Install FeynCalc and FeynHelpers

```
Import["https://raw.githubusercontent.com/FeynCalc/feyncalc/master/install.m"]
InstallFeynCalc[]
```

```
Downloading FeynCalc from https://github.com/FeynCalc/feyncalc/archive/hotfix-stable.zip .
FeynCalc zip file was saved to /tmp/m000002320481.
Extracting FeynCalc zip file to /tmp/m000002320481.dir ...done!
Copying FeynCalc to /home/vs/.Mathematica/Applications/FeynCalc ...done!
Setting up the help system... done!
Downloading FeynArts from https://github.com/FeynCalc/feynarts-mirror/archive/master.zip .
FeynArts zip file was saved to /tmp/m000005320481.
Extracting FeynArts zip file to /home/vs/.Mathematica/Applications/FeynCalc/FeynArts ...do
Copying FeynArts to /home/vs/.Mathematica/Applications/FeynCalc/FeynArts ...done!

Installation complete! Loading FeynCalc...
Patching FeynArts... done!
```

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

FeynArts 3.9 patched for use with FeynCalc, for documentation use the manual or visit www.feynarts.de.

```
Import["https://raw.githubusercontent.com/FeynCalc/feynhelpers/master/install.m"]
InstallFeynHelpers[]
```

```
Downloading FeynHelpers from https://github.com/FeynCalc/feynhelpers/archive/stable.zip ..
FeynHelpers zip file was saved to /tmp/m000007320481.
Extracting FeynHelpers zip file to /tmp/m000007320481.dir ...done!
Copying FeynHelpers to /home/vs/.Mathematica/Applications/FeynCalc/AddOns/FeynHelpers ...d
done!
Downloading Package-X from http://www.hepforge.org/archive/packagex/X-2.0.3.zip ...done!
Package-X zip file was saved to /tmp/m000008320481.
Extracting Package-X zip file to /tmp/m000008320481.dir ...done!
Copying Package-X to /home/vs/.Mathematica/Applications/X ...done!

Installation complete! To load FeynHelpers, restart Mathematica and evaluate

 $LoadAddOns={"FeynHelpers"};

 before you load FeynCalc;
```

/tmp/m000007320481.dir/feynhelpers−stable

```
Options[InstallFeynHelpers]
```

{AutoInstallPackageX → None, AutoInstallFIRE → None, AutoOverwriteFeynHelpersDirectory → None, FeynHelpersDevelopmentVersionLink → https://github.com/FeynCalc/feynhelpers/archive/master.zip, FeynHelpersStableVersionLink → https://github.com/FeynCalc/feynhelpers/archive/stable.zip, InstallFeynHelpersDevelopmentVersion → False, InstallFeynHelpersTo → /home/vs/.Mathematica/Applications/FeynCalc/AddOns/FeynHelpers}

```
Quit[]
```

Load FeynCalc and FeynHelpers

```
$LoadAddOns = {"FeynHelpers"};
<< FeynCalc`
```

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

FeynHelpers 1.0.0 loaded.

Have a look at the supplied examples. If you use FeynHelpers in your research, please cite

- V. Shtabovenko, "FeynHelpers: Connecting
  FeynCalc to FIRE and Package–X", TUM–EFT 75/15, arXiv:1611.06793

Furthermore, remember to cite the authors of the tools that you are calling from FeynHelpers, which are

- FIRE by A. Smirnov, if you are using the function FIREBurn.
- Package – X by H. Patel, if you are using the function PaXEvaluate.

Internal vs. External Representations

FeynCalcInternal

The internal representation (FeynCalcInterral or FCI) is how FeynCalc internally “sees” the objects.
For example, a 4-dimensional 4-vector is represented by

```
Pair[LorentzIndex[μ], Momentum[p]]
```

$\overline{p}^{\mu}$

Pair is one of the most basic FeynCalc objects. Depending on its arguments, it can represent
a 4-vector, a metric tensor

```
Pair[LorentzIndex[μ], LorentzIndex[ν]]
```

$\overline{g}^{\mu\nu}$

or a scalar product of two 4-vectors

```
Pair[Momentum[p], Momentum[q]]
```

$\overline{p}\cdot\overline{q}$

Another essential object is **DiracGamma** that is used to represent Dirac matrices. An uncontracted Dirac matrix is

```
DiracGamma[LorentzIndex[μ]]
```

$\overline{\gamma}^{\mu}$

and for a Feynman slash we use

```
DiracGamma[Momentum[p]]
```

$\overline{\gamma}\cdot\overline{p}$

The Levi-Civita-Tensor is

```
Eps[LorentzIndex[μ], LorentzIndex[ν], LorentzIndex[ρ], LorentzIndex[σ]]
```

$\epsilon^{\mu\nu\rho\sigma}$

or, when contracted with 4-momenta

```
Eps[Momentum[p1], Momentum[p2], Momentum[q1], Momentum[q2]]
```

$\epsilon^{\overline{\text{p1}}\,\overline{\text{p2}}\,\overline{\text{q1}}\,\overline{\text{q2}}}$

This notation (momenta in the index slots) is also used in many other tools (e.g. FORM). The advantage is, that we do not need to canonicalize the indices of the Levi-Civita-Tensor, e.g. to ensure that

```
Eps[LorentzIndex[μ], Momentum[p2], Momentum[q1], Momentum[q2]]
   Pair[LorentzIndex[μ], Momentum[p1]] -
  Eps[LorentzIndex[ν], Momentum[p2], Momentum[q1], Momentum[q2]]
   Pair[LorentzIndex[ν], Momentum[p1]]
```

$\overline{\text{p1}}^{\mu}\,\epsilon^{\mu\,\overline{\text{p2}}\,\overline{\text{q1}}\,\overline{\text{q2}}} - \overline{\text{p1}}^{\nu}\,\epsilon^{\nu\,\overline{\text{p2}}\,\overline{\text{q1}}\,\overline{\text{q2}}}$

is zero.

FeynCalcExternal

The internal representation is useful for the internal programming FeynCalc, but obviously too cumbersome
for the user input. This is why FeynCalc also has an external representation (FeynCalcExternal or FCE), that
is concise and convenient.

Let us start with the 4-vector. In the FCE-notation it is just **FV** (“FourVector”)

```
FV[p, μ]
```

$\overline{p}^{\mu}$

It is not hard to guess that the scalar product is **SP**

```
SP[p, q]
```
$\overline{p}\cdot\overline{q}$

while for the metric tensor we write **MT**

```
MT[μ, ν]
```
$\overline{g}^{\mu\nu}$

To input a Dirac matrix or a Feynman slash, use **GA** and **GS** respectively

```
GA[μ]
```
$\overline{\gamma}^{\mu}$

```
GS[p]
```
$\overline{\gamma}\cdot\overline{p}$

The Levi-Civita tensor is **LC**

```
LC[μ, ν, ρ, σ]
```
$\epsilon^{\mu\nu\rho\sigma}$

The fully contracted form is entered via

```
LC[][p1, p2, q1, q2]
```
$\epsilon^{\overline{\text{p1}}\,\overline{\text{p2}}\,\overline{\text{q1}}\,\overline{\text{q2}}}$

It is also possible to enter a mixed form

```
LC[μ][p1, p2, q]
```
$\epsilon^{\mu\,\overline{\text{p1}}\,\overline{\text{p2}}\,\overline{q}}$

```
LC[μ, ν][p1, p2]
```
$\epsilon^{\mu\nu\,\overline{\text{p1}}\,\overline{\text{p2}}}$

Switching between the representations

To convert between the two representations we use the functions **FCI** and **FCE**, which are shortcuts for **FeynCalcInternal** and **FeynCalcExternal**.

One cannot distinguish between the notations using the typesetting, i.e. when we see a typesetted objected in the **TraditionalForm**, we cannot really tell if it is in the FCI or FCE notation.

```
ex1 = FV[p, μ]
ex2 = Pair[Momentum[p], LorentzIndex[μ]]
```
$\overline{p}^{\mu}$

$\overline{p}^{\mu}$

However, we can always use **StandardForm** to see the difference

```
ex1 // StandardForm
ex2 // StandardForm
```

```
FV[p, μ]
```

```
Pair[LorentzIndex[μ], Momentum[p]]
```

StandardForm shows us how Mathematica “sees” the expressions that we enter.
Notice that the index μ is still typeset. The more “advanced” version of **StandardForm** is **FullForm**

```
ex1 // FullForm
ex2 // FullForm
```

```
FV[p, \[Mu]]
```

```
Pair[LorentzIndex[\[Mu]], Momentum[p]]
```

For most cases, **StandardForm** is fully sufficient yo analyze the input and output of FeynCalc and Mathematica.
Now using **FCI** and **FCE**, we can see how the output of **StandardForm** changes

This is the original expression

```
ex1 // StandardForm
```

```
FV[p, μ]
```

Nothing changes, since ex1 is already FCE

```
ex1 // FCE // StandardForm
```

```
FV[p, μ]
```

Now ex1 becomes FCI

```
ex1 // FCI // StandardForm
```

```
Pair[LorentzIndex[μ], Momentum[p]]
```

but we can easily make it FCE again

```
ex1 // FCI // FCE // StandardForm
```

```
FV[p, μ]
```

Why it matters

All FeynCalc functions that are meant for users will automatically convert the user input in the FCE notation into the FCI notation. You do not have to do it by yourself.

On the other hand, virtually all FeynCalc functions produce their output in the FCI form.
So when you have an expression that was obtained from FeynCalc and want to apply some replacement rules to it, we have to use the FCI form in the rule

```
ex = Pair[Momentum[p], Momentum[q]]
```

$\overline{p}\cdot\overline{q}$

No surprise that following does not work

```
ex /. SP[p, q] → 1
```

$\overline{p}\cdot\overline{q}$

But if we wrap the r.h.s of the rule with **FCI**, then everything is fine

```
ex /. FCI[SP[p, q]] → 1
```

1

Contractions

Now that we have some basic understanding of FeynCalc objects, let us do something with them. Contractions of Lorentz indices are one of the most essential operations in symbolic QFT calculations. In FeynCalc the corresponding function is called **Contract**

```
FV[p, μ] MT[μ, ν]
Contract[%]
```

$\overline{p}^{\mu}\,\overline{g}^{\mu\nu}$

$\overline{p}^{\nu}$

```
FV[p, α] FV[q, α]
Contract[%]
```

$\overline{p}^{\alpha}\,\overline{q}^{\alpha}$

$\overline{p}\cdot\overline{q}$

Notice that when we enter noncommutative objects, such as Dirac matrices, we use **Dot** (“.”) and not **Times** (“*”)

```
FV[p, α] MT[β, γ] GA[α].GA[β].GA[γ]
Contract[%]
```

$\overline{p}^{\alpha}\,\overline{\gamma}^{\alpha}.\overline{\gamma}^{\beta}.\overline{\gamma}^{\gamma}\,\overline{g}^{\beta\gamma}$

$(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\gamma}.\overline{\gamma}^{\gamma}$

This is because **Times** is commutative, so writing something like

```
GA[δ] GA[β] GA[α]
```

$\overline{\gamma}^{\alpha}\,\overline{\gamma}^{\beta}\,\overline{\gamma}^{\delta}$

will give you completely wrong results. It is also a very common beginner’s mistake!

You might have wondered why FeynCalc does not seem to distinguish between upper and lower Lorentz indices.
Well, FeynCalc assumes that all your expressions with Lorentz indices are manifestly Lorentz covariant and
respect Einstein’s summation. This implies that

- In an equality, if a free Lorentz index appears upstairs on the one hand side, it must also appear upstairs
  on the left hand side. Something like $p^\mu = c\, q_\mu$ would violate manifest Lorentz covariance. Hence,

```
FV[p, μ] == c FV[q, μ]
```

$\overline{p}^\mu = c\,\overline{q}^\mu$

  could equally stand for $p^\mu = c\, q^\mu$ or $p_\mu = c\, q_\mu$

- Each term may not contain more than two equal indices. Those are interpreted as dummy indices, i.e. one of them is an
  upper index and the other is the lower one. Which one is which does not matter. Dummy indices will be contracted.

  Still, it is your task to ensure, that your input expression respect Einstein's summation. Checking this by FeynCalc would cost too much performance.

Hence, FeynCalc will not complain about obviously incorrect expressions, but the corresponding result will be apparently
nonsensical

```
MT[μ, ν] FV[p, μ] FV[q, μ]
Contract[%]
```

$\overline{p}^\mu \overline{q}^\mu \overline{g}^{\mu\nu}$

$\overline{p}^\nu \overline{q}^\mu$

Good to know: Since the upcoming FeynCalc 9.3 will introduce support for Cartesian indices (**CartesianIndex**), where we may have the same index appearing
upstairs and downstairs in an equality (e. g. $\boldsymbol{p}^i = -\boldsymbol{p}_i$), we have to explicitly specify the position of the indices. The choice we make
is the following

- Every free Lorentz or Cartesian index is an upper index, i. e. **FV**[p,μ] corresponds to p^μ and **CV**[p,i] to $\boldsymbol{p}^i$ which also agrees with the typesetting.
- A pair of dummy Lorentz indices consists of an upper and a lower index.
- A pair of dummy Cartesian indices consists of **two upper** indices.

  Thus, while **FV**[p,μ] **FV**[q,μ] may stand for $p^\mu\, q_\mu$ or $q^\mu\, p_\mu$, **CV**[p,i] **CV**[q,i] always implies $\boldsymbol{p}^i\, \boldsymbol{q}^i$

When it comes to products of **Eps** tensors, **Contract** will by default apply the product formula with the determinant
of metric tensors

```
LC[μ, ν][p, q] LC[ρ, σ][r, s] FV[x, μ]
Contract[%]
```

$\overline{x}^{\mu} \epsilon^{\mu \nu \overline{p} \overline{q}} \epsilon^{\rho \sigma \overline{r} \overline{s}}$

$$-\overline{x}^{\rho} \overline{g}^{\nu \sigma} (\overline{p} \cdot \overline{r}) (\overline{q} \cdot \overline{s}) + \overline{x}^{\sigma} \overline{g}^{\nu \rho} (\overline{p} \cdot \overline{r}) (\overline{q} \cdot \overline{s}) + \overline{q}^{\rho} \overline{g}^{\nu \sigma} (\overline{p} \cdot \overline{r}) (\overline{s} \cdot \overline{x}) - \overline{q}^{\sigma} \overline{g}^{\nu \rho} (\overline{p} \cdot \overline{r}) (\overline{s} \cdot \overline{x}) +$$
$$\overline{x}^{\rho} \overline{g}^{\nu \sigma} (\overline{p} \cdot \overline{s}) (\overline{q} \cdot \overline{r}) - \overline{x}^{\sigma} \overline{g}^{\nu \rho} (\overline{p} \cdot \overline{s}) (\overline{q} \cdot \overline{r}) - \overline{q}^{\rho} \overline{g}^{\nu \sigma} (\overline{p} \cdot \overline{s}) (\overline{r} \cdot \overline{x}) + \overline{q}^{\sigma} \overline{g}^{\nu \rho} (\overline{p} \cdot \overline{s}) (\overline{r} \cdot \overline{x}) +$$
$$\overline{p}^{\rho} \overline{g}^{\nu \sigma} (\overline{q} \cdot \overline{s}) (\overline{r} \cdot \overline{x}) - \overline{p}^{\sigma} \overline{g}^{\nu \rho} (\overline{q} \cdot \overline{s}) (\overline{r} \cdot \overline{x}) - \overline{p}^{\rho} \overline{g}^{\nu \sigma} (\overline{q} \cdot \overline{r}) (\overline{s} \cdot \overline{x}) + \overline{p}^{\sigma} \overline{g}^{\nu \rho} (\overline{q} \cdot \overline{r}) (\overline{s} \cdot \overline{x}) +$$
$$\overline{q}^{\sigma} \overline{s}^{\nu} \overline{x}^{\rho} (\overline{p} \cdot \overline{r}) - \overline{q}^{\rho} \overline{s}^{\nu} \overline{x}^{\sigma} (\overline{p} \cdot \overline{r}) - \overline{q}^{\sigma} \overline{r}^{\nu} \overline{x}^{\rho} (\overline{p} \cdot \overline{s}) + \overline{q}^{\rho} \overline{r}^{\nu} \overline{x}^{\sigma} (\overline{p} \cdot \overline{s}) - \overline{p}^{\sigma} \overline{s}^{\nu} \overline{x}^{\rho} (\overline{q} \cdot \overline{r}) + \overline{p}^{\rho} \overline{s}^{\nu} \overline{x}^{\sigma} (\overline{q} \cdot \overline{r}) +$$
$$\overline{p}^{\sigma} \overline{r}^{\nu} \overline{x}^{\rho} (\overline{q} \cdot \overline{s}) - \overline{p}^{\rho} \overline{r}^{\nu} \overline{x}^{\sigma} (\overline{q} \cdot \overline{s}) + \overline{p}^{\sigma} \overline{q}^{\rho} \overline{s}^{\nu} (\overline{r} \cdot \overline{x}) - \overline{p}^{\rho} \overline{q}^{\sigma} \overline{s}^{\nu} (\overline{r} \cdot \overline{x}) - \overline{p}^{\sigma} \overline{q}^{\rho} \overline{r}^{\nu} (\overline{s} \cdot \overline{x}) + \overline{p}^{\rho} \overline{q}^{\sigma} \overline{r}^{\nu} (\overline{s} \cdot \overline{x})$$

This is, however, not always what we want and can be inhibited via the option **EpsContract**

```
LC[μ, ν][p, q] LC[ρ, σ][r, s] FV[x, μ]
Contract[%, EpsContract → False]
```

$\overline{x}^{\mu} \epsilon^{\mu \nu \overline{p} \overline{q}} \epsilon^{\rho \sigma \overline{r} \overline{s}}$

$\epsilon^{\overline{x} \nu \overline{p} \overline{q}} \epsilon^{\rho \sigma \overline{r} \overline{s}}$

Dimensions

You might have wondered why 4-vectors, scalar products and Dirac matrices all have a bar, like $\overline{p}^{\mu}$ or $\overline{p} \cdot \overline{q}$
The bar is there to specify that they are 4-dimensional objects.

Objects that live in D-dimensions do not have a bar, c. f.

```
FVD[p, μ]
% // FCI // StandardForm
```

p^{μ}

```
Pair[LorentzIndex[μ, D], Momentum[p, D]]
```

```
MTD[μ, ν]
% // FCI // StandardForm
```

$g^{\mu \nu}$

```
Pair[LorentzIndex[μ, D], LorentzIndex[ν, D]]
```

```
SPD[p, q]
% // FCI // StandardForm
```

$p \cdot q$

```
Pair[Momentum[p, D], Momentum[q, D]]
```

```
GAD[μ]
% // FCI // StandardForm
```

γ^{μ}

```
DiracGamma[LorentzIndex[μ, D], D]
```

```
GSD[p]
% // FCI // StandardForm
```

$\gamma\cdot p$

```
DiracGamma[Momentum[p, D], D]
```

This origin of this notation is the publication of Breitenlohner and Maison (1977, link) on the treatment of γ_5 in D-dimensions
in the t'Hooft-Veltman scheme. The main idea was that we can decompose indexed objects in 4 and D-4 dimensional pieces, e.g.
$p^{\mu} = \overline{p}^{\mu} + \hat{p}^{\mu}$

Consequently, in FeynCalc we can also enter D-4 dimensional objects

```
FVE[p, μ]
% // FCI // StandardForm
```

$\hat{p}^{\mu}$

```
Pair[LorentzIndex[μ, -4 + D], Momentum[p, -4 + D]]
```

```
MTE[p, q]
% // FCI // StandardForm
```

$\hat{g}^{p q}$

```
Pair[LorentzIndex[p, -4 + D], LorentzIndex[q, -4 + D]]
```

```
SPE[p, q]
% // FCI // StandardForm
```

$\hat{p}\cdot\hat{q}$

```
Pair[Momentum[p, -4 + D], Momentum[q, -4 + D]]
```

```
GAE[μ]
% // FCI // StandardForm
```

$\hat{\gamma}^{\mu}$

```
DiracGamma[LorentzIndex[μ, -4 + D], -4 + D]
```

```
GSE[p]
% // FCI // StandardForm
```

$\hat{\gamma}\cdot\hat{p}$

```
DiracGamma[Momentum[p, -4 + D], -4 + D]
```

When we contract Lorentz tensors from different dimensions, the contractions are resolved according to the rules from the paper of Breitenlohner and Maison, e.g.

```
FVD[p, μ] FV[q, μ]
Contract[%]
```

$p^{\mu}\, \overline{q}^{\mu}$

$\overline{p} \cdot \overline{q}$

```
FV[p, μ] FVE[q, μ]
Contract[%]
```

$\hat{q}^{\mu}\, \overline{p}^{\mu}$

0

```
(FVD[p, μ] + FVE[p, μ]) (FVD[q, μ] + FVE[q, μ])
Contract[%]
```

$(p^{\mu} + \hat{p}^{\mu})\,(q^{\mu} + \hat{q}^{\mu})$

$3\,(\hat{p} \cdot \hat{q}) + p \cdot q$

Sometimes we need to switch from one dimension to another, e.g. to convert a 4-dimensional object to a D-dimensional one
or vice versa. This is done via

```
FVD[p, μ]
ChangeDimension[%, 4]
```

p^{μ}

$\overline{p}^{\mu}$

The second argument of ChangeDImension is the new dimension . The most common choices are 4, D or D-4

```
FVD[p, μ]
ChangeDimension[%, D - 4]
```

p^{μ}

$\hat{p}^{\mu}$

```
SP[p, q]
ChangeDimension[%, D]
```

$\overline{p} \cdot \overline{q}$

$p \cdot q$

Kinematics

FeynCalc allows us to specify the values of scalar products before doing the calculation.

```
SP[p, q] = s;
```

```
SP[p, q]
```

s

To clear the previously set values, use

```
FCClearScalarProducts[]
```

```
SP[p, q]
```

$\overline{p}\cdot\overline{q}$

A good habit is to always apply **FCClearScalarProducts**[] before setting the values, like in

```
FCClearScalarProducts[];
SP[p1, p1] = m1^2;
SP[p2, p2] = m2^2;
```

Setting up the kinematics in advance improves performance of FeynCalc and leads to more compact results. The
results with the fully arbitrary kinematics are the most complicated and the longest ones.

