Unitarized EFT for a strongly interacting BSM Electroweak Sector coupled with $t\bar{t}$

Rafael L. Delgado
In colaboration with:
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Presented at: XXXVI Reunin Bienal de la RSEF, Santiago de Compostela

EPJC 77 (2017) no.7, EPJC 77 no.4 (2017) 205
1 Introduction
   - Motivation of the low-energy effective Lagrangian
   - Effective Lagrangian

2 Partial wave decomposition and unitarization procedures
   - Partial waves
   - Unitarization procedures
   - Methods of choice

3 Analysis of the parameter space

4 Conclusions
The gauge bosons $W^\pm$ and $Z$ are massive.

This is problematic: the massive terms are not gauge invariant. Gauge boson scattering amplitudes diverge with $s$ at LO.

Standard Model solution: Higgs-mechanism, which predicts the SM Higgs boson. Global symmetry breaking pattern: $SU(2)_L \times SU(2)_R \to SU(2)_C$.

In 2012, ATLAS and CMS find a 125-126 GeV scalar resonance $h$, compatible with the Higgs of the SM.
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New physics? 600 GeV

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  - Some issues: mass of neutrinos, gravity explanation (naturalness problem), astrophysical observation (dark matter, dark energy),...
  - Four scalar light modes, a strong gap.
  - Natural: further spontaneous symmetry breaking at $f > v = 246$ GeV?
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Top quark loop in the SM: VV states from gluon fusion
Studied framework

- We consider a strongly interacting EWSBS, in contrast to the weakly interacting one of the SM.
- We study the processes $VV ightarrow VV$, $VV ightarrow hh$ and $hh ightarrow hh$, and extend the result to include $t\bar{t}$ states.
- Our LO scattering amplitudes within the EWSBS diverge, but are controlled by strongly interacting dynamics which respect unitarity. This situation is similar to low-energy QCD (hadron physics).
- In order to minimize our assumptions over the (hypothetical) underlying theory, we will use dispersion relations over a partial wave decomposition (the so-called unitarization procedures);
  extend these unitarization procedures to the coupled-channels case;
  and consider an Effective Field Theory, computed at the NLO level (within the limits of the Equivalence Theorem), with three would-be Goldstone bosons $\omega$ and a Higgs-like boson $h$. 
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Chiral Perturbation Theory plus Dispersion Relations.

Simultaneous description of $\pi\pi \rightarrow \pi\pi$ and $\pi K\pi K \rightarrow \pi K\pi K$ up to 800-1000 MeV including resonances.

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Non-linear Electroweak Chiral Lagrangian

We have no clue of what, how or if new physics...

Non-linear EFT\(^1\) for \(VV\) scattering at NLO level, minimally coupled to \(hh\),

\[
\mathcal{L} = \frac{v^2}{4} g(h/f) \text{Tr}[(D_\mu U)\dagger D^\mu U] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h),
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where

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g(h/v) = 1 + 2a \frac{h}{v} + b \left(\frac{h}{v}\right)^2 + \ldots
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V(h) = V_0 + \frac{M_h^2}{2} h^2 + \sum_{n=3}^{\infty} \lambda_n h^n
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\(M_h\) and \(\lambda_n\) are subleading in chiral counting.

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EFT for $VV$ scattering, minimally coupled to $hh$

Using the spherical parameterization for the $SU(2)$ coset and neglecting the couplings with photons and quarks, we have the next Lagrangian describing $VV \rightarrow VV$, $VV \rightarrow hh$ and $hh \rightarrow hh$ processes:

$$\mathcal{L} = \left[1 + 2a \frac{h}{v} + b \left(\frac{h}{v}\right)^2\right] \frac{\partial_\mu \omega^a \partial_\mu \omega^b}{2} \left(\delta^{ab} + \frac{\omega^a \omega^b}{v^2}\right) + \frac{4a_4}{v^4} \partial_\mu \omega^a \partial_\nu \omega^a \partial_\mu \omega^b \partial_\nu \omega^b + \frac{4a_5}{v^4} \partial_\mu \omega^a \partial_\nu \omega^a \partial_\nu \omega^b \partial_\nu \omega^b + \frac{2d}{v^4} \partial_\mu h \partial_\nu h \partial_\nu \omega^a \partial_\nu \omega^a + \frac{2e}{v^4} \partial_\mu h \partial_\nu \omega^a \partial_\nu h \partial_\nu \omega^a + \frac{1}{2} \partial_\mu h \partial_\mu h + \frac{g}{v^4} \left(\partial_\mu h \partial_\mu h\right)^2$$
Extension to $t\bar{t}$ states

