Does a deformation of special relativity imply energy dependent photon time delays?

José Javier Relancio Martínez

Universidad de Zaragoza

Motivation

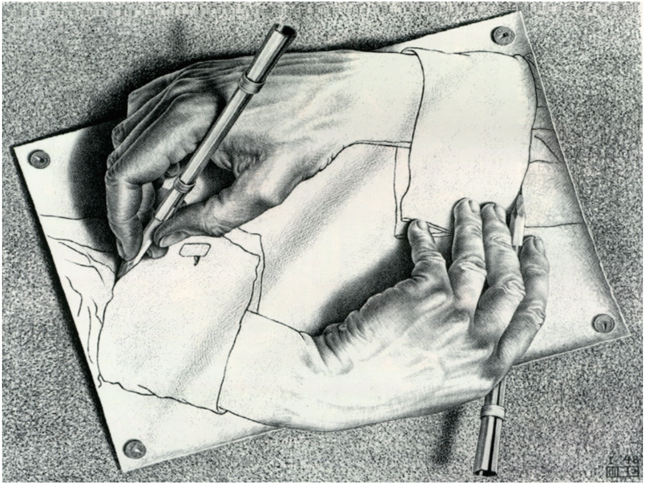
Space-time: the last frontier

- Any answer has to include the matter and also the space-time structure $\rightarrow$ Gravity
- If fundamental constituents of matter exist, does the same happen for spacetime?
- Do space «atoms» exist?

QFT and GR: incompatibilities

- One of the challenges for the nowadays physics is the unification of GR and QFT $\rightarrow$ QGT.
- In QFT, one assumes a given spacetime and one studies in detail the properties and motion of particles in it.
- In GR, one assumes that the properties of matter and radiation are given and one describes in detail the resultant space-time (curvature).
- For a QGT we mean a theory at any energy and an interaction mediated by spin-2 particle is not renormalizable.

QFT y GR: incompatibilities



Why do we need a QGT?

- Study of the first moments of the universe.
- Black holes: Singularity, information?
- Answers$\rightarrow$ QGT.

Quantum Gravity Theories

- Attempts of unification: string theory, quantum loop gravity, supergravity, causal set theory...
- In most of them a minimal length appears $\implies$ Planck length (l_P)??
- This is closely related to a energy scale $\implies$ Planck energy (Λ)??
- There are no experimental evidences of a fundamental QGT.
- New approach: study the low energy theory of QGT. **Deformed Special Relativity (DSR)** $\rightarrow$ posible experimental evidences.

Doubly Special Relativity (DSR)

- Two invariants in every inertial frame: speed of light c and Planck length l_P.
- We can obtain l_P, t_P, M_P and Λ

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\,10^{-35}\,\mathrm{m}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.4\,10^{-44}\,\mathrm{s}$$

$$\frac{\Lambda}{c^2} = M_P = \sqrt{\frac{\hbar c}{G}} = 2.2\,10^{-8}\,\mathrm{kg} = 1.2\,10^{19}\,\mathrm{GeV}/c^2$$

Consequences of a minimal length

- Quantized space-time: discrete and noncontinuous.
- Modified commutation rules

$$\Delta x \, \Delta t \geq l_P \, t_P$$

- We are unable to determine the metric at these scales: the notion of curvature is lost.
- All fundamental symmetries of SR and GR are only valid aproximations at distances larger than the Planck length.
- Naive sense of dimensions is lost.

Consequences of a minimal length

- Non continuum space-time $\implies$ observables do not vary continuously.
- The concept of point particle disappears.
- The size of particles is always bigger than Planck length $\implies m < M_P$
- Distinctions between real or virtual particle, matter-antimatter, matter-radiation,... disappears at Planck scales.

Quantum Gravity Phenomenology

- Planck energy $\rightarrow 10^{19}$ GeV
- Particle accelerators $\rightarrow 1.3\times\ 10^{4}$ GeV
- Cosmic rays$\rightarrow 10^{11}$ GeV.
- Phenomenology?$\rightarrow$ Amplifications at low energies.

