Heavy-to-light currents at NNLO in SCET

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Heavy-to-light currents $\bar{q}\,\Gamma_i\,b$ play an important role:

✓govern the hadronic dynamics in semi-leptonic and radiative B decay: e.g., $b \to u\, l^+ \nu$, $b \to s\, \gamma$, $b \to s\, l^+ l^-$

✓their matrix elements, parameterized by several form factors, also are inputs for hadronic B decays.

In shape function region (i.e., E_X large but M_X small):

✓SCET is the appropriate theoretical framework;

✓transparent factorization formulae can be derived;

⇒How to represent $\bar{q}\,\Gamma_i\,b$ in SCET accurately?

Matching heavy-to-light current from QCD to SCET:

2-body operators | 3-body operators

$$\bar{q}\,\Gamma b \;=\; \sum_i \int ds\, \tilde{C}_i^{(A)}(s)\, O_i^{(A)}(s) \;+\; \sum_i \int ds_1 ds_2\, \tilde{C}_i^{(B)}(s_1,s_2)\, O_i^{(B)}(s_1,s_2) \;+\; \dots$$

Inclusive decays	leading contribution $\langle O_i^{(A)} \rangle \;\to\; J \otimes S$	power-correction $\langle O_i^{(B)} \rangle \;\to\; \sum_i j_i \otimes s_i$
Exclusive decays	non-factorizable in SCET_{II} $\langle O_i^{(A)} \rangle \;\to\; \xi_P$ soft-overlap contribution	factorizable in SCET_{II} $\langle O_i^{(B)} \rangle \;\to\; \phi_B \otimes J_{\parallel} \otimes \phi_M$ hard spectator scattering

$C_i^{(A)}$ and $C_i^{(B)}$: the matching coefficients from QCD to SCET.

⇒*Here we report the 2-loop calculation of* $C_i^{(A)}$ *and a few applications*

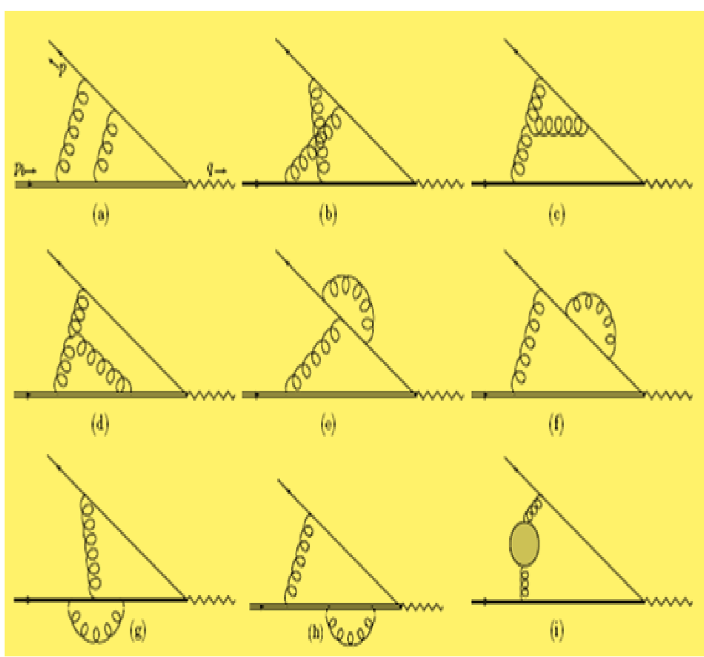
2-loop Feynman diagrams and calculation strategy:

✓automatic reduction based on IBPs;

✓combine DEs and MB techniques to calculate resulting MIs analytically;

