Is Unimodular Gravity equivalent to General Relativity?

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Details in

*PLB 773(2017)585 by S.G-M & Carmelo P. Martín

*arXiv:1711.08009 [hep-th] by S.G-M & Carmelo P. Martín

OUTLINE

1. Introduction
2. The quartic and Yukawa beta functions
3. The UV behaviour of some S-matrix elements
4. Conclusions

Introduction

- Unimodular Gravity (UG) is a truncation of General Relativty (GR) where the spacetime metric is unimodular,

$$\tilde{g} \equiv \det \tilde{g}_{\mu\nu} = -1$$

- It has the nice property that the vacuum energy does not couple to gravitation.

$$S \equiv \int d^n x \left(-\frac{1}{2\kappa^2} R[\tilde{g}] + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - V(\psi) \right)$$

- Hence, additions of the type

$$\Lambda_0 \int d^n x \sqrt{-\hat{g}}$$

are physically irrelevant.

UG Eq. of Motion

- EM = Trace-free equations (TFE)

$$R_{\mu\nu} - \frac{1}{n} R \hat{g}_{\mu\nu} = M_P^{2-n} (T_{\mu\nu} - \frac{1}{n} T \hat{g}_{\mu\nu})$$

- Now, the 2nd Bianchi identity $\nabla_\mu R^{\mu\nu} = \frac{1}{2} \nabla^\nu R$ and TFE imply

$$\nabla_\mu ((n-2)R + 2M_P^{2-n} T) = 0 \implies (n-2)R + 2M_P^{2-n} T = -2nC$$

- TFE and the previous consistency condition imply

$$R_{\mu\nu} - \frac{1}{2} R \hat{g}_{\mu\nu} - C \hat{g}_{\mu\nu} = M_P^{2-n} T_{\mu\nu}$$

ie, Einstein equations with a cosmological constant term BUT it is only an integration constant.

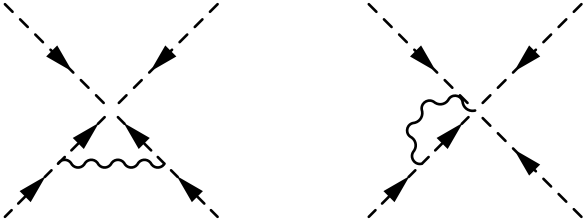
Motivation: Why Unimodular Gravity?

1) Solves in a Wilsonian way the huge disparity between the QFT “prediction" for the vacuum energy and the experimentally observed cosmological constant: Vacuum energy is not seen by gravity in UG.
2) Same classical results: solar system tests, inflation...
3) The full diffeomorphism invariance is broken to a subgroup with unit jacobian. It is enough to kill the three extra polarizations of a massive spin two theory. J. J. van der Bij, H. van Dam and Y. J. Ng, “The Exchange of Massless Spin Two Particles,” Physica **116A**, 307 (1982).

   Generalizes to curved space: C. Barcelo, R. Carballo-Rubio & L.J.Garay, PRD 89 (2014) 124019.

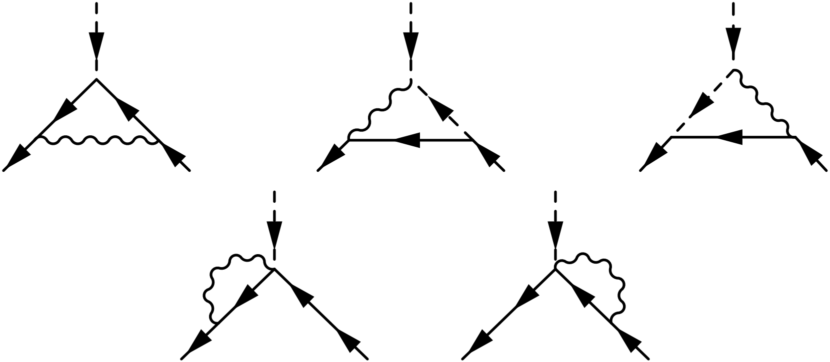
The quartic and Yukawa beta functions I

- In peturbatively renormalizable field theories the coupling constant beta functions have invaluable physical information.
- In Phys.Rev.Lett. 104 (2010) 081301 the GR corrections to the beta functions for the scalar λ and Yukawa g couplings are computed in the MS scheme in the de Donder gauge. These are given by the diagrams



Contributions to the ϕ^4 vertex.

The quartic and Yukawa beta functions I



Contributions to the Yukawa vertex.

Giving the result

$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2\,\lambda, \quad \beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2\left\{m_\phi^2\left(\frac{1}{2}\right) + m_\Psi^2\left(-1\right)\right\}$$

m_ϕ = mass of the Scalar, m_Ψ = mass of the fermion

The quartic and Yukawa beta functions III

- It is an interesting result due to their NEGATIVE value leading to an asymptotically free theory.
- So, we decided to carry out the same computations in UG and found

$$\beta_\lambda^{\text{UG}} = 0, \quad \beta_g^{\text{UG}} = \frac{1}{16\pi^2}\kappa^2 m_\Psi^2 \frac{3}{16}$$

- One is tempted to conclude that GR $\neq$ UG at the quantum level.
  - WRONG CONCLUSION!

