Universidad de Oviedo

SPECTRAL PROPERTIES IN M-THEORY

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MOTIVATION

- Semiclassical analysis is not enough to characterize theories at High Energies. One needs to use full non-perturbative approaches.

- We restrict ourselves to nonperturbative analysis of supersymmetric and non-supersymmetric matrix models (Quantum theories with matrix degrees of freedom) of theories coming from dimensional reduction of gauge theories to (1+0)Dimensions and String/M-theory matrix model regularizations.

- Physical theories like YM or SYM among others, when put in a box should have discrete spectrum but in the continuous limit it is expected that they will exhibit a continuous spectrum probably with some bound state that define a mass gap. These theories should be well defined when regularized.

- The first few bound states provide information about the potential in neighborhoods of the origin while the nature of the spectrum is related to the behavior of the potential at large distances in configuration space.

- Discreteness of the spectrum of a theory with accumulation point at infinity guarantees the existence of a complete set of eigenfunctions which can be used to decompose the action of the operator in low/high frequency expansions.

- Discreteness of the spectrum also guarantees the study of eigenvalues asymptotics for the resolvent (Heat kernel) by means of Schatten-von- Neuman ideals.

- These two last properties are not guaranteed for hamiltonians with non-empty essential spectrum, (i.e. continuous sectors on it).

- Perturbative computations of interactions in hamiltonians of these type (i.e. hamiltonians with continuous spectrum (BFSS-like)) MAY not converge to the path integral value, and in that sense the perturbative results MAY happen to be physically meaningless, when analyzed beyond semiclassical approximation.

Common False Friends

Believings	Counterexample
-Iff the bosonic spectrum of a regularized theory is discrete, supersymmetry will not change this behavior.	- The 11D regularized supermembrane has classical instabilities, discrete bosonic spectrum at quantum level, and continuous spectrum at supersymmetric level
-A theory with mass terms automatically has no flat directions.	-It depends on the higher order term contribution also not only on the mass.
-A theory without flat directions has purely discrete spectrum.	-Flat directions are not enough to gurantee discreteness since it may happen to be a a direction in configuration space constant far away from the origin that renders the spectrum continuous. See our toy model with a gap.
-A model with flat directions neccesarily have continuous spectrum.	-The bosonic membrane has flat directions but due to quantum effects the spectrum is purely discrete
-A supersymmetric theory with mass terms has purely discrete spectrum.	-Our toy model with a mass gap. It always depends on the rest of higher order interactions

Sufficient Criteria for Spectral Discreteness

L. Boulton, M.P. Garcia del Moral, A. Restuccia 2002

Lemma 1. *Let $v_k(Q)$ be the eigenvalues of the $d \times d$ matrix $V(Q)$. If all $v_k(Q) \to +\infty$ as $|Q| \to \infty$, then the spectrum of H consists of a set of isolated eigenvalues of finite multiplicity accumulating at $+\infty$.*

Lemma 2. *Let V_B be a continuous bosonic potential of the configuration space. Let V_F be a fermionic matrix potential with continuous entries v_{ij} of the configuration space. Suppose that there exist constants $b_B, b_F, R_0, p_B, p_F > 0$ independent of $Q \in \mathbb{R}^N$ satisfying the following conditions*

$$V_B \geq b_B |Q|^{p_B} \qquad \text{and} \qquad |v_{ij}| \leq b_F |Q|^{p_F}$$

for all $|Q| > R_0$. If $p_B > p_F$, then the Hamiltonian $H = P^2 + V(Q)$ of the quantum system associated with $V = V_B I + V_F$ has spectrum consisting exclusively of isolated eigenvalues of finite multiplicity.