Lagrangian additions$^2$:

$$\mathcal{L}' = i\bar{Q}\partial Q - vG(h) \left[ \bar{Q}'_L U H_Q Q'_R + h.c. \right].$$

This expression, for the heaviest quark generation, expands to$^3$

$$\mathcal{L}_Y = -G(h) \left\{ \sqrt{1 - \frac{\omega^2}{v^2}} (M_t t\bar{t} + M_b b\bar{b}) + \frac{i\omega^0}{v} (M_t \bar{t}\gamma^5 t - M_b \bar{b}\gamma^5 b) \
+ \frac{i\sqrt{2}\omega^+}{v} (M_b \bar{t}_L b_R - M_t \bar{t}_R b_L) + \frac{i\sqrt{2}\omega^-}{v} (M_t \bar{b}_L t_R - M_b \bar{b}_R t_L) \right\}$$

Two NLO counterterms needed for renormalization,

$$\mathcal{L}_{4''} = g_t \frac{M_t}{v^4} \partial_\mu \omega^a \partial^\mu \omega^b t\bar{t} + g'_t \frac{M_t}{v^4} \partial_\mu h \partial^\mu h t\bar{t}$$

$^2$Work in collaboration with A.Castillo, arXiv:1607.01158 [hep-ph], accepted in EPJC.

$^3$\(G(h) = 1 + c_1(h/v) + c_2(h/v)^2 + \ldots\), \(V_{tb}\) very close to unity.
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$^5$Eur. Phys. J. **C75** (2015), 212

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Experimental bounds on low-energy constants, NLO $a_4-a_5$

Direct constraint over $a_4$-$a_5$ from ATLAS Collaboration\(^8\)

\(^8\)Taken from ref. [PRL\textbf{113} (2014) 141803]. Note that CMS [PRL\textbf{114} (2015) 051801] gives a constraint in terms of $F_{S0}/\Lambda^4$ and $F_{S1}/\Lambda^4$ parameters, which have no direct translation to the $a_4$ and $a_5$ ones [arXiv:1310.6708, [hep-ph]].
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$$A_{IJ}(s) = A_{IJ}^{(0)} + A_{IJ}^{(1)} + O \left[ \frac{s}{v^2} \right]^3.$$  

Which will be decomposed as

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$$Q_{ij}(s) = Q_{ij}^{(0)} + Q_{ij}^{(1)} + \mathcal{O} \left[ M_t s^2 \sqrt{s}/\nu^6 \right] + \mathcal{O} \left[ M_t^2 s/\nu^4 \right],$$

which will be decomposed as

$$Q_{ij}^{(0)} = K^Q \sqrt{s} M_t$$

$$Q_{ij}^{(1)} = \left( B^Q(\mu) + E^Q \log \frac{-s}{\mu^2} \right) s \sqrt{s} M_t.$$ 

As $A_{ij}(s)$ must be scale independent,

$$B^Q(\mu) = B^Q(\mu_0) + E^Q \log \frac{\mu^2}{\mu_0^2} = B_0 + p_g g_t(\mu).$$

Based on a collaboration with A. Castillo, EPJC 77 (2017) no.7.
Unitarity for partial waves

- Unit. cond. for $S$ – matrix:
  \[ SS^\dagger = 1, \]
  plus analytical properties of matrix elements,
  plus time reversal invariance,

\[
\text{Im} \ A_{IJ,p_i \rightarrow k_1}(s) = \sum_{\{a,b\}} \sqrt{1 - \frac{4m_q^2}{s}} [A_{IJ,p_i \rightarrow q_i,ab}(s)][A_{IJ,q_i,ab \rightarrow k_i}(s)]^* \]
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The IAM method cannot be used when $A^{(0)} = 0$, because it would give a vanishing value.

