F. Nowaki, A.Poves and K. Sieja,
arXiv:1009.3813, Phys. Rev. C in press

The N=32 subshell closures



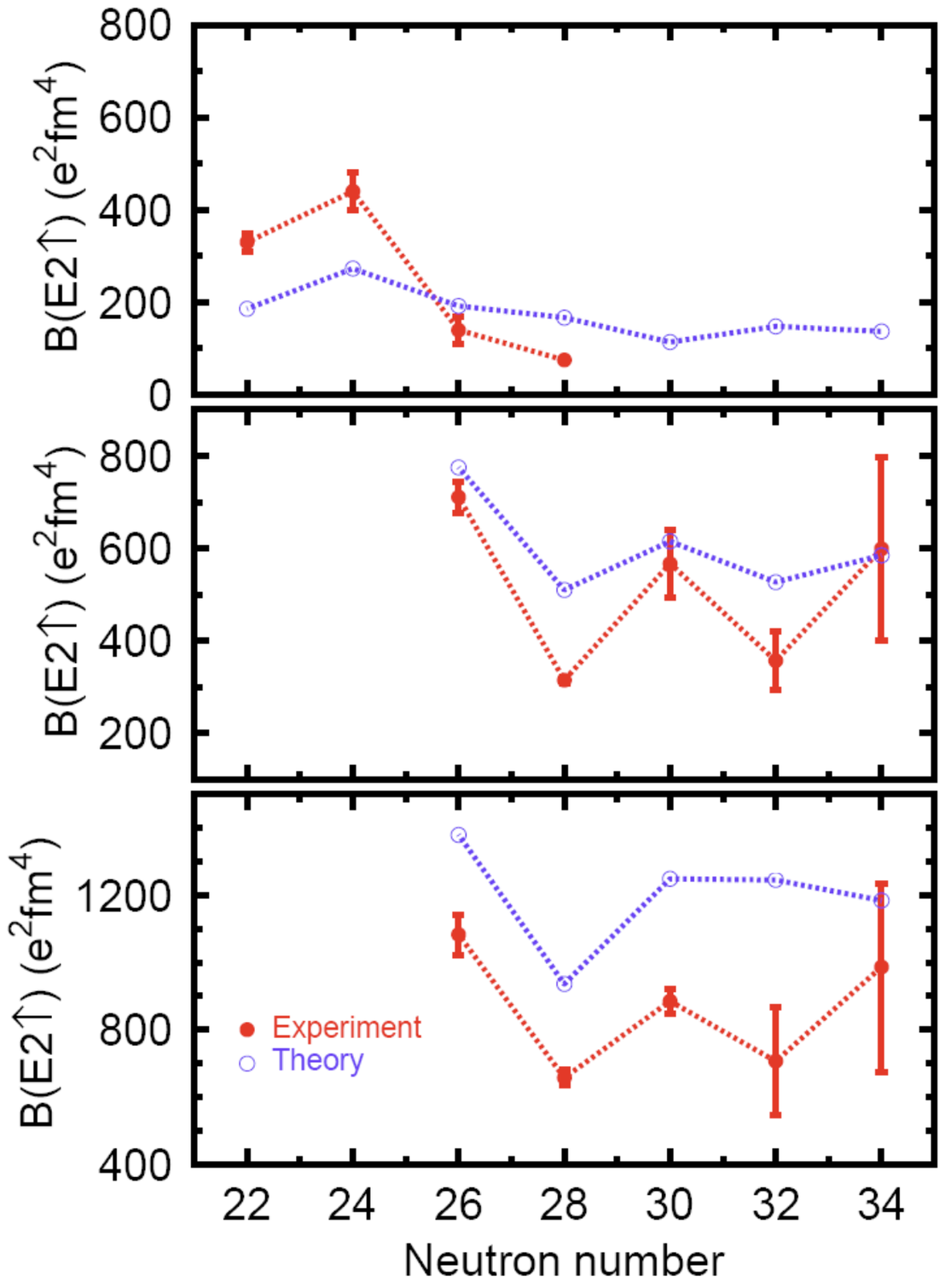
$_{20}$Ca

$_{22}$Ti

$_{24}$Cr

T.R. Rodriguez and J.L.E.
Phys. Rev. Lett. 99, 062501 (2007)