Before continuing, we clear the previously set values

```
FCClearScalarProducts[]
```

Expanding and undoing expansions

FeynCalc offers further useful functions for the manipulations of Lorentz tensors and Dirac matrices

To expand scalar products

```
ex1 = SP[p + q + r, s + t]
```

$(\overline{p}+\overline{q}+\overline{r})\cdot(\overline{s}+\overline{t})$

or expressions like

```
ex2 = FV[p + q + r, μ]
```

$(\overline{p}+\overline{q}+\overline{r})^{\mu}$

we can use

```
ExpandScalarProduct[ex1]
```

$\overline{p}\cdot\overline{s}+\overline{p}\cdot\overline{t}+\overline{q}\cdot\overline{s}+\overline{q}\cdot\overline{t}+\overline{r}\cdot\overline{s}+\overline{r}\cdot\overline{t}$

```
ExpandScalarProduct[ex2]
```

$\overline{p}^{\mu}+\overline{q}^{\mu}+\overline{r}^{\mu}$

Notice that **ExpandScalarProduct** can also do expansions only for the given momentum, while leaving the rest of the expression untouched, e.g.

```
x SP[p1 + p2, q1 + q2] + y SP[p3 + p4, q3 + q4] + z SP[p5 + p6, q5 + q6]
ExpandScalarProduct[%, Momentum → {p1}]
```

$x\left((\overline{\text{p1}}+\overline{\text{p2}})\cdot(\overline{\text{q1}}+\overline{\text{q2}})\right)+y\left((\overline{\text{p3}}+\overline{\text{p4}})\cdot(\overline{\text{q3}}+\overline{\text{q4}})\right)+z\left((\overline{\text{p5}}+\overline{\text{p6}})\cdot(\overline{\text{q5}}+\overline{\text{q6}})\right)$

$y\left((\overline{\text{p3}}+\overline{\text{p4}})\cdot(\overline{\text{q3}}+\overline{\text{q4}})\right)+z\left((\overline{\text{p5}}+\overline{\text{p6}})\cdot(\overline{\text{q5}}+\overline{\text{q6}})\right)+x\left(\overline{\text{p1}}\cdot\overline{\text{q1}}+\overline{\text{p1}}\cdot\overline{\text{q2}}+\overline{\text{p2}}\cdot\overline{\text{q1}}+\overline{\text{p2}}\cdot\overline{\text{q2}}\right)$

For the expansion of **Eps** tensors, we use

```
LC[][p1 + p2, q, r, s]
EpsEvaluate[%]
```

$\epsilon^{\overline{\text{p1}}+\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

$\epsilon^{\overline{\text{p1}}\,\overline{q}\,\overline{r}\,\overline{s}}+\epsilon^{\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

EpsEvaluate also reorders the arguments of **Eps** according to its antisymmetric properties

```
LC[μ, σ, ρ, ν]
EpsEvaluate[%]
```

$\epsilon^{\mu\sigma\rho\nu}$

$-\epsilon^{\mu\nu\rho\sigma}$

Secretly, **ExpandScalarProduct** can also handle Levi-Civitas, but this is disabled by default

```
LC[][p1 + p2, q, r, s]
ExpandScalarProduct[%]
```

$\epsilon^{\overline{\text{p1}}+\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

$\epsilon^{\overline{\text{p1}}+\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

```
LC[][p1 + p2, q, r, s]
ExpandScalarProduct[%, EpsEvaluate → True]
```

$\epsilon^{\overline{\text{p1}}+\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

$\epsilon^{\overline{\text{p1}}\,\overline{q}\,\overline{r}\,\overline{s}}+\epsilon^{\overline{\text{p2}}\,\overline{q}\,\overline{r}\,\overline{s}}$

```
LC[μ, σ, ρ, ν]
ExpandScalarProduct[%, EpsEvaluate → True]
```

$\epsilon^{\mu\sigma\rho\nu}$

$-\epsilon^{\mu\nu\rho\sigma}$

The inverse of **ExpandScalarProduct** is called **MomentumCombine**

```
3 FV[p, μ] + 4 FV[q, μ]
MomentumCombine[%]
```

$3\,\overline{p}^{\mu}+4\,\overline{q}^{\mu}$

$(3\,\overline{p}+4\,\overline{q})^{\mu}$

```
2 SP[p, q] + 9 SP[q, r]
MomentumCombine[%]
```

$2\,(\overline{p}\cdot\overline{q}) + 9\,(\overline{q}\cdot\overline{r})$

$\overline{q}\cdot(2\,\overline{p} + 9\,\overline{r})$

For Dirac matrices the corresponding functions are **DiracGammaExpand** and **DiracGammaCombine**

```
GA[μ].GS[p + q].GA[ν].GS[r + s]
DiracGammaExpand[%]
```

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p}+\overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r}+\overline{s}))$

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}+\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}+\overline{\gamma}\cdot\overline{s})$

```
GA[μ].GS[p + q].GA[ν].GS[r + s]
DiracGammaExpand[%, Momentum → r]
```

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p}+\overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r}+\overline{s}))$

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}+\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}+\overline{\gamma}\cdot\overline{s})$

```
GA[μ].GS[p + q].GA[ν].GS[r + s]
DiracGammaExpand[%]
DiracGammaCombine[%]
```

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p}+\overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r}+\overline{s}))$

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}+\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}+\overline{\gamma}\cdot\overline{s})$

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p}+\overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r}+\overline{s}))$

Notice the **DiracGammaExpand** does not expand the whole noncommutative product.
If you need that, use **DotSimplify**

```
GA[μ].GS[p + q].GA[ν].GS[r + s]
% // DiracGammaExpand // DotSimplify
```

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p}+\overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r}+\overline{s}))$

$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{s}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{s})$

Handling indices

There are two quite recent functions that can to some further extent simplify the handling of Lorentz (and other) indices

When you square an expression with dummy indices, you must rename them first. People often do this by hand, like

```
ex1 = (FV[p, μ] + FV[q, μ]) FV[r, μ] FV[r, ν]
```

$\overline{r}^{\mu}\,\overline{r}^{\nu}\,(\overline{p}^{\mu}+\overline{q}^{\mu})$

```
ex1 (ex1 /. μ → ρ)
Contract[%]
```

$\overline{r}^{\mu} (\overline{r}^{\nu})^2 \overline{r}^{\rho} (\overline{p}^{\mu} + \overline{q}^{\mu}) (\overline{p}^{\rho} + \overline{q}^{\rho})$

$\overline{r}^2 (\overline{p} \cdot \overline{r})^2 + \overline{r}^2 (\overline{q} \cdot \overline{r})^2 + 2 \overline{r}^2 (\overline{p} \cdot \overline{r}) (\overline{q} \cdot \overline{r})$

However, since FeynCalc 9 there is a function for that

```
FCRenameDummyIndices[ex1]
```

$\overline{r}^{\nu} \overline{r}^{\$AL(\$24)} (\overline{p}^{\$AL(\$24)} + \overline{q}^{\$AL(\$24)})$

```
ex1 FCRenameDummyIndices[ex1]
Contract[%]
```

$\overline{r}^{\mu} \overline{r}^{\nu} \overline{r}^{\nu} (\overline{p}^{\mu} + \overline{q}^{\mu}) \overline{r}^{\$AL(\$25)} (\overline{p}^{\$AL(\$25)} + \overline{q}^{\$AL(\$25)})$

$\overline{r}^2 (\overline{p} \cdot \overline{r})^2 + \overline{r}^2 (\overline{q} \cdot \overline{r})^2 + 2 \overline{r}^2 (\overline{p} \cdot \overline{r}) (\overline{q} \cdot \overline{r})$

Notice that **FCRenameDummyIndices** does not canonicalize the indices

```
FV[p, ν] FV[q, ν] - FV[p, μ] FV[q, μ]
FCRenameDummyIndices[%]
```

$\overline{p}^{\nu} \overline{q}^{\nu} - \overline{p}^{\mu} \overline{q}^{\mu}$

$\overline{p}^{\$AL(\$27)} \overline{q}^{\$AL(\$27)} - \overline{p}^{\$AL(\$26)} \overline{q}^{\$AL(\$26)}$

But since FeynCalc 9.1 there is a function for that too

```
FV[p, ν] FV[q, ν] - FV[p, μ] FV[q, μ]
FCCanonicalizeDummyIndices[%]
```

$\overline{p}^{\nu} \overline{q}^{\nu} - \overline{p}^{\mu} \overline{q}^{\mu}$

0

Disclaimer

- In general, index canonicalization is a very complicated and expensive operation
- There are sophisticated algorithms for doing this in an efficient way, c. f. arXiv:1702.08114 for a good overview
- **FCCanonicalizeDummyIndices** uses an extremely naive approach that works only in very simple cases.
  It is nowhere compatible to what is done in real tensor algebra tools (e.g. xPerm, Canon, Cadabra, Redberry)
- Still, for the types of expressions that one usually encounters in FeynCalc, **FCCanonicalizeDummyIndices** turns out to work satisfactory well

Finally, often we also need to uncontract already contracted indices. This is done by **Uncontract**. By default, it handles only
contractions with Dirac matrices and Levi-Civita tensors

```
LC[][p, q, r, s]
Uncontract[%, p]
Uncontract[%%, p, q]
```

$\epsilon^{\overline{p}\,\overline{q}\,\overline{r}\,\overline{s}}$

$\overline{p}^{\$AL\$2925(1)}\,\epsilon^{\$AL\$2925(1)\,\overline{q}\,\overline{r}\,\overline{s}}$

$\overline{p}^{\$AL\$2930(1)}\,\overline{q}^{\$AL\$2929(1)}\,\epsilon^{\$AL\$2930(1)\,\$AL\$2929(1)\,\overline{r}\,\overline{s}}$

```
GS[p + m1].GA[μ].GS[q + m2]
Uncontract[%, p]
Uncontract[%%, p, q]
```

$\left(\overline{\gamma}\cdot\left(\overline{\mathrm{m1}}+\overline{p}\right)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\left(\overline{\mathrm{m2}}+\overline{q}\right)\right)$

$\left(\overline{\gamma}^{\$AL\$2939(1)}\,\overline{p}^{\$AL\$2939(1)}+\overline{\gamma}\cdot\overline{\mathrm{m1}}\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\left(\overline{\mathrm{m2}}+\overline{q}\right)\right)$

$\left(\overline{\gamma}^{\$AL\$2948(1)}\,\overline{p}^{\$AL\$2948(1)}+\overline{\gamma}\cdot\overline{\mathrm{m1}}\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}^{\$AL\$2947(1)}\,\overline{q}^{\$AL\$2947(1)}+\overline{\gamma}\cdot\overline{\mathrm{m2}}\right)$

```
SP[p, q] + SP[p, r]
Uncontract[%, p]
```

$\overline{p}\cdot\overline{q}+\overline{p}\cdot\overline{r}$

$\overline{p}\cdot\overline{q}+\overline{p}\cdot\overline{r}$

To make **Uncontract** work with scalar products, we need to specify a list of vectors, that are contracted with the vector that we wish to
uncontract

```
SP[p, q] + SP[p, r]
Uncontract[%, p, Pair → {q}]
Uncontract[%, p, Pair → {r}]
```

$\overline{p}\cdot\overline{q}+\overline{p}\cdot\overline{r}$

$\overline{p}^{\$AL\$2969(1)}\,\overline{q}^{\$AL\$2969(1)}+\overline{p}\cdot\overline{r}$

$\overline{p}^{\$AL\$2969(1)}\,\overline{q}^{\$AL\$2969(1)}+\overline{p}^{\$AL\$2977(1)}\,\overline{r}^{\$AL\$2977(1)}$

or we can uncontract all the suitable scalar products via

```
SP[p, q] + SP[p, r]
Uncontract[%, p, Pair → All]
```

$\overline{p}\cdot\overline{q}+\overline{p}\cdot\overline{r}$

$\overline{p}^{\$AL\$2988(1)}\,\overline{q}^{\$AL\$2988(1)}+\overline{p}^{\$AL\$2989(1)}\,\overline{r}^{\$AL\$2989(1)}$

Dirac algebra

The two most relevant functions for the manipulations of Dirac matrices are **DiracSimplify** and **DiracTrace**

The goal of **DiracSimplify** is to eliminate all pairs of Dirac matrices with the equal indices or contracted with the same
4-vectors

```
GA[μ].GS[p + m].GA[μ]
DiracSimplify[%]
```

$\bar{\gamma}^{\mu}.(\bar{\gamma}\cdot(\overline{m}+\overline{p})).\bar{\gamma}^{\mu}$

$-2\,\bar{\gamma}\cdot\overline{m}-2\,\bar{\gamma}\cdot\overline{p}$

```
GA[μ].GS[p + m1].GA[ν].GS[p + m2]
DiracSimplify[%]
```

$\bar{\gamma}^{\mu}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{m1}}+\overline{p}\right)\right).\bar{\gamma}^{\nu}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{m2}}+\overline{p}\right)\right)$

$\bar{\gamma}^{\mu}.\left(\bar{\gamma}\cdot\overline{\mathrm{m1}}\right).\bar{\gamma}^{\nu}.\left(\bar{\gamma}\cdot\overline{\mathrm{m2}}\right)+\bar{\gamma}^{\mu}.\left(\bar{\gamma}\cdot\overline{\mathrm{m1}}\right).\bar{\gamma}^{\nu}.(\bar{\gamma}\cdot\overline{p})+\bar{\gamma}^{\mu}.(\bar{\gamma}\cdot\overline{p}).\bar{\gamma}^{\nu}.\left(\bar{\gamma}\cdot\overline{\mathrm{m2}}\right)-\overline{p}^{2}\,\bar{\gamma}^{\mu}.\bar{\gamma}^{\nu}+2\,\overline{p}^{\nu}\,\bar{\gamma}^{\mu}.(\bar{\gamma}\cdot\overline{p})$

Of course, **DiracSimplify** also works with D-dimensional objects

```
GAD[μ].GSD[p + m].GAD[μ]
DiracSimplify[%]
```

$\gamma^{\mu}.(\gamma\cdot(m+p)).\gamma^{\mu}$

$-D\,\gamma\cdot m-D\,\gamma\cdot p+2\,\gamma\cdot m+2\,\gamma\cdot p$

Good to know:

- **DiracSimplify** is sometimes too slow on medium-sized expressions
- This is known and will be addressed in of the next versions of FeynCalc
- As of the upcoming FeynCalc 9.3, all the “old” Dirac-related functions except for **DiracSimplify** have been refactored and optimized
- Changing the inner working of **DiracSimplify** will give a measurable performance boost (as it has already happened with **DiracTrace**),
  but also requires very thorough testing.
- Stay tuned!

DiracTrace is used for the evaluation of Dirac traces. The trace is not evaluated by default

```
DiracTrace[GA[μ, ν]]
```

$\mathrm{tr}\left(\bar{\gamma}^{\mu}.\bar{\gamma}^{\nu}\right)$

To obtain the result we can either use the option **DiracTraceEvaluate**

```
DiracTrace[GA[μ, ν], DiracTraceEvaluate → True]
```

$4\,\bar{g}^{\mu\nu}$

or use **TR** instead.

```
DiracTrace[GA[μ, ν]] /. DiracTrace → TR
```

$4\,\bar{g}^{\mu\nu}$

The difference between **DiracTrace** and **TR** is that the latter also tries to do color traces (if there are color matrices in the expression)

```
SUNT[a, a] GA[μ, ν]
DiracTrace[%, DiracTraceEvaluate → True]
TR[%%]
```

$T^a.T^a \bar{\gamma}^\mu.\bar{\gamma}^\nu$

$4\, T^a.T^a \bar{g}^{\mu\nu}$

$$\frac{2(N-1)(N+1)\bar{g}^{\mu\nu}}{N}$$

We can also compute traces with γ_5

```
TR[GA[α, β, μ, ν, ρ, σ, 5]]
```

$-4\, i\left(\bar{g}^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} - \bar{g}^{\alpha\mu} \epsilon^{\beta\nu\rho\sigma} + \bar{g}^{\beta\mu} \epsilon^{\alpha\nu\rho\sigma} + \bar{g}^{\nu\rho} \epsilon^{\alpha\beta\mu\sigma} - \bar{g}^{\nu\sigma} \epsilon^{\alpha\beta\mu\rho} + \bar{g}^{\rho\sigma} \epsilon^{\alpha\beta\mu\nu}\right)$

However, by default FeynCalc refuses to compute a D-dimensional trace that contains γ_5

```
TR[GAD[α, β, μ, ν, ρ, σ].GA[5]]
```

DiracTrace: You are using naive dimensional regularization (NDR), such that in D dimensions gamma^5 anticommutes with all other Dirac matrices. In this scheme (without additional prescriptions) it is not possible to compute traces with an odd number of gamma^5 unambiguously. Evaluation aborted!

$Aborted

This is because by default FeynCalc is using anticommuting γ_5 in D-dimensions, a scheme known as Naive Dimensional Regularization (NDR)

```
DiracSimplify[GAD[μ].GA[5].GAD[ν]]
```

$-\gamma^\mu.\gamma^\nu.\bar{\gamma}^5$

In general, a chiral trace is a very ambiguous object in NDR. The results depends on the position of γ_5 inside the trace, so that we chose not to produce
results that might be potentially inconsistent.

However, FeynCalc can also be told to use the Breitenlohner-Maison-t'Hooft-Veltman scheme (BMHV), which is an algebraically consistent scheme (but has other issues,
e .g. it breaks axial Ward identities)

```
$BreitMaison = True;
```

Notice that now FeynCalc anticommutes γ_5 according to the BMHV algebra, which leads to the appearance of D-4 dimensional Dirac matrices

```
DiracSimplify[GAD[μ].GA[5].GAD[ν]]
```

$2\, \gamma^\mu.\hat{\gamma}^\nu.\bar{\gamma}^5 - \gamma^\mu.\gamma^\nu.\bar{\gamma}^5$

Also Dirac traces are not an issue now

```
TR[GAD[α, β, μ, ν, ρ, σ].GA[5]]
```

$$-4 i \left(g^{\alpha \beta } \epsilon^{\mu \nu \rho \sigma } - g^{\alpha \mu } \epsilon^{\beta \nu \rho \sigma } + g^{\alpha \nu } \epsilon^{\beta \mu \rho \sigma } - g^{\alpha \rho } \epsilon^{\beta \mu \nu \sigma } + g^{\alpha \sigma } \epsilon^{\beta \mu \nu \rho } + g^{\beta \mu } \epsilon^{\alpha \nu \rho \sigma } - g^{\beta \nu } \epsilon^{\alpha \mu \rho \sigma } + g^{\beta \rho } \epsilon^{\alpha \mu \nu \sigma } - g^{\beta \sigma } \epsilon^{\alpha \mu \nu \rho } + g^{\mu \nu } \epsilon^{\alpha \beta \rho \sigma } - g^{\mu \rho } \epsilon^{\alpha \beta \nu \sigma } + g^{\mu \sigma } \epsilon^{\alpha \beta \nu \rho } + g^{\nu \rho } \epsilon^{\alpha \beta \mu \sigma } - g^{\nu \sigma } \epsilon^{\alpha \beta \mu \rho } + g^{\rho \sigma } \epsilon^{\alpha \beta \mu \nu }\right)$$

To compute chiral traces in the BMHV scheme, FeynCalc uses West's formula (T. West, 1991, link).

Still, NDR is the default scheme in FeynCalc

```
$BreitMaison = False;
```

By the way, as compared to FeynCalc 9.0 and older, **DiracTrace** really got faster in the version 9.2. For example, consider

```
exp = (GS[k - p1 - p2] + m).GA[nu].(gL GA[6] + gR GA[7]).(GS[k - p2] + m).
   GA[rho].(gL GA[6] + gR GA[7]).(GS[k] + m).GA[mu].(gL GA[6] + gR GA[7])
```

$$\left(\overline{\gamma}\cdot\left(\overline{k}-\overline{\text{p1}}-\overline{\text{p2}}\right)+m\right).\overline{\gamma}^{\text{nu}}.\left(\text{gL}\,\overline{\gamma}^{6}+\text{gR}\,\overline{\gamma}^{7}\right).\left(\overline{\gamma}\cdot\left(\overline{k}-\overline{\text{p2}}\right)+m\right).\overline{\gamma}^{\text{rho}}.\left(\text{gL}\,\overline{\gamma}^{6}+\text{gR}\,\overline{\gamma}^{7}\right).\left(\overline{\gamma}\cdot\overline{k}+m\right).\overline{\gamma}^{\text{mu}}.\left(\text{gL}\,\overline{\gamma}^{6}+\text{gR}\,\overline{\gamma}^{7}\right)$$

```
AbsoluteTiming[res = DiracTrace[exp, DiracTraceEvaluate → True];] // First
```

