Updated Standard Model Prediction for the kaon direct CP-violating ratio ϵ'/ϵ

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What is ϵ'/ϵ?

- ϵ'/ϵ constitutes a fundamental test for our understanding of flavour-changing phenomena within SM.
- ϵ and ϵ' parametrize different sources of CP violation in K decays:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \epsilon',$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - 2\epsilon'.$$

- Dominant effect from CP violation in K mixing is contained in ϵ:

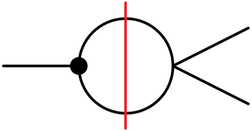
$$|\epsilon| = \frac{1}{3}|\eta_{00} + 2\eta_{+-}| = (2.228 \pm 0.011) \cdot 10^{-3}.$$

- ϵ' is a more tiny effect and accounts for direct CP violation in K decays:

$$\mathrm{Re}\left(\epsilon'/\epsilon\right) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4}$$

demostrates the existence of direct CP violation in K decays.

- The theoretical prediction of ϵ'/ϵ has a quite controversial history:
  1. First NLO calculations claimed values one order of magnitude smaller than the signal observed in 1993 by the CERN NA31 collaboration, giving support to the null result obtained by the E731 experiment.
  2. The final confirmation that $\mathrm{Re}\,(\epsilon'/\epsilon) \approx 10^{-3}$, by NA48 and KTeV, triggering a large number of NP explanations.
  3. Soon, the old SM predictions had missed completely the important role of the final pion dynamics. When these contributions were taken into account, the theoretical prediction was found to be in good agreement with the experimental value.



$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)\Big|_{\mathrm{SM}} = (1.7 \pm 0.9) \cdot 10^{-3}$$

(Pallante et al '01)

- Lattice QCD: An appropriate tool to deal with non-perturbative.

1. For many years, Lattice QCD attempts to explain
   - Enhancement of the $\Delta I = 1/2$ but remained unsuccessful.
   - ϵ'/ϵ were unreliable (Some of them even negative).
2. The situation has changed due to the development of sophisticated techniques and the increasing power of modern computers. Explaining
   - Successful $\Delta I = 3/2$ $K \to \pi\pi$: ✓

     $$\sqrt{\frac{3}{2}}\,\mathrm{Re}A_2 = (1.50 \pm 0.17) \cdot 10^{-8}\ \mathrm{GeV} \quad 0.1\,\sigma \ \text{(RBC-UKQCD '15)}$$

   - First statistically-significant signal of $\Delta I = 1/2$ enhancement: ✓

     $$\sqrt{\frac{3}{2}}\,\mathrm{Re}A_0 = (4.66 \pm 1.61) \cdot 10^{-7}\ \mathrm{GeV} \quad 1.0\,\sigma \ \text{(RBC-UKQCD '15)}$$

3. First Lattice QCD estimation of ϵ'/ϵ: ✗

   $$\mathrm{Re}\,(\epsilon'/\epsilon) = (1.4 \pm 7.0) \cdot 10^{-4} \quad 2.1\,\sigma \ \text{(RBC-UKQCD '15)}$$

- Limitations of lattice result:

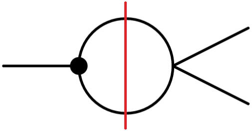
$$\text{Re}\,(\epsilon'/\epsilon) = (1.4 \pm 7.0) \cdot 10^{-4} \quad 2.1\,\sigma \ (\text{RBC-UKQCD '15})$$

  - This discrepancy has revived some of the old SM calculations predicting low values of ϵ'/ϵ (missing AGAIN the crucial pion dynamics!) and has triggered several analysis of possible contributions from NP.
  - Before claiming any evidence for NP, one should realize the technical limitations of the current lattice result:

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \quad 2.9\,\sigma \ (\text{RBC-UKQCD '15})$$
$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \quad 1.0\,\sigma \ (\text{RBC-UKQCD '15})$$

    Obviously nobody is looking for any NP contribution to $\delta_{0,2}$!
  - Still premature to derive strong implications and RBC-UKQCD collaboration is making efforts to improve the statistics.
  - Future lattice results will show if the discrepancy stays or not.

- Although in 2004, Cirigliano et al and Pallante et al within a theoretical framework which takes into account the important role of the pion dynamics obtained a theoretical value compatible with the experimental result.



$$\mathrm{Re}\,(\epsilon'/\epsilon) = (19 \pm 2^{+9}_{-2} \pm 6) \cdot 10^{-4}$$

- There have been improvements on:
  - Low energy constants (LECs),
  - Strange quark mass,
  - Strong coupling constant, etc
- Therefore, in view of the current situation, it is convenient to perform an updated of ϵ'/ϵ.