Reduced E2 transition probabilities $0^+_1 \rightarrow 2^+_1$



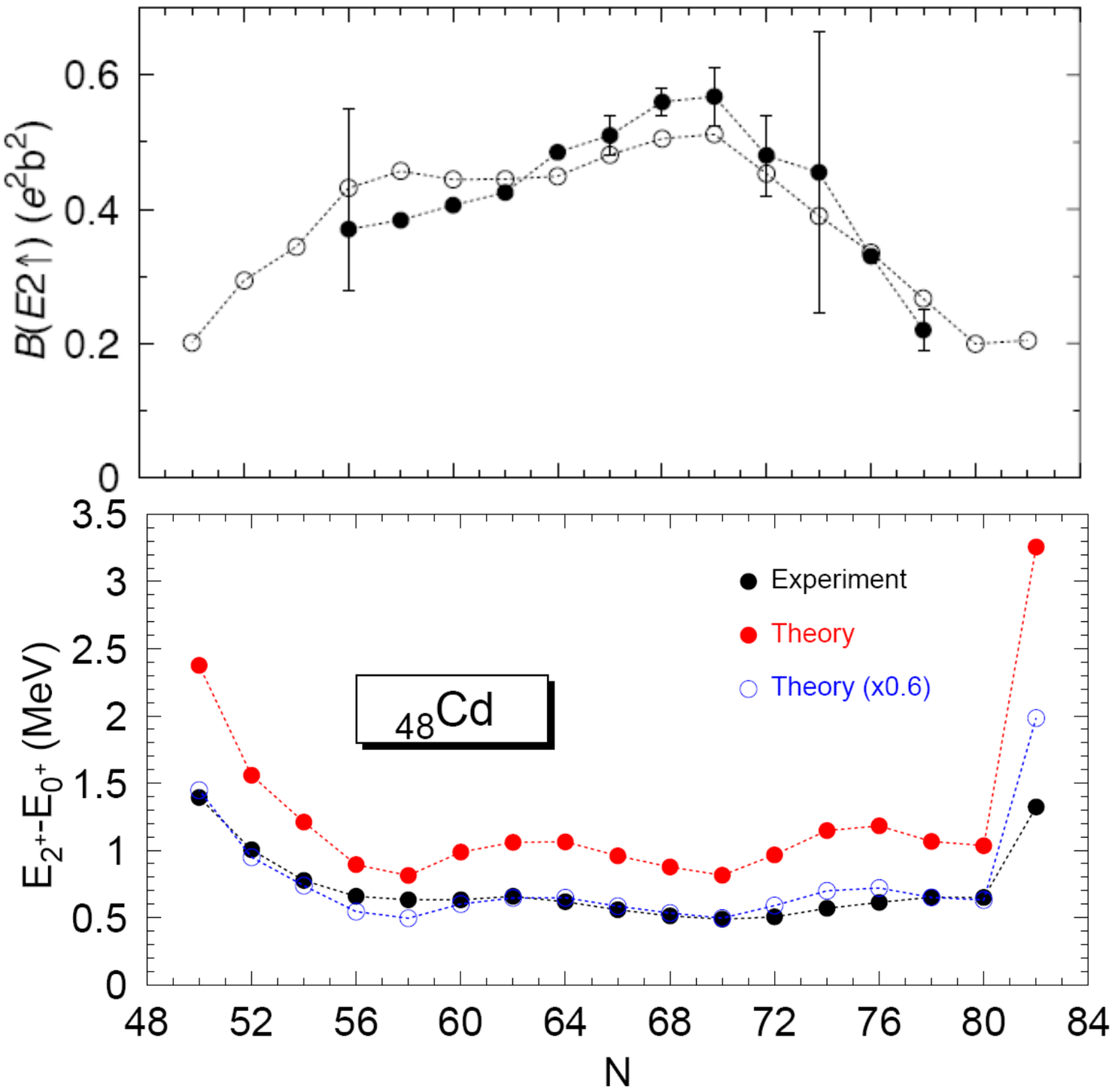
$_{20}$Ca

$_{22}$Ti

$_{24}$Cr

Beyond mean field calculation along a major shell

B(E2) and E(2^+) for all Cd's between N=50 and 82



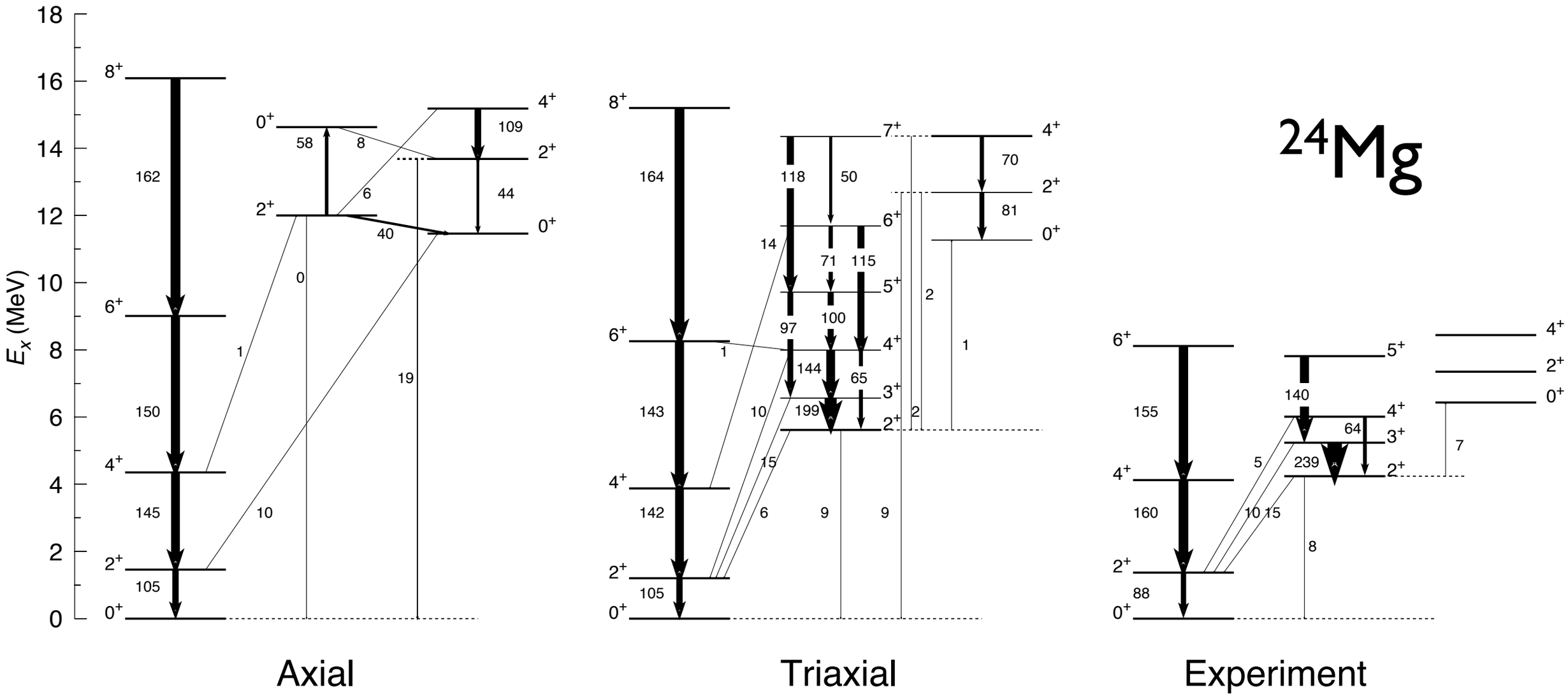
