2HDM fits with HEPfit

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- Introduction
- HEPfit
- 2HDM
  - The model
  - Parameters
- 2HDM constraints
  - Theory
  - Experiment
- Results
- Conclusions

Introduction

After the discovery of the Higgs the SM seems to be complete,

BUT several experimental and theoretical issues remain unsolved

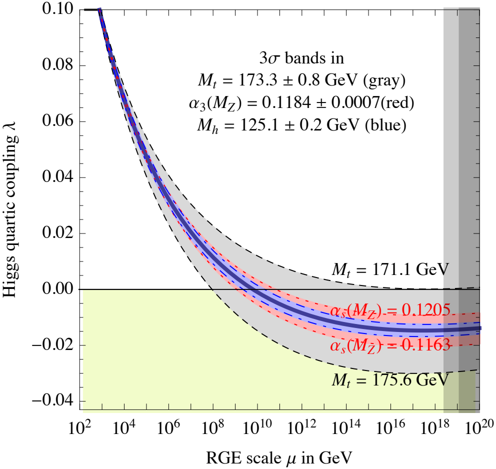
- ~~750 GeV diphoton excess~~
- Dark matter
- Origin of matter
- Neutrino masses
- Hierarchy problem
- Vacuum stability

Introduction

$$V(\Phi) = -\mu^2 \Phi\Phi^\dagger + \frac{1}{4}\lambda(\Phi\Phi^\dagger)^2$$

The SM cannot be stable

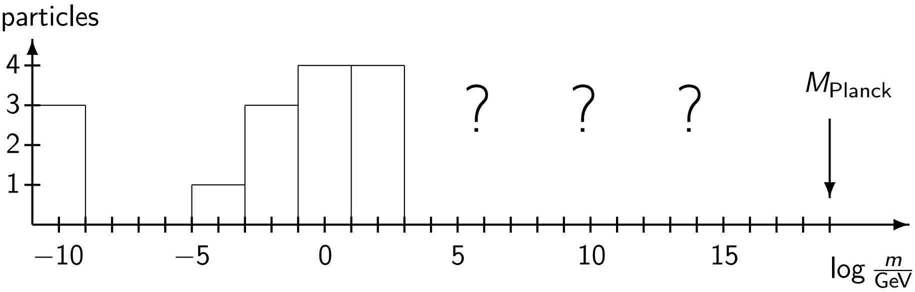
up to the Planck scale!



[Buttazzo et al. '13]

Introduction

Mass distribution of the elementary particles:



Introduction

Why another Higgs doublet ("2HDM")?

- No reason to assume only one Higgs doublet
  The 2HDM could serve as effective model,
  for instance for NHDM's or the MSSM.
- Solution to the vacuum stability problem
- Solution to the "big desert" problem
- More complicated 2HDM's are interesting for dark matter, baryogenesis, EW phase transition and could explain discrepancies between certain flavour observables

What is HEPfit?

Flexible open-source C++ code to do calculations with various observables in the SM and beyond:

- Simple user-defined models and/or observables
- Stand-alone or library modes to compute single observables
- Optional Bayesian fitting framework to do global statistical analyses
  (run-time optimized, parallelized; can be replaced by a different one)

Our goal: 4S – as much as fast and as precise as possible

Installation and usage

Depends on: ROOT, GSL, Boost, Bayesian Analysis Toolkit (BAT)

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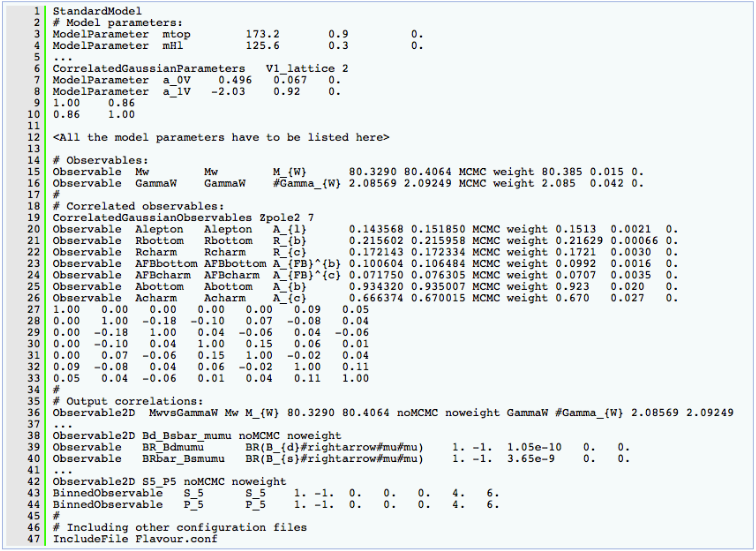
```
./analysis StandardModel.conf MonteCarlo.conf
```

StandardModel.conf

One configuration file to rule them all:

- Model (SM or extension)
- Parameter values
- Observables

StandardModel.conf



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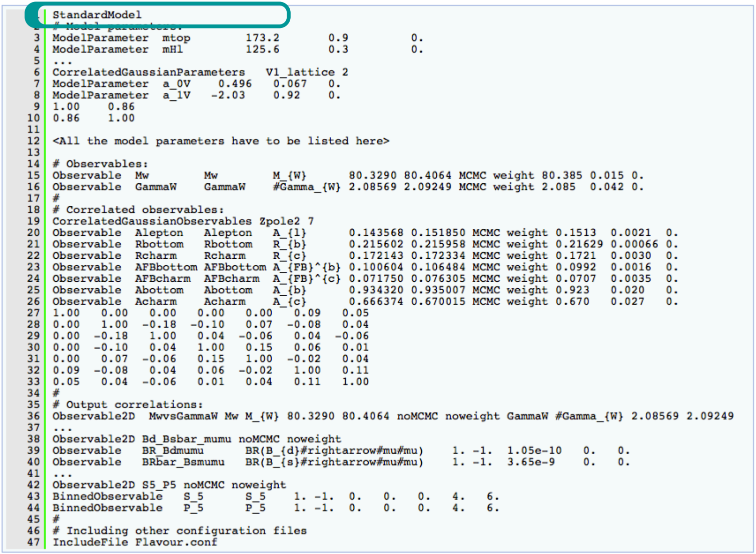