0.150735

```
res
```

$$2 \, (i\, m^2\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, \text{gL}^3 - i\, m^2\, \epsilon^{\text{mu nu rho}\,\overline{\text{p1}}}\, \text{gL}^3 - i\, \epsilon^{\text{nu}\,\overline{k}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{g}^{\text{mu rho}}\, \text{gL}^3 + i\, \epsilon^{\text{nu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{mu}}\, \text{gL}^3 + i\, \epsilon^{\text{nu rho}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{k}^{\text{mu}}\, \text{gL}^3 +$$

$$m^2\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \text{gL}^3 - m^2\, \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \text{gL}^3 - i\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{nu}}\, \text{gL}^3 - 2\, i\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{k}^{\text{nu}}\, \text{gL}^3 +$$

$$m^2\, \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, \text{gL}^3 + i\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{\text{p1}}^{\text{nu}}\, \text{gL}^3 - m^2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}}\, \text{gL}^3 + i\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{\text{p2}}^{\text{nu}}\, \text{gL}^3 +$$

$$2\, i\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{\text{p2}}^{\text{nu}}\, \text{gL}^3 - 2\, m^2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, \text{gL}^3 - i\, \epsilon^{\text{mu nu}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 - i\, \epsilon^{\text{mu nu}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 +$$

$$m^2\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 + 4\, \overline{k}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 - 2\, \overline{\text{p1}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 - 2\, \overline{\text{p2}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 - 2\, \overline{k}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 +$$

$$\overline{\text{p2}}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 - 4\, \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 + \overline{\text{p1}}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 + 2\, \overline{\text{p2}}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}}\, \text{gL}^3 +$$

$$m^2\, \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}}\, \text{gL}^3 - \overline{\text{p2}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p1}}^{\text{rho}}\, \text{gL}^3 + \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{\text{p1}}^{\text{rho}}\, \text{gL}^3 - 2\, \overline{k}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}}\, \text{gL}^3 + \overline{\text{p1}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}}\, \text{gL}^3 +$$

$$\overline{k}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}}\, \text{gL}^3 + 2\, \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}}\, \text{gL}^3 - i\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, \overline{k}^2\, \text{gL}^3 - \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{k}^2\, \text{gL}^3 + \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{k}^2\, \text{gL}^3 -$$

$$\overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, \overline{k}^2\, \text{gL}^3 + \overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^2\, \text{gL}^3 + 2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^2\, \text{gL}^3 - \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, \overline{k}^2\, \text{gL}^3 - \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}}\, \overline{k}^2\, \text{gL}^3 +$$

$$i\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{k} \cdot \overline{\text{p1}})\, \text{gL}^3 + \overline{\text{p2}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p1}})\, \text{gL}^3 - \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p1}})\, \text{gL}^3 + 2\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p1}})\, \text{gL}^3 -$$

$$\overline{g}^{\text{mu nu}}\, \overline{\text{p2}}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p1}})\, \text{gL}^3 + 2\, i\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 + 2\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 - \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 -$$

$$\overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 - 2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 + 2\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 + \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p2}})\, \text{gL}^3 -$$

$$i\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}})\, \text{gL}^3 - \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}})\, \text{gL}^3 + \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}})\, \text{gL}^3 - \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}})\, \text{gL}^3 -$$

$$i\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, \overline{\text{p2}}^2\, \text{gL}^3 - \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{\text{p2}}^2\, \text{gL}^3 + \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^2\, \text{gL}^3 - \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, \overline{\text{p2}}^2\, \text{gL}^3 - i\, \text{gR}^3\, m^2\, \epsilon^{\text{mu nu rho}\,\overline{k}} +$$

$$i\, \text{gR}^3\, m^2\, \epsilon^{\text{mu nu rho}\,\overline{\text{p1}}} + i\, \text{gR}^3\, \epsilon^{\text{nu}\,\overline{k}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{g}^{\text{mu rho}} - i\, \text{gR}^3\, \epsilon^{\text{nu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{mu}} - i\, \text{gR}^3\, \epsilon^{\text{nu rho}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{k}^{\text{mu}} +$$

$$\text{gR}^3\, m^2\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}} - \text{gR}^3\, m^2\, \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}} + i\, \text{gR}^3\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{nu}} + 2\, i\, \text{gR}^3\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{k}^{\text{nu}} +$$

$$\text{gR}^3\, m^2\, \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}} - i\, \text{gR}^3\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{\text{p1}}^{\text{nu}} - \text{gR}^3\, m^2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}} - i\, \text{gR}^3\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p1}}}\, \overline{\text{p2}}^{\text{nu}} -$$

$$2\, i\, \text{gR}^3\, \epsilon^{\text{mu rho}\,\overline{k}\,\overline{\text{p2}}}\, \overline{\text{p2}}^{\text{nu}} - 2\, \text{gR}^3\, m^2\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}} + i\, \text{gR}^3\, \epsilon^{\text{mu nu}\,\overline{k}\,\overline{\text{p1}}}\, \overline{k}^{\text{rho}} + i\, \text{gR}^3\, \epsilon^{\text{mu nu}\,\overline{\text{p1}}\,\overline{\text{p2}}}\, \overline{k}^{\text{rho}} +$$

$$\text{gR}^3\, m^2\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}} + 4\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}} - 2\, \text{gR}^3\, \overline{\text{p1}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}} - 2\, \text{gR}^3\, \overline{\text{p2}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{k}^{\text{rho}} - 2\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^{\text{rho}} +$$

$$\text{gR}^3\, \overline{\text{p2}}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^{\text{rho}} - 4\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}} + \text{gR}^3\, \overline{\text{p1}}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}} + 2\, \text{gR}^3\, \overline{\text{p2}}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^{\text{rho}} +$$

$$\text{gR}^3\, m^2\, \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}} - \text{gR}^3\, \overline{\text{p2}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p1}}^{\text{rho}} + \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{\text{p1}}^{\text{rho}} - 2\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}} + \text{gR}^3\, \overline{\text{p1}}^{\text{mu}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}} +$$

$$\text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}} + 2\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{\text{p2}}^{\text{rho}} + i\, \text{gR}^3\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, \overline{k}^2 - \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{k}^2 + \text{gR}^3\, \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{k}^2 -$$

$$\text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, \overline{k}^2 + \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}}\, \overline{k}^2 + 2\, \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, \overline{k}^2 - \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, \overline{k}^2 - \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}}\, \overline{k}^2 -$$

$$i\, \text{gR}^3\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{k} \cdot \overline{\text{p1}}) + \text{gR}^3\, \overline{\text{p2}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p1}}) - \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p1}}) + 2\, \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p1}}) -$$

$$\text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{\text{p2}}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p1}}) - 2\, i\, \text{gR}^3\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{k} \cdot \overline{\text{p2}}) + 2\, \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p2}}) - \text{gR}^3\, \overline{\text{p1}}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{k} \cdot \overline{\text{p2}}) -$$

$$\text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{\text{p1}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p2}}) - 2\, \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{\text{p2}}^{\text{nu}}\, (\overline{k} \cdot \overline{\text{p2}}) + 2\, \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p2}}) + \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{\text{p1}}^{\text{rho}}\, (\overline{k} \cdot \overline{\text{p2}}) +$$

$$i\, \text{gR}^3\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}}) - \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}}) + \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}}) - \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, (\overline{\text{p1}} \cdot \overline{\text{p2}}) +$$

$$i\, \text{gR}^3\, \epsilon^{\text{mu nu rho}\,\overline{k}}\, \overline{\text{p2}}^2 - \text{gR}^3\, \overline{k}^{\text{mu}}\, \overline{g}^{\text{nu rho}}\, \overline{\text{p2}}^2 + \text{gR}^3\, \overline{g}^{\text{mu rho}}\, \overline{k}^{\text{nu}}\, \overline{\text{p2}}^2 - \text{gR}^3\, \overline{g}^{\text{mu nu}}\, \overline{k}^{\text{rho}}\, \overline{\text{p2}}^2)$$

FeynCalc 9.0 requires roughly 6 times more time on the same machine.

Even though FeynCalc can never be as fast as FORM, there is still a lot of room for the improvement of the internal algorithms.
With some profiling one can often reach pretty good speed ups. The only problem is to find time for that...

Sometimes you may want to isolate the Dirac structure from the rest of the expression, so that you can better understand what is going on.
Consider for example the tree-level for amplitude for the q g -> q g scattering.

```
amp =
 Spinor[Momentum[p2], SMP["m_u"], 1].(-i GA[μ] SMP["g_s"] SUNTF[{Glu2}, Col3, Col5]).
    (GS[-k1 + p2] + SMP["m_u"]).(-i GA[ν] SMP["g_s"] SUNTF[{Glu4}, Col5, Col1]).
     Spinor[Momentum[p1], SMP["m_u"], 1] FAD[{k1 - p2, SMP["m_u"]}, Dimension → 4]
   FV[Polarization[k1, i, Transversality → True], μ]
   FV[Polarization[k2, -i, Transversality → True], ν] +
  Spinor[Momentum[p2], SMP["m_u"], 1].(-i GA[ν] SMP["g_s"] SUNTF[{Glu4}, Col3, Col5]).
    (GS[k2 + p2] + SMP["m_u"]).(-i GA[μ] SMP["g_s"] SUNTF[{Glu2}, Col5, Col1]).
     Spinor[Momentum[p1], SMP["m_u"], 1] FAD[{-k2 - p2, SMP["m_u"]}, Dimension → 4]
   FV[Polarization[k1, i, Transversality → True], μ]
   FV[Polarization[k2, -i, Transversality → True], ν] -
  Spinor[Momentum[p2], SMP["m_u"], 1].(-i GA[Lor3] SMP["g_s"]
      SUNTF[{Glu5}, Col3, Col1]).Spinor[Momentum[p1], SMP["m_u"], 1]
   FAD[-k1 + k2, Dimension → 4] FV[Polarization[k1, i, Transversality → True], μ]
   FV[Polarization[k2, -i, Transversality → True], ν] MT[Lor3, Lor4]
   (FV[2 k1 - k2, ν] MT[Lor4, μ] + FV[-k1 + 2 k2, μ] MT[Lor4, ν] +
     FV[-k1 - k2, Lor4] MT[μ, ν]) SMP["g_s"] SUNF[Glu2, Glu4, Glu5]
```

$$\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,\frac{1}{[(\mathrm{k1}-\mathrm{p2})^2-m_u^2]}\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\mathrm{Glu2}}_{\mathrm{Col3\,Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p2}}-\overline{\mathrm{k1}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\mathrm{Glu4}}_{\mathrm{Col5\,Col1}}\right).\left(\varphi(\overline{\mathrm{p1}},m_u)\right)+\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,\frac{1}{[(-\mathrm{k2}-\mathrm{p2})^2-m_u^2]}\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\mathrm{Glu4}}_{\mathrm{Col3\,Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\mathrm{k2}}+\overline{\mathrm{p2}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\mathrm{Glu2}}_{\mathrm{Col5\,Col1}}\right).\left(\varphi(\overline{\mathrm{p1}},m_u)\right)-g_s\,\bar{g}^{\mathrm{Lor3\,Lor4}}\,\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,f^{\mathrm{Glu2\,Glu4\,Glu5}}\left(\bar{g}^{\mathrm{Lor4}\,\mu}\left(2\,\overline{\mathrm{k1}}-\overline{\mathrm{k2}}\right)^{\nu}+\bar{g}^{\mathrm{Lor4}\,\nu}\left(2\,\overline{\mathrm{k2}}-\overline{\mathrm{k1}}\right)^{\mu}+\bar{g}^{\mu\nu}\left(-\overline{\mathrm{k1}}-\overline{\mathrm{k2}}\right)^{\mathrm{Lor4}}\right)\frac{1}{[(\mathrm{k2}-\mathrm{k1})^2]}\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\mathrm{Lor3}}\,T^{\mathrm{Glu5}}_{\mathrm{Col3\,Col1}}\right).\left(\varphi(\overline{\mathrm{p1}},m_u)\right)$$

One of the new functions in FeynCalc 9.1 and later was developed specifically for the purpose of disentangling such mess and extracting
the Dirac structures

```
ampIso = FCDiracIsolate[amp, Head → diracS]
```

$$-\left(\left(g_s^2\,\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,T^{\mathrm{Glu2}}_{\mathrm{Col5\,Col1}}\,T^{\mathrm{Glu4}}_{\mathrm{Col3\,Col5}}\,\mathrm{diracS}\left(\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\bar{\gamma}^{\nu}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{k2}}+\overline{\mathrm{p2}}\right)+m_u\right).\bar{\gamma}^{\mu}.\left(\varphi(\overline{\mathrm{p1}},m_u)\right)\right)\right)\Big/\left(\left(-\overline{\mathrm{k2}}-\overline{\mathrm{p2}}\right)^2-m_u^2\right)\right)-\left(g_s^2\,\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,T^{\mathrm{Glu4}}_{\mathrm{Col5\,Col1}}\,T^{\mathrm{Glu2}}_{\mathrm{Col3\,Col5}}\,\mathrm{diracS}\left(\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\bar{\gamma}^{\mu}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p2}}-\overline{\mathrm{k1}}\right)+m_u\right).\bar{\gamma}^{\nu}.\left(\varphi(\overline{\mathrm{p1}},m_u)\right)\right)\right)\Big/\left(\left(\overline{\mathrm{k1}}-\overline{\mathrm{p2}}\right)^2-m_u^2\right)+\frac{1}{\left(\overline{\mathrm{k2}}-\overline{\mathrm{k1}}\right)^2}i\,g_s^2\,\bar{g}^{\mathrm{Lor3\,Lor4}}\,\bar{\varepsilon}^{\mu}(\mathrm{k1})\,\bar{\varepsilon}^{*\nu}(\mathrm{k2})\,T^{\mathrm{Glu5}}_{\mathrm{Col3\,Col1}}\,f^{\mathrm{Glu2\,Glu4\,Glu5}}\left(\bar{g}^{\mu\nu}\left(-\overline{\mathrm{k1}}-\overline{\mathrm{k2}}\right)^{\mathrm{Lor4}}+\bar{g}^{\mathrm{Lor4}\,\nu}\left(2\,\overline{\mathrm{k2}}-\overline{\mathrm{k1}}\right)^{\mu}+\bar{g}^{\mathrm{Lor4}\,\mu}\left(2\,\overline{\mathrm{k1}}-\overline{\mathrm{k2}}\right)^{\nu}\right)\mathrm{diracS}\left(\left(\varphi(\overline{\mathrm{p2}},m_u)\right).\bar{\gamma}^{\mathrm{Lor3}}.\left(\varphi(\overline{\mathrm{p1}},m_u)\right)\right)$$

Now all the Dirac structures are wrapped into the Head **diracS**. So we can do something like

```
li = Cases[ampIso, _diracS, Infinity] // DeleteDuplicates // Sort
```

$$\left\{\text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{Lor3}.\left(\varphi(\overline{p1}, m_u)\right)\right), \text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\left(\overline{p2}-\overline{k1}\right)+m_u\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)\right), \text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\left(\overline{k2}+\overline{p2}\right)+m_u\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)\right)\right\}$$

which gives us a sorted list of all the unique Dirac structures in amp. This is useful for many purposes. For example, we could assign to these structures some values and then substitute them back into the expression. For simplicity, here we can use **DiracSimplify**

```
evalLi = DiracSimplify /@ (li /. diracS → Identity)
```

$$\left\{\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{Lor3}.\left(\varphi(\overline{p1}, m_u)\right), 2\,\overline{p2}^{\mu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)-\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\overline{k1}\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right), \left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\overline{k2}\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)+2\,\overline{p2}^{\nu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)\right\}$$

Now we create a substitution rule

```
repRule = Thread[Rule[li, evalLi]]
```

$$\left\{\text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{Lor3}.\left(\varphi(\overline{p1}, m_u)\right)\right)\to\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{Lor3}.\left(\varphi(\overline{p1}, m_u)\right), \text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\left(\overline{p2}-\overline{k1}\right)+m_u\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)\right)\to 2\,\overline{p2}^{\mu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)-\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\overline{k1}\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right), \text{diracS}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\left(\overline{k2}+\overline{p2}\right)+m_u\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)\right)\to\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\overline{k2}\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)+2\,\overline{p2}^{\nu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)\right\}$$

and apply it ampIso

```
ampIso /. repRule
```

$$-\left(\left(g_s^2\,\overline{\varepsilon}^{\mu}(k1)\,\overline{\varepsilon}^{*\nu}(k2)\,T^{Glu2}_{Col5\,Col1}\,T^{Glu4}_{Col3\,Col5}\left(\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\overline{k2}\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)+2\,\overline{p2}^{\nu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\varphi(\overline{p1}, m_u)\right)\right)\right)\Big/\left(\left(-\overline{k2}-\overline{p2}\right)^2-m_u^2\right)\right)-\left(g_s^2\,\overline{\varepsilon}^{\mu}(k1)\,\overline{\varepsilon}^{*\nu}(k2)\,T^{Glu4}_{Col5\,Col1}\,T^{Glu2}_{Col3\,Col5}\left(2\,\overline{p2}^{\mu}\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)-\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\overline{k1}\right).\overline{\gamma}^{\nu}.\left(\varphi(\overline{p1}, m_u)\right)\right)\right)\Big/\left(\left(\overline{k1}-\overline{p2}\right)^2-m_u^2\right)+\frac{1}{\left(\overline{k2}-\overline{k1}\right)^2}i\,g_s^2\,\overline{g}^{Lor3\,Lor4}\,\overline{\varepsilon}^{\mu}(k1)\,\overline{\varepsilon}^{*\nu}(k2)\,T^{Glu5}_{Col3\,Col1}\,f^{Glu2\,Glu4\,Glu5}\left(\overline{g}^{\mu\nu}\left(-\overline{k1}-\overline{k2}\right)^{Lor4}+\overline{g}^{Lor4\,\nu}\left(2\,\overline{k2}-\overline{k1}\right)^{\mu}+\overline{g}^{Lor4\,\mu}\left(2\,\overline{k1}-\overline{k2}\right)^{\nu}\right)\left(\varphi(\overline{p2}, m_u)\right).\overline{\gamma}^{Lor3}.\left(\varphi(\overline{p1}, m_u)\right)$$

FCDiracIsolate also has a lot of options for more fine-grained extraction

```
Options[FCDiracIsolate]
```

{ClearHeads → {FCGV(DiracChain)}, Collecting → True, DotSimplify → True, DiracGammaCombine → True, DiracSigmaExplicit → False, ExceptHeads → {}, Expanding → True, FeynCalcInternal → False, Factoring → Factor, LorentzIndex → False, Head → FCGV(DiracChain), Split → True, Isolate → False, IsolateNames → KK, IsolateFast → False, DiracGamma → True, Spinor → True, DiracTrace → True}

The introduction of this function was crucial for the recent speed ups in DiracTrace and similar routines.

Once DiracSimplify is
refactored to use FCDiracIsolate internally, it will become much faster :)

Color algebra

FeynCalc objects relevant for the color algebra are

```
SUNT[a]
```

T^{a}

```
SUNF[a, b, c]
```

$f^{a b c}$

```
SUND[a, b, c]
```

$d^{a b c}$

```
SUNDelta[a, b]
```

$\delta^{a b}$

```
SUNN
```

N

```
CA
```

C_A

```
CF
```

C_F

Currently there are two main functions to deal with colored objects: **SUNSimplify** and **SUNTrace**

```
SUNT[a, a]
SUNSimplify[%]
```

$T^{a}.T^{a}$

C_F

```
SUNT[a, b, a, b]
SUNSimplify[%]
```

$T^{a}.T^{b}.T^{a}.T^{b}$

$-\frac{1}{2} C_F (C_A - 2 C_F)$

```
SUNTrace[SUNT[a, b]]
```

$\frac{\delta^{a b}}{2}$

```
SUNTrace[SUNT[a, b, b, a]]
```

$$\frac{N^3}{4} - \frac{N}{2} + \frac{1}{4N}$$

There is also a function analogous to **FCDiracIsolate**, called **FCColorIsolate**

```
amp
```

$$\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\frac{1}{[(\text{k1}-\text{p2})^2-m_u^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\text{Glu2}}_{\text{Col3}\,\text{Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\text{p2}}-\overline{\text{k1}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\text{Glu4}}_{\text{Col5}\,\text{Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)+\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\frac{1}{[(-\text{k2}-\text{p2})^2-m_u^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\text{Glu4}}_{\text{Col3}\,\text{Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\text{k2}}+\overline{\text{p2}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\text{Glu2}}_{\text{Col5}\,\text{Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)-g_s\,\bar{g}^{\text{Lor3}\,\text{Lor4}}\,\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,f^{\text{Glu2}\,\text{Glu4}\,\text{Glu5}}\left(\bar{g}^{\text{Lor4}\,\mu}\left(2\,\overline{\text{k1}}-\overline{\text{k2}}\right)^{\nu}+\bar{g}^{\text{Lor4}\,\nu}\left(2\,\overline{\text{k2}}-\overline{\text{k1}}\right)^{\mu}+\bar{g}^{\mu\nu}\left(-\overline{\text{k1}}-\overline{\text{k2}}\right)^{\text{Lor4}}\right)\frac{1}{[(\text{k2}-\text{k1})^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\text{Lor3}}\,T^{\text{Glu5}}_{\text{Col3}\,\text{Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)$$

```
ampIso = FCColorIsolate[amp, Head → colorS]
```

$$-\left(\left(g_s^2\,\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\text{colorS}\left(T^{\text{Glu4}}_{\text{Col5}\,\text{Col1}}\,T^{\text{Glu2}}_{\text{Col3}\,\text{Col5}}\right)\left(\varphi(\overline{\text{p2}},m_u)\right).\bar{\gamma}^{\mu}.\left(\bar{\gamma}\cdot\left(\overline{\text{p2}}-\overline{\text{k1}}\right)+m_u\right).\bar{\gamma}^{\nu}.\left(\varphi(\overline{\text{p1}},m_u)\right)\right)\Big/\left(\left(\overline{\text{k1}}-\overline{\text{p2}}\right)^2-m_u^2\right)\right)-\left(g_s^2\,\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\text{colorS}\left(T^{\text{Glu2}}_{\text{Col5}\,\text{Col1}}\,T^{\text{Glu4}}_{\text{Col3}\,\text{Col5}}\right)\left(\varphi(\overline{\text{p2}},m_u)\right).\bar{\gamma}^{\nu}.\left(\bar{\gamma}\cdot\left(\overline{\text{k2}}+\overline{\text{p2}}\right)+m_u\right).\bar{\gamma}^{\mu}.\left(\varphi(\overline{\text{p1}},m_u)\right)\right)\Big/\left(\left(-\overline{\text{k2}}-\overline{\text{p2}}\right)^2-m_u^2\right)+\frac{1}{\left(\overline{\text{k2}}-\overline{\text{k1}}\right)^2}i\,g_s^2\,\bar{g}^{\text{Lor3}\,\text{Lor4}}\,\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\left(\bar{g}^{\mu\nu}\left(-\overline{\text{k1}}-\overline{\text{k2}}\right)^{\text{Lor4}}+\bar{g}^{\text{Lor4}\,\nu}\left(2\,\overline{\text{k2}}-\overline{\text{k1}}\right)^{\mu}+\bar{g}^{\text{Lor4}\,\mu}\left(2\,\overline{\text{k1}}-\overline{\text{k2}}\right)^{\nu}\right)\text{colorS}\left(T^{\text{Glu5}}_{\text{Col3}\,\text{Col1}}\,f^{\text{Glu2}\,\text{Glu4}\,\text{Glu5}}\right)\left(\varphi(\overline{\text{p2}},m_u)\right).\bar{\gamma}^{\text{Lor3}}.\left(\varphi(\overline{\text{p1}},m_u)\right)$$

```
li = Cases[ampIso, _colorS, Infinity] // DeleteDuplicates // Sort
```

$$\left\{\text{colorS}\left(T^{\text{Glu2}}_{\text{Col5}\,\text{Col1}}\,T^{\text{Glu4}}_{\text{Col3}\,\text{Col5}}\right),\ \text{colorS}\left(T^{\text{Glu4}}_{\text{Col5}\,\text{Col1}}\,T^{\text{Glu2}}_{\text{Col3}\,\text{Col5}}\right),\ \text{colorS}\left(T^{\text{Glu5}}_{\text{Col3}\,\text{Col1}}\,f^{\text{Glu2}\,\text{Glu4}\,\text{Glu5}}\right)\right\}$$

Notice that here we have color matrices in the fundamental representation with explicit fundamental indices.
Sadly, this was not available in FeynCalc until the version 9.