Realistic beyond mean field calculations across an entire major shell without any adjustable parameter

nearly constant ratio between theory and experiment

(no triaxial minima or large γ softness)

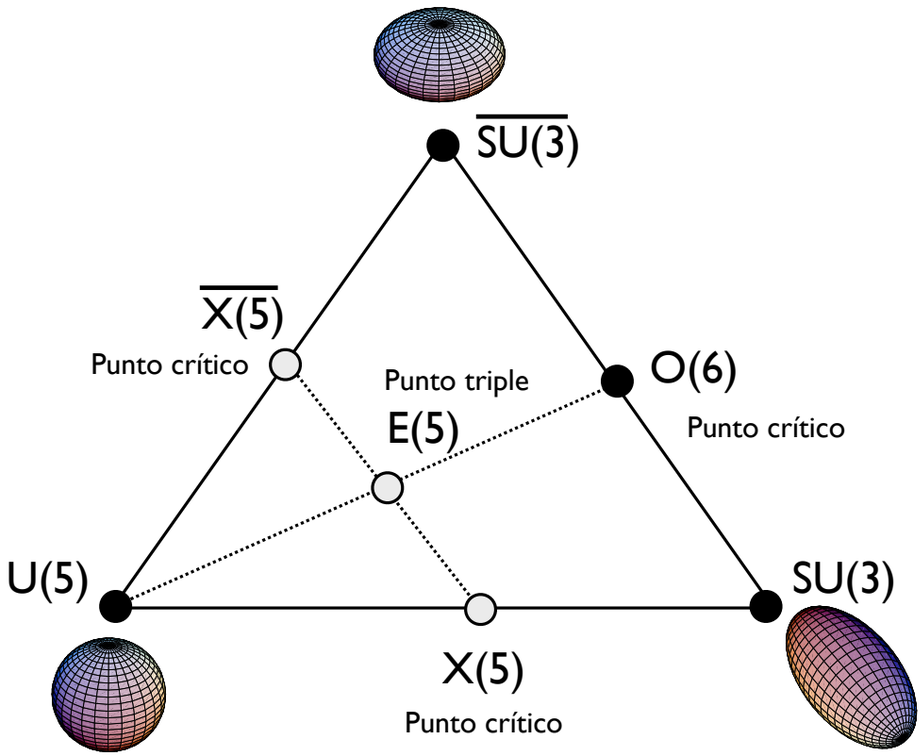
T. Rodríguez, J.L. Egido and A.Jungclaus, Phys. Lett. B668(2008)410