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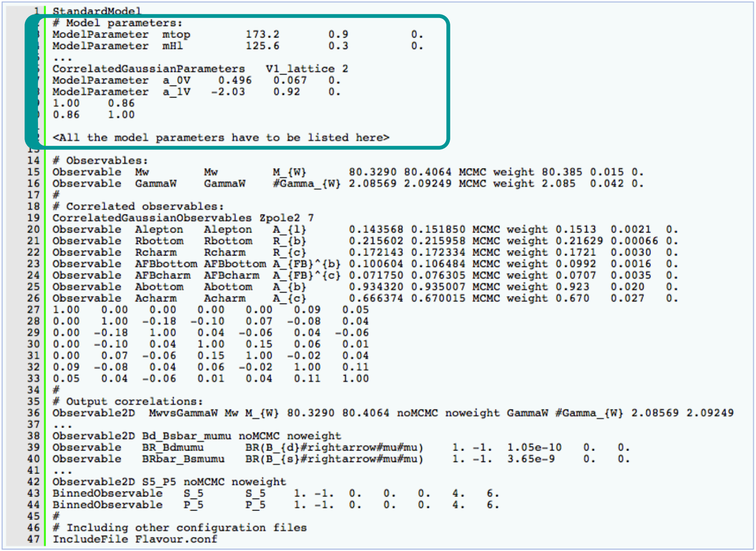

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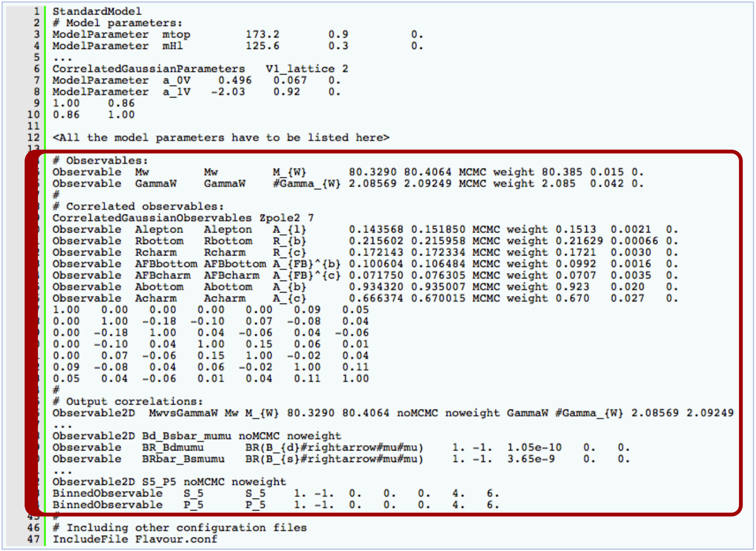

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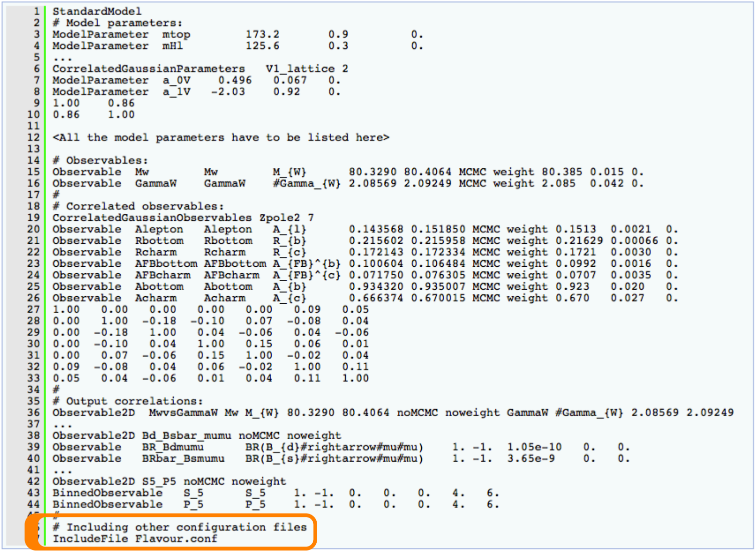

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Some examples

Observable	**SM**	**2HDM**	**SUSY**	**Dim-6**
Flavour:				
$\mathcal{B}(B_s \to \mu^+\mu^-)$	✓	✓	×	×
$\mathcal{B}(\bar{B} \to X_s\gamma)$	✓	✓	×	×
$\mathcal{B}(\tau \to \mu\gamma, 3\mu)$	–	–	✓	×
(...)				
Higgs:				
Signal strengths	✓	✓	×	✓
Direct searches	–	✓	×	–
(...)				
Electroweak precision observables:				
STU	✓	✓	×	✓
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Direct searches	–	✓	×	–
(...)				
Electroweak precision observables:				
STU	✓	✓	×	✓
(...)				

Some examples

$$\mathcal{B}(B_s \to \mu^+\mu^-) \propto (|P|^2 + |S|^2)$$

$$P = C_{10} + \frac{m_{B_s}^2}{2m_W^2}\frac{m_b}{m_b + m_s}C_P$$

$$S = \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}\frac{m_{B_s}^2}{2m_W^2}\frac{m_b}{m_b + m_s}C_S$$

For example $C_i = C_i^{\text{SM}} + C_i^{\text{2HDM}}$

$\mathcal{B}(B_s \to \mu^+\mu^-)$ is defined as Flavour observable;
the C_i are defined in StandardModel and 2HDM.