The N/D and the IK methods cannot be used if $D + E = 0$, because in this case computing $A_L(s)$ and $A_R(s)$ is not possible.

The naive K-matrix method,

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)},$$

fails because it is not analytical in the first Riemann sheet and, consequently, it is not a proper partial wave compatible with microcausality.
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Scalar-isoscalar channels

From left to right and top to bottom, elastic $\omega\omega$, elastic $hh$, and cross channel $\omega\omega \to hh$, for $a = 0.88$, $b = 3$, $\mu = 3$ TeV and all NLO parameters set to 0. PRD 114 (2015) 221803, PRD 91 (2015) 075017.
Vector-isovector channels

From our ref\textsuperscript{9}. We have taken $a = 0.88$ and $b = 1.5$, but while for the left plot all the NLO parameters vanish, for the right plot we have taken $a_4 = 0.003$, known to yield an IAM resonance according to the Barcelona group\textsuperscript{10}.

\textsuperscript{9}PRD 91 (2015) 075017
\textsuperscript{10}PRD 90 (2014) 015035
Resonance from $W_L W_L \rightarrow hh$

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Rafael L. Delgado,
Antonio Dobado,
Felipe J. Llanes-Estrada,
*Possible New Resonance from $W_L W_L$-hh Interchannel Coupling*,

PRL 114 (2015) 221803
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Resonances in $W_L W_L \rightarrow W_L W_L$ due to $a$ and $b$ parameters

- PRL & PRD 91 (2015) 075017
- From left, clockwise, $IJ = 00, 11, 20$
- Excluding resonances $M_S < 700 \text{ GeV}, M_V < 1.5 \text{ TeV}$
- Constraint over $b$ even without data about $W_L W_L \rightarrow hh$ and $hh \rightarrow hh$ scattering processes.
Video: Elastic Channels
Video: Coupling with $t\bar{t}$ states
What about the Monte Carlo implementation?

\[ a = 1; \, a_4 \cdot 10^4 = 3.5 \text{ (BP1), 1 (BP2), 0.5 (BP3)}; \]
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Conclusions: our work

- We have performed a comprehensive study of the unitarized amplitudes obtained from the non-linear Effective Chiral Lagrangian, which describes the EWSBS in the TeV region.
- We have carried out an NLO computation with a massless EFT.
- Studied $2 \rightarrow 2$ scattering processes within the EWSBS: 
  \[ \{ V_L V_L, hh \} \rightarrow \{ V_L V_L, hh \}. \]
- And (weak) couplings with $t\bar{t}$ states.
- We are ready for new strong interactions at the LHC. What are the prospects?
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Conclusions, possible extensions

- **SM → unitarity.**
  - Higgsless model (now experimentally excluded) → unitarity violation in $WW$ scattering → new physics.
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  - Depends on couplings. Ok with the present experimental bounds.
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Backup Slides
I) IAM method

This method needs a NLO computation,

\[ \tilde{t}^\omega = \frac{t_0^\omega}{1 - \frac{t_1^\omega}{t_0^\omega}}, \]

where

\[ t_1^\omega = s^2 \left( D \log \left[ \frac{s}{\mu^2} \right] + E \log \left[ \frac{-s}{\mu^2} \right] + (D + E) \log \left[ \frac{\mu^2}{\mu_0^2} \right] \right) \]
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We have checked\textsuperscript{11}, for the tree level case,

\[
\mathcal{L} = \frac{1}{2} g(\varphi/f) \partial_\mu \omega^a \partial^\mu \omega^b \left( \delta_{ab} + \frac{\omega^a \omega^b}{v^2 - \omega^2} \right) + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} M^2 \varphi^2 - \lambda_3 \varphi^3 - \lambda_4 \varphi^4 + \ldots
\]

\[
g(\varphi/f) = 1 + \sum_{n=1}^{\infty} g_n \left( \frac{\varphi}{f} \right)^n = 1 + 2\alpha \frac{\varphi}{f} + \beta \left( \frac{\varphi}{f} \right)^2 + \ldots
\]

where \( a \equiv \alpha v/f \), \( b = \beta v^2/f^2 \), and so one, the concordance with the methods

