FeynHelpers: Connecting FeynCalc to FIRE and Package-X

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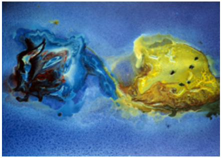


Physik-Department T30f

Outline

A generic perturbative QFT calculation may involve many different steps

- Feynman diagrams
  - Feynman rules from $\mathcal{L}$
  - Diagram generation
  - Amplitudes
  - ...
- Dirac algebra
  - Simplification of γ-matrix chains
  - Dirac traces
  - SPVAT form
  - Fierz identities
  - ...
- Loop integrals
  - Tensor reduction
  - Partial fractioning
  - Mapping of topologies
  - IBP-Reduction
  - Numerics
  - ...

...

We can automatize each step separately using standalone packages (e. g. FEYNARTS, LOOPTOOLS [Hahn & Perez-Victoria, 1999], FEYNRULES [Christensen & Duhr, 2008], QGRAF [Nogueira, 1993], TRACER [Jamin & Lautenbacher, 1993], FORMTRACER [Cyrol et al., 2016], FORCER [Ruijl et al., 2017], PYSECDEC [Borowka et al.,], ...) and self-written codes.

Or we can employ all-in-one packages that handle most of these steps in one framework.

Two big categories of all-in-one packages

- Fully-automatic (FORMCALC [Hahn & Perez-Victoria, 1999], GOSAM [Cullen et al., 2014], GRACE [Belanger et al., 2006], DIANA [Tentyukov & Fleischer, 2000], FDC [Wang, 2004], . . .)
- Semi-automatic (FEYNCALC [Mertig et al., 1991, Shtabovenko et al., 2016], HEPMATH [Wiebusch, 2014], PACKAGE-X [Patel, 2015], . . .)

Fully-automatic tools

Blackbox: Require only minimal user input and provide a small set of options. The code takes care of the rest.

- ⊕ Easy to use
- ⊕ Foolproof
- ⊕ Constantly good performance
- ⊕ Saves your time
- ⊖ Limited number of templated calculations
- ⊖ Difficult to extend/modify for your needs
- ⊖ Not easy to obtain intermediate results

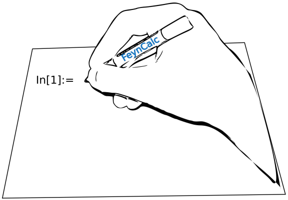
Semi-automatic tools

Toolbox: Combine different tools with many options to get the computation done in your way.

- ⊖ You must know what you are doing
- ⊖ Easy to make mistakes
- ⊖ The performance depends on your skills
- ⊖ Writing codes may take quite some time
- ⊕ Very broad range of applications
- ⊕ Extendable with user-defined objectsb
- ⊕ Intermediate results at each step

FEYNCALC

Open source (GPLv3) MATHEMATICA package for symbolic semi-automatic evaluation of Feynman diagrams and algebraic expressions in QFT.



Features

- Suitable for evaluating both single expressions and full Feynman diagrams.
- The calculation can be organized in many different ways (flexibility)
- Extensive typesetting for better readability
- Lorentz index contractions, $\mathrm{SU}(N)$ algebra, Dirac algebra, etc.
- Passarino-Veltman reduction of one-loop amplitudes to standard scalar integrals
- Basic support for manipulating multi-loop integrals
- General tools for non-commutative algebra
- BUT: Essentially only algebraic manipulations, everything else requires extra tools.

FEYNCALC developer team

- Rolf Mertig (GluonVision GmbH): original author of the package, first release 1991
- Frederik Orellana (Technical University of Denmark): joined 1997
- VS (TUM, soon Zhejiang University): joined 2014

Recent developments (since 2014)

- Large parts of the code improved or rewritten from scratch.
- Public source code repository on GITHUB: `https://github.com/FeynCalc`
- Online documentation `https://feyncalc.github.io/reference`
- Ships with many sample calculations
- Extensive unit testing framework
- New and improved functions for loop calculations.
- Big emphasis on using FEYNCALC for Effective Field Theory (EFT) calculations.
    - Original motivation for FEYNHELPERS: Matching calculations in relativistic EFTs
    - Upcoming FEYNCALC 9.3 and FEYNONIUM: Matching calculations in nonrelativistic EFTs (in particular NRQED/NRQCD [Caswell & Lepage, 1986, Bodwin et al., 1995], pNRQED/pNRQCD [Pineda & Soto, 1998b, Pineda & Soto, 1998a, Brambilla et al., 2000])

When is FEYNCALC useful?

