Quarkonium Correlators in the QGP: a *T*-matrix approach

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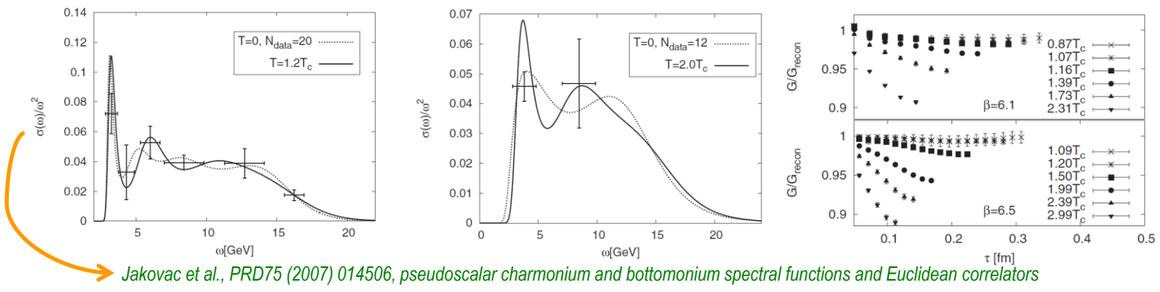
Abstract

We study the evolution of heavy quarkonium states with temperature in a Quark Gluon Plasma (QGP) by evaluating the in-medium Q anti-Q *T*-matrix within a reduced Bethe-Salpeter equation in both *S*- and *P*-wave channels. The underlying interaction potential is extracted from finite-temperature QCD lattice calculations of the singlet free energy of a Q anti-Q pair. The bound states are found to gradually move above the Q anti-Q threshold after which they rapidly dissolve in the hot system. The *T*-matrix approach provides a unified treatment of bound and scattering states, including threshold effects and the transition to the (perturbative) continuum. We apply the *T*-matrix to calculate Q anti-Q spectral functions and Euclidean-time correlators, which we compare to results from lattice QCD. Our study shows a large sensitivity to the interplay of bound and scattering states, as well as to single-particle in-medium properties such as effective quark masses. Further applications to other possible surviving states in the heavy-light quark and pure gauge (gluon gluon) sectors at finite temperature can be addressed within this framework.

Introduction

- Heavy quarkonia [Matsui-Satz '86]: spectroscopic probes of QCD matter in HIC's
  - → study of color screening, dissociation reactions, threshold effects
- Mesonic spectral functions from Lattice QCD: heavy quarkonia survive well beyond Tc
  Asakawa, Hatsuda, NPA715 (2003) 863c ; Datta et al., Phys. Proc. Suppl. 119 (2003) 487

Strongly interacting QGP (sQGP)?



Jakovac et al., PRD75 (2007) 014506, pseudoscalar charmonium and bottomonium spectral functions and Euclidean correlators

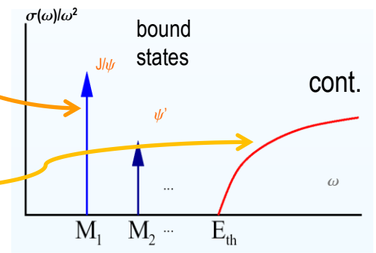
- Why use models or phenomenological approaches (vs LQCD simulations)?
  - *Euclidean correlators* accurately calculated in LQCD, not spectral functions (requires "unfolding" statistical algorithms, limited by numerical precission, e.g., lattice size)
  - Models provide *insight on physical mechanisms* responsible for the dynamics of the system (in-medium properties)
  - Once all parameters are fixed, models may be applied in different physical scenarios

Heavy quark anti-quark correlations in the sQGP

- **Previous work: potential model approach for quarkonium spectral functions**
  - Schrödinger Eq [bound states] + uncorrelated (free) continuum *Shuryak et al '04; Wong '05; Alberico et al '05; Mocsy, Petreczky '05*

$$\left[-\frac{\nabla^2}{\tilde{m}} + V_1(r,T)\right]\psi_\varepsilon(\boldsymbol{r},T) = \varepsilon(T)\psi_\varepsilon(\boldsymbol{r},T)$$

$$\sigma(\omega,T) = \sum_i 2M_i F_i^2 \delta(\omega^2 - M_i^2) + \frac{3}{8\pi}\omega^2 f(\omega, E_{\rm th})\Theta(\omega - E_{\rm th})$$