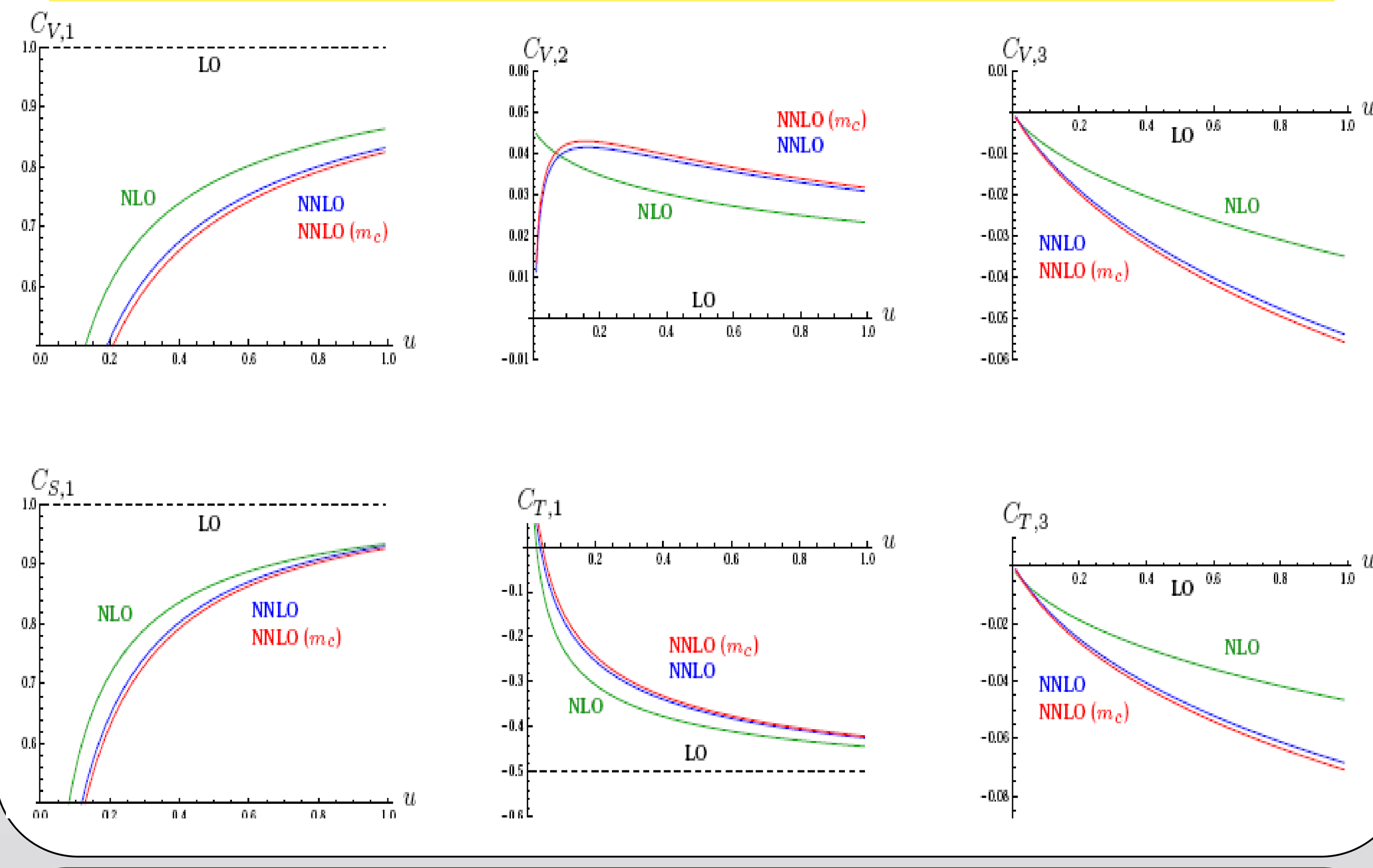
✓use standard QCD counterterms for UV renormalization;

✓for IR-subtraction, need the SCET-current renormalization constant.