Theoretical Framework (Cirigliano et al '04, Pallante et al '01)

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	↓ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	↓ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

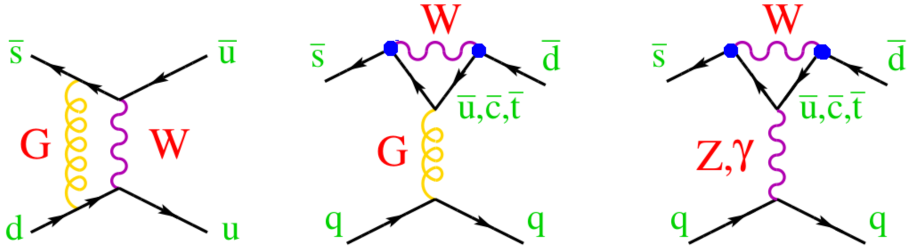
Short-distance description

$\Delta S = 1$ transitions for $K \to \pi\pi$

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu)\, Q_i(\mu)$$

$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$



Current - Current operators | **QCD - Penguins operators** | **Electroweak Penguins operators**

Q_6 Q_8

where $Q_6 \equiv (\bar{s}_\alpha d_\beta)_{\rm V-A} \sum_q (\bar{q}_\beta q_\alpha)_{\rm V+A}$, $Q_8 \equiv \frac{3}{2} (\bar{s}_\alpha d_\beta)_{\rm V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\rm V+A}$.

Chiral Perturbation Theory (χPT) description

- χPT formulation of SM is an ideal framework to describe the pseudoscalar-octet dynamics through a perturbative expansion in powers of p^2/Λ_χ^2 where $\Lambda_\chi \equiv$ chiral symmetry breaking scale $\sim$ 1GeV.
- Chiral symmetry fix the allowed χPT operators, at given order in p.
- $\mathcal{O}(G_F\, p^2)$: **Goldstone Interactions** (π, K, η)

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda\, L_\mu\, L^\mu) + G_{27}\, F^4 \left(L_{\mu\,23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu\,21}\, L^\mu_{13}\right)$$

$$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V^*_{us}\, g_{8,27} \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda_{ij} \equiv \delta_{i3}\delta_{j2}; \quad U \equiv \exp\{i\sqrt{2}\phi/F\}$$

- Short-distance dynamics encoded in Low-Energy Couplings.
- LECs can be determined at $N_c \to \infty$ **(matching)**

$K \to \pi\pi$ Isospin amplitudes

- The amplitudes for $K \to \pi\pi$ can be parametrized by

$$
\begin{aligned}
A[K^0 \to \pi^+\pi^-] &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}} (\mathcal{A}_{3/2} + \mathcal{A}_{5/2}) \\
A[K^0 \to \pi^0\pi^0] &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \sqrt{2}\, (\mathcal{A}_{3/2} + \mathcal{A}_{5/2}) \\
A[K^+ \to \pi^+\pi^0] &= \frac{3}{2} A_2^+\, e^{i\chi_2^+} = \frac{3}{2} \left(\mathcal{A}_{3/2} - \frac{2}{3} \mathcal{A}_{5/2} \right).
\end{aligned}
$$

χ_I can be identified with the S-wave $\pi\pi$ scattering phase shifts $\delta_I(M_K)$.

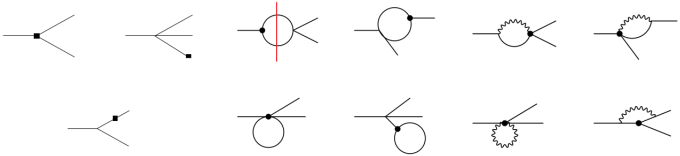
- In terms of the $K \to \pi\pi$ isospin amplitudes,

$$
\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)} \omega \left[\frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} - \frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} \right]
$$

1. $\Delta I = 1/2$ rule: ϵ' is suppresed by the ratio $\omega = \frac{\mathrm{Re} A_2}{\mathrm{Re} A_0} \approx 1/22$.
2. Strong Final States Interactions (FSI): $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$.