Photon time delay

- Due to the «foam» of space-time, stochastic variations of the speed would be produced.
- These deviations can be obtained through modified dispersion relations (MDR), that for $E \ll \Lambda$

$$m^2 = C(p) \simeq E^2 - \vec{p}^2 - \xi_n E^2 \left(\frac{E}{\Lambda}\right)^n$$

- With the Hamiltonian concept of speed

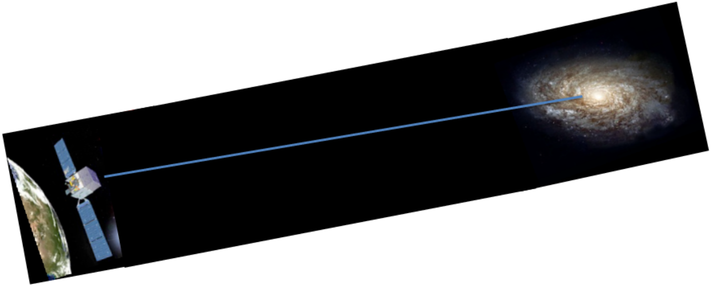
$$v = \frac{dE}{dp}$$

this causes a difference in the flight time

$$\Delta t \sim \left(\frac{d}{c}\right) \xi_n \left(\frac{E}{\Lambda}\right)^n$$

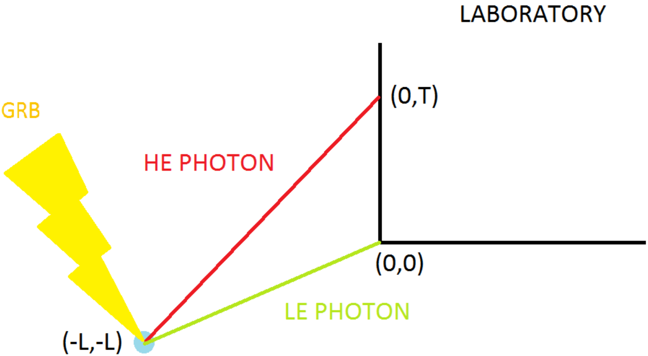
Photon time delay

- This delay can be measured for photons with different energies coming from a *gamma ray burst*.
- Recent experiments impose strong restrictions to deviations with respect to SR at first ($n = 1$)



Time delay in a classical spacetime

- Two photons with different energies are emitted simultaneously from a GRB $(-L, -L)$.
- Let us suppose a MDR for the HE photon.
- The detector would show a difference in the time of arrival T.



Construction of a noncommutative spacetime

- $l_P \implies$ noncommutative spacetime $\rightarrow$ quantum spacetime
- A noncommutative spacetime can be developed through

$$\tilde{x}^\mu = x^\nu \varphi^\mu_\nu(p)$$

with the following Poisson brackets

$$\{p_\nu, \tilde{x}^\mu\} = \varphi^\mu_\nu(p)$$

$$\{\tilde{x}^\mu, \tilde{x}^\nu\} = x^\lambda \left(\frac{\partial \varphi^\mu_\lambda(p)}{\partial p_\rho} \varphi^\nu_\rho(p) - \frac{\partial \varphi^\nu_\lambda(p)}{\partial p_\rho} \varphi^\mu_\rho(p) \right)$$

- $\lim_{p \to 0} \varphi^\mu_\nu(p) = \delta^\mu_\nu$

κ-Minkowski spacetime

- It is a very much studied model in DSR theories

$$\{\tilde{x}^0, \tilde{x}^i\} = -\frac{\tilde{x}^i}{\Lambda}$$

- The phase space Poisson brackets depend on the choice of $\varphi^\mu_\nu(p)$, which is in 1 to 1 correspondence with the different choices of a canonical phase space coordinates (x, p).
- These different choices give different MDR's and LT's.
- There are choices for which the MDR is such that $v = 1$ for photons.