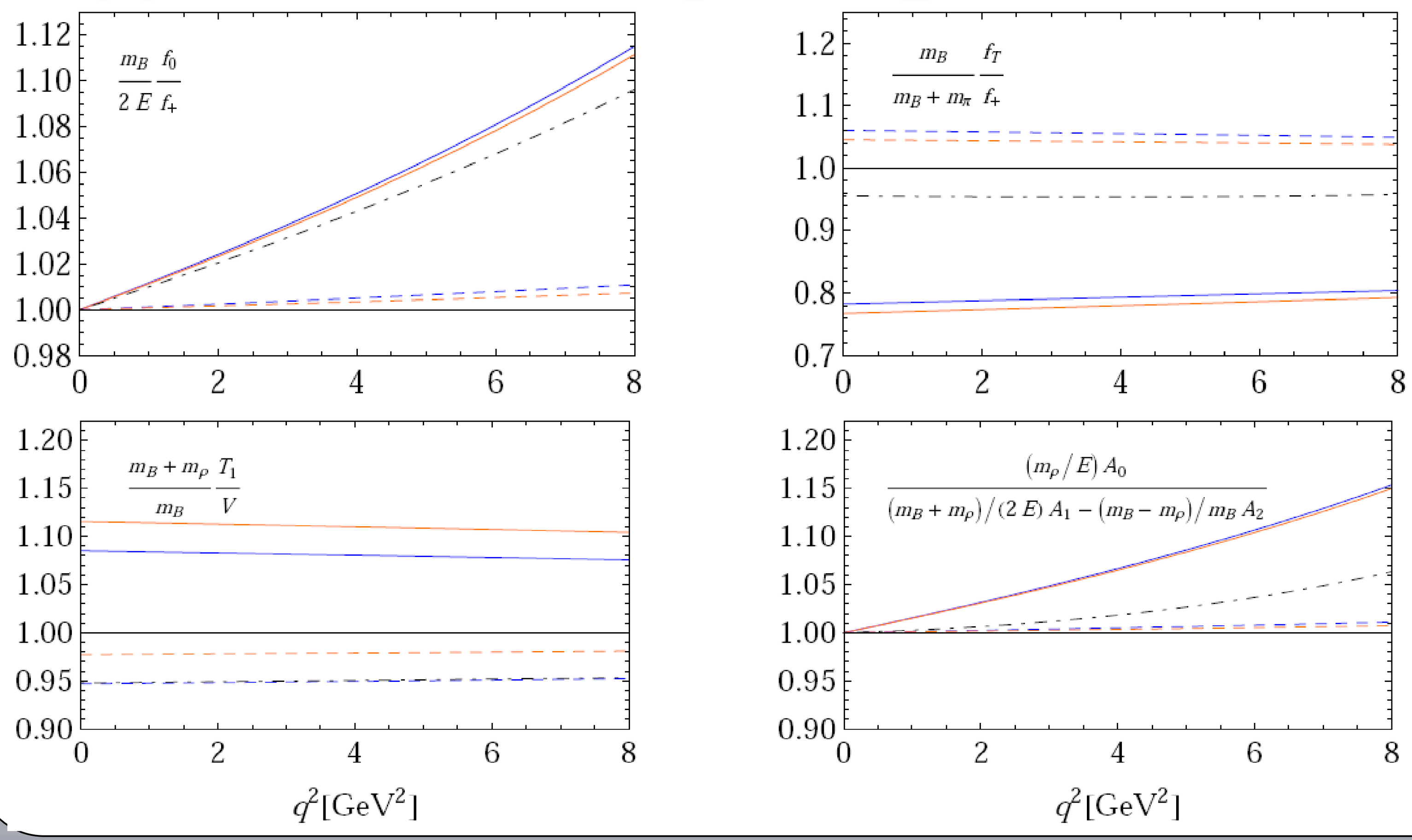


Matching coefficients at NNLO:

Γ_i	1	γ_5	γ^μ			$\gamma_5\gamma^\mu$			$i\sigma^{\mu\nu}$			
Γ'_j	1	γ_5	γ^μ	v^μ	n_-^μ	$\gamma_5\gamma^\mu$	$v^\mu\gamma_5$	$n_-^\mu\gamma_5$	$\gamma^{[\mu}\gamma^{\nu]}$	$v^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}v^{\nu]}$
C_i^j	C_S	C_P	C_V^1	C_V^2	C_V^3	C_A^1	C_A^2	C_A^3	C_T^1	C_T^2	C_T^3	C_T^4



Application I: heavy-to-light form factor ratios



Solid curves: full results at NNLO (blue) and NLO (orange);

Dashed curves: result without the spectator-scattering term;

Dash-dotted: QCDSR results

NNLO term is generally quite moderate, most significant for the ratio T1/V.

Application II: inclusive B → Xu $l^+\nu$

Method	$\Delta\mathcal{B}^{\rm exp}$ $[10^{-4}]$	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NLO	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NNLO
$E_l > 2.1\,\mathrm{GeV}$ CLEO [38]	$3.3 \pm 0.2 \pm 0.7$	$3.56 \pm 0.40^{+0.48+0.31}_{-0.27-0.26}$	$3.81 \pm 0.43^{+0.33+0.31}_{-0.21-0.26}$
$E_l > 2.0\,\mathrm{GeV}$ BABAR [39]	$5.7 \pm 0.4 \pm 0.5$	$3.97 \pm 0.22^{+0.37+0.26}_{-0.23-0.25}$	$4.30 \pm 0.24^{+0.26+0.28}_{-0.20-0.27}$
$E_l > 1.9\,\mathrm{GeV}$ BELLE [40]	$8.5 \pm 0.4 \pm 1.5$	$4.27 \pm 0.39^{+0.32+0.25}_{-0.19-0.22}$	$4.65 \pm 0.43^{+0.27+0.27}_{-0.18-0.24}$
$M_X < 1.7\,\mathrm{GeV}$ BELLE [41]	$12.3 \pm 1.1 \pm 1.2$	$3.55 \pm 0.24^{+0.22+0.21}_{-0.13-0.19}$	$3.87 \pm 0.26^{+0.21+0.21}_{-0.13-0.19}$
$M_X < 1.55\,\mathrm{GeV}$ BABAR [42]	$11.7 \pm 0.9 \pm 0.7$	$3.67 \pm 0.18^{+0.29+0.26}_{-0.17-0.24}$	$3.96 \pm 0.19^{+0.20+0.26}_{-0.13-0.24}$
$P_+ < 0.66\,\mathrm{GeV}$ BELLE [41]	$11.0 \pm 1.0 \pm 1.6$	$3.56 \pm 0.31^{+0.30+0.27}_{-0.17-0.23}$	$3.84 \pm 0.33^{+0.21+0.26}_{-0.13-0.22}$
$P_+ < 0.66\,\mathrm{GeV}$ BABAR [42]	$9.4 \pm 1.0 \pm 0.8$	$3.30 \pm 0.23^{+0.27+0.25}_{-0.16-0.22}$	$3.55 \pm 0.24^{+0.19+0.24}_{-0.13-0.21}$