REFERENCES

L.Boulton, M.P. Garcia del Moral, A. Restuccia hep-th/0211047 Nucl.Phys.B671:343-358,2003." **Discreteness of the spectrum of the compactified D = 11 supermembrane with nontrivial winding"**

L.Boulton, M.P.Garcia del Moral, A. Restuccia **The Supermembrane with central charges: (2+1)-D NCSYM, confinement and phase transition, Nucl.Phys.B795:27-51,2008.hep-th/0609054**

L.Boulton, M.P. Garcia del Moral, A. Restuccia arXiv:1011.4791; *Spectral properties in supersymmetric matrix models.*

New Results: Non-empty essential spectrum

dWLN toy model

De Wit, Luscher, Nicolai 1988

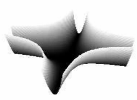

Fig. 5:The potential of the toy model

$$H = \begin{pmatrix} -\Delta + x^2y^2 & x+iy \\ x-iy & -\Delta + x^2y^2 \end{pmatrix}$$

EIGENVALUES $\lambda^{\pm}_{\text{SUSY}}(x,y) = x^2y^2 \pm \sqrt{x^2+y^2}.$

Unbounded from below

$(0,y)$ and $(x,0)$, $\lambda^{-}_{\text{SUSY}}(x,y) \to -\infty$

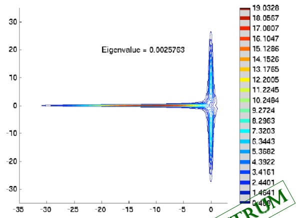

FIGURE 2. Estimation of the density function for the ground state of H restricted to $[-100,100]^2$ subject to Dirichlet boundary conditions in the case of the model of Section 6.1. The calculations are performed by means of the finite element method and an h-adaptive scheme.

CONTINUOUS SPECTRUM

A toy model with mass terms

$$H = \begin{pmatrix} -\Delta + y^2 + x^2y^2 & x+iy \\ x-iy & -\Delta + y^2 + x^2y^2 \end{pmatrix}.$$

$$\lambda^{\pm}_{\text{SUSY}}(x,y) = x^2y^2 + y^2 \pm \sqrt{x^2+(y-1)^2}$$

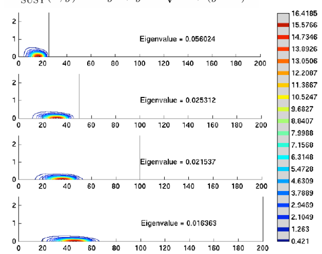

FIGURE 1. Estimation of the density function for the ground state of H restricted to a box $[-L,L]^2$, subject to Dirichlet boundary conditions, for $L =$ 25 (top), 50, 100, 200 (bottom), in the case of the model of Section 6.2. The vertical line depicts part of the boundary of the box.

Analogous to the D2-D0 system

Eigenvalue Number	$L = 25$	$L = 50$	$L = 100$	$L = 200$
1	0.0560	0.0253	0.0215	0.0164
	0.1158	0.0466	0.0386	0.0281
	0.1970	0.0745	0.0603	0.0426
	0.2980	0.1093	0.0852	0.0593
5	0.4207	0.1506	0.1097	0.0776
	0.5714	0.1980	0.1372	0.0983
	0.7543	0.2515	0.1739	0.1229
	0.9701	0.3114	0.2177	0.1511
	1.2181	0.3785	0.2661	0.1817
10	1.4974	0.4537	0.3172	0.2141
	1.8056	0.5376	0.3694	0.2492
	2.1325	0.6302	0.4249	0.2874
	2.3863	0.7313	0.4886	0.3279
	2.5953	0.8405	0.5611	0.3700
15	2.9707	0.9576	0.6401	0.4161
	3.4025	1.0828	0.7232	0.4682
	3.8690	1.2166	0.8081	0.5257
	4.3642	1.3593	0.8633	0.5873
	4.8560	1.5104	0.8980	0.6515
20	5.0743	1.6687	0.9947	0.7174

TABLE 1. Estimation of the first 20 eigenvalues of H restricted to a box $[-L,L]^2$ subject to Dirichlet boundary conditions in the case of the model of Section 6.2.

CONTINUOUS SPECTRUM

A toy model with a mass gap

$$H = \begin{pmatrix} -\Delta + V_B(x,y) & x+iy+i \\ x-iy-i & -\Delta + V_B(x,y) \end{pmatrix}$$

with $V_B(x,y) = x^2(y+1)^2 + y^2$

The eigenvalues are given by

$$\lambda^{\pm}_{\text{SUSY}}(x,y) = x^2(y+1)^2 + y^2 \pm \sqrt{x^2+(y+1)^2}.$$

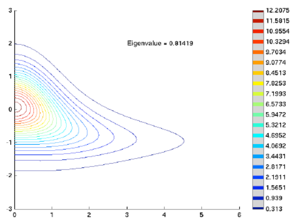

FIGURE 3. Estimation of the density function for the ground state of H restricted to $[-40,40]^2$ subject to Dirichlet boundary conditions in the case of the model of Section 6.3. The calculations are performed by means of the finite element method and an h-adaptive scheme.