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- Large spectrum of flavour observables (rare decays, non-leptonic decays etc.), most of them at the highest available precision

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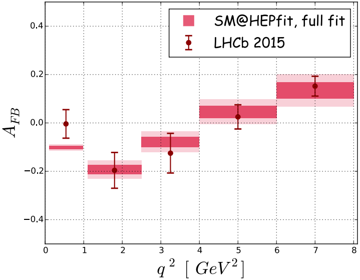

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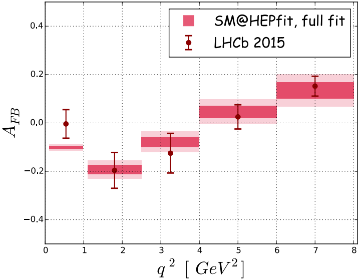

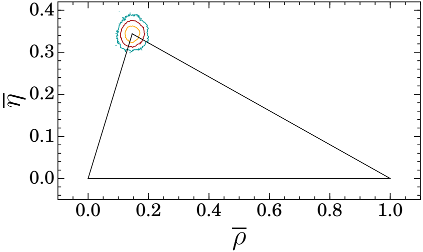

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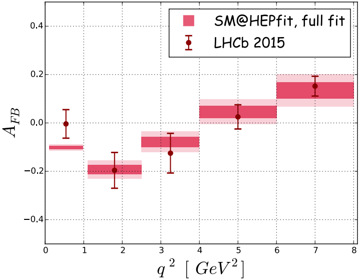
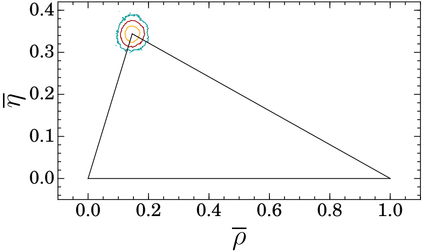
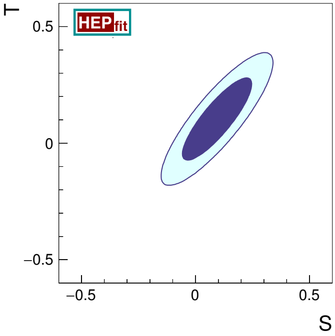
- EWPO, LEP2 observables



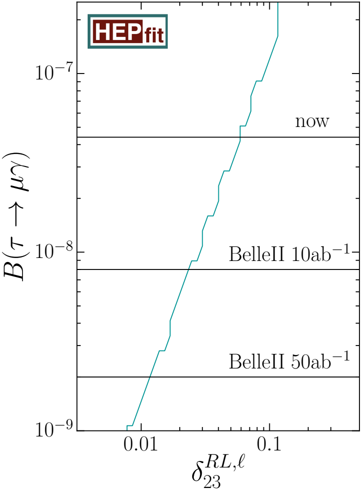


Generic SM extensions

- Oblique parameters (S, T, U and ϵ_i)
- Modified $Zb\bar{b}$ couplings ($\delta g^b_{L,R}$)
- Modified Higgs couplings ($\kappa_{V,f}$)
- Dimension 6 effective theory (C_i/Λ^2)



MSSM



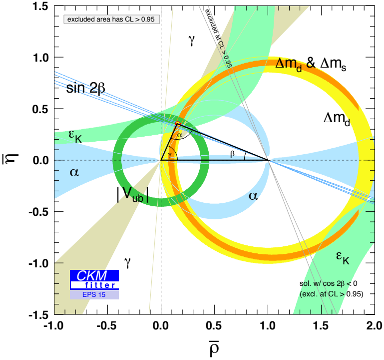
- Generic flavour structure implemented
- Lepton flavour violating decays: $\ell_i \to \ell_j$, $\ell_i \to 3\ell_j$, $(g-2)_\mu$, μ to e conversion in nuclei
- Vacuum stability constraints
- We are working on metastability and RGE.

Other models

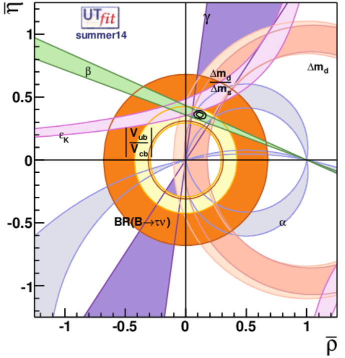
- 2HDM with softly broken Z_2 (see part 2 of this talk)
- General 2HDM
- Left-Right symmetric model
- Your model?

Run time

Unitarity triangle fits with run time of at least a few days



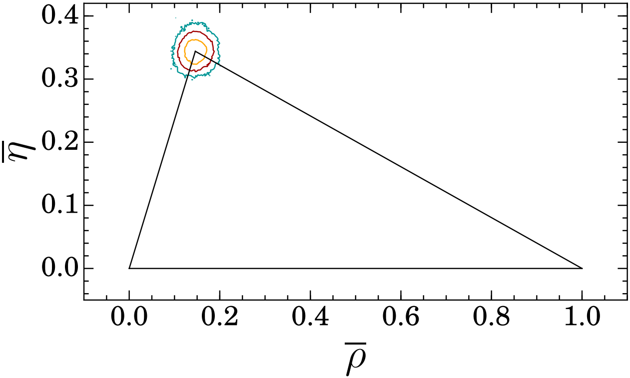
[CKMfitter '15]



[UTfit '14]

Run time

Unitarity triangle fit with `HEPfit` is possible on a laptop: about 1 hour with eight cores



The HEPfit team

HEPfit

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- Marco Ciuchini
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- Marco Fedele
- Enrico Franco
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- Giovanni Grilli di Cortona
- Mauro Valli

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- Maurizio Pierini

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Tohoku University
- Norimi Yokozaki

Lanzhou University
- Fu-Sheng Yu

Introduction

HEPfit

2HDM
- The model
- Parameters

2HDM constraints
- Theory
- Experiment

Results

Conclusions

Theory

Scalar Lagrangian:

$$\begin{aligned}\mathcal{L}_H &= (D_\mu \Phi\ )^\dagger (D^\mu \Phi\ ) \\ &\quad - m^2\ \Phi^\dagger \Phi \\ &\quad - \frac{\lambda}{2} \left(\Phi^\dagger \Phi\ \right)^2\end{aligned}$$