- Small or medium-sized calculations, too specific for fully automatic packages
- FEYNCALC as a “calculator” for QFT expressions
- Cross-check results from other people
- Extensive manipulations on the level of the amplitudes

Limitations of FEYNCALC?

- Written entirely in WOLFRAM language, cannot be used without MATHEMATICA
- Inherits MATHEMATICA’s performance problems with large number of terms
- Not really suited for large and complex calculations
- Much slower than FORM

Why not combine FEYNCALC/MATHEMATICA with FORM?

- Thomas Hahn already had a similar idea many years ago.
- FORMCALC is much faster than FEYNCALC, but also less flexible
- Performance-wise it is not so clever to constantly pass very large expressions between MATHEMATICA and FORM
- However, that would be necessary(?) to preserve the flexibility of FEYNCALC
- FEYNCALCFORMLINK employs FORM for index contractions and Dirac traces.
- FORMTRACER is a recent package that provides access to FORM from MATHEMATICA

Most used functions for loop calculations

- `ApartFF`: Partial fractioning for 1-loop and multi-loop integrals
- `FDS`: Shifts in loop momenta for 1-loop and multi-loop integrals
- `TID`: Tensor reduction for 1-loop integrals
- `ToPaVe`: Converts scalar 1-loop integrals to Passarino–Veltman scalar functions
- `PaVeReduce`: Reduction of Passarino–Veltman coefficient functions to scalar functions
- `FCMultiLoopTID`: Tensor reduction for multi-loop integrals

Less known functions

- `FCLoopBasisIncompleteQ`
- `FCLoopBasisOverDeterminedQ`
- `FCLoopBasisFindCompletion`
- `FCLoopIBPReducableQ`

Partial fractioning

- Scalar loop integrals can be often simplified even further by using partial fractioning.
- Well known identities (implemented in `SPC` and `Apart2`) are

$$q \cdot p = \frac{1}{2}[(q+p)^2 + m_2^2 - (q^2 + m_1^2) - p^2 - m_2^2 + m_1^2],$$
$$\frac{1}{(q^2 - m_1^2)(q^2 - m_2^2)} = \frac{1}{m_1^2 - m_2^2}\left(\frac{1}{q^2 - m_1^2} - \frac{1}{q^2 - m_2^2}\right).$$

- But: Many decompositions, e.g.

$$\int d^D q \, \frac{1}{q^2 (q-p)^2 (q+p)^2} = \frac{1}{p^2} \int d^D q \left(\frac{1}{q^2 (q-p)^2} - \frac{1}{(q-p)^2 (q+p)^2} \right),$$

  require more sophisticated algorithms.
- New in FeynCalc 9: `ApartFF` introduces partial fractioning algorithm from [Feng, 2012]
- Compared to the reference Mathematica implementation (`https://github.com/F-Feng/APart`), it is fully integrated into FeynCalc

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In[1]:= ApartFF[FAD[{q}, {q − p}, {q + p}], {q}]

Out[2]:= $\frac{1}{p^2 q^2 . (q-p)^2} - \frac{1}{p^2 q^2 . (q-2p)^2}$

Tensor reduction

- 1-loop tensor reduction is done via Passarino-Veltman technique: `TID`
- `TID` has received many improvements in FEYNCALC 9 and above
- Default mode: Reduce each tensor integral to PaVe scalar functions (A_0, B_0, C_0, D_0)

In[1]:= FCI[GAD[μ].(m + GSD[q]).GAD[μ] FAD[{q, m}]]

Out[1]:= $\frac{\gamma^{\mu}.(m+\gamma\cdot q).\gamma^{\mu}}{(q^2-m^2).(q-p)^2}$

In[2]:= TID[%, −p + q], q]// ToPaVe[#, q]&

Out[2]= $\frac{i\pi^2(D-2)\mathrm{A}_0\left(m^2\right)\gamma\cdot p}{2p^2} - \frac{i\pi^2\mathrm{B}_0\left(p^2,0,m^2\right)\left(Dm^2\gamma\cdot p-2Dmp^2+Dp^2\gamma\cdot p-2m^2\gamma\cdot p-2p^2\gamma\cdot p\right)}{2p^2}$

Tensor reduction

- Zero Gram determinants? Detected automatically, reductions switches to Passarino–Veltman coefficient functions (e.g. B_1, B_{00}, C_{222} etc.)