Such matrices are entered via **SUNTF**

```
SUNTF[a, b, c]
```

$$T^{a}_{b\,c}$$

SUNSimplify can automatically contract them

```
SUNTF[a1, b, c] SUNTF[a2, c, d]
SUNSimplify[%]
```

$$T^{\text{a1}}_{b\,c}\,T^{\text{a2}}_{c\,d}$$

$$\left(T^{\text{a1}}\,T^{\text{a2}}\right)_{b\,d}$$

```
SUNTF[a1, b, c] SUNTF[a2, c, b]
SUNSimplify[%]
```

$T^{a1}_{b c} \, T^{a2}_{c b}$

$\frac{\delta^{a1\,a2}}{2}$

- In general, the color algebra subsystem of FeynCalc is missing a lot of useful functionality and the performance is not satisfactory
- This is being worked on. We can do better and we will do better!

Loops

Propagators (and products thereof) are represented via **FeynAmpDenominator**

```
FeynAmpDenominator[PropagatorDenominator[Momentum[q, D], m]]
```

$\frac{1}{q^2 - m^2}$

Again, for the external input we always use a shortcut

```
FAD[{q, m}]
```

$\frac{1}{[q^2-m^2]}$

We can also have more propagators

```
FAD[{q, m0}, {q + p1, m1}, {q + p2, m1}]
```

$\frac{1}{([q^2-\mathrm{m0}^2]).([(\mathrm{p1}+q)^2-\mathrm{m1}^2]).([(\mathrm{p2}+q)^2-\mathrm{m1}^2])}$

The presence of **FeynAmpDenominator** in an expression does not automatically mean that it is a loop amplitude. **FeynAmpDenominator** can equally appear in tree level amplitudes, where it stands for the usual 4-dimensional propagator, like in amp

```
amp
Cases2[amp, FAD]
```

$$\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\frac{1}{[(\text{k1}-\text{p2})^2-m_u^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\text{Glu2}}_{\text{Col3 Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\text{p2}}-\overline{\text{k1}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\text{Glu4}}_{\text{Col5 Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)+\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,\frac{1}{[(-\text{k2}-\text{p2})^2-m_u^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\nu}\,T^{\text{Glu4}}_{\text{Col3 Col5}}\right).\left(\bar{\gamma}\cdot\left(\overline{\text{k2}}+\overline{\text{p2}}\right)+m_u\right).\left(-i\,g_s\,\bar{\gamma}^{\mu}\,T^{\text{Glu2}}_{\text{Col5 Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)-g_s\,\bar{g}^{\text{Lor3 Lor4}}\,\bar{\varepsilon}^{\mu}(\text{k1})\,\bar{\varepsilon}^{*\nu}(\text{k2})\,f^{\text{Glu2 Glu4 Glu5}}\left(\bar{g}^{\text{Lor4}\,\mu}\left(2\,\overline{\text{k1}}-\overline{\text{k2}}\right)^{\nu}+\bar{g}^{\text{Lor4}\,\nu}\left(2\,\overline{\text{k2}}-\overline{\text{k1}}\right)^{\mu}+\bar{g}^{\mu\nu}\left(-\overline{\text{k1}}-\overline{\text{k2}}\right)^{\text{Lor4}}\right)\frac{1}{[(\text{k2}-\text{k1})^2]}\left(\varphi(\overline{\text{p2}},m_u)\right).\left(-i\,g_s\,\bar{\gamma}^{\text{Lor3}}\,T^{\text{Glu5}}_{\text{Col3 Col1}}\right).\left(\varphi(\overline{\text{p1}},m_u)\right)$$

$$\left\{\frac{1}{[(\text{k2}-\text{k1})^2]},\ \frac{1}{[(\text{k1}-\text{p2})^2-m_u^2]},\ \frac{1}{[(-\text{k2}-\text{p2})^2-m_u^2]}\right\}$$

In FeynCalc there is no explicit way to distinguish between loop amplitudes and tree-level amplitudes. When you
use functions that manipulate loop integrals, you need to tell them explicitly what is your loop momentum.

Manipulations of FeynAmpDenominators

There are several functions, that are useful both for tree- and loop-level amplitudes, depending on what we want to do

For example, one can split one FeynAmpDenominator into many

```
FAD[{k1 - k2}, {k1 - p2, m}, {k2 + p2, m}]
FeynAmpDenominatorSplit[%]
% // FCE // StandardForm
```

$$\frac{1}{([(\text{k1}-\text{k2})^2]).([(\text{k1}-\text{p2})^2-m^2]).([(\text{k2}+\text{p2})^2-m^2])}$$

$$\frac{1}{(\text{k1}-\text{k2})^2\left((\text{k1}-\text{p2})^2-m^2\right)\left((\text{k2}+\text{p2})^2-m^2\right)}$$

```
FAD[k1 - k2] FAD[{k1 - p2, m}] FAD[{k2 + p2, m}]
```

or combine several into one

```
FeynAmpDenominatorCombine[FAD[k1 - k2] FAD[{k1 - p2, m}] FAD[{k2 + p2, m}]]
% // FCE // StandardForm
```

$$\frac{1}{\left((\text{k2}+\text{p2})^2-m^2\right).(\text{k1}-\text{k2})^2.\left((\text{k1}-\text{p2})^2-m^2\right)}$$

```
FAD[{k2 + p2, m}, k1 - k2, {k1 - p2, m}]
```

At the tree-level we often do not need the FeynAmpDenominators but rather want to express everything in terms
of explicit scalar products, in order to exploit kinematic simplifications. This is handled by **PropagatorDenominatorExplicit**

```
PropagatorDenominatorExplicit[FAD[{k2 + p2, m}, k1 - k2, {k1 - p2, m}]]
```

$1/((-2\,(\mathrm{k1}\cdot\mathrm{k2})+\mathrm{k1}^2+\mathrm{k2}^2)\,(-2\,(\mathrm{k1}\cdot\mathrm{p2})+\mathrm{k1}^2-m^2+\mathrm{p2}^2)\,(2\,(\mathrm{k2}\cdot\mathrm{p2})+\mathrm{k2}^2-m^2+\mathrm{p2}^2))$

ApartFF

For example, to cancel scalar products in the numerator we use **ApartFF**, a very powerful partial fractioning routine that
uses the algorithm of F. Feng (2012, arXiv:1204.2314)

```
SPD[p, q] FAD[{q, m}, p + q]
ApartFF[%, {q}]
```

$$(p\cdot q)\,\frac{1}{([q^2-m^2]).([(p+q)^2])}$$

$$\frac{1}{2\,(q^2-m^2)}-\frac{m^2+p^2}{2\,q^2.((q-p)^2-m^2)}$$

Of course, **ApartFF** can also handle much more interesting cases

```
FCClearScalarProducts[];
SPD[p, p] = 4 m^2;
int = FAD[{k1 - p / 2 + q, m}, {k1 - p / 2 + q, m}, {p / 2 + q, m}, q, {- (p / 2) + q, m}]
```

$$1\Big/\left(\left[\left(\mathrm{k1}-\frac{p}{2}+q\right)^2-m^2\right]\right)^2.\left(\left[\left(\frac{p}{2}+q\right)^2-m^2\right]\right).([q^2]).\left(\left[\left(q-\frac{p}{2}\right)^2-m^2\right]\right)$$

```
ApartFF[int, {q}]
```

$$2/(q^2-m^2).((q-\mathrm{k1})^2-m^2).((q-\mathrm{k1})^2-m^2).((q-p)^2-m^2).((q-p)^2-m^2)-$$
$$1\Big/q^2.\left(\left(q-\frac{p}{2}\right)^2-m^2\right).\left(\left(q-\frac{p}{2}\right)^2-m^2\right).\left(\left(-\mathrm{k1}+\frac{p}{2}+q\right)^2-m^2\right).\left(\left(-\mathrm{k1}+\frac{p}{2}+q\right)^2-m^2\right)$$

```
FCClearScalarProducts[];
SPD[k, k] = 0;
SPD[p, p] = m^2;
int = FAD[{q1, m}, {q1 - p}, {q1 - 2 p, m}, {q1 - k - 2 p, m}]
```

$$1/([\mathrm{q1}^2-m^2]).([(\mathrm{q1}-p)^2]).([(\mathrm{q1}-2\,p)^2-m^2]).([(-k-2\,p+\mathrm{q1})^2-m^2])$$

```
ApartFF[FAD[{q1, m}, {q1 - p}, {q1 - 2 p, m}, {q1 - k - 2 p, m}], {q1}]
```

$$\frac{2}{(\mathrm{q1}^2-m^2).((k+\mathrm{q1})^2-m^2).((\mathrm{q1}-2\,p)^2-m^2).((\mathrm{q1}-2\,p)^2-m^2)}-$$
$$\frac{1}{(\mathrm{q1}^2-m^2).(\mathrm{q1}^2-m^2).(\mathrm{q1}-p)^2.((-k-2\,p+\mathrm{q1})^2-m^2)}$$

ApartFF does not really care, if it is a 1-loop or a multi-loop integral

2-loops:

```
FCClearScalarProducts[]
int = SPD[q2, p] SPD[q1, p] FAD[{q1, m}, {q2, m}, q1 - p, q2 - p, q2 - q1]
```

$(p\cdot q1)\,(p\cdot q2)\; 1/([q1^2{-}m^2]).([q2^2{-}m^2]).([(q1-p)^2]).([(q2-p)^2]).([(q2-q1)^2])$

```
ApartFF[int, {q1, q2}]
```

$$\frac{(m^2+p^2)^2}{4\,(q1^2-m^2).(q2^2-m^2).(q2-p)^2.(q1-q2)^2.(q1-p)^2}-\frac{m^2+p^2}{2\,(q1^2-m^2).(q2^2-m^2).(q1-q2)^2.(q1-p)^2}+\frac{m^2+p^2}{2\,q2^2.q1^2.((q1-p)^2-m^2).(q1-q2)^2}-\frac{1}{2\,(q1^2-m^2).(q2-p)^2.(q1-q2)^2}+\frac{1}{4\,(q1^2-m^2).(q2^2-m^2).(q2-q1)^2}$$

3-loops?

```
int = SPD[q2, p2] SPD[q3, p1] SPD[q1, p3]
  FAD[{q1, m}, {q2, m}, {q3}, q1 - p3, q2 - p2, q2 - q1, q2 + 2 q3]
```

$(p1\cdot q3)\,(p2\cdot q2)\,(p3\cdot q1)$
$1/([q1^2{-}m^2]).([q2^2{-}m^2]).([q3^2]).([(q1-p3)^2]).([(q2-p2)^2]).([(q2-q1)^2]).([(q2+2\,q3)^2])$

```
ApartFF[int, {q1, q2, q3}]
```

$$\frac{(m^2+p2^2)\,(p1\cdot q3)}{4\,q3^2.(q1^2-m^2).(q1+2\,q3)^2.(q2-q1)^2.(q2-p3)^2.(q1-p2)^2}+\frac{(m^2+p3^2)\,(p1\cdot q3)}{4\,q3^2.(q1^2-m^2).(q2+2\,q3)^2.(q2-q1)^2.(q2-p2)^2.(q1-p3)^2}+$$
$$((m^2+p2^2)\,(m^2+p3^2)\,(p1\cdot q3))/(4\,q3^2.(q1^2-m^2).(q2^2-m^2).(q1+2\,q3)^2.(q2-q1)^2.(q2-p3)^2.(q1-p2)^2)-$$
$$\frac{p1\cdot q3}{4\,q3^2.(q1^2-m^2).(q2+2\,q3)^2.(q2-q1)^2.(q2-p2)^2}-\frac{(m^2+p2^2)\,(p1\cdot q3)}{4\,q3^2.(q1^2-m^2).(q2^2-m^2).(q1+2\,q3)^2.(q2-q1)^2.(q1-p2)^2}-$$
$$\frac{p1\cdot q3}{4\,q3^2.(q1^2-m^2).(q1+2\,q3)^2.(q2-q1)^2.(q2-p3)^2}-\frac{(m^2+p3^2)\,(p1\cdot q3)}{4\,q3^2.(q1^2-m^2).(q2^2-m^2).(q2+2\,q3)^2.(q2-q1)^2.(q1-p3)^2}+$$
$$\frac{p1\cdot q3}{4\,q3^2.(q1^2-m^2).(q2^2-m^2).(q1+2\,q3)^2.(q1-q2)^2}+\frac{p1\cdot q3}{4\,q3^2.(q1+2\,q3)^2.(q2-q1)^2.(q2-p3)^2.(q1-p2)^2}$$

FeynAmpDenominatorSimplify

Another useful function for the manipulation of loop integrals (albeit by far not as powerful as **ApartFF**) is **FDS** (**FeynAmpDenominatorSimplify**)

- **FDS** tries to simplify loop integrals by shifting the loop momenta and putting scaleless integrals to zero.
- However, since **FDS** has no information about the topology of the input expression, what it does is basically heuristics

- This is why the output is not always satisfactory. An improvement of **FDS** using is one of the top priorities for the future
  FeynCalc versions

```
FAD[q, q]
FeynAmpDenominatorSimplify[%, q]
```

$$\frac{1}{([q^2])^2}$$

0

```
FAD[q - p1, q - p2]
FeynAmpDenominatorSimplify[%, q]
```

$$\frac{1}{([(q-\text{p1})^2]).([(q-\text{p2})^2])}$$

$$\frac{1}{q^2.(-\text{p1}+\text{p2}+q)^2}$$

```
FAD[{q - k - p, m}, q] - FAD[{q, m}, q - p - k]
FeynAmpDenominatorSimplify[%, q]
```

$$\frac{1}{([(-k-p+q)^2-m^2]).([q^2])}-\frac{1}{([q^2-m^2]).([(-k-p+q)^2])}$$

0

```
FAD[{l - p, m}, {l + q, m}, l - t, l + a, l - b]
FDS[%, l]
```

$$1/([(l-p)^2-m^2]).([(l+q)^2-m^2]).([(l-t)^2]).([(a+l)^2]).([(l-b)^2])$$

$$\frac{1}{l^2.(a+l+t)^2.(-b+l+t)^2.((l-p+t)^2-m^2).((l+q+t)^2-m^2)}$$

TID

1-loop tensor reduction using Passarino-Veltman method is handled by **TID**

```
FVD[q, μ] FVD[q, ν] FAD[{q, m}]
TID[%, q]
```

$$q^{\mu}\, q^{\nu}\, \frac{1}{[q^2-m^2]}$$

$$\frac{m^2\, g^{\mu\nu}}{D\left(q^2-m^2\right)}$$

```
int = FVD[q, μ] SPD[q, p] FAD[{q, m0}, {q + p, m1}]
TID[%, q]
```

$$q^{\mu}\,(p\cdot q)\,\frac{1}{([q^2-\text{m0}^2]).([(p+q)^2-\text{m1}^2])}$$

$$\frac{p^{\mu}\,(\text{m0}^2-\text{m1}^2+p^2)^2}{4\,p^2\,(q^2-\text{m0}^2).((q-p)^2-\text{m1}^2)}-\frac{p^{\mu}\,(\text{m0}^2-\text{m1}^2+p^2)}{4\,p^2\,(q^2-\text{m0}^2)}+\frac{p^{\mu}\,(\text{m0}^2-\text{m1}^2+3\,p^2)}{4\,p^2\,(q^2-\text{m1}^2)}$$

By default, **TID** tries to reduce everything to scalar integrals with unit denominators.
However, if it encounters zero Gram determinants, it automatically switches to the coefficient functions

```
FCClearScalarProducts[]
SPD[p, p] = 0;
```

```
TID[int, q]
```

$$\frac{p^{\mu}}{2\,(q^2-\text{m1}^2)}-\frac{1}{2}\,i\,\pi^2\,(\text{m0}^2-\text{m1}^2)\,p^{\mu}\,\text{B}_1(0,\,\text{m0}^2,\,\text{m1}^2)$$

If we want the result to be express entirely in terms of Passarino-Veltman function, i. e. without FAD's, we
can use **ToPaVe**

```
TID[int, q, ToPaVe → True]
```

$$\frac{1}{2}\,i\,\pi^2\,p^{\mu}\,\text{A}_0(\text{m1}^2)-\frac{1}{2}\,i\,\pi^2\,(\text{m0}^2-\text{m1}^2)\,p^{\mu}\,\text{B}_1(0,\,\text{m0}^2,\,\text{m1}^2)$$

ToPaVe is actually also a standalone function, so it can be used independently of **TID**

```
FCClearScalarProducts[]
FAD[q, {q + p1}, {q + p2}]
ToPaVe[%, q]
```

$$\frac{1}{([q^2]).([(\text{p1}+q)^2]).([(\text{p2}+q)^2])}$$

$$i\,\pi^2\,\text{C}_0(\text{p1}^2,\,\text{p2}^2,\,\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2,\,0,\,0,\,0)$$

Even if there are no Gram determinants, for some tensor integrals the full result in terms of scalar integrals
is just too large

```
int = FVD[q, μ] FVD[q, ν] FAD[q, {q + p1}, {q + p2}]
res = TID[int, q]
```

$$q^{\mu}\,q^{\nu}\,\frac{1}{([q^2]).([(\text{p1}+q)^2]).([(\text{p2}+q)^2])}$$

$$-\frac{1}{4\,(2-D)\,q^2.(q-\text{p1})^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2}$$
$$\big(2\,(D-1)\,\text{p2}^{\mu}\,\text{p2}^{\nu}\,(\text{p1}\cdot\text{p2})\,\text{p1}^4-(D-1)\,\text{p2}^{\mu}\,\text{p2}^{\nu}\,(\text{p1}\cdot\text{p2}+\text{p2}^2)\,\text{p1}^4+2\,(D-1)\,\text{p1}^{\mu}\,\text{p1}^{\nu}\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,\text{p1}^2+$$
$$(D-1)\,\text{p2}^{\mu}\,\text{p1}^{\nu}\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}+\text{p2}^2)\,\text{p1}^2+(D-1)\,\text{p1}^{\mu}\,\text{p2}^{\nu}\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}+\text{p2}^2)\,\text{p1}^2+$$
$$2\,g^{\mu\nu}\,(\text{p1}\cdot\text{p2})\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,\text{p1}^2-g^{\mu\nu}\,(\text{p1}\cdot\text{p2}+\text{p2}^2)\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,\text{p1}^2-$$

$$\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2-\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)$$
$$\mathrm{p1}^2+\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\left(\mathrm{p1}\cdot\mathrm{p2}+\mathrm{p2}^2\right)\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\Big)-$$
$$\frac{1}{4\,(2-D)\,q^2.(q-\mathrm{p2})^2\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)^2}\Big(2\,(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4-$$
$$(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\left(\mathrm{p1}^2+\mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^4+2\,(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\,\mathrm{p1}^2\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2+$$
$$(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2+\mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^2+(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2+\mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^2+$$
$$2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2-g^{\mu\nu}\left(\mathrm{p1}^2+\mathrm{p1}\cdot\mathrm{p2}\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2-$$
$$\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2-\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2+$$
$$\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\left(\mathrm{p1}^2+\mathrm{p1}\cdot\mathrm{p2}\right)\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\Big)+$$
$$\frac{1}{4\,(2-D)\,q^2.(q-\mathrm{p1})^2.(q-\mathrm{p2})^2\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)^2}$$
$$\Big(-(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,\mathrm{p2}^4\,\mathrm{p1}^4-(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\,\mathrm{p2}^4\,\mathrm{p1}^4+(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4+$$
$$(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4+2\,(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4-$$
$$g^{\mu\nu}\,\mathrm{p2}^2\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^4+\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^4+$$
$$2\,(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2+(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2+$$
$$(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2-g^{\mu\nu}\,\mathrm{p2}^4\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2+$$
$$2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2-\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,\mathrm{p2}^2\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2-$$
$$\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,\mathrm{p2}^2\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2+D\,\mathrm{p1}^2\,\mathrm{p2}^2-2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2+$$
$$\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,\mathrm{p2}^4\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\Big)-$$
$$\Big(-(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\left(-3\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)+\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-\right.$$
$$\left.2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)\right)\mathrm{p1}^4-$$
$$4\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left(\mathrm{p1}^2\,\mathrm{p2}^2-(\mathrm{p1}\cdot\mathrm{p2})^2\right)\mathrm{p1}^2+(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})$$
$$\left(-3\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)+\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)\right)$$
$$\mathrm{p1}^2+(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(-3\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)+\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-\right.$$
$$\left.2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)\right)\mathrm{p1}^2-g^{\mu\nu}\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)$$
$$\left(-3\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)+\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)\right)$$
$$\mathrm{p1}^2-2\,(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(-\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\right.$$
$$\left.\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left(\mathrm{p1}^2+2\,\mathrm{p2}^2\right)\right)\mathrm{p1}^2+4\,g^{\mu\nu}\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)^2-$$
$$4\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)-$$
$$4\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)+$$
$$4\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)+(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,\mathrm{p2}^4\left(-\mathrm{p1}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\right.$$
$$\left.2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)-\mathrm{p2}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\left(\mathrm{p1}^2+2\,(\mathrm{p1}\cdot\mathrm{p2})\right)\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\right)-$$
$$(D-1)\,\mathrm{p2}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\left(-\mathrm{p1}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)-\right.$$
$$\left.\mathrm{p2}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\left(\mathrm{p1}^2+2\,(\mathrm{p1}\cdot\mathrm{p2})\right)\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\right)-$$
$$(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p2}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\left(-\mathrm{p1}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)-\right.$$
$$\left.\mathrm{p2}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\left(\mathrm{p1}^2+2\,(\mathrm{p1}\cdot\mathrm{p2})\right)\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\right)+$$
$$g^{\mu\nu}\,\mathrm{p2}^2\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\left(-\mathrm{p1}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)-\right.$$
$$\left.\mathrm{p2}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\left(\mathrm{p1}^2+2\,(\mathrm{p1}\cdot\mathrm{p2})\right)\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\right)-$$
$$\mathrm{p2}^{\mu}\,\mathrm{p2}^{\nu}\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\left(-\mathrm{p1}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)-\right.$$
$$\left.\mathrm{p2}^2\left(\mathrm{p1}\cdot\mathrm{p2}-\mathrm{p1}^2\right)+\left(\mathrm{p1}^2+2\,(\mathrm{p1}\cdot\mathrm{p2})\right)\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\right)+$$
$$\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2+2\,(\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\left(-3\,\mathrm{p2}^2\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)+\right.$$
$$\left.\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)\right)-$$
$$2\,(D-1)\,\mathrm{p1}^{\mu}\,\mathrm{p1}^{\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\left(-\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-\right.$$
$$\left.\mathrm{p2}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+\left(\mathrm{p1}^2-2\,(\mathrm{p1}\cdot\mathrm{p2})+\mathrm{p2}^2\right)\left(\mathrm{p1}^2+2\,\mathrm{p2}^2\right)\right)-$$
$$2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left((\mathrm{p1}\cdot\mathrm{p2})^2-\mathrm{p1}^2\,\mathrm{p2}^2\right)\left(-\mathrm{p1}^2\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)+2\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p2}^2-\mathrm{p1}\cdot\mathrm{p2}\right)-\right.$$

$$p2^2\,(p2^2 - p1\cdot p2) + (p1^2 - 2\,(p1\cdot p2) + p2^2)\,(p1^2 + 2\,p2^2)) +$$
$$p2^\mu\, p1^\nu\,(D\,(p1\cdot p2)^2 + D\,p1^2\,p2^2 - 2\,p1^2\,p2^2)\,(-p1^2\,(p2^2 - p1\cdot p2) + 2\,(p1\cdot p2)\,(p2^2 - p1\cdot p2) -$$
$$p2^2\,(p2^2 - p1\cdot p2) + (p1^2 - 2\,(p1\cdot p2) + p2^2)\,(p1^2 + 2\,p2^2)) +$$
$$p1^\mu\, p2^\nu\,(D\,(p1\cdot p2)^2 + D\,p1^2\,p2^2 - 2\,p1^2\,p2^2)\,(-p1^2\,(p2^2 - p1\cdot p2) + 2\,(p1\cdot p2)\,(p2^2 - p1\cdot p2) -$$
$$p2^2\,(p2^2 - p1\cdot p2) + (p1^2 - 2\,(p1\cdot p2) + p2^2)\,(p1^2 + 2\,p2^2)))\Big/$$
$$\left(4\,(2 - D)\,q^2.(-p1 + p2 + q)^2\,(p1^2 - 2\,(p1\cdot p2) + p2^2)\,((p1\cdot p2)^2 - p1^2\,p2^2)^2\right)$$