\textsuperscript{11}\textsuperscript{See J.Phys. G41 (2014) 025002.}
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II) K matrix

\[ \tilde{T} = T(1 - J(s)T)^{-1}, \quad J(s) = -\frac{1}{\pi} \log \left[ \frac{-s}{\Lambda^2} \right], \]

so that, for \( \tilde{t}_\omega \),

\[ \tilde{t}_\omega = \frac{t_\omega - J(t_\omega t_\varphi - t_{\omega \varphi}^2)}{1 - J(t_\omega + t_\varphi) + J^2(t_\omega t_\varphi - t_{\omega \varphi}^2)}, \]

for \( \beta = \alpha^2 \) (elastic case),

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$N \to \infty$, with $v^2/N$ fixed. The amplitude $A_N$ to order $1/N$ is a Lippmann-Schwinger series,

$$A_N = A - A \frac{N!}{2} A + A \frac{N!}{2} A \frac{N!}{2} A - \ldots$$

$$I(s) = \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2(q + p)^2} = \frac{1}{16\pi^2} \log \left[ \frac{-s}{\Lambda^2} \right] = -\frac{1}{8\pi} J(s)$$

Note: actually, $N = 3$. For the (iso)scalar partial wave (chiral limit, $I = J = 0$),

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(elastic scattering at tree level only $\beta = \alpha^2$. See ref. J.Phys. G41 (2014) 025002). Ansatz

$$\tilde{t}_\omega(s) = \frac{N(s)}{D(s)},$$

where $N(s)$ has a left hand cut (and $\Im N(s > 0) = 0$) $D(s)$ has a right hand cut (and $\Im D(s < 0) = 0$);

$$D(s) = 1 - \frac{s}{\pi} \int_{0}^{\infty} \frac{ds' N(s')}{s'(s' - s - i\epsilon)},$$

$$N(s) = \frac{s}{\pi} \int_{-\infty}^{0} \frac{ds' \Im N(s')}{s'(s' - s - i\epsilon)}$$
Coupled channels, tree level amplitudes

\[ f = 2v, \beta = \alpha^2 = 1, \lambda_3 = \frac{M_\varphi^2}{f}, \lambda_4 = \frac{M_\varphi^2}{f^2}. \]

OX axis: s in TeV^2.
Tree level, modulus of $\tilde{t}_\omega$, $K$ matrix

- All units in TeV.
- From top to bottom, $f = 1.2, 0.8, 0.4$ TeV
- $\Lambda = 3$ TeV
- $\mu = 100$ GeV
Im $t_\omega$ in the N/D method, $f = 1 \text{ TeV}, \beta = 1, m = 150 \text{ GeV}$
$\text{Re } t_\omega$ and $\text{Im } t_\omega$, large $N$, $f = 400 \text{ GeV}$
Re $t_\omega$ and Im $t_\omega$, large $N$, $f = 4$ TeV
Tree level, motion of the pole position of $t_\omega$ K–matrix, $M_\phi = 125$ GeV, $f \in (250$ GeV, $6$ TeV))
Comparison between the full LO $\omega\omega \rightarrow hh$ ($\cos \theta = 3$) and that computed through the ET. The SM is used here. Work in collaboration with S. Moretti, to test a modified version of MadGraph.
Comparison between the full LO \( \omega\omega \rightarrow hh \) (\( \cos \theta = 6 \)) and that computed through the ET. The SM is used here. Work in collaboration with S. Moretti, to test a modified version of MadGraph.
Motion of the resonance mass and width

Dependence on $b$ with $a^2 = 1$ fixed (upper curve) and for $a = 1 - \xi$ and $b = 1 - 2\xi$ with $\xi = v/f$ as in the MCHM (lower blue curve).