  Consider $\int \frac{d^D q}{(2\pi)^D} \frac{\gamma^\mu (m + \not{q}) \gamma^\mu}{(q^2 - m^2)(q-p)^2}$ with $p^2 = 0$

In[1]:= SPD[p, p] = 0;
TID[GAD[μ].(m + GSD[q]).GAD[μ] FAD[{q, m}, −p + q], q];

Out[2]:= $i\pi^2 \mathrm{B}_0\left(0,0,m^2\right)(Dm - D\gamma\cdot p + 2\gamma\cdot p) - i\pi^2(D-2)\gamma\cdot p\mathrm{B}_1\left(0,0,m^2\right)$

- Useful options:
  - `UsePaVeBasis`: Enforces reduction into coefficient functions for any kinematics.
  - `GenPaVe`: Allows define PaVe functions in a different way (standard is the LOOPTOOLS convention)
  - `Isolate`: Kinematic coefficients in front of the loop inetgrals will be abbreviated. Use `FRH` to recover the original form.

How about multi-loop tensor reduction?

- In general, not very useful above 1-loop, many scalar products in the denominators can't be cancelled against propagators in the numerators.
- Still practical for loop momenta contracted with Dirac matrices and Levi-Civita tensors. FEYNCALC 9 features `FCMultiLoopTID`:
  - uses the same PaVe algorithm as for 1-loop.
  - currently no proper way to handle zero Gram determinants.

In [1]:= FCI[FVD[q1, μ] FVD[q2, ν] FAD[q1, q2, {q1 − p1}, {q2 − p1}, {q1 − q2}]]

Out[1]:= $$\frac{q1^{\mu} q2^{\nu}}{q1^2.q2^2.(q1-p1)^2.(q2-p1)^2.(q1-q2)^2}$$

In [2]:= FCMultiLoopTID[%, {q1, q2}]

Out[2]:= $$\frac{D\,p1^{\mu} p1^{\nu} - p1^2 g^{\mu\nu}}{4(D-1)q2^2.q1^2.(q2-p1)^2.(q1-q2)^2.(q1-p1)^2} - \frac{p1^2 g^{\mu\nu} - p1^{\mu} p1^{\nu}}{2(D-1)p1^2 q2^2.q1^2.(q2-p1)^2.(q1-p1)^2} + \frac{p1^2 g^{\mu\nu} - p1^{\mu} p1^{\nu}}{(D-1)p1^2 q2^2.q1^2.(q1-q2)^2.(q1-p1)^2} - \frac{D\,p1^{\mu} p1^{\nu} - p1^2 g^{\mu\nu}}{2(D-1)p1^4 q1^2.(q2-p1)^2.(q1-q2)^2}$$

EXTRACTION OF LOOP INTEGRALS

- To evaluate the loop integrals outside of FEYNCALC, we need to extract all the unique integrals from the given expression
- New in FeynCalc 9: `FCLoopIsolate`

In[1]:= gse = FCI[FAD[q, −p + q] MTD[Lor3, Lor4] (FVD[−p − q, Lor5] MTD[Lor1, Lor3] + FVD[2 p − q, Lor3] MTD[Lor1, Lor5] + FVD[−p + 2 q, Lor1] MTD[Lor3, Lor5]) (FVD[p + q, Lor6] MTD[Lor2, Lor4] + FVD[−2 p + q, Lor4] MTD[Lor2, Lor6] + FVD[p − 2 q, Lor2] MTD[Lor4, Lor6]) MTD[Lor5, Lor6]]

Out[1]:= $$\frac{g^{\text{Lor3Lor4}} g^{\text{Lor5Lor6}} \left(g^{\text{Lor3Lor5}}(2q-p)^{\text{Lor1}} + g^{\text{Lor1Lor5}}(2p-q)^{\text{Lor3}} + g^{\text{Lor1Lor3}}(-p-q)^{\text{Lor5}}\right)}{q^2.(q-p)^2} \left(g^{\text{Lor4Lor6}}(p-2q)^{\text{Lor2}} + g^{\text{Lor2Lor6}}(q-2p)^{\text{Lor4}} + g^{\text{Lor2Lor4}}(p+q)^{\text{Lor6}}\right)$$