Theory

Scalar Lagrangian:

$$\begin{aligned}\mathcal{L}_H^{\text{2HDM}} = {} & (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) \\ & - m_1^2\ \Phi_1^\dagger\Phi_1 - m_2^2\ \Phi_2^\dagger\Phi_2 \\ & - \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 - \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2\end{aligned}$$

Theory

Scalar Lagrangian:

$$\begin{aligned}\mathcal{L}_H^{\text{2HDM}} = & (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) \\ & - m_{11}^2 \Phi_1^\dagger \Phi_1 - m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ & - \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 - \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 - \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\ & - \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) - \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right] \\ & - \left\{ \left[\lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \right] \left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right\}\end{aligned}$$

Theory

Scalar Lagrangian:

$$
\begin{aligned}
\mathcal{L}_H^{\text{2HDM}} = & \left(D_\mu \Phi_1\right)^\dagger \left(D^\mu \Phi_1\right) + \left(D_\mu \Phi_2\right)^\dagger \left(D^\mu \Phi_2\right) \\
& - m_{11}^2 \Phi_1^\dagger \Phi_1 - m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) \\
& - \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1\right)^2 - \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2\right)^2 - \lambda_3 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_2^\dagger \Phi_2\right) \\
& - \lambda_4 \left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_2^\dagger \Phi_1\right) - \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right] \\
& - \left\{\left[\lambda_6 \left(\Phi_1^\dagger \Phi_1\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right)\right] \left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.}\right\}
\end{aligned}
$$

assuming Z_2 symmetry $\Phi_2 \to -\Phi_2$: 8 parameters

Theory

Scalar Lagrangian:

$$\begin{aligned}\mathcal{L}_H^{\text{2HDM}} = & (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) \\ & - m_{11}^2\Phi_1^\dagger\Phi_1 - m_{22}^2\Phi_2^\dagger\Phi_2 + m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) \\ & - \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 - \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 - \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\ & - \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) - \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right] \\ & - \left\{\left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\}\end{aligned}$$

assuming Z_2 symmetry $\Phi_2 \to -\Phi_2$: 8 parameters

assuming no CP violation: 8 real parameters

Theory

Yukawa Lagrangian:

$$
\begin{aligned}
\mathcal{L}_Y \quad = - \sum_{j,k=1}^{3} \Big[& Y^d_{jk} \ \left(\overline{Q}_j \Phi \ \right) d_k \\
& + Y^u_{jk} \ \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi^* \right) u_k \\
& + Y^\ell_{jk} \ \left(\bar{L}_j \Phi \ \right) \ell_k \qquad\qquad\qquad\qquad + \text{h.c.} \Big]
\end{aligned}
$$

Theory

Yukawa Lagrangian:

$$\begin{aligned}\mathcal{L}_Y^{2\mathrm{HDM}} = -\sum_{j,k=1}^{3} \Big[& Y_{jk}^{d,1} \left(\overline{Q}_j \Phi_1\right) d_k + Y_{jk}^{d,2} \left(\overline{Q}_j \Phi_2\right) d_k \\ & + Y_{jk}^{u,1} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_1^*\right) u_k + Y_{jk}^{u,2} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_2^*\right) u_k \\ & + Y_{jk}^{\ell,1} (\bar{L}_j \Phi_1)\, \ell_k + Y_{jk}^{\ell,2} (\bar{L}_j \Phi_2)\, \ell_k + \mathrm{h.c.} \Big]\end{aligned}$$

2HDM of type III (no additional symmetry)

Theory

Yukawa Lagrangian:

$$\mathcal{L}_Y^{\text{2HDM}} = -\sum_{j,k=1}^{3} \Big[Y_{jk}^{d,1} \left(\overline{Q}_j \Phi_1\right) d_k + Y_{jk}^{d,2} \left(\overline{Q}_j \Phi_2\right) d_k + Y_{jk}^{u,1} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_1^*\right) u_k + Y_{jk}^{u,2} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_2^*\right) u_k + Y_{jk}^{\ell,1} \left(\bar{L}_j \Phi_1\right) \ell_k + Y_{jk}^{\ell,2} \left(\bar{L}_j \Phi_2\right) \ell_k + \mathrm{h.c.} \Big]$$

2HDM of type I (additional Z_2: $\Phi_2 \to -\Phi_2$)

Theory

Yukawa Lagrangian:

$$\mathcal{L}_Y^{\text{2HDM}} = -\sum_{j,k=1}^{3} \Big[Y_{jk}^{d,1} \left(\overline{Q}_j \Phi_1\right) d_k + Y_{jk}^{d,2} \left(\overline{Q}_j \Phi_2\right) d_k + Y_{jk}^{u,1} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_1^*\right) u_k + Y_{jk}^{u,2} \left(\overline{Q}_j \mathrm{i}\sigma_2 \Phi_2^*\right) u_k + Y_{jk}^{\ell,1} \left(\bar{L}_j \Phi_1\right) \ell_k + Y_{jk}^{\ell,2} \left(\bar{L}_j \Phi_2\right) \ell_k + \mathrm{h.c.} \Big]$$

2HDM of type II (additional Z_2: $\Phi_2 \to -\Phi_2,\ u_k \to -u_k$)

Theory

Re-parametrisation:

λ_1		$v \approx 246$ GeV	v.e.v.
λ_2		$m_{h^0} \approx 126$ GeV	light CP-even Higgs mass
λ_3		m_{H^0}	heavy CP-even Higgs mass
λ_4	$\rightarrow$	m_A	CP-odd Higgs mass
λ_5		m_{H^+}	charged Higgs mass
m_{11}^2		α	diagonalisation angle of the CP-even mass matrix
m_{22}^2		β	diagonalisation angle of the CP-odd and charged mass matrix
m_{12}^2		m_{12}^2	soft Z_2 breaking parameter

Theory

We will use $\tan\beta$ and $\beta-\alpha$ instead of α and β.

Theory

We will use $\tan\beta$ and $\beta-\alpha$ instead of α and β.