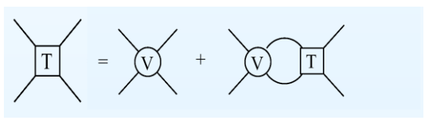
(later improved for rescattering, ex: non-rel Green's Function method)
Mocsy et al '06; Laine '07; Wong et al '07; Alberico et al '07

Schematic spectral function: "delta"-like bound states + pQCD-inspired continuum

- **Our proposal: Lipmann-Schwinger Eq for the Q anti-Q *T*-matrix** *Mannarelli, Rapp '05; Cabrera, Rapp '06*
  - 3D-reduced Bethe Salpeter Eq. (static interaction) *Blakenbeckler-Sugar '66*
  - Partial-wave expansion: *S*- and *P*- wave scattering considered

$$T_l(E;q',q) = V_l(q',q) + \frac{2}{\pi}\int_0^\infty dk\, k^2\, V_l(q',k)\, G_{\bar{Q}Q}(E;k)\, T_l(E;k,q)\,[1-2f(\omega_k)]$$

$$G_{\bar{Q}Q}(E;k) = \frac{m^2}{\omega_k}\frac{1}{E^2/4-\omega_k{}^2-2i\,\omega_k\,{\rm Im}\,\Sigma(\omega_k,k)}$$



Single-particle properties: encoded in the 2-(quasi)particle propagator
- Σ → quark selfenergy
- Relativistic dispersion relations

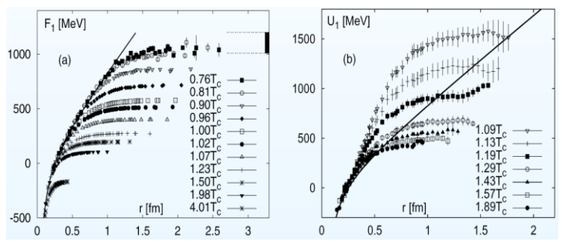
Q, Q̄ many-body properties can be implemented!

  - The *T*-matrix accounts for bound+scattering states on the same footing ("soft" transition to continuum)
  - Solving the *T*-matrix equation: inversion algorithm *Haftel-Tabakin '70*

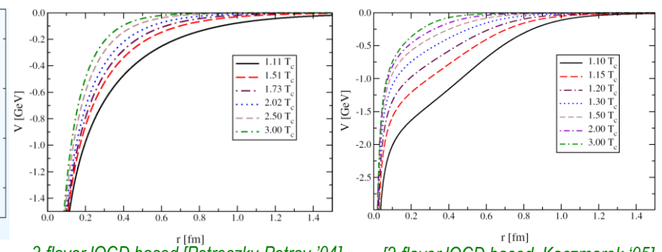
$$T = V + V\,G\,T \rightarrow \sum_{k=1}^{N}\mathcal{F}(E)_{ik}\,T(E)_{kj} = V_{ij} \quad \Longrightarrow \quad \det\mathcal{F}(E) = 0\ ,\ \ E < E_{th}$$

Bound-state condition
(equivalent to finding the zeroes of the Jost function in scattering theory)

- **Heavy quark anti-quark potential:**
  - Input from Lattice-QCD simulations of the singlet free energy, F_1, of a static heavy quark anti-quark pair
  - Potential identified with the singlet internal energy; $U_1 = F_1 - T\,(\,dF_1/dT\,)$ *[as suggested by study of survival temperatures extracted from Lattice-QCD analysis of quarkonium spectral functions, Asakawa-Hatsuda '03]*
  - Highest uncertainty of the calculation [F_1 vs U_1 discussion, numerical precission of thermal derivatives]



Kaczmarek, Zantow '05



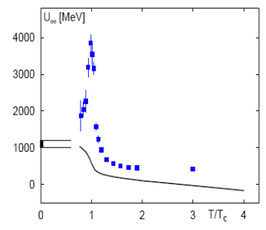
3-flavor lQCD based [Petreczky-Petrov '04] *[2-flavor lQCD based, Kaczmarek '05]*

  - Long-distance assymptotic value: identified with *in-medium heavy-quark mass*:

$$V_1(r,T) = U_1(r,T) - U_1^\infty(T);\quad m_Q^*(T) = m_Q + U_1^\infty(T)/2;\quad E_\Psi \approx 2\,m_Q^* - E_B$$