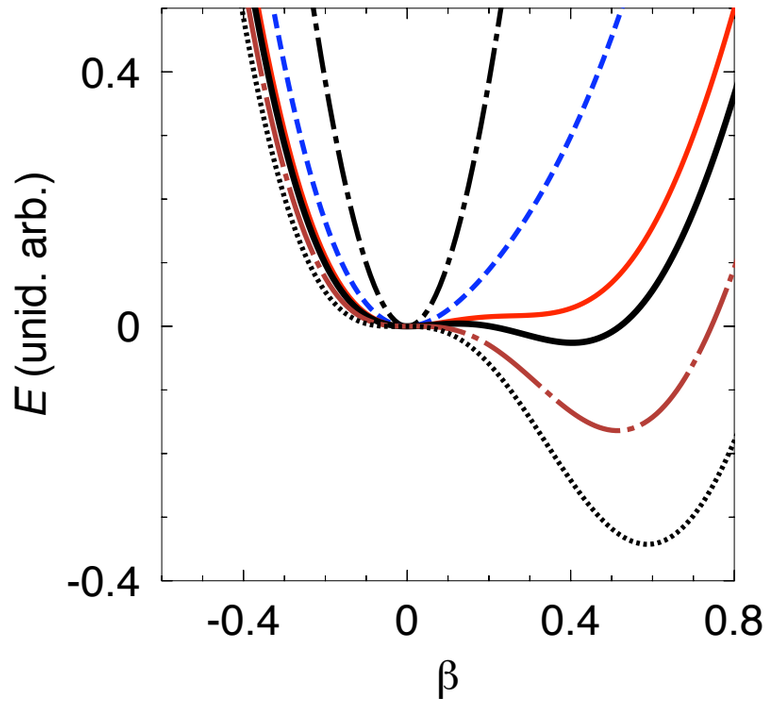
Particle number and Triaxial Angular Momentum Projection