[Greub, Neubert, Pecjak 09] *shift |Vub| upwards by ~10%*

Application III: semi-inclusive B → Xs $l^+ l^-$ decay

$$\frac{q_0^2}{2m_b(m_B - \langle p_X^+\rangle)} = -\frac{\mathrm{Re}\,[C_7^{\rm incl}(q_0^2)]}{\mathrm{Re}\,[C_9^{\rm incl}(q_0^2)]}\,\frac{c_1^7(u_0)}{c_1^9(u_0)}, \qquad u_0 \equiv 1 - q_0^2/(m_b(m_B - \langle p_X^+\rangle))$$

$\Longrightarrow$ *this is our main formula to determine the FBA zero.*

$$\sum_{i=1}^{10} C_i(\mu_0)\,O_i(\mu_0) \;=\; \frac{e^2}{4\pi^2}\Big[C_7^{\rm incl}(q^2,\mu_0)\,J_7^\mu\,\bar\ell\gamma_\mu\ell + C_9^{\rm incl}(q^2,\mu_0)\,J_9^\mu\,\bar\ell\gamma_\mu\ell + C_{10}^{\rm incl}(q^2,\mu_0)\,J_9^\mu\,\bar\ell\gamma_\mu\gamma_5\ell\Big],$$

$$J_9^\mu = \bar{s}\gamma^\mu P_L b\,, \qquad J_7^\mu = \frac{2\,m_b}{q^2}\,\bar{s}\,iq_\nu\sigma^{\nu\mu}P_R b\Big|_{\mu=m_b}$$

$$J_9^\mu = \sum_{i=1,2,3} c_i^9(u,\mu)\,[\bar\xi W_{hc}]\,\Gamma^\mu_{9,i}\,h_v\,, \qquad \Gamma^\mu_{9,i} = P_R\{\gamma^\mu, v^\mu, q^\mu\}\,,$$

$$J_7^\mu = \frac{2m_b}{q^2}\sum_{i=1,2} c_i^7(u,\mu)\,[\bar\xi W_{hc}]\,\Gamma^\mu_{7,i}\,h_v\,, \quad \Gamma^\mu_{7,i} = P_R\{iq_\nu\sigma^{\nu\mu}, q_\nu(q^\nu v^\mu - q^\mu v^\nu)\}$$

$$q_0^2\Big|_{R_\perp=1} = (3.62\,\dots\,3.69)\,\mathrm{GeV}^2 \quad \text{for} \quad m_X^{\rm cut} = (2.0\,\dots\,1.8)\,\mathrm{GeV}$$

$$q_0^2\Big|_{R_\perp\,\mathrm{NLO}} = (3.55\,\dots\,3.61)\,\mathrm{GeV}^2 \quad \text{for} \quad m_X^{\rm cut} = (2.0\,\dots\,1.8)\,\mathrm{GeV}$$

$$q_0^2\Big|_{R_\perp\,\mathrm{NNLO}} = (3.44\,\dots\,3.50)\,\mathrm{GeV}^2 \quad \text{for} \quad m_X^{\rm cut} = (2.0\,\dots\,1.8)\,\mathrm{GeV}$$

✓ the sensitivity of the zero on $m_X^{\rm cut}$ is only $\pm 0.03\,\mathrm{GeV}^2$ and hence below 1%;

✓ the impact of the NLO correction to $R_\perp$ is to shift the zero by -2.2%;

✓ the size of the NNLO correction to $R_\perp$ is significant; it amounts to a shift of the NLO zero by another -3% and hence is larger than the NLO shift;

✓ the total shift induced by $R_\perp$ through NNLO therefore amounts to -5%;

For more details:

G.Bell, M. Beneke, T. Huber, and X.Q. Li, Nucl.Phys.B843 (2011) 143;

M. Beneke, T. Huber and X.Q. Li, Nucl.Phys.B811 (2009) 77.

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Valencia, 29 Nov-1Dec 2010