In[2]:= FCLoopIsolate[Contract[gse], {q}, **Head** −> loop] // Cases2[#, loop] &

Out[2]:= $$\left\{\text{loop}\left(\frac{1}{q^2.(q-p)^2}\right), \text{loop}\left(\frac{q^{\text{Lor1}}}{q^2.(q-p)^2}\right), \text{loop}\left(\frac{q^{\text{Lor2}}}{q^2.(q-p)^2}\right), \text{loop}\left(\frac{q^{\text{Lor1}} q^{\text{Lor2}}}{q^2.(q-p)^2}\right), \text{loop}\left(\frac{pq\cdot}{q^2.(q-p)^2}\right), \text{loop}\left(\frac{q^2}{q^2.(q-p)^2}\right)\right\}$$

- Furthermore:
  - `FCLoopSplit` to separate different types of loop integrals (free of loops, scalar integrals with and without scalar products in the numerators, tensor integrals)
  - `FCLoopExtract` for combined application of `FCLoopIsolate` and `FCLoopSplit`

TOOLS FOR IBP-REDUCTION

- Reduction of scalar loop integrals using integration-by-parts (IBP) identities [Chetyrkin & Tkachov, 1981] is a standard technique in modern loop calculations.
- Many publicly available IBP-packages on the market: FIRE [Smirnov & Smirnov, 2013], LITERED [Lee, 2012], REDUZE [Studerus, 2009], AIR [Anastasiou & Lazopoulos, 2004], ...
- Expected input: loop integrals with propagators that form a basis.
- What about integrals with an incomplete or overdetermined basis?
  - `FCLoopBasisIncompleteQ` detects integrals that require a basis completion
  - `FCLoopBasisFindCompletion` gives a list of propagators (with zero exponents) required to complete the basis
  - `FCLoopBasisOverdeterminedQ` checks if the propagators are linearly dependent. Such integrals can be decomposed further using `ApartFF`.

In[1]:= FCI[FAD[{q1, m, 2}, {q1 + q3, m}, {q2 − q3}, q2]]

Out[1]:= $\frac{1}{(\text{q1}^2 - m^2) . (\text{q1}^2 - m^2) . ((\text{q1} + \text{q3})^2 - m^2) . (\text{q2} - \text{q3})^2 . \text{q2}^2}$

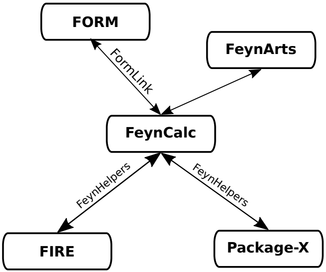
In[2]:= FCLoopBasisIncompleteQ[%, {q1, q2, q3}]
Out[2]:= **True**

In[3]:= FCLoopBasisFindCompletion[%%, {q1, q2, q3}][[2]]
Out[3]:= $\{(\text{q1} \cdot \text{q2}), (\text{q1} \cdot \text{q3})\}$

Motivation

- The field of automatic calculations appears to be a very competitive environment.
- Some groups do not share their codes at all
- Others make them available to collaborators only.
- People behind similar software regarded as competitors.
- It is more efficient to combine useful tools together than to compete.

- Useful tools to be used with FEYNCALC for the evaluation of 1-loop integrals:
  - FIRE [Smirnov, 2015]
  - PACKAGE X [Patel, 2015]
- Challenges: Need to convert between the conventions used in each package and avoid variable shadowing.
- Solution: FEYNHELPERS [Shtabovenko, 2016] seamlessly integrates both tools into FeynCalc.



Tensor reduction a la Passarino–Veltman

- Very old technique for dealing with tensor 1-loop integrals [Passarino & Veltman, 1979]
- Still widely used in many loop calculations.
- Main idea: convert all the tensor integrals into scalar ones (Passarino–Veltman coefficient functions)
- Evaluation of any 1-loop integral can be reduced to the evaluation of the resulting coefficient functions
- A lot of tools for numerical evaluation: FF [van Oldenborgh, 1991], LOOPTOOLS [Hahn & Perez-Victoria, 1999], QCDLOOP [Carrazza et al., 2016], ONELOOP [van Hameren, 2011], GOLEM95C [Cullen et al., 2011], PJFRY [Fleischer & Riemann, 2011], COLLIER [Denner et al., 2017], . . .
- Where to get analytic results for singular kinematics or zero Gram determinants? Often needed for renormalization, EFTs, . . .
- Most of the results can be found somewhere in the literature.