Of course we collect with respect to FAD's and isolate the prefactors, but the full result still remains messy

```
Collect2[res, FeynAmpDenominator, IsolateNames → KK]
```

$$\frac{KK(665)}{4\,q^2.(q-p1)^2.(q-p2)^2} - \frac{KK(668)}{4\,q^2.(-p1+p2+q)^2} - \frac{KK(670)}{4\,q^2.(q-p1)^2} - \frac{KK(672)}{4\,q^2.(q-p2)^2}$$

In such cases, we can get a much more compact results , if we stick to coefficient functions and do not demand the full reduction to scalars. To do so, use the option **UsePaVeBasis**

```
res = TID[int, q, UsePaVeBasis → True]
```

$$i\,\pi^2\,g^{\mu\nu}\,C_{00}(p1^2,\,-2\,(p1\cdot p2) + p1^2 + p2^2,\,p2^2,\,0,\,0,\,0) +$$
$$i\,\pi^2\,p1^\mu\,p1^\nu\,C_{11}(p1^2,\,-2\,(p1\cdot p2) + p1^2 + p2^2,\,p2^2,\,0,\,0,\,0) +$$
$$i\,\pi^2\,(p1^\nu\,p2^\mu + p1^\mu\,p2^\nu)\,C_{12}(p1^2,\,-2\,(p1\cdot p2) + p1^2 + p2^2,\,p2^2,\,0,\,0,\,0) +$$
$$i\,\pi^2\,p2^\mu\,p2^\nu\,C_{22}(p1^2,\,-2\,(p1\cdot p2) + p1^2 + p2^2,\,p2^2,\,0,\,0,\,0)$$

The resulting coefficient functions can be further reduced with **PaVeReduce**

```
PaVeReduce[res]
```

$$
\begin{aligned}
&\frac{1}{4\,(2-D)\left((p1\cdot p2)^2-p1^2\,p2^2\right)^2} \\
&\quad i\,\pi^2\,\big(D\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3-2\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3-g^{\mu\nu}\,p1^2\,(p1\cdot p2)^3+p2^{\mu}\,p1^{\nu}\,p1^2\,(p1\cdot p2)^2+ \\
&\qquad p1^{\mu}\,p2^{\nu}\,p1^2\,(p1\cdot p2)^2+D\,p1^{\mu}\,p1^{\nu}\,p2^2\,(p1\cdot p2)^2-2\,p1^{\mu}\,p1^{\nu}\,p2^2\,(p1\cdot p2)^2+ \\
&\qquad g^{\mu\nu}\,p1^2\,p2^2\,(p1\cdot p2)^2-D\,p2^{\mu}\,p2^{\nu}\,p1^4\,(p1\cdot p2)+p2^{\mu}\,p2^{\nu}\,p1^4\,(p1\cdot p2)+g^{\mu\nu}\,p1^4\,p2^2\,(p1\cdot p2)- \\
&\qquad 2\,D\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+3\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-D\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+ \\
&\qquad p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-D\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)- \\
&\qquad g^{\mu\nu}\,p1^4\,p2^4+p1^{\mu}\,p1^{\nu}\,p1^2\,p2^4+D\,p2^{\mu}\,p1^{\nu}\,p1^4\,p2^2-2\,p2^{\mu}\,p1^{\nu}\,p1^4\,p2^2+ \\
&\qquad D\,p1^{\mu}\,p2^{\nu}\,p1^4\,p2^2-2\,p1^{\mu}\,p2^{\nu}\,p1^4\,p2^2+D\,p2^{\mu}\,p2^{\nu}\,p1^4\,p2^2-p2^{\mu}\,p2^{\nu}\,p1^4\,p2^2\big)\,B_0\big(p1^2,\,0,\,0\big)-
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{4\,(2-D)\left((p1\cdot p2)^2-p1^2\,p2^2\right)^2}\,i\,\pi^2\,\big(-D\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3+2\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3+g^{\mu\nu}\,p2^2\,(p1\cdot p2)^3- \\
&\qquad D\,p2^{\mu}\,p2^{\nu}\,p1^2\,(p1\cdot p2)^2+2\,p2^{\mu}\,p2^{\nu}\,p1^2\,(p1\cdot p2)^2-p2^{\mu}\,p1^{\nu}\,p2^2\,(p1\cdot p2)^2- \\
&\qquad p1^{\mu}\,p2^{\nu}\,p2^2\,(p1\cdot p2)^2-g^{\mu\nu}\,p1^2\,p2^2\,(p1\cdot p2)^2+D\,p1^{\mu}\,p1^{\nu}\,p2^4\,(p1\cdot p2)-p1^{\mu}\,p1^{\nu}\,p2^4\,(p1\cdot p2)- \\
&\qquad g^{\mu\nu}\,p1^2\,p2^4\,(p1\cdot p2)+D\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+ \\
&\qquad D\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+2\,D\,p2^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)- \\
&\qquad 3\,p2^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+g^{\mu\nu}\,p1^4\,p2^4-D\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^4+p1^{\mu}\,p1^{\nu}\,p1^2\,p2^4-D\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^4+ \\
&\qquad 2\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^4-D\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^4+2\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^4-p2^{\mu}\,p2^{\nu}\,p1^4\,p2^2\big)\,B_0\big(p2^2,\,0,\,0\big)-
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{4\,(2-D)\left((p1\cdot p2)^2-p1^2\,p2^2\right)^2}\,i\,\pi^2\,\big(2\,g^{\mu\nu}\,(p1\cdot p2)^4+D\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3-2\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3+ \\
&\qquad D\,p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3-4\,p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)^3+D\,p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3-4\,p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3+ \\
&\qquad D\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3-2\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^3-g^{\mu\nu}\,p1^2\,(p1\cdot p2)^3-g^{\mu\nu}\,p2^2\,(p1\cdot p2)^3+ \\
&\qquad p2^{\mu}\,p1^{\nu}\,p1^2\,(p1\cdot p2)^2+p1^{\mu}\,p2^{\nu}\,p1^2\,(p1\cdot p2)^2+2\,p2^{\mu}\,p2^{\nu}\,p1^2\,(p1\cdot p2)^2+ \\
&\qquad 2\,p1^{\mu}\,p1^{\nu}\,p2^2\,(p1\cdot p2)^2+p2^{\mu}\,p1^{\nu}\,p2^2\,(p1\cdot p2)^2+p1^{\mu}\,p2^{\nu}\,p2^2\,(p1\cdot p2)^2- \\
&\qquad 2\,g^{\mu\nu}\,p1^2\,p2^2\,(p1\cdot p2)^2-D\,p2^{\mu}\,p2^{\nu}\,p1^4\,(p1\cdot p2)+p2^{\mu}\,p2^{\nu}\,p1^4\,(p1\cdot p2)-D\,p1^{\mu}\,p1^{\nu}\,p2^4\,(p1\cdot p2)+ \\
&\qquad p1^{\mu}\,p1^{\nu}\,p2^4\,(p1\cdot p2)+g^{\mu\nu}\,p1^2\,p2^4\,(p1\cdot p2)+g^{\mu\nu}\,p1^4\,p2^2\,(p1\cdot p2)-2\,D\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+ \\
&\qquad 3\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-3\,D\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+6\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)- \\
&\qquad 3\,D\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+6\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)-2\,D\,p2^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+ \\
&\qquad 3\,p2^{\mu}\,p2^{\nu}\,p1^2\,p2^2\,(p1\cdot p2)+2\,D\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^4-4\,p1^{\mu}\,p1^{\nu}\,p1^2\,p2^4+D\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^4- \\
&\qquad 2\,p2^{\mu}\,p1^{\nu}\,p1^2\,p2^4+D\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^4-2\,p1^{\mu}\,p2^{\nu}\,p1^2\,p2^4+D\,p2^{\mu}\,p1^{\nu}\,p1^4\,p2^2-2\,p2^{\mu}\,p1^{\nu}\,p1^4\,p2^2+ \\
&\qquad D\,p1^{\mu}\,p2^{\nu}\,p1^4\,p2^2-2\,p1^{\mu}\,p2^{\nu}\,p1^4\,p2^2+2\,D\,p2^{\mu}\,p2^{\nu}\,p1^4\,p2^2-4\,p2^{\mu}\,p2^{\nu}\,p1^4\,p2^2\big) \\
&\quad B_0\big(p1^2-2\,(p1\cdot p2)+p2^2,\,0,\,0\big)+\frac{1}{4\,(2-D)\left((p1\cdot p2)^2-p1^2\,p2^2\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&i\,\pi^2\,\big(g^{\mu\nu}\,p2^4\,p1^6-p2^{\mu}\,p2^{\nu}\,p2^2\,p1^6+g^{\mu\nu}\,p2^6\,p1^4-D\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^2\,p1^4+2\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)^2\,p1^4- \\
&\qquad D\,p1^{\mu}\,p1^{\nu}\,p2^4\,p1^4+p1^{\mu}\,p1^{\nu}\,p2^4\,p1^4-D\,p2^{\mu}\,p1^{\nu}\,p2^4\,p1^4+2\,p2^{\mu}\,p1^{\nu}\,p2^4\,p1^4-D\,p1^{\mu}\,p2^{\nu}\,p2^4\,p1^4+ \\
&\qquad 2\,p1^{\mu}\,p2^{\nu}\,p2^4\,p1^4-D\,p2^{\mu}\,p2^{\nu}\,p2^4\,p1^4+p2^{\mu}\,p2^{\nu}\,p2^4\,p1^4-2\,g^{\mu\nu}\,(p1\cdot p2)\,p2^4\,p1^4- \\
&\qquad g^{\mu\nu}\,(p1\cdot p2)^2\,p2^2\,p1^4+D\,p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4-p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4+ \\
&\qquad D\,p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4-p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4+2\,D\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4- \\
&\qquad 2\,p2^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^2\,p1^4-p1^{\mu}\,p1^{\nu}\,p2^6\,p1^2-g^{\mu\nu}\,(p1\cdot p2)^2\,p2^4\,p1^2+ \\
&\qquad 2\,D\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2-2\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2+D\,p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2- \\
&\qquad p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2+D\,p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2-p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)\,p2^4\,p1^2+ \\
&\qquad 2\,g^{\mu\nu}\,(p1\cdot p2)^3\,p2^2\,p1^2-D\,p2^{\mu}\,p1^{\nu}\,(p1\cdot p2)^2\,p2^2\,p1^2-D\,p1^{\mu}\,p2^{\nu}\,(p1\cdot p2)^2\,p2^2\,p1^2- \\
&\qquad D\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^2\,p2^4+2\,p1^{\mu}\,p1^{\nu}\,(p1\cdot p2)^2\,p2^4\big)\,C_0\big(p1^2,\,p2^2,\,p1^2-2\,(p1\cdot p2)+p2^2,\,0,\,0,\,0\big)
\end{aligned}
$$

FCMultiLoopTID

TID works only at 1-loop. For multi-loop integrals we can use **FCMultiLoopTID**

```
FVD[q1, mu] FVD[q2, nu] SPD[q1, q2] FAD[{q1, m1}, {q2, m2}, {q1 - q2}]
```

$$q1^{mu}\, q2^{nu}\, (q1\cdot q2)\, \frac{1}{([q1^2-m1^2]).([q2^2-m2^2]).([(q1-q2)^2])}$$

```
FCMultiLoopTID[%, {q1, q2}]
```

$$\frac{(m1^2+m2^2)^2\, g^{mu\,nu}}{4\,D\,(q1^2-m1^2).(q2^2-m2^2).(q2-q1)^2} - \frac{(m1^2+m2^2)\, g^{mu\,nu}}{4\,D\,(q1^2-m1^2).(q2^2-m2^2)} - \frac{g^{mu\,nu}\,(q1\cdot q2)}{2\,D\,(q1^2-m2^2).(q2^2-m1^2)}$$

```
FVD[q1, mu] FVD[q2, nu] FAD[q1, q2, {p1 - q2}, {q1 - p1}, {q2 - p1}]
```

$$q1^{mu}\, q2^{nu}\, \frac{1}{([q1^2]).([q2^2]).([(p1-q2)^2]).([(q1-p1)^2]).([(q2-p1)^2])}$$

```
FCMultiLoopTID[%, {q1, q2}]
```

$$\frac{D\,p1^{mu}\,p1^{nu}-p1^2\,g^{mu\,nu}}{4\,(D-1)\,q2^2.q1^2.(q2-p1)^2.(q1-p1)^2.(q1-p1)^2} - \frac{D\,p1^{mu}\,p1^{nu}-p1^2\,g^{mu\,nu}}{4\,(D-1)\,p1^2\,q2^2.q1^2.(q2-p1)^2.(q1-p1)^2} + \frac{(q1\cdot q2)\,(p1^2\,g^{mu\,nu}-p1^{mu}\,p1^{nu})}{(D-1)\,p1^2\,q2^2.q1^2.(q2-p1)^2.(q1-p1)^2.(q2-p1)^2}$$

One should keep in mind, that these multi-loop decompositions may become very large

```
int = FVD[q1, mu] FVD[q1, rho] FVD[q2, nu] FAD[{q1 + p2, m}, {q2 - p1, m}, {q1 - q2}]
```

$$q1^{mu}\, q2^{nu}\, q1^{rho}\, \frac{1}{([(p2+q1)^2-m^2]).([(q2-p1)^2-m^2]).([(q1-q2)^2])}$$

```
res = FCMultiLoopTID[int, {q1, q2}]
```

$$\Big((p1\cdot q1)^2\,(p1\cdot q2)\,\big(D\,p2^{mu}\,p2^{nu}\,p2^{rho}\,(p1\cdot p2)^3 - 2\,p2^{mu}\,p2^{nu}\,p2^{rho}\,(p1\cdot p2)^3 + p2^{mu}\,g^{nu\,rho}\,p2^2\,(p1\cdot p2)^3 +$$
$$g^{mu\,rho}\,p2^{nu}\,p2^2\,(p1\cdot p2)^3 + g^{mu\,nu}\,p2^{rho}\,p2^2\,(p1\cdot p2)^3 - p1^{mu}\,g^{nu\,rho}\,p2^4\,(p1\cdot p2)^2 -$$
$$g^{mu\,rho}\,p1^{nu}\,p2^4\,(p1\cdot p2)^2 - g^{mu\,nu}\,p1^{rho}\,p2^4\,(p1\cdot p2)^2 - D\,p2^{mu}\,p2^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^2 -$$
$$D\,p2^{mu}\,p1^{nu}\,p2^{rho}\,p2^2\,(p1\cdot p2)^2 - D\,p1^{mu}\,p2^{nu}\,p2^{rho}\,p2^2\,(p1\cdot p2)^2 +$$
$$D\,p2^{mu}\,p1^{nu}\,p1^{rho}\,p2^4\,(p1\cdot p2) + p2^{mu}\,p1^{nu}\,p1^{rho}\,p2^4\,(p1\cdot p2) + D\,p1^{mu}\,p2^{nu}\,p1^{rho}\,p2^4\,(p1\cdot p2) +$$
$$p1^{mu}\,p2^{nu}\,p1^{rho}\,p2^4\,(p1\cdot p2) + D\,p1^{mu}\,p1^{nu}\,p2^{rho}\,p2^4\,(p1\cdot p2) + p1^{mu}\,p1^{nu}\,p2^{rho}\,p2^4\,(p1\cdot p2) -$$
$$p2^{mu}\,g^{nu\,rho}\,p1^2\,p2^4\,(p1\cdot p2) - g^{mu\,rho}\,p2^{nu}\,p1^2\,p2^4\,(p1\cdot p2) - g^{mu\,nu}\,p2^{rho}\,p1^2\,p2^4\,(p1\cdot p2) +$$
$$3\,p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^2\,p2^2\,(p1\cdot p2) - D\,p1^{mu}\,p1^{nu}\,p1^{rho}\,p2^6 - p1^{mu}\,p1^{nu}\,p1^{rho}\,p2^6 +$$
$$p1^{mu}\,g^{nu\,rho}\,p1^2\,p2^6 + g^{mu\,rho}\,p1^{nu}\,p1^2\,p2^6 + g^{mu\,nu}\,p1^{rho}\,p1^2\,p2^6 -$$
$$p2^{mu}\,p2^{nu}\,p1^{rho}\,p1^2\,p2^4 - p2^{mu}\,p1^{nu}\,p2^{rho}\,p1^2\,p2^4 - p1^{mu}\,p2^{nu}\,p2^{rho}\,p1^2\,p2^4\big)\Big)\Big/$$
$$\Big((D-2)\,((p2+q1)^2-m^2).((q2-p1)^2-m^2).(q1-q2)^2\,\big((p1\cdot p2)^2-p1^2\,p2^2\big)^3\Big) +$$
$$\Big((p1\cdot q1)^2\,\big(p2^{mu}\,g^{nu\,rho}\,(p1\cdot p2)^4 + g^{mu\,nu}\,p2^{rho}\,(p1\cdot p2)^4 - p2^{mu}\,p2^{nu}\,p1^{rho}\,(p1\cdot p2)^3 -$$
$$D\,p2^{mu}\,p1^{nu}\,p2^{rho}\,(p1\cdot p2)^3 - p1^{mu}\,p2^{nu}\,p2^{rho}\,(p1\cdot p2)^3 - p1^{mu}\,g^{nu\,rho}\,p2^2\,(p1\cdot p2)^3 -$$
$$g^{mu\,rho}\,p1^{nu}\,p2^2\,(p1\cdot p2)^3 - g^{mu\,nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^3 + D\,p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)^2 +$$
$$D\,p2^{mu}\,p1^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^2 + p2^{mu}\,p1^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^2 +$$
$$2\,p1^{mu}\,p2^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^2 + D\,p1^{mu}\,p1^{nu}\,p2^{rho}\,p2^2\,(p1\cdot p2)^2 +$$
$$p1^{mu}\,p1^{nu}\,p2^{rho}\,p2^2\,(p1\cdot p2)^2 - p2^{mu}\,g^{nu\,rho}\,p1^2\,p2^2\,(p1\cdot p2)^2 + g^{mu\,rho}\,p2^{nu}\,p1^2\,p2^2\,(p1\cdot p2)^2 -$$