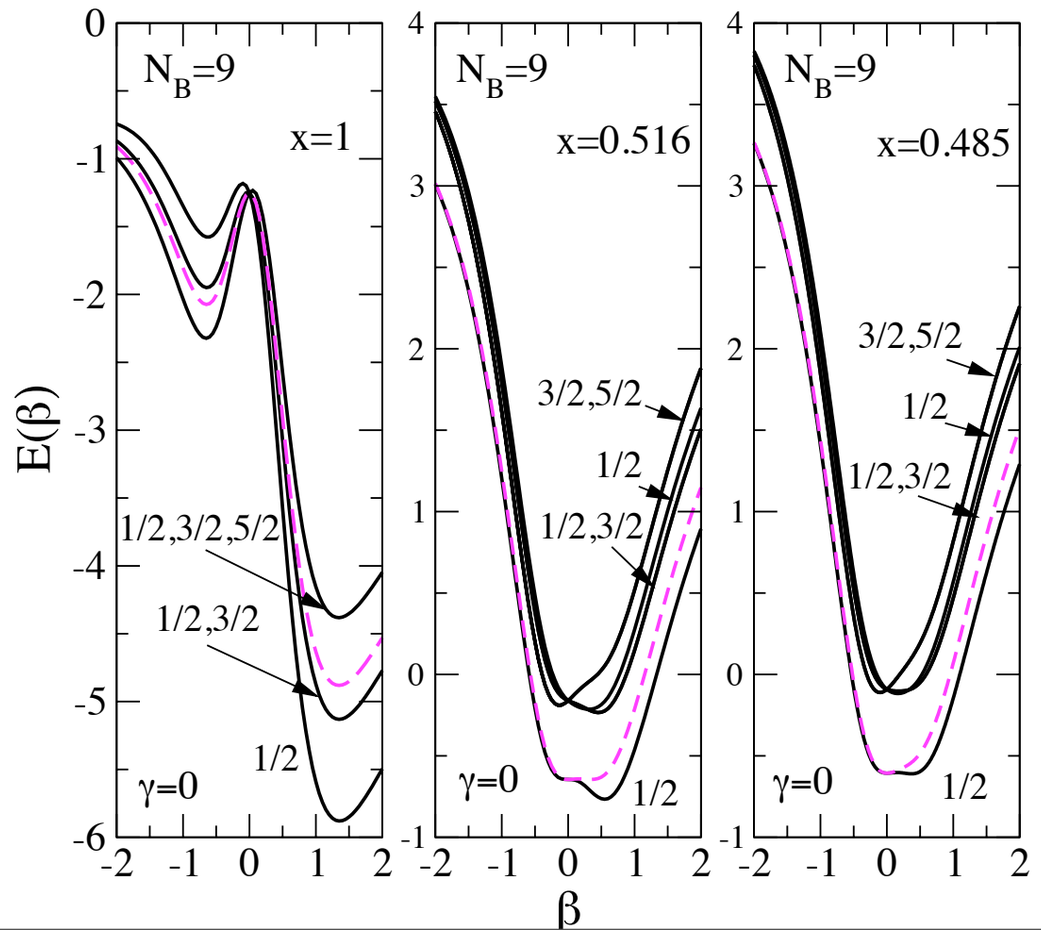


T. R. Rodriguez and J.L.E., Phys. Rev. **C81**, 064323(2010)

Shape transitions in algebraic models

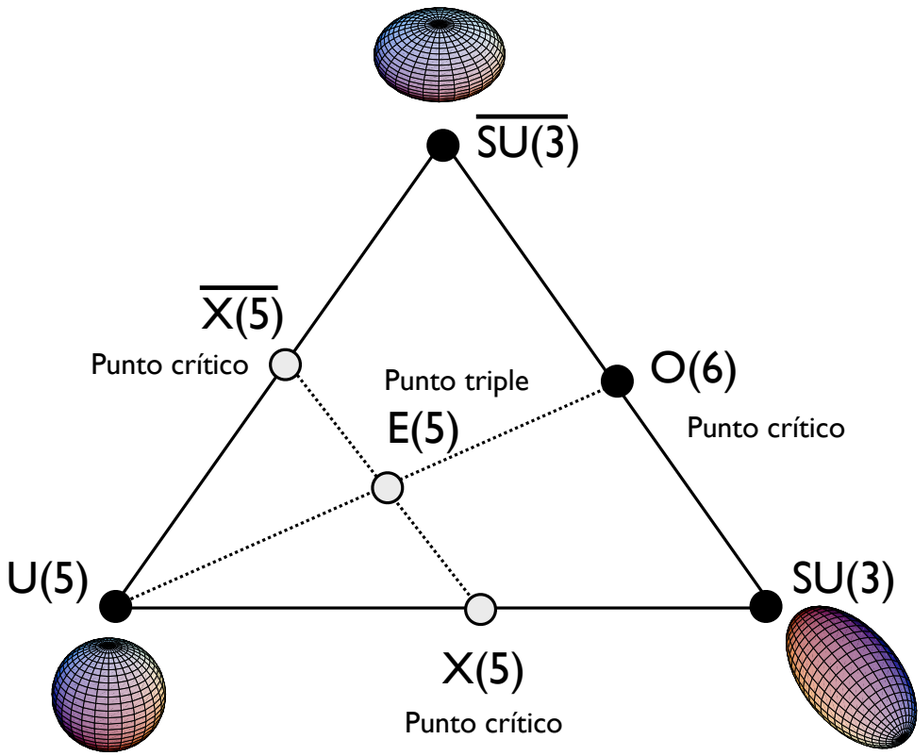