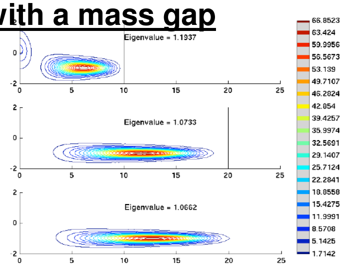

FIGURE 4. Estimation of the density function for the first exited state of H restricted to a box $[-L,L]^2$, subject to Dirichlet boundary conditions, for $L =$ 10 (top), 20, 40 (bottom) in the case of the model of Section 6.3. The vertical line depicts part of the boundary of the box.

Eigenvalue Number	$L = 10$	$L = 20$	$L = 40$
1	0.8218	0.8142	0.8142
2	1.1937	1.0733	1.0662
3	1.5474	1.1707	1.1511
4	2.0405	1.3190	1.2803
5	2.5313	1.5173	1.4515
6	3.0876	1.7618	[illegible]

TABLE 2. Estimation of the first 6 eigenvalues of H restricted to a box $[-L,L]^2$ subject to Dirichlet boundary conditions in the case of the model of Section 6.3.

CONTINUOUS SPECTRUM

New Results: Discreteness!!!!!!

BMN model

Beyond semiclassical approx.

$$L_{BMN} = T - V_B - V_F$$

$$V_B = \text{Tr}\left[\frac{\mu^2}{36R}\sum_{i=1,2,3}(X^i)^2 + \frac{\mu^2}{144R}\sum_{i=4}^{9}(X^i)^2 + \frac{i\mu}{3}\sum_{i,j,k=1}^{3}\epsilon_{ijk}X^iX^jX^k - \frac{R}{2}\sum_{i,j=1}^{9}[X^i,X^j]^2\right]$$

$$V_F = \text{Tr}\left[\frac{\mu}{4}\Psi^T\gamma_{123}\Psi - 2iR\sum_{i=1}^{9}(\Psi^T\gamma_i[\Psi,X^i])\right]$$

By analyzing

$$V_{B1} = \frac{1}{2}\text{Tr}\left[-i\sqrt{R}[X^i,X^j] - \frac{\mu}{6\sqrt{R}}\epsilon^{ijk}X^k\right]^2 = \frac{\mu^2}{36R}\rho^2 P(\rho,\varphi)$$

We prove *Theorem 3. Let $R_0 > \frac{\mu}{3R}\sqrt{C_2(N)N}$ where $C_2(N) = \frac{N^2-1}{4}$ and μ, R different from zero. Then $P(\rho,\varphi) > C > 0$ for all $\rho > R_0$ and $\varphi \in S^{3N^2}$.*

In virtue of Lemma1 and 2 we prove that the spectrum of the full hamiltonian is purely discrete

The large N Limit: **Our bound diverges**, this means this limit cannot be analyzed by means of these theorems, although we conjecture a nonempty essential spectrum in the continuum

The Supermembrane with Central charges

We provide two different regularizations of the supersymmetric hamiltonian. We rigorously prove that each one has purely discrete spectrum and accumulation point at infinity.