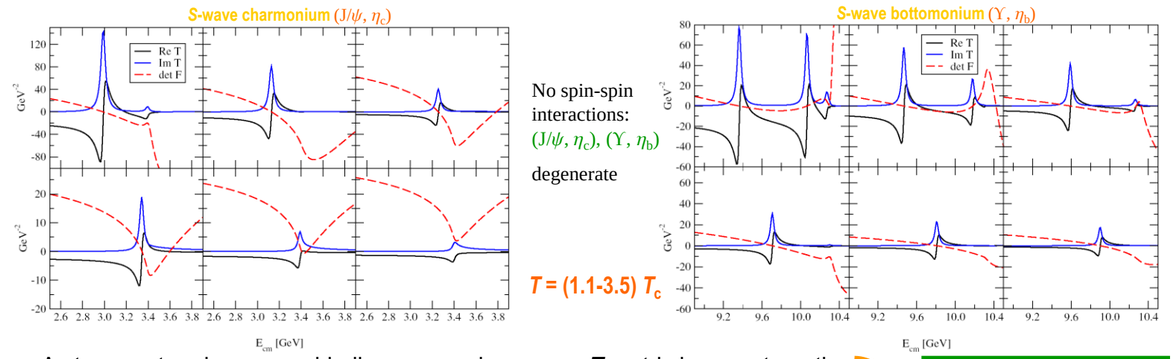
  - Scrödinger Eq. vs *T*- matrix approach:

In-med. quark mass modifies continuum threshold and enters non-linearly in scattering equation



Results: *T*-matrix for Q anti-Q scattering

- Baseline results: m_c=1.7GeV, m_b=5.1GeV (ReΣ=0); Γ_Ψ small (ImΣ=−10MeV); only *V* is modified with temperature



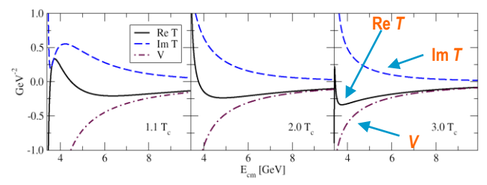
No spin-spin interactions: $(J/\psi, \eta_c)$, (Υ, η_b) degenerate

$T = (1.1\text{-}3.5)\ T_c$

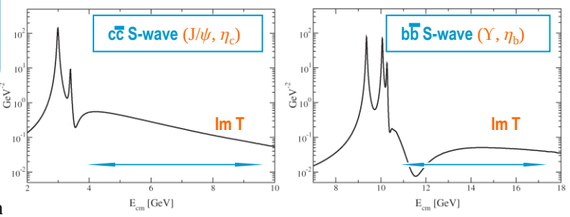
- As temperature increases, binding energy decreases, *T*-matrix looses strength:
  - J/ψ survives up to $T \approx 2.5\text{-}2.8\ T_c$, then dissolves; Υ survives beyond $T > 3.5\ T_c$

Bound states turn into resonances beyond threshold and melt in the continuum

- Presence of correlations above the heavy quark anti-quark threshold:



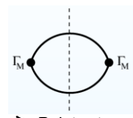
Non-perturbative strength in the continuum:
Important role in HQ diffusion: provides [color Coulomb potential](#) (both Q,q)



  - Compare full amplitude vs. Born approximation (*V*)
  - $Q\bar{Q}$ system highly correlated above E_{th}: **resonance-like structures**
  - These effects ignored in "schematic" models of the spectral function

Spectral function and Euclidean correlators from *T*-matrix

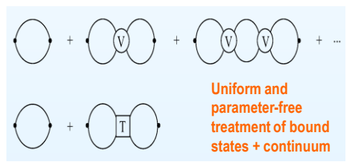
- Euclidean-time mesonic correlation function:
  - In our *T*-matrix approach



$$\Gamma_M = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5 \longrightarrow \text{S, PS, V, AV}$$

$$G(\tau,T) = \int_0^\infty d\omega\,\sigma(\omega,T)\,\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh(\omega\,\beta/2)}$$

Relates to spectral function in Minkowski space via a *thermal transformation*



- $G(E) = \int G_{\bar{Q}Q} + \int G_{\bar{Q}Q}\,T\,G_{\bar{Q}Q}$
- $\sigma(E) \propto {\rm Im}\,G(E)$ (doubly off-shell *T*-matrix)

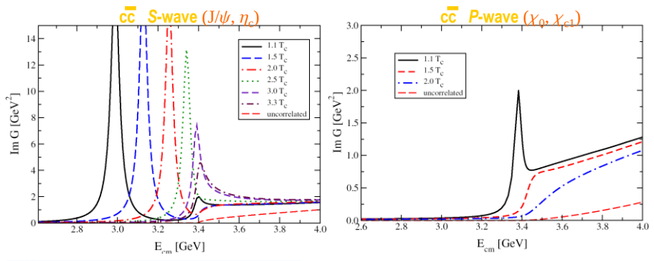
We build up the Green's function, *G*(*E*), of the *Q* anti-*Q* pair from the *T*-matrix. The imaginary part of *G* provides the spectral function of the system