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Video, $(a,b)$ param. space
Resonances in $W_L W_L \to W_L W_L$ due to $a$ and $a_4$ parameters

- $b = a^2$
- PRD 91 (2015) 075017
- From left, clockwise, $IJ = 00, 11, 20$
- Excluding resonances $M_S < 700$ GeV, $M_V < 1.5$ TeV
Resonances in $\mathcal{W}_L \mathcal{W}_L \rightarrow \mathcal{W}_L \mathcal{W}_L$ due to $b$, $g$, $d$ and $e$ parameters

Effective Theory, PRD 91 (2015) 075017, isoscalar channels ($I = J = 0$).
Equivalence Theorem

- For \( s \gg M_h^2, M_W^2, M_Z^2 \approx (100 \text{ GeV})^2 \), longitudinal modes of gauge bosons can be identified with the would-be Goldstones. For instance,

\[
T(W_L^a W_L^b \rightarrow W_L^c W_L^d) = T(\omega^a \omega^b \rightarrow \omega^c \omega^d) + \mathcal{O}(M_W/\sqrt{s})
\]

- The EWSBS behaves as if the would-be Goldstone bosons were physical states. The non-gauged Lagragian can be used directly to compute scattering amplitudes.

- During the 90’s, the limits of applicability of this theorem were studied in detail, leading to the conclusion that it is valid for chiral Lagrangians, like those used in this presentation:

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Partial wave decomposition: $\gamma\gamma$ states

The form of the partial wave is

$$P_{IJ,\Lambda}(s) = P_{IJ,\Lambda}^{(0)} + \mathcal{O}(\alpha_{em}^2) + \mathcal{O}(\alpha_{em} s^2).$$

Note that $\gamma\gamma$ with $J = 2$, $\Lambda = \pm 2$ also couples with the EWSBS, following

$$P_{10,0}^{(0)} \propto \alpha s, \quad P_{12,\pm 2}^{(0)} \propto \alpha.$$

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Note that $\gamma\gamma$ with $J = 2, \Lambda = \pm 2$ also couples with the EWSBS, following

$$P^{(0)}_{10,0} \propto \alpha s \quad P^{(0)}_{12,\pm 2} \propto \alpha$$

Partial wave decomposition: $\gamma\gamma$ states

The form of the partial wave is

$$P_{IJ,\Lambda}(s) = P_{IJ,\Lambda}^{(0)} + \mathcal{O}(\alpha_{em}^2) + \mathcal{O}(\alpha_{em}s^2).$$

Note that $\gamma\gamma$ with $J = 2$, $\Lambda = \pm 2$ also couples with the EWSBS, following

$$P_{I0,0}^{(0)} \propto \alpha s \quad P_{I2,\pm 2}^{(0)} \propto \alpha$$

Scalar-isotensor channels \((IJ = 20)\)

From our ref\(^\text{12}\). From left to right, \(a = 0.88\), \(a = 1.15\). We have taken \(b = a^2\) and the NLO parameters set to zero. Both real and imaginary part shown. Real ones correspond to bottom lines at left and upper at low \(E\) at right.

\(^{12}\)PRD 91 (2015) 075017
Isotensor-scalar channels ($IJ = 02$)

$a = 0.88$, $b = a^2$, $a_4 = -2a_5 = 3/(192\pi)$, all the other NLO param. set to zero. PRD 91 (2015) 075017.
Reson. in $W_L W_L \rightarrow W_L W_L$ due to $a_4$ and $a_5$, ours

- $a = 0.90$, $b = a^2$
- PRD 91 (2015) 075017
- From left, clockwise, $IJ = 00, 11, 20$
- Excluding resonances $M_S < 700$ GeV, $M_V < 1.5$ TeV
CROSS-CHECK: Espriu, Yencho, Mescia
PRD88, 055002
PRD90, 015035
At right, exclusion regions include resonances with $M_{S,V} < 600$ GeV.
Production of $W^+ W^-$ (blue) vs. $W_L^+ W_L^-$ (red) in the SM. x-axis in GeV and y-axis in events/33.3 GeV. $\sqrt{s} = 13$ TeV, $L = 10$ fb$^{-1}$. 