PACKAGE-X

- Recent [Patel, 2015] MATHEMATICA package for semi-automatic 1-loop calculations (closed-source freeware)
- Unique feature: Library of analytic expressions for Passarino–Veltman functions with up to 4 legs and almost arbitrary kinematics.
- Can also extract UV- and IR-parts and expand coefficient functions in their arguments.
- Someone indeed has collected all those results from the literature!

Interface to PACKAGE-X

- Main function: `PaXEvaluate`
- Works: on scalar 1-loop integrals (unit numerators) and Passarino–Veltman coefficient functions A, B, C and D
- Takes two arguments (plus options): input expression, loop momentum.
- Use `PaXEvaluateUV`(`PaXEvaluateIR`) to get the UV(IR)-divergent part of the result
- `PaXEvaluateUVIRSplit` returns the full result with the explicit distinction between ϵ_{UV} and ϵ_{IR}.
- All four functions share the same set of options

Let us compute $\int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m^2}$

In[1]:= **int=PaXEvaluate[FAD[{q,m}],q,PaXImplicitPrefactor→1/(2Pi)^D]**

Out[1]= $\frac{i m^2}{16\pi^2 \varepsilon} - \frac{i m^2 \left(-\log\left(\frac{\mu^2}{m^2}\right) + \gamma - 1 - \log(4\pi)\right)}{16\,\pi^2}$

Make the result look more compact ($\Delta \equiv 1/\epsilon - \gamma_E + \log(4\pi)$) using `FCHideEpsilon`

In[2]:= **int//FCHideEpsilon**

Out[2]= $\frac{i \Delta m^2}{16\pi^2} + \frac{i m^2 \left(\log\left(\frac{\mu^2}{m^2}\right) + 1\right)}{16\pi^2}$

Evaluation of Passarino–Veltman functions:

In[3]:= **PaXEvaluate[B0[SPD[p,p],0,m^2]]**

Out[3]= $\frac{1}{\varepsilon} + \log\left(\frac{\mu^2}{\pi m^2}\right) - \frac{m^2 \log\left(\frac{m^2}{m^2 - p^2}\right)}{p^2} + \log\left(\frac{m^2}{m^2 - p^2}\right) - \gamma + 2$

We can also expand coefficient functions in their parameters (masses or external momenta). To expand $B_0(p^2, 0, m^2)$ around $p^2 = m^2$ up to first order with `PaXEvaluate` we first need to assign an arbitrary symbolic value to the scalar product p^2, e.g. `pp`

In[4]:= **SPD[p,p]=pp;**

Then use the option `PaXSeries` to specify the expansion parameters and activate the option `PaXAnalytic`

In[5]:= **PaXEvaluate[B0[SPD[p,p],0,m^2],PaXSeries→{{pp,m^2,1}},PaXAnalytic→True]**

Out[5]= $\frac{3\,\mathrm{m}^2-\mathrm{pp}}{2\varepsilon \mathrm{m}^2}-\frac{(3\,\mathrm{m}^2-\mathrm{pp})\left(-\log\left(\frac{\mu^2}{\mathrm{m}^2}\right)+\gamma-2+\log(\pi)\right)}{2\mathrm{m}^2}$

Get only in the UV-part of this series: `PaXEvaluate` with `PaXEvaluateUV`

In[6]:= **PaXEvaluateUV[B0[SPD[p,p],0,m^2],PaXSeries→{{pp,m^2,1}},PaXAnalytic→True]**

Out[6]= $\frac{1}{\varepsilon_{\mathrm{UV}}}$

The IR-part is equally easy

In[7]:= **PaXEvaluateIR[B0[SPD[p,p],0,m^2],PaXSeries→{{pp,m^2,1}},PaXAnalytic→True]**

Out[7]= $\frac{\mathrm{m}^2-\mathrm{pp}}{2\mathrm{m}^2\varepsilon_{\mathrm{IR}}}$

Full result with the explicit distinction between UV and IR singularities

In[8]:= **PaXEvaluateUVIRSplit[B0[SPD[p,p],0,m^2],PaXSeries→{{pp,m^2,1}},PaXAnalytic→True]**