Snyder spacetime

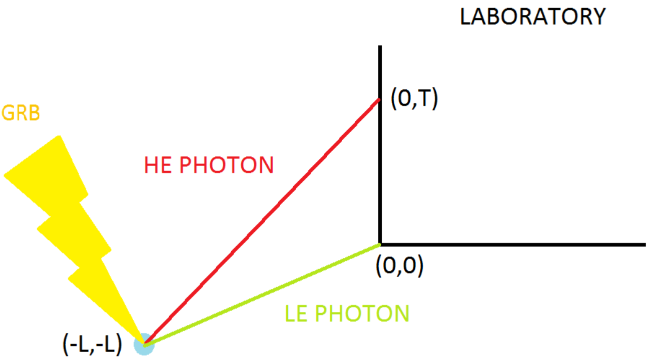
- It is a covariant model

$$\{\tilde{x}^\mu, \tilde{x}^\nu\} = \frac{J^{\mu\nu}}{\Lambda^2}$$

  proposed by Snyder in 1947 in order to avoid divergences in QFT.
- Due to covariance, the DR and LT are those of SR.

Naive (and incorrect) model

- Two photons with different energies are emitted simultaneously from a GRB.
- Let us suppose a MDR and a noncommutative spacetime for the HE photon.
- The detector would show a difference in the time of arrival.



Naive (and incorrect) model

- This model does not take into account that translations act non trivially, i.e., depend on the momentum.

Presentation of the model

- The LE photon lives in a commutative spacetime, so the emission point is the same $(-L, -L)$.
- As the HE photon lives in a noncommutative spacetime, for an observer in the laboratory the emission has coordinates

$$\tilde{t}^B = \varphi^0_0 t^B + \varphi^0_1 x^B = \tilde{t}^A - L(\varphi^0_0 + \varphi^0_1)$$
$$\tilde{x}^B = \varphi^1_0 t^B + \varphi^1_1 x^B = \tilde{x}^A - L(\varphi^1_0 + \varphi^1_1)$$

- For the observer in the laboratory the emission is not simultaneous.
- TD would be

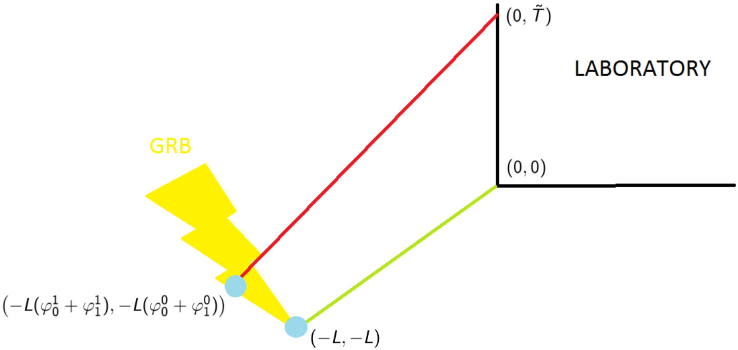
$$\tilde{T} = \tilde{v}^{-1} L(\varphi^1_0 + \varphi^1_1) - L(\varphi^0_0 + \varphi^0_1)$$

where

$$\tilde{v} = \frac{\varphi^1_0(\partial C/\partial E) - \varphi^1_1(\partial C/\partial p)}{\varphi^0_0(\partial C/\partial E) - \varphi^0_1(\partial C/\partial p)}$$

Correct model

- Two photons with different energies are emitted simultaneously from a GRB for an observer at the GRB, but not for the laboratory one.
- The TD depends on the noncommutativity of the spacetime.



Results

- In different models of noncommutativity, the only contribution at leading order to TD's comes from the MDR.
- This can be understood reorganizing the time delay expression

$$\tilde{T} = L\frac{\varphi_0^0\varphi_1^1 - \varphi_0^1\varphi_1^0}{\varphi_1^1 + \varphi_0^1/v}\left(\frac{1}{v} - 1\right)$$

- When $v = 1$ there is no time delay (as in the commutative case).
- But $v = 1$ for photons $\not\Rightarrow$ DR of SR.
- It is possible to go BSR with a noncommutative spacetime with no observation of TD.

Conclusions

- We study the TD of photons in a generic noncommutative spacetime with a MDR.
- The main result is that one can have a modification of SR without a observation of TD if the MDR is such that $v = 1$ for photons.
- Bad news: one can not test a modification of SR.
- Good news: the high energy scale parametrizing the departures of SR could be orders of magnitude less than the Planck one.