Amplitudes at NLO of $K \to \pi\pi$ $\mathcal{O}(p^4,\ (m_u - m_d)p^2,\ e^2p^0,\ e^2p^2)$

Including strong isospin violation and electromagnetic corrections (Cirigliano et al '04)



$$\mathcal{A}_n = G_{27}\, F_\pi\, (M_K^2 - M_\pi^2)\, \mathcal{A}_n^{(27)}$$
$$+\ G_8\, F_\pi \left\{ (M_K^2 - M_\pi^2) \left[\mathcal{A}_n^{(8)} + \varepsilon^{(2)}\, \mathcal{A}_n^{(\varepsilon)} \right] - e^2\, {F_\pi}^2 \left[\mathcal{A}_n^{(\gamma)} + Z\, \mathcal{A}_n^{(Z)} + g_{\rm ewk}\, \mathcal{A}_n^{(g)} \right] \right\}$$

where $\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right]$; $X = 27,\ 8,\ \varepsilon,\ \gamma,\ Z,\ g.$

$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^*\, g_{8,27}$, $\varepsilon^{(2)} = (\sqrt{3}/4)(m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (M_{\pi^\pm}^2 - M_{\pi_0}^2)/(2e^2F_\pi^2) \approx 0.8$

Estimation of the LECs

- From phenomenological data or with additional input from theory.
- Principle of calculation LECs: perform a matching between two EFTs.
- In the large-N_c limit, the T-product of two colour singlet currents factorizes:

$$\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle \left\{ 1 + \mathcal{O}\left(\frac{1}{N_c}\right) \right\}$$

Since quark currents have a well-known representation in χPT, the matching between the EFTs can be done at leading order in $1/N_c$.

- Weak couplings of $\mathcal{O}(G_F\, p^2)$ and $\mathcal{O}(e^2\, G_8\, p^0)$:

$$\begin{aligned} g_8^\infty &= -\frac{2}{5}\, C_1(\mu_{SD}) + \frac{3}{5}\, C_2(\mu_{SD}) + C_4(\mu_{SD}) - 16\, L_5\, B(\mu_{SD})\, C_6(\mu_{SD}), \\ g_{27}^\infty &= \frac{3}{5}\left[C_1(\mu_{SD}) + C_2(\mu_{SD})\right], \\ (e^2\, g_8\, g_{ewk})^\infty &= -3\, B(\mu_{SD})\, C_8(\mu_{SD}) - \frac{16}{3}\, B(\mu_{SD})\, C_6(\mu_{SD})\, e^2\, (K_9 - 2\, K_{10}). \end{aligned}$$

where $B(\mu_{SD}) \equiv \frac{\langle \bar{q}q \rangle}{F_\pi^3} = \left[\frac{M_K^2}{(m_s+m_d)(\mu_{SD})F_\pi}\right]^2 \left[1 - \frac{16 M_K^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{8M_\pi^2}{F_\pi^2}\, L_5\right]$

$$\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2-\chi_0)} \omega_+ \left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}} (1 - \Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}} \right]$$

Strong cancelation: Q_6 $-$ Q_8

1. **$O(p^4)$ χPT Loops: Large correction $\Delta_L \mathcal{A}_n^{(X)}$**

$$\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right]$$

Pallante-Pich-Scimemi

$\Delta_L \mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47\, i$;	
$\Delta_L \mathcal{A}_{1/2}^{(27)} = 1.02 \pm 0.60 + 0.47\, i$;	$\Delta_L \mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21\, i$
$\Delta_L \mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.05 + 0.47\, i$;	$\Delta_L \mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - 0.21\, i$

2. **$O(p^4)$ LECs fixed at $N_c \to \infty$: Small correction $\Delta_C \mathcal{A}_n^{(X)}$**

3. **Isospin Breaking $O((m_u - m_d)p^2, e^2p^2)$: Sizeable correction**

$$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$$ (Cirigliano-Ecker-Neufeld-Pich)

4. **$\mathrm{Re}(g_8)$, $\mathrm{Re}(g_{27})$, $\chi_0 - \chi_2$ fitted to data** (Cirigliano et al '12)

Final result

- **Successful SM prediction for ϵ'/ϵ:**

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)\Big|_{\mathrm{SM}} = \left(22 \pm 3_{(\mu)} \pm 3_{(m_s)} \pm 7_{(\frac{1}{N_c})}\right) \cdot 10^{-4}$$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)\Big|_{\mathrm{exp}} = (16.6 \pm 2.4) \cdot 10^{-4}$$

- **In good agreement with the experimental measurement!**

THANKS FOR YOUR ATTENTION