Out[8]= $\frac{\mathrm{m}^2-\mathrm{pp}}{2\mathrm{m}^2\varepsilon_{\mathrm{IR}}}-\frac{(3\,\mathrm{m}^2-\mathrm{pp})\left(-\log\left(\frac{\mu^2}{\mathrm{m}^2}\right)+\gamma-2+\log(\pi)\right)}{2\,\mathrm{m}^2}+\frac{1}{\varepsilon_{\mathrm{UV}}}$

Interface to FIRE

- Main function: `FIREBurn`
- Reduces scalar multi-loop integrals to simpler ones using IBP-techniques.
- Takes three arguments (plus options): input expression, list of loop momenta and the list of external momenta.
- Automatically adds propagators to integrals with incomplete bases of propagators
- Automatically detects integrals with linearly dependent propagators

Current limitations

- No recognition of integral families
- Each loop integral is evaluated separately
- Hence, rather inefficient ...

IBP-reduce the 1-loop integral $\int \frac{d^D l}{[l^2]^2[(l-p)^2-m^2]^2}$

In[9]:= **FIREBurn[FAD[{l,0,2},{l−p,m,2}],{l},{p}]**

Out[9]= $\frac{(D-2)(2\,D\,m^2-9\,m^2-pp)}{2m^2(m^2-pp)^3((l-p)^2-m^2)} - \frac{(D-3)(D\,m^2+D\,pp-4m^2-6\,pp)}{(m^2-pp)^3 l^2.((l-p)^2-m^2)}$

No dependence on external momenta → supply an empty list for the third argument. For example, for $\int \frac{d^D q_1 d^D q_2 d^D q_3}{[q_1^2-m^2]^2[(q_1+q_3)^2-m^2][(q_2-q_3)^2][q_2^2]^2}$

In[10]:= **FIREBurn[FAD[{q1,m,2},{q1+q3,m},{q2−q3},{q2,0,2}],{q1,q2,q3},{}]**

Out[10]= $-\frac{(D-3)(3\,D-10)(3\,D-8)}{16(2\,D-7)m^4(q1^2-m^2).q2^2.(q2-q3)^2.((q1+q3)^2-m^2)}$

My favourite example: Calculation of the QCD on-shell vertex for QCD/NRQCD matching [Manohar, 1997]

The HQET/NRQCD Lagrangian to order α/m^3

Aneesh V. Manohar

Department of Physics, University of California at San Diego,

9500 Gilman Drive, La Jolla, CA 92093-0319

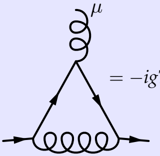
(January 1997)

Abstract

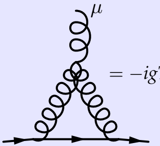
The HQET/NRQCD Lagrangian is computed to order α/m^3. The computation is performed using dimensional regularization to regulate the ultraviolet and infrared divergences. The results are consistent with reparametrization invariance to order $1/m^3$. Some subtleties in the matching conditions for NRQCD are discussed.

Reproducing results of Manohar

- QCD side of the matching: The on-shell vertex function is evaluated using background field formalism [Abbott, 1981, Abbott, 1982] and expanded up to the first order in the relative momentum squared.
- The abelian and non-abelian diagrams can be parametrized as



$$= -igT^a \bar{u}(p_2) \left(F_1^{(V)}(q^2)\gamma^\mu + iF_2^{(V)}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} \right) u(p_1), \qquad (1)$$



$$= -igT^a \bar{u}(p_2) \left(F_1^{(g)}(q^2)\gamma^\mu + iF_2^{(g)}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} \right) u(p_1), \qquad (2)$$

where $q \equiv p_2 = p_1$. Our goal is to compute the form-factors $F_{1/2}^{(V)}(q^2)$ and $F_{1/2}^{(g)}(q^2)$ expanded up to $\mathcal{O}(q^2/m^2)$.