$g^{mu\,nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2)^2 - D\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^4\, (p1 \cdot p2) -$
$p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^4\, (p1 \cdot p2) + p1^{mu}\, g^{nu\,rho}\, p1^2\, p2^4\, (p1 \cdot p2) + g^{mu\,rho}\, p1^{nu}\, p1^2\, p2^4\, (p1 \cdot p2) +$
$g^{mu\,nu}\, p1^{rho}\, p1^2\, p2^4\, (p1 \cdot p2) - D\, p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) -$
$p2^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - D\, p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - g^{mu\,rho}\, p2^{nu}\, p1^4\, p2^4 +$
$D\, p1^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^4 - p1^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^4 + p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, p2^2\big)\, (p2 \cdot q2)\big) \Big/$
$\Big((D-2)\, \big((p2+q1)^2 - m^2\big).\big((q2-p1)^2 - m^2\big).(q1-q2)^2\, \big(p1^2\, p2^2 - (p1 \cdot p2)^2\big)^3\Big) +$
$\big((p1 \cdot q1)\, (p1 \cdot q2)\, \big(p2^{mu}\, g^{nu\,rho}\, (p1 \cdot p2)^4 + 2\, g^{mu\,rho}\, p2^{nu}\, (p1 \cdot p2)^4 + g^{mu\,nu}\, p2^{rho}\, (p1 \cdot p2)^4 -$
$D\, p2^{mu}\, p2^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 - p2^{mu}\, p2^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 - 2\, p2^{mu}\, p1^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 -$
$D\, p1^{mu}\, p2^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 - p1^{mu}\, p2^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 - 2\, p1^{mu}\, g^{nu\,rho}\, p2^2\, (p1 \cdot p2)^3 -$
$2\, g^{mu\,rho}\, p1^{nu}\, p2^2\, (p1 \cdot p2)^3 - 2\, g^{mu\,nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^3 + 2\, D\, p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^2 +$
$D\, p2^{mu}\, p1^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 + 3\, p2^{mu}\, p1^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 +$
$2\, D\, p1^{mu}\, p2^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 + 2\, p1^{mu}\, p2^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 +$
$D\, p1^{mu}\, p1^{nu}\, p2^{rho}\, p2^2\, (p1 \cdot p2)^2 + 3\, p1^{mu}\, p1^{nu}\, p2^{rho}\, p2^2\, (p1 \cdot p2)^2 -$
$2\, g^{mu\,rho}\, p2^{nu}\, p1^2\, p2^2\, (p1 \cdot p2)^2 - 2\, D\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^4\, (p1 \cdot p2) -$
$2\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^4\, (p1 \cdot p2) + 2\, p1^{mu}\, g^{nu\,rho}\, p1^2\, p2^4\, (p1 \cdot p2) + 2\, g^{mu\,rho}\, p1^{nu}\, p1^2\, p2^4\, (p1 \cdot p2) +$
$2\, g^{mu\,nu}\, p1^{rho}\, p1^2\, p2^4\, (p1 \cdot p2) - D\, p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) -$
$p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - 2\, D\, p2^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) -$
$D\, p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - p2^{mu}\, g^{nu\,rho}\, p1^4\, p2^4 -$
$g^{mu\,nu}\, p2^{rho}\, p1^4\, p2^4 + D\, p2^{mu}\, p1^{nu}\, p1^{rho}\, p1^2\, p2^4 - p2^{mu}\, p1^{nu}\, p1^{rho}\, p1^2\, p2^4 +$
$D\, p1^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^4 - p1^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^4 + 2\, p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, p2^2\big)\, (p2 \cdot q1)\big) \Big/$
$\Big((D-2)\, \big((p2+q1)^2 - m^2\big).\big((q2-p1)^2 - m^2\big).(q1-q2)^2\, \big(p1^2\, p2^2 - (p1 \cdot p2)^2\big)^3\Big) -$
$\big((p1 \cdot q1)\, \big(-p1^{mu}\, g^{nu\,rho}\, (p1 \cdot p2)^4 - 2\, g^{mu\,rho}\, p1^{nu}\, (p1 \cdot p2)^4 - g^{mu\,nu}\, p1^{rho}\, (p1 \cdot p2)^4 +$
$D\, p2^{mu}\, p1^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 + p2^{mu}\, p1^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 + 2\, p1^{mu}\, p2^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 +$
$D\, p1^{mu}\, p1^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 + p1^{mu}\, p1^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 + 2\, p2^{mu}\, g^{nu\,rho}\, p1^2\, (p1 \cdot p2)^3 +$
$2\, g^{mu\,rho}\, p2^{nu}\, p1^2\, (p1 \cdot p2)^3 + 2\, g^{mu\,nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^3 - D\, p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, (p1 \cdot p2)^2 -$
$3\, p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, (p1 \cdot p2)^2 - 2\, D\, p2^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^2 -$
$2\, p2^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^2 - D\, p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^2 -$
$3\, p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2)^2 - 2\, D\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 + 2\, g^{mu\,rho}\, p1^{nu}\, p1^2$
$p2^2\, (p1 \cdot p2)^2 + 2\, D\, p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, (p1 \cdot p2) + 2\, p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, (p1 \cdot p2) -$
$2\, p2^{mu}\, g^{nu\,rho}\, p1^4\, p2^2\, (p1 \cdot p2) - 2\, g^{mu\,rho}\, p2^{nu}\, p1^4\, p2^2\, (p1 \cdot p2) - 2\, g^{mu\,nu}\, p2^{rho}\, p1^4\, p2^2\, (p1 \cdot p2) +$
$D\, p2^{mu}\, p1^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) + p2^{mu}\, p1^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) +$
$2\, D\, p1^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) + D\, p1^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) +$
$p1^{mu}\, p1^{nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) + p1^{mu}\, g^{nu\,rho}\, p1^4\, p2^4 + g^{mu\,nu}\, p1^{rho}\, p1^4\, p2^4 -$
$2\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p1^2\, p2^4 - D\, p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^4\, p2^2 + p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^4\, p2^2 -$
$D\, p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, p2^2 + p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^4\, p2^2\big)\, (p2 \cdot q1)\, (p2 \cdot q2)\big) \Big/$
$\Big((D-2)\, \big((p2+q1)^2 - m^2\big).\big((q2-p1)^2 - m^2\big).(q1-q2)^2\, \big(p1^2\, p2^2 - (p1 \cdot p2)^2\big)^3\Big) +$
$\big((p1 \cdot q1)\, \big(p2^{mu}\, g^{nu\,rho}\, (p1 \cdot p2)^3 + g^{mu\,nu}\, p2^{rho}\, (p1 \cdot p2)^3 - p2^{mu}\, p2^{nu}\, p1^{rho}\, (p1 \cdot p2)^2 -$
$2\, p2^{mu}\, p1^{nu}\, p2^{rho}\, (p1 \cdot p2)^2 - p1^{mu}\, p2^{nu}\, p2^{rho}\, (p1 \cdot p2)^2 - p1^{mu}\, g^{nu\,rho}\, p2^2\, (p1 \cdot p2)^2 -$
$g^{mu\,nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2)^2 + 2\, p2^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, (p1 \cdot p2) + 2\, p2^{mu}\, p1^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2) +$
$2\, p1^{mu}\, p2^{nu}\, p1^{rho}\, p2^2\, (p1 \cdot p2) + 2\, p1^{mu}\, p1^{nu}\, p2^{rho}\, p2^2\, (p1 \cdot p2) - p2^{mu}\, g^{nu\,rho}\, p1^2\, p2^2\, (p1 \cdot p2) -$
$g^{mu\,nu}\, p2^{rho}\, p1^2\, p2^2\, (p1 \cdot p2) - 2\, p1^{mu}\, p1^{nu}\, p1^{rho}\, p2^4 + p1^{mu}\, g^{nu\,rho}\, p1^2\, p2^4 +$
$g^{mu\,nu}\, p1^{rho}\, p1^2\, p2^4 - p2^{mu}\, p2^{nu}\, p1^{rho}\, p1^2\, p2^2 - p1^{mu}\, p2^{nu}\, p2^{rho}\, p1^2\, p2^2\big)\, (q1 \cdot q2)\big) \Big/$
$\Big((D-2)\, \big((p2+q1)^2 - m^2\big).\big((q2-p1)^2 - m^2\big).(q1-q2)^2\, \big(p1^2\, p2^2 - (p1 \cdot p2)^2\big)^2\Big) -$
$\big((p1 \cdot q2)\, \big(-p1^{mu}\, g^{nu\,rho}\, (p1 \cdot p2)^4 - g^{mu\,nu}\, p1^{rho}\, (p1 \cdot p2)^4 + p2^{mu}\, p1^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 +$
$D\, p1^{mu}\, p2^{nu}\, p1^{rho}\, (p1 \cdot p2)^3 + p1^{mu}\, p1^{nu}\, p2^{rho}\, (p1 \cdot p2)^3 + p2^{mu}\, g^{nu\,rho}\, p1^2\, (p1 \cdot p2)^3 +$

$$g^{mu\,rho}\,p2^{nu}\,p1^2\,(p1\cdot p2)^3+g^{mu\,nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)^3-D\,p2^{mu}\,p2^{nu}\,p1^{rho}\,p1^2\,(p1\cdot p2)^2-$$
$$p2^{mu}\,p2^{nu}\,p1^{rho}\,p1^2\,(p1\cdot p2)^2-2\,p2^{mu}\,p1^{nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)^2-D\,p1^{mu}\,p2^{nu}\,p2^{rho}$$
$$p1^2\,(p1\cdot p2)^2-p1^{mu}\,p2^{nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)^2-D\,p1^{mu}\,p1^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)^2+$$
$$p1^{mu}\,g^{nu\,rho}\,p1^2\,p2^2\,(p1\cdot p2)^2-g^{mu\,rho}\,p1^{nu}\,p1^2\,p2^2\,(p1\cdot p2)^2+g^{mu\,nu}\,p1^{rho}\,p1^2\,p2^2\,(p1\cdot p2)^2+$$
$$D\,p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^4\,(p1\cdot p2)+p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^4\,(p1\cdot p2)-p2^{mu}\,g^{nu\,rho}\,p1^4\,p2^2\,(p1\cdot p2)-$$
$$g^{mu\,rho}\,p2^{nu}\,p1^4\,p2^2\,(p1\cdot p2)-g^{mu\,nu}\,p2^{rho}\,p1^4\,p2^2\,(p1\cdot p2)+D\,p2^{mu}\,p1^{nu}\,p1^{rho}\,p1^2\,p2^2\,(p1\cdot p2)+$$
$$p1^{mu}\,p2^{nu}\,p1^{rho}\,p1^2\,p2^2\,(p1\cdot p2)+D\,p1^{mu}\,p1^{nu}\,p2^{rho}\,p1^2\,p2^2\,(p1\cdot p2)+g^{mu\,rho}\,p1^{nu}\,p1^4\,p2^4-$$
$$p1^{mu}\,p1^{nu}\,p1^{rho}\,p1^2\,p2^4-D\,p2^{mu}\,p1^{nu}\,p2^{rho}\,p1^4\,p2^2+p2^{mu}\,p1^{nu}\,p2^{rho}\,p1^4\,p2^2\Big)\,(p2\cdot q1)^2\Big)\Big/$$
$$\Big((D-2)\,\big((p2+q1)^2-m^2\big).\big((q2-p1)^2-m^2\big).(q1-q2)^2\,\big(p1^2\,p2^2-(p1\cdot p2)^2\big)^3\Big)+$$
$$\Big(\Big(D\,p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^6+p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^6-p2^{mu}\,g^{nu\,rho}\,p2^2\,p1^6-g^{mu\,rho}\,p2^{nu}\,p2^2\,p1^6-$$
$$g^{mu\,nu}\,p2^{rho}\,p2^2\,p1^6+p2^{mu}\,g^{nu\,rho}\,(p1\cdot p2)^2\,p1^4+g^{mu\,rho}\,p2^{nu}\,(p1\cdot p2)^2\,p1^4+$$
$$g^{mu\,nu}\,p2^{rho}\,(p1\cdot p2)^2\,p1^4-D\,p2^{mu}\,p2^{nu}\,p1^{rho}\,(p1\cdot p2)\,p1^4-p2^{mu}\,p2^{nu}\,p1^{rho}\,(p1\cdot p2)\,p1^4-$$
$$D\,p2^{mu}\,p1^{nu}\,p2^{rho}\,(p1\cdot p2)\,p1^4-p2^{mu}\,p1^{nu}\,p2^{rho}\,(p1\cdot p2)\,p1^4-D\,p1^{mu}\,p2^{nu}\,p2^{rho}\,(p1\cdot p2)\,p1^4-$$
$$p1^{mu}\,p2^{nu}\,p2^{rho}\,(p1\cdot p2)\,p1^4+p2^{mu}\,p1^{nu}\,p1^{rho}\,p2^2\,p1^4+p1^{mu}\,p2^{nu}\,p1^{rho}\,p2^2\,p1^4+$$
$$p1^{mu}\,p1^{nu}\,p2^{rho}\,p2^2\,p1^4+p1^{mu}\,g^{nu\,rho}\,(p1\cdot p2)\,p2^2\,p1^4+g^{mu\,rho}\,p1^{nu}\,(p1\cdot p2)\,p2^2\,p1^4+$$
$$g^{mu\,nu}\,p1^{rho}\,(p1\cdot p2)\,p2^2\,p1^4-p1^{mu}\,g^{nu\,rho}\,(p1\cdot p2)^3\,p1^2-g^{mu\,rho}\,p1^{nu}\,(p1\cdot p2)^3\,p1^2-$$
$$g^{mu\,nu}\,p1^{rho}\,(p1\cdot p2)^3\,p1^2+D\,p2^{mu}\,p1^{nu}\,p1^{rho}\,(p1\cdot p2)^2\,p1^2+D\,p1^{mu}\,p2^{nu}\,p1^{rho}\,(p1\cdot p2)^2\,p1^2+$$
$$D\,p1^{mu}\,p1^{nu}\,p2^{rho}\,(p1\cdot p2)^2\,p1^2-3\,p1^{mu}\,p1^{nu}\,p1^{rho}\,(p1\cdot p2)\,p2^2\,p1^2-$$
$$D\,p1^{mu}\,p1^{nu}\,p1^{rho}\,(p1\cdot p2)^3+2\,p1^{mu}\,p1^{nu}\,p1^{rho}\,(p1\cdot p2)^3\Big)\,(p2\cdot q1)^2\,(p2\cdot q2)\Big)\Big/$$
$$\Big((D-2)\,\big((p2+q1)^2-m^2\big).\big((q2-p1)^2-m^2\big).(q1-q2)^2\,\big(p1^2\,p2^2-(p1\cdot p2)^2\big)^3\Big)+$$
$$\Big((p1\cdot q2)\,\big(p2^{nu}\,(p1\cdot p2)-p1^{nu}\,p2^2\big)\,\big(g^{mu\,rho}\,(p1\cdot p2)^2-p2^{mu}\,p1^{rho}\,(p1\cdot p2)-$$
$$p1^{mu}\,p2^{rho}\,(p1\cdot p2)+p2^{mu}\,p2^{rho}\,p1^2+p1^{mu}\,p1^{rho}\,p2^2-g^{mu\,rho}\,p1^2\,p2^2\big)\,q1^2\Big)\Big/$$
$$\Big((D-2)\,\big((p2+q1)^2-m^2\big).\big((q2-p1)^2-m^2\big).(q1-q2)^2\,\big(p1^2\,p2^2-(p1\cdot p2)^2\big)^2\Big)-$$
$$\Big(\big(p2^{nu}\,p1^2-p1^{nu}\,(p1\cdot p2)\big)\,\big(g^{mu\,rho}\,(p1\cdot p2)^2-p2^{mu}\,p1^{rho}\,(p1\cdot p2)-p1^{mu}\,p2^{rho}\,(p1\cdot p2)+$$
$$p2^{mu}\,p2^{rho}\,p1^2+p1^{mu}\,p1^{rho}\,p2^2-g^{mu\,rho}\,p1^2\,p2^2\big)\,(p2\cdot q2)\,q1^2\Big)\Big/$$
$$\Big((D-2)\,\big((p2+q1)^2-m^2\big).\big((q2-p1)^2-m^2\big).(q1-q2)^2\,\big(p1^2\,p2^2-(p1\cdot p2)^2\big)^2\Big)-$$
$$\Big(\Big(-p1^{mu}\,g^{nu\,rho}\,(p1\cdot p2)^3-g^{mu\,nu}\,p1^{rho}\,(p1\cdot p2)^3+p2^{mu}\,p1^{nu}\,p1^{rho}\,(p1\cdot p2)^2+$$
$$2\,p1^{mu}\,p2^{nu}\,p1^{rho}\,(p1\cdot p2)^2+p1^{mu}\,p1^{nu}\,p2^{rho}\,(p1\cdot p2)^2+p2^{mu}\,g^{nu\,rho}\,p1^2\,(p1\cdot p2)^2+$$
$$g^{mu\,nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)^2-2\,p2^{mu}\,p2^{nu}\,p1^{rho}\,p1^2\,(p1\cdot p2)-2\,p2^{mu}\,p1^{nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)-$$
$$2\,p1^{mu}\,p2^{nu}\,p2^{rho}\,p1^2\,(p1\cdot p2)-2\,p1^{mu}\,p1^{nu}\,p1^{rho}\,p2^2\,(p1\cdot p2)+p1^{mu}\,g^{nu\,rho}\,p1^2\,p2^2\,(p1\cdot p2)+$$
$$g^{mu\,nu}\,p1^{rho}\,p1^2\,p2^2\,(p1\cdot p2)+2\,p2^{mu}\,p2^{nu}\,p2^{rho}\,p1^4-p2^{mu}\,g^{nu\,rho}\,p1^4\,p2^2-$$
$$g^{mu\,nu}\,p2^{rho}\,p1^4\,p2^2+p2^{mu}\,p1^{nu}\,p1^{rho}\,p1^2\,p2^2+p1^{mu}\,p1^{nu}\,p2^{rho}\,p1^2\,p2^2\Big)\,(p2\cdot q1)\,(q1\cdot q2)\Big)\Big/$$
$$\Big((D-2)\,\big((p2+q1)^2-m^2\big).\big((q2-p1)^2-m^2\big).(q1-q2)^2\,\big(p1^2\,p2^2-(p1\cdot p2)^2\big)^2\Big)$$

Again, if the kinematics is known in advance, one should specify it beforehand.
In this case we might obtain a much shorter result. For example, if $p_1 \cdot p_1 = p_2 \cdot p_2 = 0$,
then instead of doing

```
res /. {FCI[SPD[p1, p1]] | FCI[SPD[p2, p2]] → 0}
```

$$
\begin{aligned}
&\big(\big(D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - 2\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p1}\cdot\mathrm{q1})^2\,(\mathrm{p1}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) -\\
&\big(\big(\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 + g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 - \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 -\\
&\qquad D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p1}\cdot\mathrm{q1})^2\,(\mathrm{p2}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) -\\
&\big(\big(\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 + 2\,g^{\mathrm{mu\,rho}}\,\mathrm{p2}^{\mathrm{nu}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 + g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 -\\
&\qquad D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - 2\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 -\\
&\qquad D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{p1}\cdot\mathrm{q2})\,(\mathrm{p2}\cdot\mathrm{q1})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) +\\
&\big(\big(-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 - 2\,g^{\mathrm{mu\,rho}}\,\mathrm{p1}^{\mathrm{nu}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 - g^{\mathrm{mu\,nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 +\\
&\qquad D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + 2\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 +\\
&\qquad D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{p2}\cdot\mathrm{q1})\,(\mathrm{p2}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) +\\
&\big(\big(\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 -\\
&\qquad 2\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2\big)\,(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{q1}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^4\big) +\\
&\big(\big(-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 - g^{\mathrm{mu\,nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^4 + \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 +\\
&\qquad D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p1}\cdot\mathrm{q2})\,(\mathrm{p2}\cdot\mathrm{q1})^2\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) -\\
&\big(\big(2\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3\big)\,(\mathrm{p2}\cdot\mathrm{q1})^2\,(\mathrm{p2}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^6\big) +\\
&\big(\mathrm{p2}^{\mathrm{nu}}\,\big(g^{\mathrm{mu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2}) - \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p1}\cdot\mathrm{q2})\,\mathrm{q1}^2\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\big) +\\
&\big(\mathrm{p1}^{\mathrm{nu}}\,\big(g^{\mathrm{mu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2}) - \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p2}\cdot\mathrm{q2})\,\mathrm{q1}^2\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\big) -\\
&\big(\big(-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 - g^{\mathrm{mu\,nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^3 + \mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 +\\
&\qquad 2\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2 + \mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})^2\big)\,(\mathrm{p2}\cdot\mathrm{q1})\,(\mathrm{q1}\cdot\mathrm{q2})\big)\big/\\
&\quad\big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^4\big)
\end{aligned}
$$

One should better do

```
FCClearScalarProducts[]
SPD[p1, p1] = 0;
SPD[p2, p2] = 0;
```

```
FCMultiLoopTID[int, {q1, q2}]
```

$$\frac{\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{q1})^2\,(\mathrm{p1}\cdot\mathrm{q2})}{((\mathrm{p2}+\mathrm{q1})^2-m^2).((\mathrm{q2}-\mathrm{p1})^2-m^2).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3}+$$

$$\Big(\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})-g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)$$
$$(\mathrm{p1}\cdot\mathrm{q1})^2\,(\mathrm{p2}\cdot\mathrm{q2})\Big)\Big/\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\Big)+$$

$$\Big(\big(D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+2\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}+D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-$$
$$\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})-2\,g^{\mathrm{mu\,rho}}\,\mathrm{p2}^{\mathrm{nu}}\,(\mathrm{p1}\cdot\mathrm{p2})-g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{p1}\cdot\mathrm{q2})\,(\mathrm{p2}\cdot\mathrm{q1})\Big)\Big/$$
$$\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\Big)+$$

$$\Big(\big(D\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+2\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}-g^{\mathrm{mu\,nu}}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p1}^{\mathrm{rho}}+D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}+$$
$$\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})-2\,g^{\mathrm{mu\,rho}}\,\mathrm{p1}^{\mathrm{nu}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{p2}\cdot\mathrm{q1})\,(\mathrm{p2}\cdot\mathrm{q2})\Big)\Big/$$
$$\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\Big)-$$

$$\Big(\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+2\,\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-\mathrm{p2}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})-g^{\mathrm{mu\,nu}}\,\mathrm{p2}^{\mathrm{rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)$$
$$(\mathrm{p1}\cdot\mathrm{q1})\,(\mathrm{q1}\cdot\mathrm{q2})\Big)\Big/\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^2\Big)+$$

$$\Big(\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+D\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}-g^{\mathrm{mu\,nu}}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)$$
$$(\mathrm{p1}\cdot\mathrm{q2})\,(\mathrm{p2}\cdot\mathrm{q1})^2\Big)\Big/\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3\Big)+$$

$$\frac{\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}\,(\mathrm{p2}\cdot\mathrm{q1})^2\,(\mathrm{p2}\cdot\mathrm{q2})}{((\mathrm{p2}+\mathrm{q1})^2-m^2).((\mathrm{q2}-\mathrm{p1})^2-m^2).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^3}-$$

$$\Big(\mathrm{p2}^{\mathrm{nu}}\,\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{rho}}-g^{\mathrm{mu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p1}\cdot\mathrm{q2})\,\mathrm{q1}^2\Big)\Big/$$
$$\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^2\Big)-$$

$$\Big(\mathrm{p1}^{\mathrm{nu}}\,\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{rho}}-g^{\mathrm{mu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)\,(\mathrm{p2}\cdot\mathrm{q2})\,\mathrm{q1}^2\Big)\Big/$$
$$\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^2\Big)-$$

$$\Big(\big(\mathrm{p2}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}+2\,\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p2}^{\mathrm{nu}}\,\mathrm{p1}^{\mathrm{rho}}-g^{\mathrm{mu\,nu}}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p1}^{\mathrm{rho}}+\mathrm{p1}^{\mathrm{mu}}\,\mathrm{p1}^{\mathrm{nu}}\,\mathrm{p2}^{\mathrm{rho}}-\mathrm{p1}^{\mathrm{mu}}\,g^{\mathrm{nu\,rho}}\,(\mathrm{p1}\cdot\mathrm{p2})\big)$$
$$(\mathrm{p2}\cdot\mathrm{q1})\,(\mathrm{q1}\cdot\mathrm{q2})\Big)\Big/\Big((D-2)\,\big((\mathrm{p2}+\mathrm{q1})^2-m^2\big).\big((\mathrm{q2}-\mathrm{p1})^2-m^2\big).(\mathrm{q1}-\mathrm{q2})^2\,(\mathrm{p1}\cdot\mathrm{p2})^2\Big)$$

which is much smarter and much faster.

```
FCClearScalarProducts[]
```

Auxiliary functions

There are also some auxiliary functions related to loop integrals, that can be useful in many ways

