Non-Abelian Microstate Geometries

Alejandro Ruipérez Vicente

IFT-UAM/CSIC

October 24, 2017

Outline

Motivation

- Black Hole Thermodynamics
- Fuzzball proposal
- Classical description of microstate geometries

SEYM theories

- Timelike supersymmetric solutions w/ one additional spacelike isometry
- Reduced BPS equations

Microstate geometries

- Multicenter solutions
- Ambipolar Gibbons-Hawking spaces
- Asymptotic charges
- Bubble equations
- Closed timelike curves

Black Hole Thermodynamics

Classical Black Holes

- Laws of black hole mechanics suggest: $T \sim \kappa$ and $S \sim A_H$
- Bekenstein (1972). To preserve the 2^{nd} law of thermodynamics, BHs must have entropy: $S_{Bek} \sim A_H$.
- BH=ensemble of geometries (microstates)
- Expect $N \sim e^{S_{Bek}}$ microstates (in contradiction w/ the no-hair conjeture)
- Classical description of these microstate geometries?

Quantum Black Holes

- Hawking (1974). BHs radiate energy away (and therefore evaporate)
- Mechanism: Schwinger particle pair production at the horizon
- Unitarity is not preserved. Pure states evolve into mixed states. QM fails.
- If assume that HR does not carry any information about the matter that entered the hole $\rightsquigarrow$ information loss paradox

Fuzzball proposal

Resolution of the paradox. Several proposals:

1. Quantum Gravity violates unitarity (Hawking)
2. BH remnants. QG may prevent BHs from collapse. HR is entangled with the hole.
3. QG corrections modify Hawking's computation and HR carries all the information about the initial state.
   - 3.1 Modifications near the singularity ($\sim l_p$: Planck scale)$\rightarrow$ Subleading contributions
   - 3.2 Modiciations at the horizon scale

[Mathur, Lunin]

Fuzzball proposal: The region inside the horizon is substituted by something (a ball of strings?) in such a way that

- The fuzzball is sensitive to what falls inside (no information paradox)
- The fuzzball is a degenerate state (microscopic interpretation of the BH entropy)
- Each microstate would correspond to a String Theory configuration
- Some of these admit a classical description in supergravity theories

[Bena, Warner]

Classical description of microstate geometries

Microstate geometries are solutions of supergravity w/ some nice properties:

1. Asymptotically, they look like BH solutions
2. They are smooth (the singularity is resolved)
3. Absence of event horizons

Solution generating technique

- Bena and Warner (2005). "Recipe" to construct these solutions in SUGRA.
- Later generalization to non-Abelian microstate geometries in the context of SEYM theories

[P. F. Ramírez]

- Physically ill-defined solutions (CTCs). No systematic way to construct them

SEYM theories

Action (Bosonic sector)

$$S = \int d^5x \sqrt{|g|} \left\{ R + \tfrac{1}{2} g_{xy} \mathfrak{D}_\mu \phi^x \mathfrak{D}^\mu \phi^y - \tfrac{1}{4} a_{IJ} F^{I\,\mu\nu} F^J{}_{\mu\nu} - \tfrac{1}{4} C_{IJK} \frac{\varepsilon^{\mu\nu\rho\sigma\lambda}}{\sqrt{|g|}} \left[F^I{}_{\mu\nu} F^J{}_{\rho\sigma} A^K{}_\lambda \right.\right.$$
$$\left.\left. - \tfrac{1}{2} g f_{LM}{}^I F^J{}_{\mu\nu} A^K{}_\rho A^L{}_\sigma A^M{}_\lambda + \tfrac{1}{10} g^2 f_{LM}{}^I f_{NP}{}^J A^K{}_\mu A^L{}_\nu A^M{}_\rho A^N{}_\sigma A^P{}_\lambda \right] \right\}$$

- The SUGRA model is completely characterized by the symmetric tensor C_{IJK}
- $ST[2,6]$ model $\rightsquigarrow$ $C_{0xy} = \frac{1}{6} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$
- Spectrum:
  - Supergravity multiplet: $(e^a{}_\mu, A^0_\mu, \psi^i_\mu)$
  - 5 vector multiplets $(\phi^x, A^x_\mu, \lambda^{xi})$
- Non-Abelian gauging: three of the vector fields (A^α w/ $\alpha = 3, 4, 5$) gauge a $SU(2)$ subgroup of the isometries of the scalar manifold