Conclusions: experimental issues, $W_L^+ W_L^-$ prod. in the SM.
Note the usage of the chiral counting from \(^{13}\).

Chiral counting

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EFT for $VV$ scattering, minimally coupled to $hh$

Since we are considering scattering processes within the EWSBS, the covariant derivate reduces to

$$D_\mu U = \partial_\mu U.$$ 

Define

$$V_\mu \equiv (D_\mu U)U^\dagger.$$ 

The next counterterms are needed for the NLO computation of the $VV$ scattering, minimally coupled to $hh$

$$\mathcal{L}_4 = a_4[\text{Tr}(V_\mu V_\nu)][\text{Tr}(V_\mu V_\nu)] + a_5[\text{Tr}(V_\mu V_\mu)][\text{Tr}(V_\nu V_\nu)]$$

$$+ \frac{d}{v^2}(\partial_\mu h\partial_\mu h)\text{Tr}[(D_\nu U)^\dagger D^\nu U] + \frac{e}{v^2}(\partial_\mu h\partial_\nu h)\text{Tr}[(D_\mu U)^\dagger D_\nu U]$$

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Particular cases of the theory

\[ a^2 = b = 0 \]

Higgsless ECL, now experimentally discarded.

\[ a^2 = 1 - \frac{v^2}{f^2}, \quad b = 1 - 2\frac{v^2}{f^2} \]

SO(5)/SO(4) Minimal Composite Higgs Model (MCHM)
S.De Curtis, S.Moretti, K.Yagyu, E.Yildirim, JHEP1204 (2012) 042

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Dilaton models
E.Halyo, Mod.Phys.Lett.A8, 275; W.D.Goldberg et al, PRL100 111802

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Standard Model: without non-perturbative interactions
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Unitarization procedures for elastic processes

\[ A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)}, \]

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\[ A^{IK}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + g(s)A_L(s)}, \]

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where

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Extension to $\gamma\gamma$ and $t\bar{t}$ scattering

**Basic assumption**

- **EWSBS is strongly interacting. $\gamma\gamma$ and $t\bar{t}$ are perturbative.**
- Coupling with photons, controlled by $\alpha = e^2/4\pi \ll s/v^2$.
- Coupling with top quarks, controlled by $M_t\sqrt{s}/v^2 \ll s/v^2$.

**Perturbative unitarization:** $\omega \omega \to \{\gamma\gamma, t\bar{t}\}$

\[
\tilde{P} = \frac{\tilde{A}_{IJ}}{A_{IJ}^{(0)}} P^{(0)}
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Partial wave decomposition

EWSBS alone (+eventually $t\bar{t}$)

$$A_{IJ}(s) = \frac{1}{32\pi K} \int_{-1}^{1} dx \ P_J(x) A_I[s, t(s, x), u(s, x)]$$

Matrix element from partial wave decomposition

$$A_I(s, t, u) = 16\pi K \sum_{J=0}^{\infty} (2J+1) P_J(x(s, t)) A_{IJ}(s)$$

Helicity partial waves for EWSBS+$\gamma\gamma$

$$F_{IJ}^{\lambda_1\lambda_2}(s) = \frac{1}{64\pi^2 K} \sqrt{\frac{4\pi}{2J+1}} \int d\Omega \ A_I^{\lambda_1\lambda_2}(s, \Omega) Y_{J,\lambda_1-\lambda_2}(\Omega)$$
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We need the parameterization of the $U(\omega^a) \in SU(2)_L \times SU(2)_R/SU(2)_C$ coset. In either case, whatever the non–linear term is,

$$U(x) = 1 + i \frac{\tau^a \omega^a(x)}{v} + O(\omega^2).$$

Two choices have been used:

- Spherical parameterization
  $$U(x) = 1 \sqrt{1 - \frac{\omega^2(x)}{v^2}} + i \frac{\tau^a \omega^a(x)}{v}$$

- Exponential parameterization (here, a cross-check for EWSBS+$\gamma \gamma$)
  $$U(x) = \exp \left( i \frac{\tau^a \pi^a(x)}{v} \right)$$
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