Y_A

$$V = \langle |D_1A_2 + i\{A_1,A_2\}|^2 + |D_2A_1|^2 \rangle$$
$$= \rho^2\left(\left|\frac{iB_{(0,1)} - A_{(1,0)}B_{(-1,1)} + A_{(-1,0)}B_{(1,1)}}{\rho}\right|^2 + \left|\frac{iB_{(1,1)} - A_{(1,0)}B_{(0,1)}}{\rho}\right|^2\right)$$
$$+ \rho^2\left(\left|\frac{iB_{(1-,1)} + A_{(1,0)}B_{(0,-1)}}{\rho}\right|^2 + \left|\frac{A_{(1,0)}B_{(1,1)}}{\rho}\right|^2 + \left|\frac{A_{(-1,0)}B_{(-1,1)}}{\rho}\right|^2 + \left|\frac{A_{(1,0)}}{\rho}\right|^2\right),$$

wehre $\rho^2 = |A_{(1,0)}|^2 + |B_{(0,1)}|^2 + |B_{(1,1)}|^2 + |B_{(-1,1)}|^2.$

SU(N)

$$\begin{aligned} H = \text{Tr}\Big(&\frac{1}{2N^3}((P_m)^2 + (\Pi_r)^2) + \\ &+ \frac{n^2}{16\pi^2N^3}(i[X^m,X^n])^2 + \frac{n^2}{8\pi^2N^3}\left(\frac{1}{N}[T_{V_r},X^m]T_{-V_r} + i[\mathcal{A}_r,X^m]\right)^2 + \\ &+ \frac{n^2}{16\pi^2N^3}\left(i[\mathcal{A}_r,\mathcal{A}_s] - \frac{1}{N}([T_{V_s},\mathcal{A}_r]T_{-V_s} - [T_{V_r},\mathcal{A}_s]T_{-V_r})\right)^2 + \frac{1}{8}n^2 + \\ &+ \frac{n}{4\pi N^3}\Lambda\left([X^m,P_m] - \frac{i}{N}[T_{V_r},\Pi_r]T_{-V_r} + [\mathcal{A}_r,\Pi_r]\right) + \\ &+ \frac{in}{4\pi N^3}(\overline{\Psi}\Gamma_-\Gamma_m[X^m,\Psi] - \overline{\Psi}\Gamma_-\Gamma_r[\mathcal{A}_r,\Psi] + \Lambda[\overline{\Psi}\Gamma_-,\Psi] + \\ &- \frac{i}{N}\overline{\Psi}\Gamma_-\Gamma_r[T_{V_r},\Psi]T_{-V_r})\Big) \end{aligned}$$

The large N Limit: the bound is convergent!!!

MIM2 as a fundamental d.o.f in distinction with the M2 in 11D

Summary of Results 1011.4791

- We provide some lemmas that give a sufficient criteria for Discreteness on SUSY and NON-SUSY theories in Quantum mechanics with matrix degrees of freedom. They are useful to perform a nonperturbative analysis of gauge theories dimensionaly reduced to (1+0)D and also to matrix models obtained in M-theory /String theories.

- We prove rigurously that BMN matrix model has a purely discrete spectrum for any finite value of N beyond semiclassical approximation. The bound we naturally find diverges in the large N limit. It does not exclude the possibility to find a better one that could not diverge. A rigurous proof in the large N limit will be needed. We conjecture a non-empty essential spectrum at the continuum limit. Iff that would be the case, the interpretation in terms of coincident gravitons should be revised.

- We prove rigorously that the supermembrane with central charges in two different regularizations has purely discrete spectrum. The large N limit of our bound converge to the value we already found in 2005 in the large N limit. The regularized semiclassical eigenvalues converge properly to the semiclassical ones in the continuous. The bosonic potential of the full theory also satisfy a analogous type of bound that the regularized one (2005). For all these evidences it seems plausible that the large N limit of this theory will have a purely discrete spectrum with finite multiplicity with accumulation point at infinity. Iff that is the case, the supermembrane with central charges could be interpreted as a fundamental supermembrane describing microscopical degrees of freedom of (at least part) of M-theory.

- We prove rigorously in our paper beyond semiclassical approximation that the system D2-D0 (N,K) irrespectively to the numbers of N D2´s, K D0´s has continuous spectrum starting from a valued determined by the monopole contribution. We also show some results of toy models with nonempty essential spectrum to illustrate the different behaviors in simple models with a fermionic contribution. We compute also the eigenvalues numerically to see the existence of bound states below or inside the continuous spectrum.

- Perturbative computation of interactions from BFSS model are **delicate** since the original hamiltonian operator has continuous spectrum so the expansion MAY NOT converge to the value of the path integral. These interactions should be checked carefully.

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