Alignment limit: $(\beta-\alpha)-\frac{\pi}{2}\to 0$ $\Rightarrow$All h couplings SM-like

Decoupling limit: $(\beta-\alpha)-\frac{\pi}{2}\ll 1$ and $m_H \approx m_A \approx m_{H^+} \gg m_h$

[Gunion, Haber '02]

Introduction

HEPfit

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Theory

Positivity of the Higgs potential

The Higgs potential should be bounded from below to yield $v < \infty$.

For $\Phi_1, \Phi_2 \to \infty$ we demand for the simplified potential

$$\lambda_1 x^2 + \lambda_2 y^2 + f(\lambda_3, \lambda_4, \lambda_5) xy > 0$$

$$\Rightarrow \qquad \lambda_1 > 0, \qquad \lambda_2 > 0,$$
$$\lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \qquad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}$$

[Deshpande, Ma '78]

Theory

Vacuum stability

We must avoid that our model contains an unstable vacuum.
Excluding metastability:

$$m_{12}^2 \left(m_{11}^2 - \sqrt{\frac{\lambda_1}{\lambda_2}} m_{22}^2 \right) \left(\tan\beta - \sqrt[4]{\frac{\lambda_1}{\lambda_2}} \right) > 0$$

[Barroso, Ferreira, Ivanov, Santos '13]

Theory

$\boxed{\text{Perturbativity of the quartic couplings}}$

Perturbativity in the SM: $\lambda = \frac{m_h^2}{2v^2} \approx 0.13 \ll 1$

Before m_h was known, there were upper limits from $\boxed{\text{RGE}}$ and $\boxed{\text{unitarity}}$.

Theory

RGE in the SM

$$\frac{\mathrm{d}\lambda(Q^2)}{\mathrm{d}(\ln Q^2)} = \frac{3}{4\pi^2}\left(\lambda^2 + \lambda Y_t - Y_t^2\right) + \mathcal{O}(\lambda g_i^2, g_i^4)$$

For $\lambda > Y_t \approx 1$:

$$\lambda(Q^2) \approx \frac{\lambda(m_Z^2)}{1 - \frac{3\lambda(m_Z^2)}{4\pi^2}\ln\frac{Q^2}{m_Z^2}}$$

$$\Rightarrow \boxed{\lambda(m_Z^2) \lesssim 5}$$

Theory

RGE in the SM

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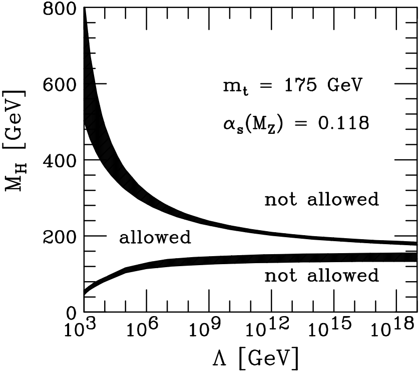
$$\Rightarrow \boxed{\lambda(m_Z^2) \lesssim 5}$$

NLO RGE: $\boxed{\lambda(m_Z^2) \lesssim 2.2}$ [Nierste, Riesselmann '96]



[Hambye, Riesselmann '97]

Theory

Unitarity in quantum mechanics

The S-matrix is required to be unitary:

$$S^\dagger S = \mathbb{1}$$

Partial wave decomposition:

$$|2a_\ell^{2\to 2} - i|^2 + \sum_{n>2} |2a_\ell^{2\to n}|^2 = 1$$

For large $s = (p_1 + p_2)^2$, we can neglect the $a_\ell^{2\to n}$ part and set $\ell = 0$.

$$\Rightarrow |\mathrm{Re}(a_0^{2\to 2})| \leq \frac{1}{2}$$

Theory

Unitarity in the SM

$$\mathbf{a_0} = \frac{1}{16\pi s} \int_{-s}^{0} \mathcal{M}(s,t)\mathrm{d}t$$

Theory

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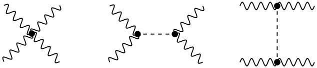

For only one initial and final state:

$$\mathcal{M}_{w^+w^-\to w^+w^-} = -4\lambda - (-2i\lambda v)^2 \left(\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right)$$
$$= -2\lambda \left(\frac{s}{s-m_h^2} + \frac{t}{t-m_h^2}\right)$$

Theory

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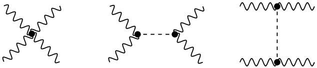

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$$a_0^{w^+w^-\to w^+w^-} \approx \frac{1}{16\pi s} \cdot (-2\lambda \cdot 2s) = -\frac{\lambda}{4\pi}$$

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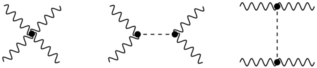

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For all scalar 2 → 2 states:

$$a_0^{2\to 2} = -\frac{3\lambda}{8\pi}$$

Theory

Unitarity in the SM

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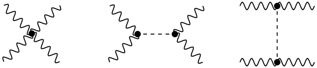

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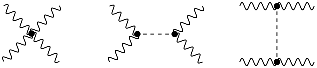
For all scalar 2 → 2 states:

$$a_0^{2\to 2} = -\frac{3\lambda}{8\pi} \Rightarrow \boxed{\lambda \lesssim 4}$$

Theory

Unitarity in the SM

$$\mathbf{a_0} = \frac{1}{16\pi s}\int_{-s}^{0}\mathcal{M}(s,t)\mathrm{d}t$$



For only one initial and final state:

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$$= -2\lambda\left(\frac{s}{s-m_h^2}+\frac{t}{t-m_h^2}\right)$$
$$a_0^{w^+w^-\to w^+w^-} \approx \frac{1}{16\pi s}\cdot(-2\lambda\cdot 2s) = -\frac{\lambda}{4\pi}$$

For all scalar 2 → 2 states:

$$a_0^{2\to 2} = -\frac{3\lambda}{8\pi} \Rightarrow \boxed{\lambda \lesssim 4}$$

NLO unitarity: $\boxed{\lambda \lesssim 2.2}$ [Durand, Johnson, Lopez '92]