Not so simple to do with software

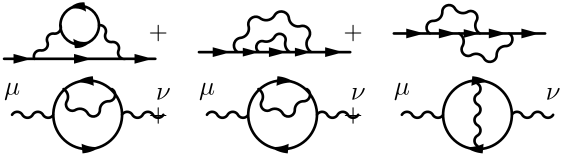
- It is not a total cross-section/decay rate, so fully automatic tools are not useful.
- Need to expand Passarino–Veltman integrals in the relative momentum.
- Distinguish between UV and IR singularities in DR using different regulators ϵ_{UV} and ϵ_{IR}.
- Since this is a matching, we want analytic results.
- With FEYNHELPERS this computation is straight-forward. We use the abbreviation $\Delta \equiv 1/\epsilon - \gamma_E + \log(4\pi)$ and use $D = 4 - 2\epsilon$
- To compare to the literature we need to switch to $D = 4 - \epsilon$ via $1/\epsilon \to 2/\epsilon$ and eliminate γ_E and $\log(4\pi)$ by substituting μ^2 with $\mu^2 \frac{e^{\gamma_E}}{4\pi}$ (following the conventions of Manohar).

$$F_1^{(g)} = \frac{\alpha_s}{8\pi} C_2(ad) \int_0^1 dx \int_0^{1-x} dy \Big\{ -\Gamma(1+\epsilon/2) \big[2q^2 (x+y) + 2m^2 (1-x-y)(2(x+y) + (4-\epsilon)(1-x-y)) \big] \left(m^2 (x+y-1)^2 - q^2 xy \right)^{-1-\epsilon/2} + (2-\epsilon)\Gamma(\epsilon/2) \left(m^2 (x+y-1)^2 - q^2 xy \right)^{-\epsilon/2} \Big\}$$
$$= \frac{\alpha_s}{8\pi} C_2(ad) \left[\frac{2}{\epsilon_{UV}} + \frac{4}{\epsilon_{IR}} + 4 - 6 \log \frac{m}{\mu} + \frac{q^2}{m^2} \left(-\frac{3}{\epsilon_{IR}} - 1 + 3 \log \frac{m}{\mu} \right) + \ldots \right], \quad (31)$$

$$F_2^{(g)} = -\frac{\alpha_s}{4\pi} C_2(ad)\, m^2 \Gamma(1+\epsilon/2) \int_0^1 dx \int_0^{1-x} dy\, (1-x-y) \times (\epsilon + (2-\epsilon)(x+y)) \left(m^2 (x+y-1)^2 - q^2 xy \right)^{-1-\epsilon/2}$$
$$= \frac{\alpha_s}{8\pi} C_2(ad) \left[\frac{4}{\epsilon_{IR}} + 6 - 4 \log \frac{m}{\mu} + \frac{q^2}{m^2} \left(\frac{4}{\epsilon_{IR}} + 1 - 4 \log \frac{m}{\mu} \right) + \ldots \right]. \quad (32)$$

$$F_1^{(V)} = \frac{\alpha}{\pi} \left(C_2(Q) - \frac{1}{2} C_2(ad) \right) \left[\frac{1}{2\epsilon_{UV}} + \frac{1}{\epsilon_{IR}} + 1 - \frac{3}{2} \log \frac{m}{\mu} + \frac{q^2}{m^2} \left(-\frac{1}{3} \frac{1}{\epsilon_{IR}} - \frac{1}{8} + \frac{1}{3} \log \frac{m}{\mu} \right) \right], \quad (29)$$
$$F_2^{(V)} = \frac{\alpha}{\pi} \left(C_2(Q) - \frac{1}{2} C_2(ad) \right) \left[\frac{1}{2} + \frac{q^2}{12 m^2} \right]. \quad (30)$$

Another example: photon and electron self-energies (with full gauge dependence) in massless QED at 2-loops. Requires evaluation of six 2-loop diagrams



$$\equiv i\not{p}\Sigma_{2V}(p^2),$$

$$\equiv -i(p^2 g^{\mu\nu} - p^\mu p^\nu)\Pi_2(p^2),$$

- Final results contain only two master integrals
- Need to use `FCMultiLoopTID` instead of `TID`

As expected, the vacuum polarization amplitude is gauge invariant, while the electron self-energy depends on the gauge parameter ξ. These results precisely agree with the literature, e.g. Eq. 5.18 and Eq. 5.51 from [Grozin, 2005].

- With FeynHelpers many types of calculations that were difficult or hardly feasible with FeynCalc previously become very simple.
- Goals for future development: improve the integration with Package-X and FIRE but also to add new interfaces to interesting and useful HEP tools.
- FeynHelpers comes with many examples. Highlight: 1-loop QED renormalization in MS, $\overline{\mathrm{MS}}$ and on-shell schemes with full gauge dependence (also useful for teaching).

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