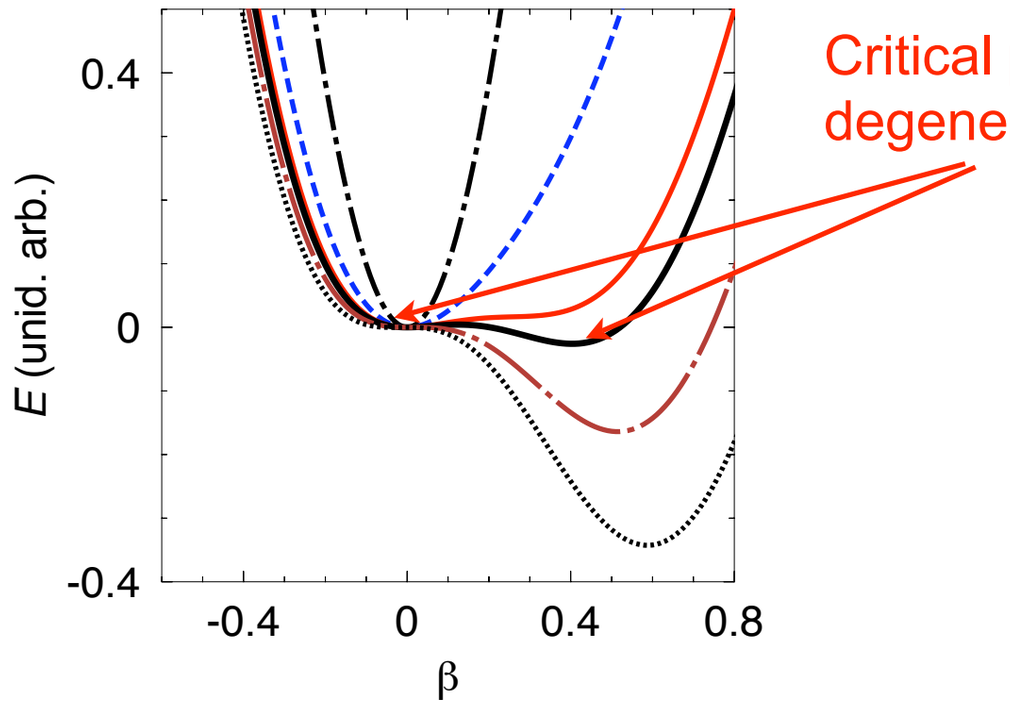




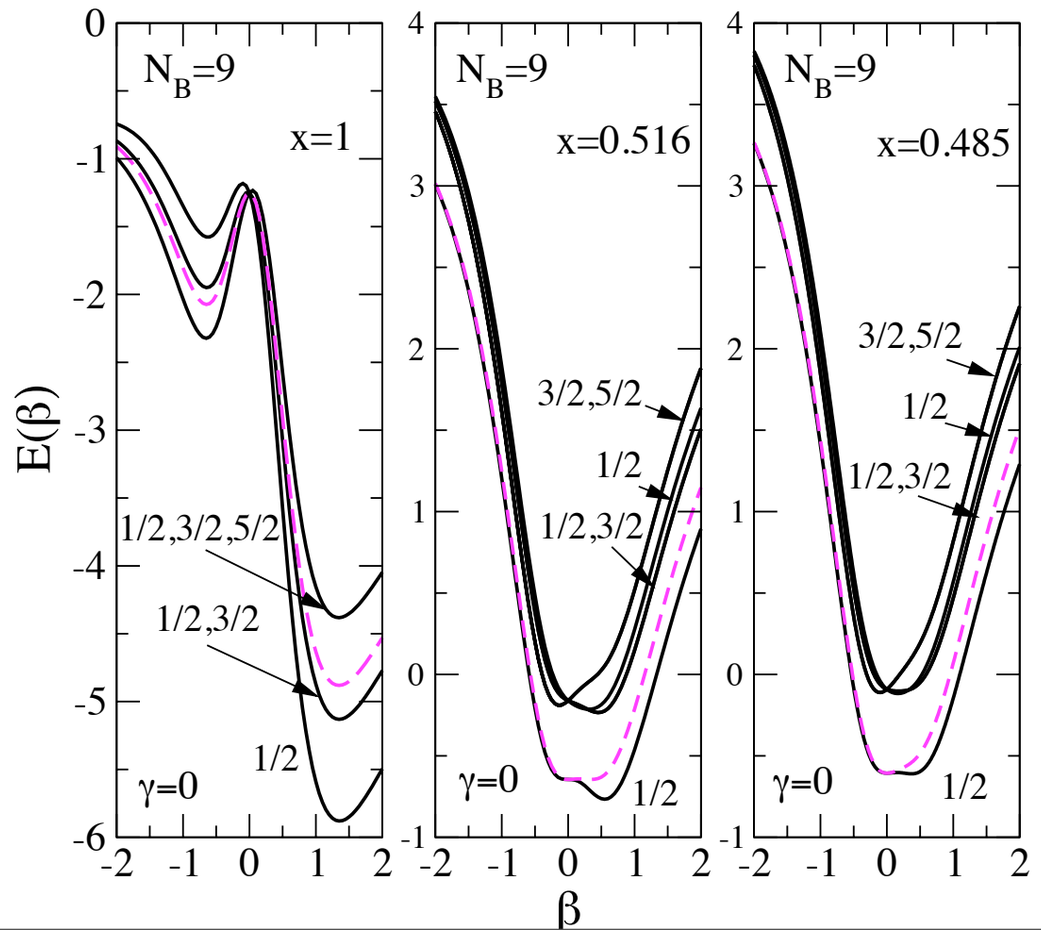
C.E. Alonso, J.M. Arias and A. Vitturi
Phys. Rev. Lett. 98, 052501(2007)