The quartic and Yukawa beta functions IV

- Indeed, the beta functions defined as in Phys.Rev.Lett. 104 (2010) 081301 (ie, by a standard multiplicative renormalization) lack intrinsic physical meaning, for they turn out to be gauge dependent.
- We decided to compute the same functions –for GR– using a generalized de Donder gauge

$$\int d^n x\, \alpha \left(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\right)^2,$$

getting

$$\beta_\lambda^{GR} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2 \left(\frac{3}{2}+\alpha\right)\lambda$$
$$\beta_g^{GR} = \frac{1}{16\pi^2}\kappa^2 \left\{ m_\phi^2 \left[\frac{1}{2} - \left(\frac{1}{2}+\alpha\right)\right] + m_\Psi^2 \left[-1 - \left(\frac{1}{2}+\alpha\right)\frac{85}{16}\right]\right\}$$

The quartic and Yukawa beta functions V

- β_g^{GR} and β_g^{GR} do not have intrinsic physical meaning.
- By introducing a non-multiplicative (but local) wave function renormalization (as did, in the YM case, J.Ellis & N. Mavromatos Phys.Lett. B711 (2012) 139)

$$
\begin{aligned}
g_0 &= \mu^{-\epsilon} Z_g Z_\psi^{-1} Z_\phi^{-1/2} g, & \phi_0 &= \phi + \frac{1}{2}\delta Z_\phi \phi, \\
\Psi_0 &= \Psi + \frac{1}{2}\delta Z_\Psi \Psi + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \phi \Psi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \phi \Psi, & m_{\Psi_0} &= (1 + \delta Z_{m_\Psi}) m_\Psi, \\
\bar{\Psi}_0 &= \bar{\Psi} + \frac{1}{2}\delta Z_\Psi \bar{\Psi} + \frac{1}{2} a_1 \kappa^2 m_\Psi^2 \bar{\Psi}\phi + \frac{1}{2} b_1 \kappa^2 m_\phi^2 \bar{\Psi}\phi, & m_{\phi_0} &= (1 + \delta Z_{m_\phi}) m_\phi .
\end{aligned}
$$

one obtains that

$$\beta_g^{\text{GR}} = 0 = \beta_g^{\text{UG}}$$

- So that β_g^{GR} and β_g^{UG} have no intrinsic physical meaning.

The quartic and Yukawa beta functions VI

- Analogous analysis for $\beta_\lambda^{\mathrm{GR}}$ and $\beta_\lambda^{\mathrm{UG}}$. See PLB 773 (2017) 585.
- In this regard there is no disagreement between GR and UG (ie, both contributions to β_g and β_λ can be set to zero by nonmultiplicative field renormalizations), but this does not settle the question of the physical equivalence between GR and UG coupled to the $\lambda\phi^4$ and Yukawa theories.
- Similar to what happens in GR with the gauge couplings: J. Ellis & N. Mavromatos PLB711(2012)139.

The UV behaviour of S-matrix elements I

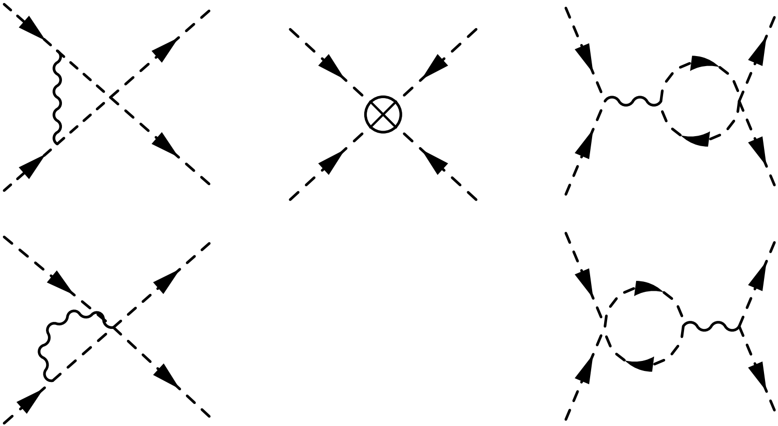
- To check whether UV divergent behaviour of the GR contributions to the S-matrix elements of the $\lambda\phi^4$ and Yukawa theory agree with those of UG, we decided to compute such behaviour for the scattering processes

$$\phi+\phi\rightarrow\phi+\phi \qquad \& \qquad \Psi+\Psi\rightarrow\Psi+\Psi,$$

  at one-loop
- We have shown that the GR and UG contributions agree, although this agreement is achieved only after summing over all Feynman diagrams.

The UV behaviour of S-matrix elements II

- The one-loop contributions to the S-matrix of order κ^2 are



(a) 1PI diagrams **(b)** Counterterm **(c)** N1PI diagrams

The UV behaviour of S-matrix elements III

- For GR, the (on-shell) divergences read

$$\begin{aligned}
\text{D1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda \left(1+[\tfrac{1}{2}+\alpha]\right)(-2)\\
\text{Count} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda \left(1+[\tfrac{1}{2}+\alpha]\right)(-2)\\
\text{DN1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda (-5/6)\\
\text{FINAL RESULT} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda (-5/6)
\end{aligned}$$

- For UG, the (on-shell) divergences read

$$\begin{aligned}
\text{D1PI} &= 0\\
\text{Count} &= 0\\
\text{DN1PI} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda (-5/6)\\
\text{FINAL RESULT} &= -\frac{i}{16\pi^2\varepsilon}\kappa^2 m_\phi^2 \lambda (-5/6)
\end{aligned}$$

- There is a complete agreement between GR and UG.
- The same conclusion for $\Psi+\Psi\to\Psi+\Psi$ (details in S. G-M and Carmelo Pérez Martín, forthcoming paper).

CONCLUSION

- The gravitational contributions to the beta funcions have no physical meaning.
- As far as we can tell there is no difference between quantum GR and quantum UG when the Cosmological Constant vanishes.
- Plenty of work still to be done:
  - eg, does UG come from String Theory? Recall the evidence that UG and GR have the same S-matrix.
  - Goroff and Sagnotti computation.