Theory

Unitarity in the 2HDM $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ instead of one λ

Tree-level unitarity matrix:

$$\frac{1}{8\pi}\begin{pmatrix}
\lambda_1 & \lambda_5 & & & & & & & & & & \\
\lambda_5 & \lambda_2 & & & & & & & & & & \\
 & & \lambda_3+\lambda_4 & & & & & & & & & \\
 & & & \lambda_3-\lambda_4 & & & & & & & & \\
 & & & & \lambda_1 & \lambda_4 & & & & & & \\
 & & & & \lambda_4 & \lambda_2 & & & & & & \\
 & & & & & & \lambda_3 & \lambda_5 & & & & \\
 & & & & & & \lambda_5 & \lambda_3 & & & & \\
 & & & & & & & & 3\lambda_1 & 2\lambda_3+\lambda_4 & & \\
 & & & & & & & & 2\lambda_3+\lambda_4 & 3\lambda_2 & & \\
 & & & & & & & & & & \lambda_3+2\lambda_4 & 3\lambda_5 \\
 & & & & & & & & & & 3\lambda_5 & \lambda_3+2\lambda_4
\end{pmatrix}$$

[Ginzburg, Ivanov '05]

Theory

Unitarity in the 2HDM $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ instead of one λ

Tree-level unitarity eigenvalues:

$$a_0^{(1,2)} = \frac{1}{16\pi}\left(\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}\right)$$

$$a_0^{(3,4)} = \frac{1}{16\pi}\left(\lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}\right)$$

$$a_0^{(5,6)} = \frac{1}{16\pi}\left(3(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + 4(2\lambda_3 + \lambda_4)^2}\right)$$

$$a_0^{(7,8)} = \frac{1}{8\pi}(\lambda_3 \pm \lambda_4)$$

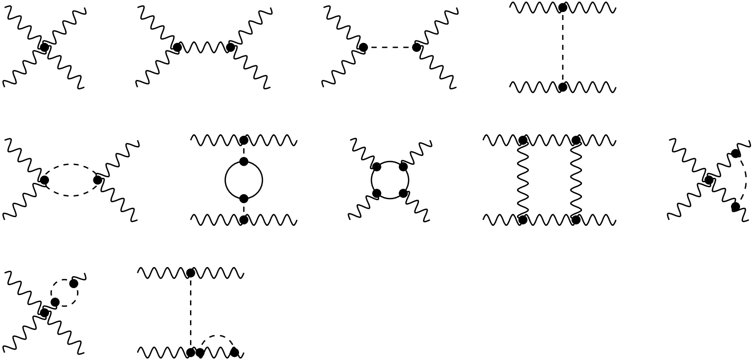
$$a_0^{(9,10)} = \frac{1}{8\pi}(\lambda_3 \pm \lambda_5)$$

$$a_0^{(11,12)} = \frac{1}{8\pi}(\lambda_3 + 2\lambda_4 \pm 3\lambda_5)$$

[Ginzburg, Ivanov '05]

Theory

NLO unitarity in the 2HDM



[Grinstein, Murphy, Uttayarat '15]

Theory

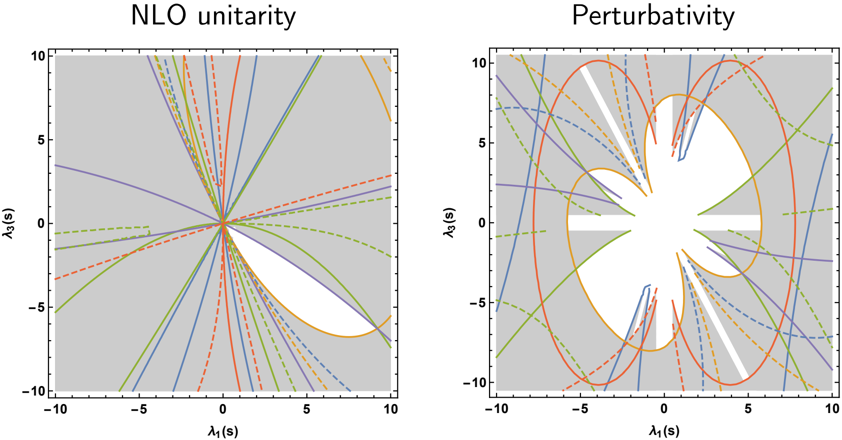
Perturbativity of the NLO contributions

$$R_1' = \frac{|a_0^{\text{NLO}}|}{|a_0^{\text{LO}}|} \leq 1$$

We apply this only if $|a_0^{\text{LO}}| > 0.02 \approx \frac{1}{16\pi}$.

[Grinstein, Murphy, Uttayarat '15]

Theory



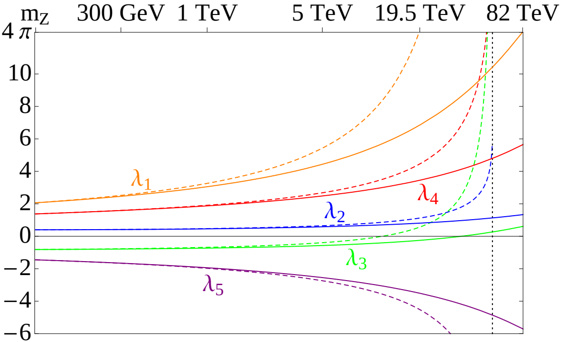
$$\lambda_2 = \lambda_1, \lambda_4 = -\lambda_1 - \lambda_3, \lambda_5 = 0$$

[Grinstein, Murphy, Uttayarat '15]

Theory

We obtained the NLO RGE using PyR@TE.