Shape transitions in algebraic models





Critical points:
degenerated minima



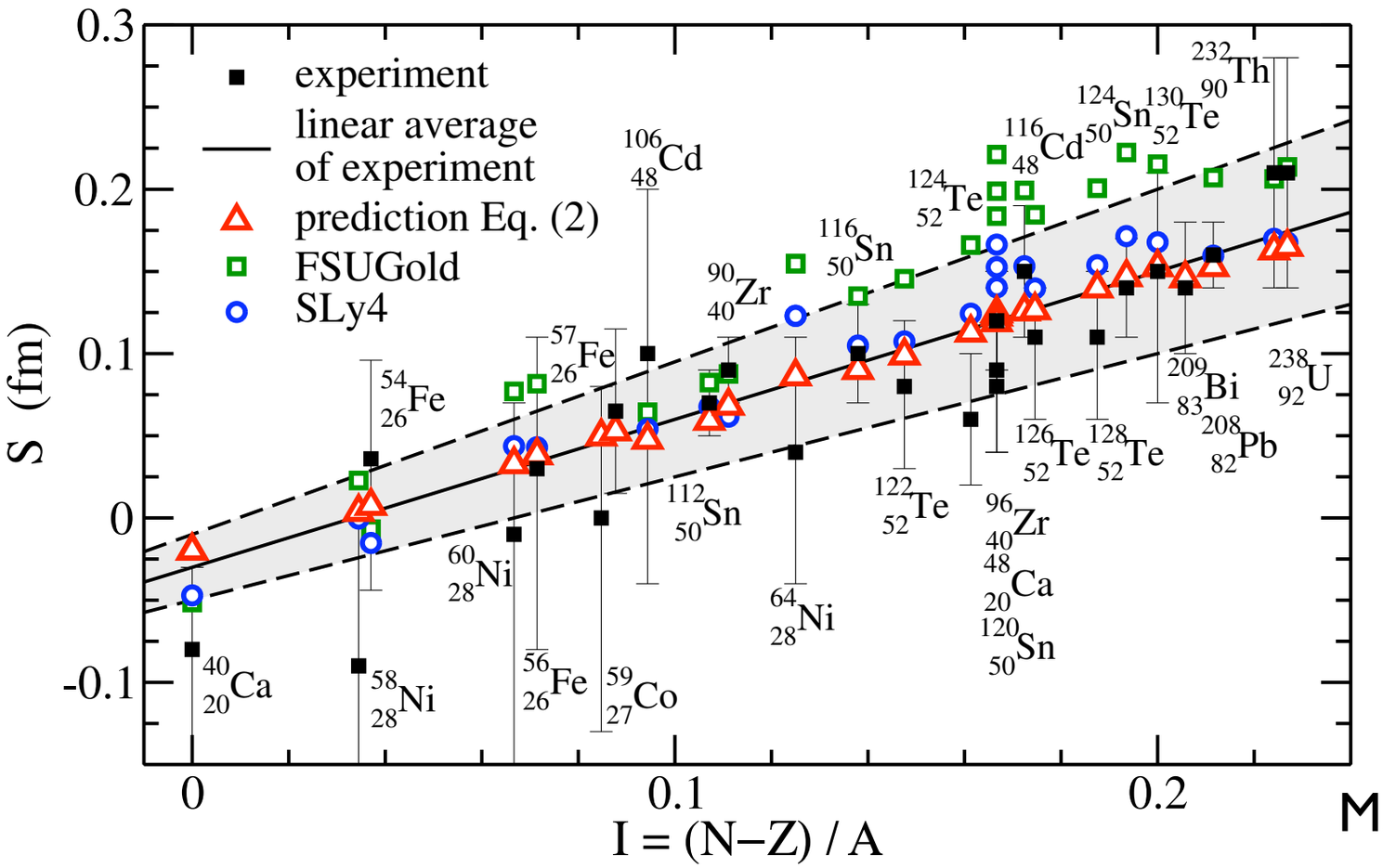
C.E. Alonso, J.M. Arias and A. Vitturi
Phys. Rev. Lett. 98, 052501(2007)