Timelike supersymmetric solutions w/ one additional isometry

- Metric:

$$
\begin{aligned}
ds^2 &= f^2(dt+\omega)^2 - f^{-1} d\hat{s}^2_{GH} \\
d\hat{s}^2_{GH} &= H^{-1}(d\psi+\chi)^2 + H dx^s dx^s
\end{aligned}
$$

- Vector fields:

$$
\begin{aligned}
A^I &= h^I f(dt+\omega) + \hat{A}^I \\
\hat{A}^I &= -H^{-1}\Phi^I(d\psi+\chi) + \breve{A}^I
\end{aligned}
$$

- Scalars:

$$
Z_I = L_I + 3C_{IJK}\Phi^J\Phi^K H^{-1}
$$

- 1-form ω:

$$
\omega = \omega_5(dz+\chi) + \breve{\omega}
$$

All functions defining the fields are independent of the time coordinate t and the isometric coordinate ψ

Reduced BPS equations

The supersymmetry eqs are reduced to

$$\begin{aligned}
\star_3 dH &= d\chi \\
\star_3 \breve{\mathfrak{D}}\Phi^I &= \breve{F}^I \qquad \text{(Bogomol'nyi eqs)} \\
\breve{\mathfrak{D}}^2 L_I &= g^2 f_{IJ}{}^L f_{KL}{}^M \Phi^J \Phi^K L_M \\
\star_3 d\breve{\omega} &= HdM - MdH + \frac{1}{2}(\Phi^I \breve{\mathfrak{D}} L_I - L_I \breve{\mathfrak{D}}\Phi^I) \\
\omega_5 &= M + \frac{1}{2} L_I \Phi^I H^{-1} + C_{IJK}\Phi^I\Phi^J\Phi^K H^{-2}
\end{aligned}$$

- The Abelian sector (H, Φ^i, L_i) is solved by harmonic functions
- Surprisingly, the non-Abelian sector (Φ^α, L_α) is also solved by harmonic functions!

$$\Phi^\alpha = -\frac{1}{gP}\frac{\partial P}{\partial x^s}\delta^\alpha_s\,, \qquad L_\alpha = -\frac{1}{gP}\frac{\partial Q}{\partial x^s}\delta^s_\alpha$$

- We must ensure that the integrability condition of $\breve{\omega}$ is also satisfied (see later)

Multicenter solutions

We use the simplest harmonic functions:

$$
\begin{aligned}
H &= \sum_a \frac{q_a}{r_a}, \quad q_a \in \mathbb{Z} \\
M &= m_0 + \frac{m_a}{r_a} \\
L_i &= l_0^i + \sum_a \frac{l_a^i}{r_a} \\
\Phi^i &= \sum_a \frac{k_a^i}{r_a} \\
P &= 1 + \sum_a \frac{\lambda_a}{r_a} \\
Q &= \sum_a \frac{\sigma_a \lambda_a}{r_a}
\end{aligned}
$$

Comments:

- The base spaces is an ambipolar Gibbons-Hawking space
- If we want to describe asymptotically flat microstate geometries, not all the parameters are free and we must impose some relations among themselves.
- The non-Abelian vector fields represent a multicenter instanton configuration

Ambipolar GH spaces

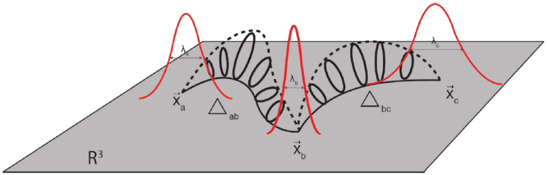
The base space is an ambipolar Gibbons-Hawking space:

$$d\hat{s}^2 = H^{-1}\left(d\psi + \chi\right)^2 + H dx^s dx^s \qquad\qquad H = \sum_a \frac{q_a}{r_a}$$

Properties:

- Alternate from $+$ to $-$ signature
- Asymptotically flat if $\sum_a q_a = 1$
- Near the centers: $d\hat{s}^2 \sim d\rho_a^2 + \rho_a^2 d\Omega^2_{S^3/q_a}$
- Non-trivial topology (non-contractible 2-cycles Δ_{ab}) $\rightsquigarrow$ asymptotic charges

$$\Pi^i_{ab} = \int_{\Delta_{ab}} F^i = \frac{k^i_b}{q_b} - \frac{k^i_a}{q_a}$$



Asymptotic charges

Abelian charges:

$$\mathcal{Q}_0 = -\sum_{a,b,c} q_a q_b q_c \Pi^1_{ab} \Pi^2_{ac} + \frac{1}{2g^2} \sum_a \frac{1}{q_a}$$
$$\mathcal{Q}_1 = -\sum_{a,b,c} q_a q_b q_c \Pi^0_{ab} \Pi^2_{ac}$$
$$\mathcal{Q}_2 = -\sum_{a,b,c} q_a q_b q_c \Pi^0_{ab} \Pi^1_{ac}$$

Angular momentum:

$$J = -\frac{1}{2} \sum_{a,b,c,d} q_a q_b q_c q_d \Pi^0_{ab} \Pi^1_{ac} \Pi^2_{ad} + \frac{1}{4g^2} \sum_{a,b} \frac{q_b \Pi^0_{ab}}{q_a}$$

Mass:

$$\mathcal{M} = \frac{\pi}{G_N^{(5)}} \left(\frac{\mathcal{Q}_0}{l_0^0} + \frac{\mathcal{Q}_1}{l_0^1} + \frac{\mathcal{Q}_2}{l_0^2} \right)$$

Entropy (of the corresponding BH solution):

$$S = \frac{\pi^2}{2G_N^{(5)}} \sqrt{\mathcal{Q}_0 \mathcal{Q}_1 \mathcal{Q}_2 - J^2}$$

Bubble equations

Bubble equations $\sim$ integrability condition of $\breve{\omega}$

$$\sum_{b\neq a} \frac{q_a q_b}{r_{ab}} \Pi^0_{ab} \left(\Pi^1_{ab}\Pi^2_{ab} - \frac{1}{2g^2}\mathbb{T}_{ab} \right) = \sum_{b,i} q_a q_b l^i_0 \Pi^i_{ab}$$

where

$$\Pi^i_{ab} = \frac{k^i_b}{q_b} - \frac{k^i_a}{q_a} \qquad \mathbb{T}_{ab} = \frac{1}{q_a^2} + \frac{1}{q_b^2}$$

- Only $n-1$ independent eqs
- Traditionally solved for the distances (more complicated)
- But there is no reason to do so... if instead take Π^i_{ab} as unknowns the above eqs reduce to

$$\mathcal{M}X = B$$

- Trivially solved (if $\det \mathcal{M} \neq 0$) $\rightsquigarrow X = \mathcal{M}^{-1}B$

Even when the bubble equations are solved, the solution is not physical (in the major part of the cases) since it presents **closed timelike curves**...

Closed timelike curves (CTCs)

Let's rewrite the metric in a convenient form...

$$ds^2 = f^2 dt^2 + 2f^2 dt\omega - \frac{\mathcal{I}}{f^{-2H^2}} \left(d\varphi + \chi - \frac{\omega_5 H^2}{\mathcal{I}} \breve{\omega} \right)^2 - f^{-1} H \left(dx^s dx^s - \frac{\breve{\omega}^2}{\mathcal{I}} \right)$$

$$\mathcal{I} = f^{-3} H - \omega_5^2 H^2$$

- The condition $\mathcal{I} > 0$ is enough to guarantee the solution doesn't have CTCs
- Only a very small region of the whole parameter space satisfy this constraint
- It makes the explicit construction of these solution very difficult

Conjeture:

A given configuration is free of CTCs if and only if all the eigenvalues of the matrix $\mathcal{M}$ are positive

[J. Ávila, P.F. Ramírez, AR]

- Strong evidence in favour
- Very powerful tool to construct solutions

Thanks for your attention