[Lyonnet, Schienbein, Staub, Wingerter '13]



[Chowdhury, OE '15]

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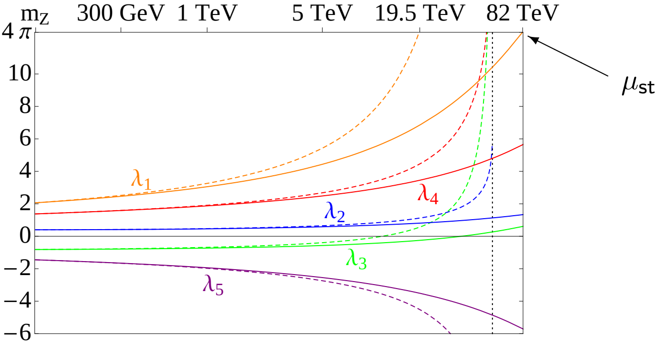
[Lyonnet, Schienbein, Staub, Wingerter '13]



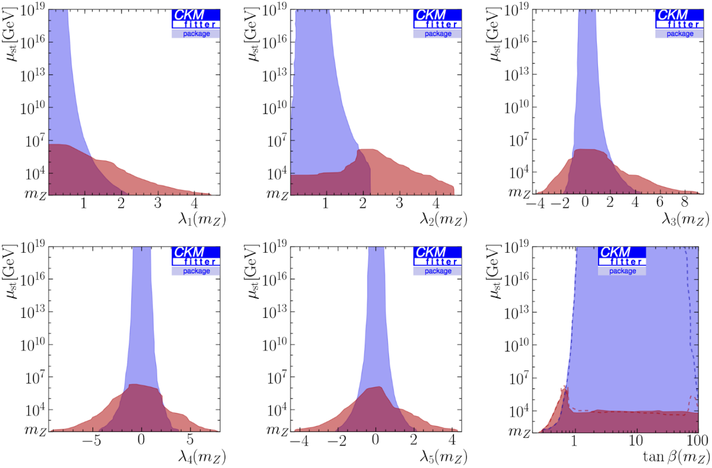
[Chowdhury, OE '15]

Theory

NLO RGE

$1/8 < |a_0| < 1/4$

$|a_0| < 1/8$

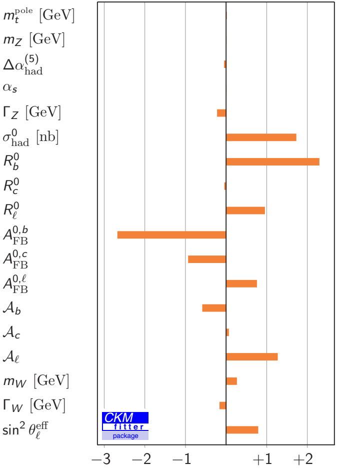


[Chowdhury, OE '15]

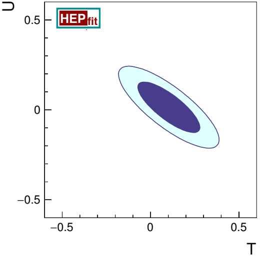
Experiment

W and Z pole observables – Two different approaches:

EWPO



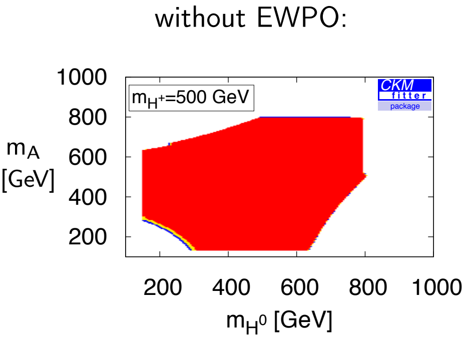
STU



	Fit result	Correlations		
S	0.09±0.10	1.00		
T	0.10±0.12	0.86	1.00	
U	0.01±0.09	−0.54	−0.81	1.00

[OE '13; HEPfit '16]

Experiment



[OE '13]

Experiment

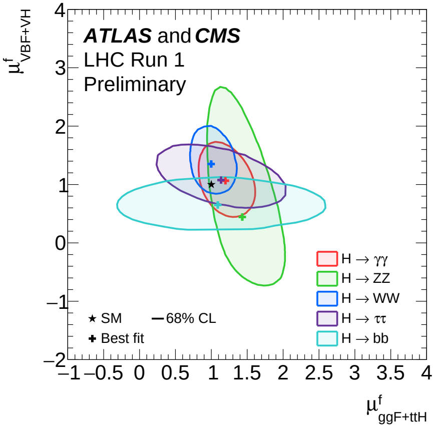
Higgs signal strengths

$$\mu^{\mathrm{f}}_{\mathrm{ggF+tth}} = \frac{[(\sigma_{\mathrm{ggF}} + \sigma_{\mathrm{tth}}) \cdot B(h \to f)]_{\mathrm{2HDM}}}{[(\sigma_{\mathrm{ggF}} + \sigma_{\mathrm{tth}}) \cdot B(h \to f)]_{\mathrm{SM}}}$$

$$\mu^{\mathrm{f}}_{\mathrm{VBF+Vh}} = \sin^2(\beta - \alpha) \cdot \frac{[B(h \to f)]_{\mathrm{2HDM}}}{[B(h \to f)]_{\mathrm{SM}}}$$

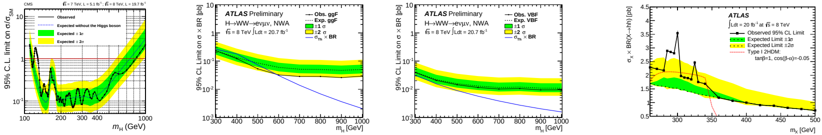
Experiment

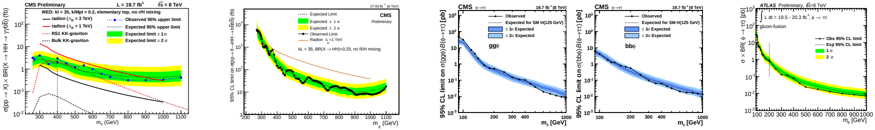
Higgs signal strengths

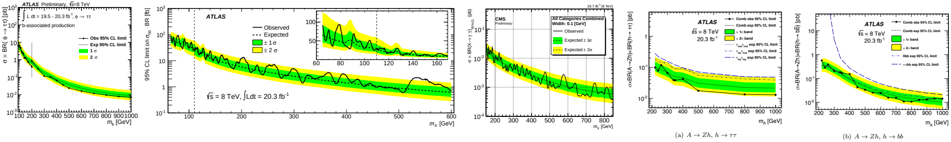


[ATLAS & CMS '15]