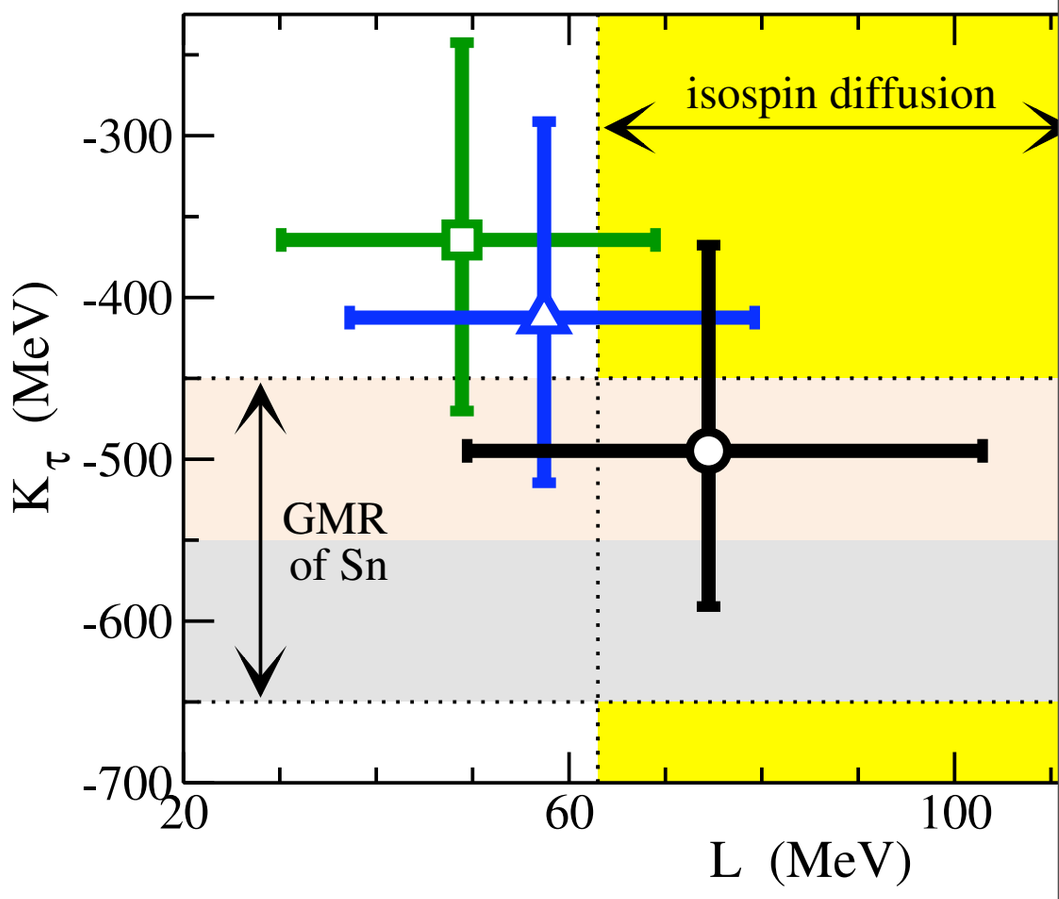
Nuclear Symmetry Energy and the neutron skins

$$e(\rho,\delta) = e(\rho,0) + c_{sym}(\rho)\delta^2 + \Theta(\delta^4), \quad \rho = \rho_p + \rho_n,\ \delta = \frac{\rho_n - \rho_p}{\rho}$$

$$c_{sym}(\rho) = 32 - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2, \quad \epsilon = \frac{\rho_0 - \rho}{3\rho_0}, \quad \rho_0 = \text{saturation density}$$

$$K_\tau \approx K_{sym} - 6L$$





M. Centelles, X. Roca-Maza, X. Viñas, M. Warda
Phys. Rev. Lett. 102, 122502 (2009)

Conclusions

- **A renaissance of nuclear physics has taken place with the study of exotic nuclei.**

- **Strong progress in theoretical nuclear physics in the last years in many fronts.**

- **In Spain, in particular, many groups in most nuclear issues are contributing to this progress and collaborating with our experimental groups.**

- **A very exciting long way is still awaiting to reach a better understanding of atomic nuclei.**