Consider for example the sum of the four 1-loop gluon self-energy diagrams

```
amp = DiracTrace[(MQ - GSD[q]).(-ⅈ GAD[ν] SMP["g_s"] SUNTF[{b}, Col3, Col4]).
     (MQ + GSD[p - q]).(-ⅈ GAD[μ] SMP["g_s"] SUNTF[{a}, Col4, Col3])]
   FAD[{q, MQ}, {-p + q, MQ}] - FAD[q, -p + q] FVD[q, ν] FVD[-p + q, μ]
   SMP["g_s"]^2 SUNF[a, c, d] SUNF[b, c, d] - 1/2 (FVD[2 p - q, ρ] MTD[Lor5, μ] +
      FVD[-p + 2 q, μ] MTD[Lor5, ρ] + FVD[-p - q, Lor5] MTD[μ, ρ])
   (FVD[-2 p + q, σ] MTD[Lor6, ν] + FVD[p - 2 q, ν] MTD[Lor6, σ] + FVD[p + q, Lor6] MTD[ν, σ])
   (- (1 - GaugeXi)^2 FAD[q, q, -p + q, -p + q] FVD[p - q, Lor6] FVD[q, ρ] FVD[q, σ] FVD[-p + q,
        Lor5] - (1 - GaugeXi) FAD[q, q, -p + q] FVD[q, ρ] FVD[q, σ] MTD[Lor5, Lor6] +
      (1 - GaugeXi) FAD[q, -p + q, -p + q] FVD[p - q, Lor6] FVD[-p + q, Lor5] MTD[ρ, σ] +
      FAD[q, -p + q] MTD[Lor5, Lor6] MTD[ρ, σ]) SMP["g_s"]^2 SUNF[a, c, d] SUNF[b, c, d] -
  1/2 ⅈ (- (1 - GaugeXi) FAD[q, q] FVD[q, ρ] FVD[q, σ] + FAD[q] MTD[ρ, σ])
   (ⅈ MTD[μ, σ] MTD[ν, ρ] SMP["g_s"]^2 SUNF[a, c, $AL$2586] SUNF[b, c, $AL$2586] -
      ⅈ MTD[μ, ν] MTD[ρ, σ] SMP["g_s"]^2 (SUNF[a, c, $AL$2587] SUNF[b, c, $AL$2587] +
         SUNF[a, c, $AL$2588] SUNF[b, c, $AL$2588]) +
      ⅈ MTD[μ, ρ] MTD[ν, σ] SMP["g_s"]^2 SUNF[a, c, $AL$2590] SUNF[b, c, $AL$2590])
```

$$\frac{1}{([q^2-\mathrm{MQ}^2]).([(q-p)^2-\mathrm{MQ}^2])}\,\mathrm{tr}\big((\mathrm{MQ}-\gamma\cdot q).(-i\,\gamma^\nu\, g_s\, T^b_{\mathrm{Col3\,Col4}}).(\mathrm{MQ}+\gamma\cdot(p-q)).(-i\,\gamma^\mu\, g_s\, T^a_{\mathrm{Col4\,Col3}})\big)-$$
$$\frac{1}{2}\, g_s^2\, f^{a c d} f^{b c d}\left(g^{\mathrm{Lor5}\,\mu}(2p-q)^\rho+g^{\mathrm{Lor5}\,\rho}(2q-p)^\mu+g^{\mu\rho}(-p-q)^{\mathrm{Lor5}}\right)$$
$$\left(g^{\mathrm{Lor6}\,\nu}(q-2p)^\sigma+g^{\mathrm{Lor6}\,\sigma}(p-2q)^\nu+g^{\nu\sigma}(p+q)^{\mathrm{Lor6}}\right)$$
$$\Bigg(-(1-\xi)\, q^\rho q^\sigma g^{\mathrm{Lor5\,Lor6}}\frac{1}{([q^2])^2.([(q-p)^2])}+(1-\xi)\, g^{\rho\sigma}(q-p)^{\mathrm{Lor5}}(p-q)^{\mathrm{Lor6}}\frac{1}{([q^2]).([(q-p)^2])^2}+$$
$$g^{\mathrm{Lor5\,Lor6}}\, g^{\rho\sigma}\frac{1}{([q^2]).([(q-p)^2])}+(1-\xi)^2 q^\rho q^\sigma (q-p)^{\mathrm{Lor5}}(p-q)^{\mathrm{Lor6}}\left(-\frac{1}{([q^2])^2.([(q-p)^2])^2}\right)\Bigg)+$$
$$g_s^2\, q^\nu (q-p)^\mu f^{a c d} f^{b c d}\left(-\frac{1}{([q^2]).([(q-p)^2])}\right)-\frac{1}{2}\, i\left(g^{\rho\sigma}\frac{1}{[q^2]}+(\xi-1)\, q^\rho q^\sigma\frac{1}{([q^2])^2}\right)$$
$$\Big(i\, g_s^2\, g^{\mu\sigma} g^{\nu\rho} f^{a c \$AL\$2586} f^{b c \$AL\$2586}-$$
$$i\, g_s^2\, g^{\mu\nu} g^{\rho\sigma}\left(f^{a c \$AL\$2587} f^{b c \$AL\$2587}+f^{a c \$AL\$2588} f^{b c \$AL\$2588}\right)+i\, g_s^2\, g^{\mu\rho} g^{\nu\sigma} f^{a c \$AL\$2590} f^{b c \$AL\$2590}\Big)$$

Is there a simple way to extract all the unique loop integrals that appear in this expression, like we can do for Dirac and Color strcutures.
Yes, if we use **FCLoopIsolate**

```
ampIso = FCLoopIsolate[amp, {q}, Head → loopInt]
```

$$-\mathrm{loopInt}\left(\frac{(q-p)^\mu q^\nu}{q^2.(q-p)^2}\right) f^{a c d} f^{b c d} g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\mathrm{loopInt}\left(\frac{(q-p)^{\mathrm{Lor5}}(p-q)^{\mathrm{Lor6}}(2p-q)^\rho q^\rho q^\sigma (q-2p)^\sigma}{q^2.q^2.(q-p)^2.(q-p)^2}\right) g^{\mathrm{Lor5}\,\mu} g^{\mathrm{Lor6}\,\nu} f^{a c d} f^{b c d} g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\mathrm{loopInt}\left(\frac{(2p-q)^\rho q^\rho q^\sigma (q-2p)^\sigma}{q^2.q^2.(q-p)^2}\right) g^{\mathrm{Lor5\,Lor6}} g^{\mathrm{Lor5}\,\mu} g^{\mathrm{Lor6}\,\nu} f^{a c d} f^{b c d} g_s^2+$$

$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\nu}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\nu}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor6}\,\nu}\,g^{\mu\,\rho}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor6}\,\nu}\,g^{\mu\,\rho}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor6}\,\sigma}\,g^{\mu\,\rho}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor6}\,\sigma}\,g^{\mu\,\rho}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\mu}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\mu}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)^2\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)g^{\mu\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p+q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\mu\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(2\,p-q)^{\rho}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2-$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2+$$

$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\mu}\,g^{\text{Lor6}\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\text{Lor6}\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor6}\,\nu}\,g^{\mu\,\rho}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor6}\,\nu}\,g^{\mu\,\rho}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor6}\,\sigma}\,g^{\mu\,\rho}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor6}\,\sigma}\,g^{\mu\,\rho}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\mu}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\mu}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\text{Lor5}\,\rho}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\text{Lor5}\,\rho}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}}{q^2.(q-p)^2.(q-p)^2}\right)g^{\mu\,\rho}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p+q)^{\text{Lor6}}}{q^2.(q-p)^2}\right)g^{\text{Lor5}\,\text{Lor6}}\,g^{\mu\,\rho}\,g^{\nu\,\sigma}\,g^{\rho\,\sigma}\,f^{a\,c\,d}\,f^{b\,c\,d}\,g_s^2 -$$
$$\frac{1}{2}\,\text{loopInt}\left(\frac{1}{q^2}\right)g^{\rho\,\sigma}\left(-g^{\mu\,\sigma}\,g^{\nu\,\rho}\,f^{a\,c\,\$\text{AL}\$2586}\,f^{b\,c\,\$\text{AL}\$2586}+g^{\mu\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,\$\text{AL}\$2587}\,f^{b\,c\,\$\text{AL}\$2587}+\right.$$
$$\left.g^{\mu\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,\$\text{AL}\$2588}\,f^{b\,c\,\$\text{AL}\$2588}-g^{\mu\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,\$\text{AL}\$2590}\,f^{b\,c\,\$\text{AL}\$2590}\right)g_s^2 +$$
$$\frac{1}{2}(\xi-1)\,\text{loopInt}\left(\frac{q^{\rho}\,q^{\sigma}}{q^2.q^2}\right)\left(g^{\mu\,\sigma}\,g^{\nu\,\rho}\,f^{a\,c\,\$\text{AL}\$2586}\,f^{b\,c\,\$\text{AL}\$2586}-g^{\mu\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,\$\text{AL}\$2587}\,f^{b\,c\,\$\text{AL}\$2587}-\right.$$
$$\left.g^{\mu\,\nu}\,g^{\rho\,\sigma}\,f^{a\,c\,\$\text{AL}\$2588}\,f^{b\,c\,\$\text{AL}\$2588}+g^{\mu\,\rho}\,g^{\nu\,\sigma}\,f^{a\,c\,\$\text{AL}\$2590}\,f^{b\,c\,\$\text{AL}\$2590}\right)g_s^2 +$$
$$\text{loopInt}\Big(\text{tr}\Big(-\text{MQ}^2\,\gamma^{\nu}.\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2-\text{MQ}\,\gamma^{\nu}.(\gamma\cdot p).\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2+$$
$$\text{MQ}\,\gamma^{\nu}.(\gamma\cdot q).\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2+\text{MQ}\,(\gamma\cdot q).\gamma^{\nu}.\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2+(\gamma\cdot q).\gamma^{\nu}.(\gamma\cdot p).\gamma^{\mu}$$
$$T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2-(\gamma\cdot q).\gamma^{\nu}.(\gamma\cdot q).\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2\Big)\Big/\big(q^2-\text{MQ}^2\big).\big((q-p)^2-\text{MQ}^2\big)\Big)$$

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Cases2[ampIso, loopInt]
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$$\Big\{\text{loopInt}\left(\frac{1}{q^2}\right),$$
$$\text{loopInt}\Big(\text{tr}\Big(-\text{MQ}^2\,\gamma^{\nu}.\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2-\text{MQ}\,\gamma^{\nu}.(\gamma\cdot p).\gamma^{\mu}\,T^a_{\text{Col4 Col3}}\,T^b_{\text{Col3 Col4}}\,g_s^2+\text{MQ}\,\gamma^{\nu}.(\gamma\cdot q).\gamma^{\mu}\,T^a_{\text{Col4 Col3}}$$

$T^{b}_{\text{Col3 Col4}}\, g_s^2 + \text{MQ}\, (\gamma\cdot q).\gamma^{\nu}.\gamma^{\mu}\, T^{a}_{\text{Col4 Col3}}\, T^{b}_{\text{Col3 Col4}}\, g_s^2 + (\gamma\cdot q).\gamma^{\nu}.(\gamma\cdot p).\gamma^{\mu}\, T^{a}_{\text{Col4 Col3}}\, T^{b}_{\text{Col3 Col4}}\, g_s^2 -$
$(\gamma\cdot q).\gamma^{\nu}.(\gamma\cdot q).\gamma^{\mu}\, T^{a}_{\text{Col4 Col3}}\, T^{b}_{\text{Col3 Col4}}\, g_s^2)\big/\big(q^2 - \text{MQ}^2\big).\big((q-p)^2 - \text{MQ}^2\big)\big),$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p+q)^{\text{Lor6}}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}}{q^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(q-p)^{\mu}\,q^{\nu}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{q^{\rho}\,q^{\sigma}}{q^2.q^2}\right),\ \text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p+q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(p-2\,q)^{\nu}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p+q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(p-2\,q)^{\nu}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right),$$

$$\text{loopInt}\left(\frac{(2\,p-q)^{\rho}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,(q-2\,p)^{\sigma}}{q^2.(q-p)^2.(q-p)^2}\right),$$
$$\text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(-p-q)^{\text{Lor5}}\,(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$
$$\text{loopInt}\left(\frac{(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,q-p)^{\mu}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right),$$
$$\text{loopInt}\left(\frac{(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2}\right),\ \text{loopInt}\left(\frac{(q-p)^{\text{Lor5}}\,(p-q)^{\text{Lor6}}\,(2\,p-q)^{\rho}\,q^{\rho}\,q^{\sigma}\,(q-2\,p)^{\sigma}}{q^2.q^2.(q-p)^2.(q-p)^2}\right)\Big\}$$

The number of list elements will be much smaller if we first contract all the free Lorentz indices

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ampIso = FCLoopIsolate[Contract[amp], {q}, Head → loopInt];
Cases2[ampIso, loopInt]
```

$$\Big\{\text{loopInt}\Big(\frac{1}{q^2}\Big), \text{loopInt}\Big(\frac{1}{q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\text{tr}\big(-\text{MQ}^2\, \gamma^\nu.\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2 - \text{MQ}\, \gamma^\nu.(\gamma\cdot p).\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2 + \text{MQ}\, \gamma^\nu.(\gamma\cdot q).\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2 + \text{MQ}\, (\gamma\cdot q).\gamma^\nu.\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2 + (\gamma\cdot q).\gamma^\nu.(\gamma\cdot p).\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2 - (\gamma\cdot q).\gamma^\nu.(\gamma\cdot q).\gamma^\mu\, T^a_{\text{Col4 Col3}}\, T^b_{\text{Col3 Col4}}\, g_s^2\big)\big/\big(q^2-\text{MQ}^2\big).\big((q-p)^2-\text{MQ}^2\big)\Big),$$

$$\text{loopInt}\Big(\frac{1}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu}{q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.(q-p)^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(q-p)^\mu\, q^\nu}{q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{p\cdot q}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.q^2.(q-p)^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.(q-p)^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{(p\cdot q)^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(p\cdot q)^2}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.q^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^2}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.(q-p)^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu\, q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu\, q^2}{q^2.q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu\, q^2}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(p\cdot q)\, q^2}{q^2.q^2.(q-p)^2}\Big),$$

$$\text{loopInt}\Big(\frac{q^4}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^4}{q^2.(q-p)^2.(q-p)^2}\Big)\Big\}$$

What about the loop momenta inside the Dirac trace?
The trick is to use **FCTraceExpand** before applying **FCLoopIsolate**

```
ampIso = FCLoopIsolate[amp // Contract // FCTraceExpand, {q}, Head → loopInt];
Cases2[ampIso, loopInt]
```

$$\Big\{\text{loopInt}\Big(\frac{1}{q^2}\Big), \text{loopInt}\Big(\frac{1}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{1}{(q^2-\text{MQ}^2).((q-p)^2-\text{MQ}^2)}\Big),$$
$$\text{loopInt}\Big(\frac{\text{tr}(\gamma^\nu.(\gamma\cdot q).\gamma^\mu)}{(q^2-\text{MQ}^2).((q-p)^2-\text{MQ}^2)}\Big), \text{loopInt}\Big(\frac{\text{tr}((\gamma\cdot q).\gamma^\nu.\gamma^\mu)}{(q^2-\text{MQ}^2).((q-p)^2-\text{MQ}^2)}\Big),$$
$$\text{loopInt}\Big(\frac{\text{tr}((\gamma\cdot q).\gamma^\nu.(\gamma\cdot p).\gamma^\mu)}{(q^2-\text{MQ}^2).((q-p)^2-\text{MQ}^2)}\Big), \text{loopInt}\Big(\frac{\text{tr}((\gamma\cdot q).\gamma^\nu.(\gamma\cdot q).\gamma^\mu)}{(q^2-\text{MQ}^2).((q-p)^2-\text{MQ}^2)}\Big), \text{loopInt}\Big(\frac{1}{q^2.(q-p)^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^\mu}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(q-p)^\mu\, q^\nu}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{p\cdot q}{q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\,(p\cdot q)}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu\,(p\cdot q)}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(p\cdot q)^2}{q^2.q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{(p\cdot q)^2}{q^2.q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.q^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^2}{q^2.q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^2}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\nu\, q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^\mu\, q^\nu\, q^2}{q^2.q^2.(q-p)^2}\Big),$$
$$\text{loopInt}\Big(\frac{q^\mu\, q^\nu\, q^2}{q^2.(q-p)^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{(p\cdot q)\, q^2}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^4}{q^2.q^2.(q-p)^2}\Big), \text{loopInt}\Big(\frac{q^4}{q^2.(q-p)^2.(q-p)^2}\Big)\Big\}$$

FCTraceExpand expands Dirac traces using linearity, without computing anything

```
DiracTrace[GA[i] + GA[j]]
FCTraceExpand[%]
```

$\text{tr}(\overline{\gamma}^i + \overline{\gamma}^j)$

$\text{tr}(\overline{\gamma}^i) + \text{tr}(\overline{\gamma}^j)$

FCLoopIsolate is equally useful for any number of loops. However, if we are explicitly interested in the multi-loop integrals,
we should also know the FCLoopBasis* functions

FCLoopBasisIncompleteQ acts on a single loop integral and returns True, if the propagators of this integral do not form a basis.

```
FAD[{q1, m1}, {q1 - l + p, m}]
FCLoopBasisIncompleteQ[%, {q1}]
```

$$\frac{1}{([\text{q1}^2-\text{m1}^2]).([(-l+p+\text{q1})^2-m^2])}$$

True

```
FCI[FAD[{q1, m1}, {q1 - l + p, m}]]
FCLoopBasisIncompleteQ[%, {q1}]
```

$$\frac{1}{(\text{q1}^2-\text{m1}^2).((-l+p+\text{q1})^2-m^2)}$$

True

What is missing in the propagator basis of this integral? **FCLoopBasisFindCompletion** can tell

```
FCLoopBasisFindCompletion[FCI[FAD[{q1, m1}, {q1 - l + p, m}]], {q1}]
```

$$\left\{\frac{1}{(\text{q1}^2-\text{m1}^2).((-l+p+\text{q1})^2-m^2)}, \{l\cdot \text{q1}\}\right\}$$

Let us go to two 2-loops

```
SPD[q1, q2] FAD[{q1, m1}, {q2, m2}]
FCLoopBasisIncompleteQ[%, {q1, q2}]
```

$$(\text{q1}\cdot \text{q2})\,\frac{1}{([\text{q1}^2-\text{m1}^2]).([\text{q2}^2-\text{m2}^2])}$$

False

```
FAD[q1, q2, {q1 - l1, m1}, {q2 - l2, m2}]
FCLoopBasisIncompleteQ[%, {q1, q2}]
```

$$\frac{1}{([\text{q1}^2]).([\text{q2}^2]).([(\text{q1}-\text{l1})^2-\text{m1}^2]).([(\text{q2}-\text{l2})^2-\text{m2}^2])}$$

True

And the missing propagators are

```
FCLoopBasisFindCompletion[FAD[q1, q2, {q1 - l1, m1}, {q2 - l2, m2}], {q1, q2}]
```

$$\left\{\frac{1}{\text{q1}^2.\text{q2}^2.((\text{q1}-\text{l1})^2-\text{m1}^2).((\text{q2}-\text{l2})^2-\text{m2}^2)}, \{\text{l1}\cdot \text{q2}, \text{l2}\cdot \text{q1}, \text{q1}\cdot \text{q2}\}\right\}$$

How about integrals that have an overdetermined basis of propagators?

```
SPD[q1, q2] SPD[q1, l2] SPD[q2, l1] FAD[q1, q2, {q1 - l1, m1}, {q2 - l2, m2}]
FCLoopBasisOverdeterminedQ[%, {q1, q2}]
```

$$(\text{l1}\cdot \text{q2})\,(\text{l2}\cdot \text{q1})\,(\text{q1}\cdot \text{q2})\,\frac{1}{([\text{q1}^2]).([\text{q2}^2]).([(\text{q1}-\text{l1})^2-\text{m1}^2]).([(\text{q2}-\text{l2})^2-\text{m2}^2])}$$

False

```
int = FAD[q1, q2, {q1 + l, m1}, {q1 - l, m1}, {q2 + l, m1}, {q2 - l, m1}]
FCLoopBasisOverdeterminedQ[int, {q1, q2}]
```

$$1/([\text{q1}^2]).([\text{q2}^2]).([(l+\text{q1})^2-\text{m1}^2]).([(\text{q1}-l)^2-\text{m1}^2]).([(l+\text{q2})^2-\text{m1}^2]).([(\text{q2}-l)^2-\text{m1}^2])$$

True

Overdetermined basis means that the integral can partial-fractioned with **ApartFF**

```
res = ApartFF[int, {q1, q2}]
```

$$-\left(2\Big/\left(\left(\text{m1}^2-l^2\right)^2\,\text{q1}^2.\left((\text{q1}-l)^2-\text{m1}^2\right).\left((\text{q2}-l)^2-\text{m1}^2\right).\left((l+\text{q2})^2-\text{m1}^2\right)\right)\right)+$$
$$1\Big/\left(\left(\text{m1}^2-l^2\right)^2\left((\text{q1}-l)^2-\text{m1}^2\right).\left((l+\text{q1})^2-\text{m1}^2\right).\left((\text{q2}-l)^2-\text{m1}^2\right).\left((l+\text{q2})^2-\text{m1}^2\right)\right)+$$
$$\frac{1}{\left(\text{m1}^2-l^2\right)^2\,\text{q2}^2.\text{q1}^2.\left((\text{q1}-l)^2-\text{m1}^2\right).\left((\text{q2}-l)^2-\text{m1}^2\right)}$$

Notice that for every single integral in the output of **ApartFF** the basis is not anymore overdetermined

```
FCLoopBasisOverdeterminedQ[#, {q1, q2}] & /@ (List @@ res)
```

{False, False, False}

FeynHelpers

Now the have developed some understanding for the manipulations of loop integrals in FeynCalc, we can start using FeynHelpers to make our life much easier

The syntax of FeynHelpers is, in fact, very simple. There are essentially only two commands (**PaXEvalute** and **FIREBurn**)
that need minimal user input and handle the whole communication between FeynCalc and Package-X or FIRE

Of course, these functions also have some options, but most of them are rarely needed.

PaXEvaluate

Let us start with something simple, e. g. the A0 function in the standard normalization of Denner (used in LoopTools and many other packages)

```
A0[m^2]
PaXEvaluate[%]
```

$$\text{A}_0\left(m^2\right)$$

$$\frac{m^2}{\varepsilon}-m^2\left(-\log\left(\frac{\mu^2}{\pi\,m^2}\right)+\gamma-1\right)$$

Want to be sure that it is a UV-divergent integral?