- Normalization of the Euclidean correlation function: take ratio with a **"reconstructed" correlator**, calculated at the temperature of interest using a reference spectral function at a different temperature (e.g., *T*=0). Necessary to regularize and remove the "artificial" temperature dependence of the thermal kernel. Deviations of the ***normalized correlator*** from unity imply genuine changes in the spectral function, $\sigma(E)$, due to medium effects.

$$G_r(\tau,T) = \int_0^\infty d\omega\,\sigma(\omega,T=0)\,\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh(\omega\,\beta/2)} \qquad R_G(\tau,T) \equiv \frac{G(\tau,T)}{G_r(\tau,T)}$$

- Spectral functions from the *T*-matrix: m_c=1.7GeV, m_b=5.1GeV (ReΣ=0); Γ_Ψ small (ImΣ=−10MeV); only *V* changes with temp. as above

• Exhibit considerable ***non-perturbative enhancement*** around and beyond threshold, particularly when a bound state crosses into the continuum (see *T*-matrix results above)
• Normalized correlators from LQCD simulations are ***fairly well reproduced*** as long as the following additional medium effects are consistently accounted for:



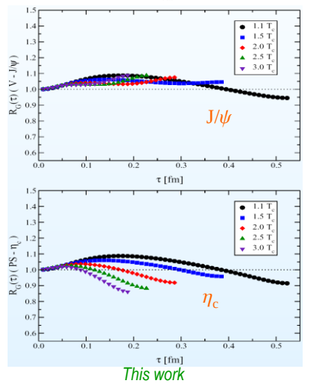
  - In medium heavy-quark masses *[e.g., T-dependent Q anti-Q thresholds]*

$$m_Q^*(T) = m_Q + U_1^\infty(T)/2$$

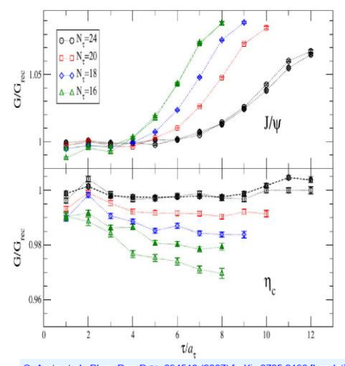
  - Diffusive contributions to σ from zero-modes

$$\sigma^{zero-mode}(\omega) = \omega\,\delta(\omega)\alpha_M\,\chi(T)$$
$$\chi(T) = -2\,N_c\,\frac{1}{\pi^2}\int dk\,k^2\,\frac{m_Q^2}{\omega_k^2}\,\frac{\partial n_F}{\partial\omega_k}$$

  - correlators from LQCD incorporate these contributions
  - related to the heavy-quark diffusion constant
  - PS channel protected, all others (S, V, AV) sensitive
  - Break degeneracy between (PS, V) and (S, AV) channels



This work



G. Aarts et al., Phys. Rev. D 76, 094513 (2007) [arXiv:0705.2198 [hep-lat]]

P-wave (scalar and axial-vector channel: χ_{c0}, χ_{c1}) correlators: also fair agreement with LQCD results [not shown].
See also Jakovac et al., PRD75 (2007) 014506

Summary and References

- We propose a *T*-matrix approach to heavy quark anti-quark scattering in a QGP, with the input of a *Q* anti-*Q* potential extracted from LQCD calculations of the internal energy.
- Both charmonium and bottomonium bound states are obtained in *S*- and *P*-wave scattering. These states gradually move to higher energies (with a decreasing binding energy) as temperature is increased, and eventually dissolve into (wide) resonances beyond the heavy-quark pair threshold energy.
- Spectral functions and Euclidean correlators are sensitive to the evolution of heavy quarkonium states with temperature, to non-perturbative quark anti-quark rescattering in the continuum and to threshold effects.
- In-medium heavy-quark masses (selfenergy) and diffusive (zero-mode) contributions are necessary to reproduce the size, shape and temperature dependence of LQCD correlators.
- The *T*-matrix approach is an appropriate scheme to study strong correlations in other interacting channels: *Qq* sector (non-perturbative rescattering leads to efficient HQ difussion via excitation of *D*- and *B*-like meson resonances) and *gg* sector (glueball abundance in gluon-glueball gas may reflect strong correlations beyond the temperature of dissociation of glueball states).

References:

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- F. Brau, F. Buisseret, Phys. Rev. D79, 114007 (2009)

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