```
res = PaXEvaluateUVIRSplit[A0[m^2]]
```

$$\frac{m^2}{\varepsilon_{\text{UV}}}-m^2\left(-\log\left(\frac{\mu^2}{\pi\,m^2}\right)+\gamma-1\right)$$

We can also abbreviate the singularity structure with **SMP**[“Delta_UV”]

```
FCHideEpsilon[res]
```

$$m^2\,\Delta_{\text{UV}}-m^2\left(-\log\left(\frac{\mu^2}{\pi\,m^2}\right)-1+\log(4\,\pi)\right)$$

or make it explicit again

```
FCShowEpsilon[%]
```

$$m^2\left(\frac{1}{\varepsilon_{\text{UV}}}-\gamma+\log(4\pi)\right)-m^2\left(-\log\left(\frac{\mu^2}{\pi m^2}\right)-1+\log(4\pi)\right)$$

To evaluate FAD-type integrals, we need to specify the loop momentum explicitly.
Furthermore, when we enter loop integrals as FAD*SPD, we usually also imply that they have the standard normalization with $\frac{1}{(2\pi)^D}$
We do not have to write the prefactor out explicitly, but we must of course include it when we are evaluating our master integrals
symbolically or numerically.

In **PaXEvaluate** this can be taken into account via the option **PaXImplicitPrefactor**

```
FAD[{q, m}]
PaXEvaluate[%, q, PaXImplicitPrefactor → 1 / (2 Pi) ^ (4 - 2 Epsilon)]
```

$$\frac{1}{[q^2-m^2]}$$

$$\frac{i\, m^2}{16\pi^2\varepsilon}-\frac{i\, m^2\left(-\log\left(\frac{\mu^2}{m^2}\right)+\gamma-1-\log(4\pi)\right)}{16\pi^2}$$

When it comes to the evaluation of functions with complicated kinematics, Package-X is sometimes a bit stubborn

```
PaXEvaluate[PaVe[0, 0, 1, {SP[p, p], SP[p, p], m^2}, {m^2, m^2, m^2}]]
```

$$-\frac{9 m^2 \overline{p}^4 \mathrm{C}_0(m^2, \overline{p}^2, \overline{p}^2, m^2, m^2, m^2)}{2 (m^2 - 4 \overline{p}^2)^2} + \frac{\overline{p}^6 \mathrm{C}_0(m^2, \overline{p}^2, \overline{p}^2, m^2, m^2, m^2)}{(m^2 - 4 \overline{p}^2)^2} - \frac{m^6 \mathrm{C}_0(m^2, \overline{p}^2, \overline{p}^2, m^2, m^2, m^2)}{2 (m^2 - 4 \overline{p}^2)^2} + \frac{3 m^4 \overline{p}^2 \mathrm{C}_0(m^2, \overline{p}^2, \overline{p}^2, m^2, m^2, m^2)}{(m^2 - 4 \overline{p}^2)^2} - \frac{\sqrt{3}\, \pi m^2 \overline{p}^2}{(m^2 - 4 \overline{p}^2)^2} - \frac{11 m^2}{36 (m^2 - 4 \overline{p}^2)} + \frac{\pi \overline{p}^4}{\sqrt{3}\, (m^2 - 4 \overline{p}^2)^2} + \frac{19 \overline{p}^2}{18 (m^2 - 4 \overline{p}^2)} - \frac{7 m^2 \sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} \log\left(\frac{\sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} - \overline{p}^2 + 2 m^2}{2 m^2}\right)}{3 (m^2 - 4 \overline{p}^2)^2} - \frac{\overline{p}^2 \sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} \log\left(\frac{\sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} - \overline{p}^2 + 2 m^2}{2 m^2}\right)}{3 (m^2 - 4 \overline{p}^2)^2} + \frac{\sqrt{3}\, \pi m^4}{4 (m^2 - 4 \overline{p}^2)^2} + \frac{2 m^4 \sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} \log\left(\frac{\sqrt{\overline{p}^2 (\overline{p}^2 - 4 m^2)} - \overline{p}^2 + 2 m^2}{2 m^2}\right)}{3 \overline{p}^2 (m^2 - 4 \overline{p}^2)^2} - \frac{1}{12 \varepsilon} + \frac{1}{36}\left(-3 \log\left(\frac{\mu^2}{m^2}\right) + 3 \gamma - 3 \log(4 \pi) + 6 \log(\pi) + \log(64)\right)$$

It knows that there is no simple way to express the full result for the C_0 function, so it prefers to keep in the implicit form.

Using the option **PaXC0Expand** we can obtain (admittedly quite large and complicated) result nonetheless

```
PaXEvaluate[
 PaVe[0, 0, 1, {SP[p, p], SP[p, p], m^2}, {m^2, m^2, m^2}], PaXC0Expand → True]
```

$$\text{ConditionalExpression}\Big[\frac{1}{12}\left(-\log(4\pi)+\gamma-\frac{1}{\varepsilon}\right)-\frac{1}{12}\log\left(\frac{\mu^2}{m^2}\right)+$$

$$\Big(\log\left(\frac{2m^2-\overline{p}^2+\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}}{2m^2}\right)\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\,(2m^4-7\overline{p}^2m^2-\overline{p}^{2\,2})\Big)\Big/\left(3(m^2-4\overline{p}^2)^2\,\overline{p}^2\right)+$$

$$\frac{\pi(3m^4-12\overline{p}^2m^2+4\overline{p}^{2\,2})}{4\sqrt{3}\,(m^2-4\overline{p}^2)^2}-$$

$$\frac{1}{2(m^2-4\overline{p}^2)^2}\Big(\text{Li}_2\Big(\Big(\big(i(i+\sqrt{3})(\sqrt{m^2-4\overline{p}^2}-m)m^3+2(1-i\sqrt{3})(\sqrt{m^2-4\overline{p}^2}-2m)\overline{p}^2m+4\overline{p}^{2\,2}\big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(m^2-2\overline{p}^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)+$$

$$\text{Li}_2\Big(\Big(\big(-i(-i+\sqrt{3})(\sqrt{m^2-4\overline{p}^2}-m)m^3+2(1+i\sqrt{3})(\sqrt{m^2-4\overline{p}^2}-2m)\overline{p}^2m+4\overline{p}^{2\,2}\big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(-m^2+2\overline{p}^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)-$$

$$\text{Li}_2\Big(\Big(\big((1+i\sqrt{3})(m+\sqrt{m^2-4\overline{p}^2})m^3-2i(-i+\sqrt{3})(2m+\sqrt{m^2-4\overline{p}^2})\overline{p}^2m+4\overline{p}^{2\,2}\big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(m^2-2\overline{p}^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)-$$

$$\text{Li}_2\Big(\Big(\big((1-i\sqrt{3})(m+\sqrt{m^2-4\overline{p}^2})m^3+2i(i+\sqrt{3})(2m+\sqrt{m^2-4\overline{p}^2})\overline{p}^2m+4\overline{p}^{2\,2}\big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(-m^2+2\overline{p}^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)-$$

$$\Big(2\,\text{Li}_2\Big(\Big(m\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\,(m-\sqrt{m^2-4\overline{p}^2})+\overline{p}^2\big(m^2-\sqrt{m^2-4\overline{p}^2}\,m-4\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\big)\Big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(\sqrt{m^4-4m^2\overline{p}^2}-m^2\big)\epsilon\Big)\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)+$$

$$\Big(2\,\text{Li}_2\Big(\Big(m\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\,(m+\sqrt{m^2-4\overline{p}^2})+\overline{p}^2\big(m^2+\sqrt{m^2-4\overline{p}^2}\,m-4\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\big)\Big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(m^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)-$$

$$\Big(2\,\text{Li}_2\Big(\Big(m\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\,(\sqrt{m^2-4\overline{p}^2}-m)+\overline{p}^2\big(m^2-\sqrt{m^2-4\overline{p}^2}\,m+4\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\big)\Big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(m^2+\sqrt{m^4-4m^2\overline{p}^2}\big)\epsilon\Big)\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)+$$

$$\Big(2\,\text{Li}_2\Big(\Big(\overline{p}^2\big(m^2+\sqrt{m^2-4\overline{p}^2}\,m+4\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\big)-m(m+\sqrt{m^2-4\overline{p}^2})\sqrt{\overline{p}^2(\overline{p}^2-4m^2)}\Big)\Big/$$

$$\big(4(m^4-4\overline{p}^2m^2+\overline{p}^{2\,2})\big)+i\big(\sqrt{m^4-4m^2\overline{p}^2}-m^2\big)\epsilon\Big)\Big)\Big/\Big(\sqrt{m^2(m^2-4\overline{p}^2)}\Big)\Big)$$

$$(m^6-6\overline{p}^2m^4+9\overline{p}^{2\,2}m^2-2\overline{p}^{2\,3})-\frac{11m^2-38\overline{p}^2}{36(m^2-4\overline{p}^2)}+\frac{\log(\pi)}{6}+\frac{\log(2)}{6},\ m^4-4$$

$$m^2$$

$$\overline{p}^2>0\Big]$$

Notice that if we are interested only in the UV-divergent piece, we can get it rather easily without going through the full result

```
PaXEvaluateUV[
 PaVe[0, 0, 1, {SP[p, p], SP[p, p], m^2}, {m^2, m^2, m^2}], PaXC0Expand → True]
```

$$-\frac{1}{12\,\varepsilon_{\mathrm{UV}}}$$

Package-X can also expand coefficient functions in their arguments, which is also very useful for many applications.

In **PaXEvaluate** the corresponding option is called **PaXSeries**

```
B0[SPD[p1, p1], m1^2, m2^2]
```

$$\mathrm{B}_0(\mathrm{p1}^2, \mathrm{m1}^2, \mathrm{m2}^2)$$

```
PaXEvaluate[%, PaXSeries → {{m1, 0, 2}}]
```

X`LoopRefineSeries: Taylor series of X`PVB does not exist near m1 = 0.

PaXEvaluate: PaXEvaluate has encountered an error and must abort the evaluation. The error description reads: resultX is not a List.

$Aborted

By default Package-X tries to do expansion around the final results, which is usually not what we want. With the option
PaXAnalytic the expansion will be done on the level of Feynman parameters, as one would do it by hand

```
PaXEvaluate[B0[SPD[p1, p1], m1^2, m2^2], PaXSeries → {{m1, 0, 2}}, PaXAnalytic → True]
```

$$\frac{\mathrm{m1}^2+\mathrm{m2}^2}{\varepsilon\,\mathrm{m2}^2-\varepsilon\,\mathrm{p1}^2}-\frac{1}{\mathrm{m2}^2-\mathrm{p1}^2}\left(\mathrm{m1}^2\left(-\log\left(\frac{\mu^2}{\mathrm{m2}^2}\right)\right)+\gamma\,\mathrm{m1}^2+\mathrm{m1}^2\log(\pi)-\mathrm{m2}^2\log\left(\frac{\mu^2}{\mathrm{m2}^2}\right)+\gamma\,\mathrm{m2}^2-2\,\mathrm{m2}^2+\mathrm{m2}^2\log(\pi)\right)+$$
$$\frac{(\mathrm{m1}^2+2\,\mathrm{m2}^2)\log\left(\frac{\mathrm{m2}^2}{\mathrm{m2}^2-\mathrm{p1}^2}\right)}{\mathrm{m2}^2-\mathrm{p1}^2}-\frac{\mathrm{m2}^2\,(\mathrm{m2}-\mathrm{m1})\,(\mathrm{m1}+\mathrm{m2})\log\left(\frac{\mathrm{m2}^2}{\mathrm{m2}^2-\mathrm{p1}^2}\right)}{\mathrm{p1}^2\,(\mathrm{m2}^2-\mathrm{p1}^2)}-$$
$$\frac{\mathrm{p1}^2}{\varepsilon\,\mathrm{m2}^2-\varepsilon\,\mathrm{p1}^2}+\frac{\mathrm{p1}^2\left(-\log\left(\frac{\mu^2}{\mathrm{m2}^2}\right)+\gamma-2+\log(\pi)\right)}{\mathrm{m2}^2-\mathrm{p1}^2}-\frac{\mathrm{p1}^2\log\left(\frac{\mathrm{m2}^2}{\mathrm{m2}^2-\mathrm{p1}^2}\right)}{\mathrm{m2}^2-\mathrm{p1}^2}$$

We can also do a double expansion in m1 and m2

```
PaXEvaluate[B0[SPD[p1, p1], m1^2, m2^2],
 PaXSeries → {{m1, 0, 2}, {m2, 0, 2}}, PaXAnalytic → True]
```

$$\frac{1}{\varepsilon}-\frac{2\,m1^2\,m2^2}{\varepsilon\,p1^4}-\frac{m1^2+m2^2}{\varepsilon\,p1^2}-\frac{2\,m1^2\,m2^2\log\left(-\frac{\mu^2}{p1^2}\right)}{p1^4}-\frac{\left(m1^2+m2^2\right)\log\left(-\frac{\mu^2}{p1^2}\right)}{p1^2}+\frac{2\,m1^2\,m2^2\,(-1+\gamma+\log(\pi))}{p1^4}+\frac{(\gamma+\log(\pi))\left(m1^2+m2^2\right)}{p1^2}+\log\left(-\frac{\mu^2}{p1^2}\right)-\gamma+2-\log(\pi)$$

Where do the poles come form?

```
PaXEvaluateUVIRSplit[B0[SPD[p1, p1], m1^2, m2^2],
 PaXSeries → {{m1, 0, 2}}, PaXAnalytic → True]
```

$$-\frac{1}{\varepsilon_{IR}}+\frac{m1^2+m2^2-p1^2}{m2^2\,\varepsilon_{IR}-p1^2\,\varepsilon_{IR}}-\frac{1}{p1^2\left(m2^2-p1^2\right)}\left(-m1^2\,p1^2\log\left(\frac{\mu^2}{m2^2}\right)-m1^2\,m2^2\log\left(\frac{m2^2}{m2^2-p1^2}\right)-m1^2\,p1^2\log\left(\frac{m2^2}{m2^2-p1^2}\right)+\gamma\,m1^2\,p1^2+m1^2\,p1^2\log(\pi)-m2^2\,p1^2\log\left(\frac{\mu^2}{m2^2}\right)+p1^4\log\left(\frac{\mu^2}{m2^2}\right)+\gamma\,m2^2\,p1^2-2\,m2^2\,p1^2-2\,m2^2\,p1^2\log\left(\frac{m2^2}{m2^2-p1^2}\right)+m2^2\,p1^2\log(\pi)+p1^4\log\left(\frac{m2^2}{m2^2-p1^2}\right)+m2^4\log\left(\frac{m2^2}{m2^2-p1^2}\right)-\gamma\,p1^4+2\,p1^4-p1^4\log(\pi)\right)+\frac{1}{\varepsilon_{UV}}$$

```
PaXEvaluateUVIRSplit[B0[SPD[p1, p1], m1^2, m2^2],
 PaXSeries → {{m1, 0, 2}, {m2, 0, 2}}, PaXAnalytic → True]
```

$$-\frac{2\,m1^2\,m2^2+m1^2\,p1^2+m2^2\,p1^2}{p1^4\,\varepsilon_{IR}}+\frac{1}{p1^4}\left(-2\,m1^2\,m2^2\log\left(-\frac{\mu^2}{p1^2}\right)+2\,\gamma\,m1^2\,m2^2-2\,m1^2\,m2^2+2\,m1^2\,m2^2\log(\pi)-m1^2\,p1^2\log\left(-\frac{\mu^2}{p1^2}\right)+\gamma\,m1^2\,p1^2+m1^2\,p1^2\log(\pi)-m2^2\,p1^2\log\left(-\frac{\mu^2}{p1^2}\right)+\gamma\,m2^2\,p1^2+m2^2\,p1^2\log(\pi)+p1^4\log\left(-\frac{\mu^2}{p1^2}\right)-\gamma\,p1^4+2\,p1^4-p1^4\log(\pi)\right)+\frac{1}{\varepsilon_{UV}}$$

FIREBurn

FIREBurn is still somewhat incomplete, because it uses only a small amount of capabilities of FIRE. There are
definitely many things that can be improved here (especially the performance)

Nevertheless, for small calculations **FIREBurn** (even in its current form) turns out to be extremely useful.

The syntax is very simple: We just need to specify the loop and the external momenta, while the inter- face takes
care of the rest

Let us first start with something simple

```
FAD[{q, m, 4}]
```

$$\frac{1}{([q^2-m^2])^4}$$

```
FIREBurn[%, {q}, {}]
```

```
FIREBurn: Processing integral 1 of 1; IBP-reduction done, timing: 1.627
```

$$\frac{(D-6)(D-4)(D-2)}{48\,m^6\,(q^2-m^2)}$$

```
FAD[{q, m1, 2}, {q + p, m2, 2}]
```

$$\frac{1}{([q^2-\text{m1}^2])^2.([(p+q)^2-\text{m2}^2])^2}$$

```
res = FIREBurn[%, {q}, {p}]
```

```
FIREBurn: Processing integral 1 of 1; IBP-reduction done, timing: 1.193
```

$$\left((D-2)\left(2\,D\,\text{m1}^4-2\,D\,\text{m1}^2\,\text{m2}^2-2\,D\,\text{m1}^2\,p^2-9\,\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2+8\,\text{m1}^2\,p^2+\text{m2}^4-2\,\text{m2}^2\,p^2+p^4\right)\right)\Big/ \left(2\,\text{m1}^2\,(q^2-\text{m1}^2)\,(\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\right)+ \left((D-2)\left(-2\,D\,\text{m1}^2\,\text{m2}^2+2\,D\,\text{m2}^4-2\,D\,\text{m2}^2\,p^2+\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2-2\,\text{m1}^2\,p^2-9\,\text{m2}^4+8\,\text{m2}^2\,p^2+p^4\right)\right)\Big/ \left(2\,\text{m2}^2\,(\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,((p+q)^2-\text{m2}^2)\right)- \left((D-3)\left(D\,\text{m1}^4-2\,D\,\text{m1}^2\,\text{m2}^2+D\,\text{m2}^4-D\,p^4-4\,\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2-2\,\text{m1}^2\,p^2-4\,\text{m2}^4-2\,\text{m2}^2\,p^2+6\,p^4\right)\right)\Big/ \left((\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(q^2-\text{m1}^2).((p+q)^2-\text{m2}^2)\right)$$

Notice that the output of **FIREBurn** can be often simplified further with **FDS**

```
FDS[res, q]
```

$$\left((D-2)\left(2\,D\,\text{m1}^4-2\,D\,\text{m1}^2\,\text{m2}^2-2\,D\,\text{m1}^2\,p^2-9\,\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2+8\,\text{m1}^2\,p^2+\text{m2}^4-2\,\text{m2}^2\,p^2+p^4\right)\right)\Big/ \left(2\,\text{m1}^2\,(q^2-\text{m1}^2)\,(\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\right)+ \left((D-2)\left(-2\,D\,\text{m1}^2\,\text{m2}^2+2\,D\,\text{m2}^4-2\,D\,\text{m2}^2\,p^2+\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2-2\,\text{m1}^2\,p^2-9\,\text{m2}^4+8\,\text{m2}^2\,p^2+p^4\right)\right)\Big/ \left(2\,\text{m2}^2\,(q^2-\text{m2}^2)\,(\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\right)- \left((D-3)\left(D\,\text{m1}^4-2\,D\,\text{m1}^2\,\text{m2}^2+D\,\text{m2}^4-D\,p^4-4\,\text{m1}^4+8\,\text{m1}^2\,\text{m2}^2-2\,\text{m1}^2\,p^2-4\,\text{m2}^4-2\,\text{m2}^2\,p^2+6\,p^4\right)\right)\Big/ \left((\text{m1}^2-2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(\text{m1}^2+2\,\text{m1}\,\text{m2}+\text{m2}^2-p^2)^2\,(q^2-\text{m1}^2).((q-p)^2-\text{m2}^2)\right)$$

Furthermore, FIREBurn measures the time required to process each integral with FIRE. If you are not interested
to know this, use the option **Timing**

```
FIREBurn[FAD[{q, m, 4}], {q}, {}, Timing → False]
```

$$\frac{(D-6)(D-4)(D-2)}{48\,m^6\,(q^2-m^2)}$$

The following integral actually has an incomplete basis of propagators

```
int = FAD[{q, 0, 2}, {q - p - k}]
```

$$\frac{1}{([q^2])^2.([(-k-p+q)^2])}$$

FIREBurn can handle it nonetheless

```
FIREBurn[int, {q}, {p, k}]
```

```
FIREBurn: Processing integral 1 of 1; IBP-reduction done, timing: 1.234
```

$$-\frac{D-3}{q^2.(-k-p+q)^2\left(2\,(k\cdot p)+k^2+p^2\right)}$$

This is because FIREBurn internally uses **FCLoopBasisFindCompletion**

```
FCLoopBasisFindCompletion[int, {q}]
```

$$\left\{\frac{1}{q^2.q^2.(-k-p+q)^2}, \{k\cdot q\}\right\}$$

However, we also may tell it explicitly, which propagators should be added. This is done via the option **FIREAddPropagators**

```
FIREBurn[FAD[{q, 0, 2}, {q - p - k}], {q}, {p, k}, FIREAddPropagators → {SPD[q, k] - m}]
```

```
FIREBurn: Processing integral 1 of 1; IBP-reduction done, timing: 1.212
```

$$-\frac{D-3}{q^2.(-k-p+q)^2\left(2\,(k\cdot p)+k^2+p^2\right)}$$

Here the result is the same. However for multi-loop integrals the choice of the missing propagators might influence the final result.

Again, we can use FIRE also with multi-loop integrals

```
FIREBurn[FAD[{q1, m, 2}, {q1 + q3, m}, {q2 - q3}, {q2, 0, 2}], {q1, q2, q3}, {p}]
```

```
FIREBurn: Processing integral 1 of 1; IBP-reduction done, timing: 2.299
```

$$-\frac{(D-3)\,(3\,D-10)\,(3\,D-8)}{16\,(2\,D-7)\,m^4\left(\text{q1}^2-m^2\right).\text{q2}^2.(\text{q2}-\text{q3})^2.\left((\text{q1}+\text{q3})^2-m^2\right)}$$

Keep in mind that FeynHelpers currently can use only the Mathematica version of FIRE.

Which means that the performance of FIRE on complicated integrals will be not as good, as if we were using the C++ back-end