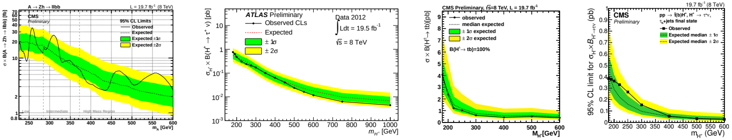
Experiment







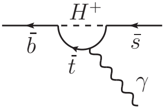
(a) $A \to Zh$, $h \to \tau\tau$ (b) $A \to Zh$, $h \to bb$



[LHC '15]

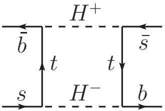
Experiment

Flavour



$\mathcal{B}(b \to s\gamma)$

[Hermann, Misiak, Steinhauser '12; Misiak et al. '15; HFAG '14]

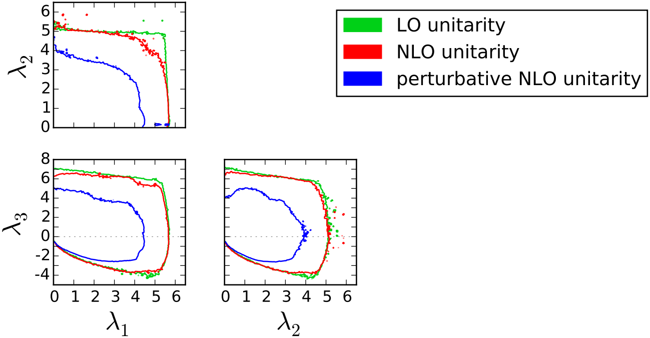


Δm_{B_s}

[Deschamps et al. '09; LHCb '13]

Quartic couplings

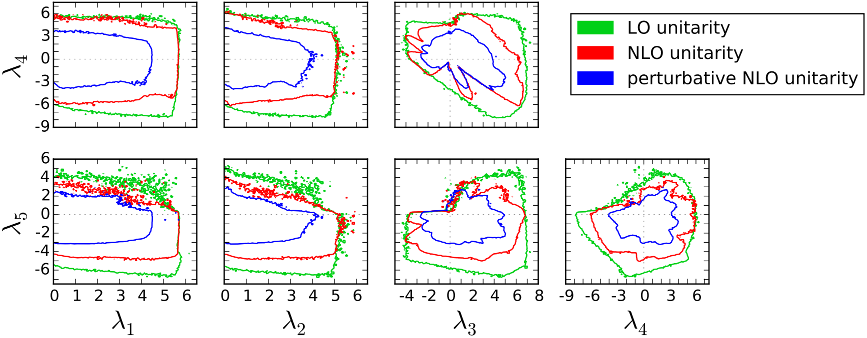
NLO unitarity bounds combined with stability up to 750 GeV:



[Cacchio, Chowdhury, OE, Murphy in preparation]

Quartic couplings

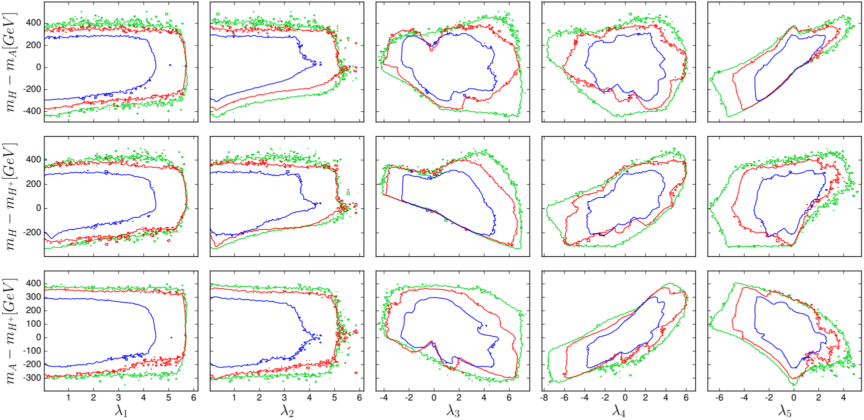
NLO unitarity bounds combined with stability up to 750 GeV:



[Cacchio, Chowdhury, OE, Murphy in preparation]

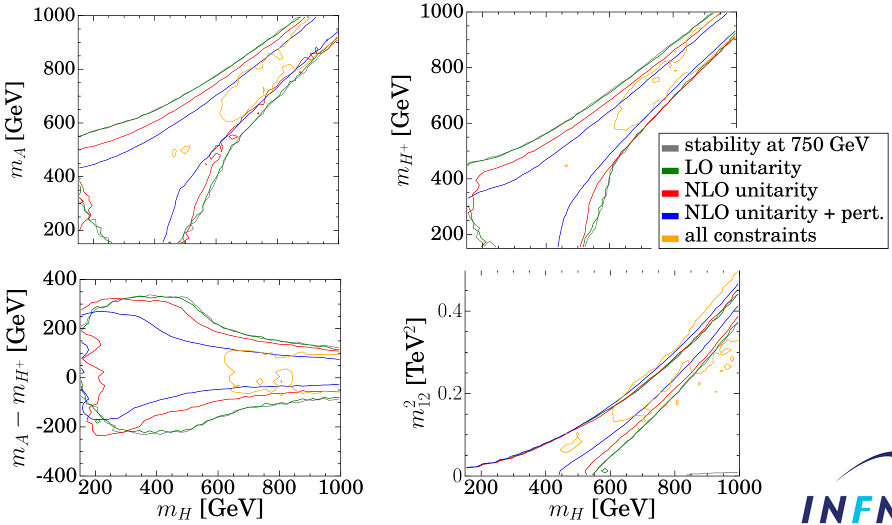
Mass differences

NLO unitarity bounds combined with stability up to 750 GeV:



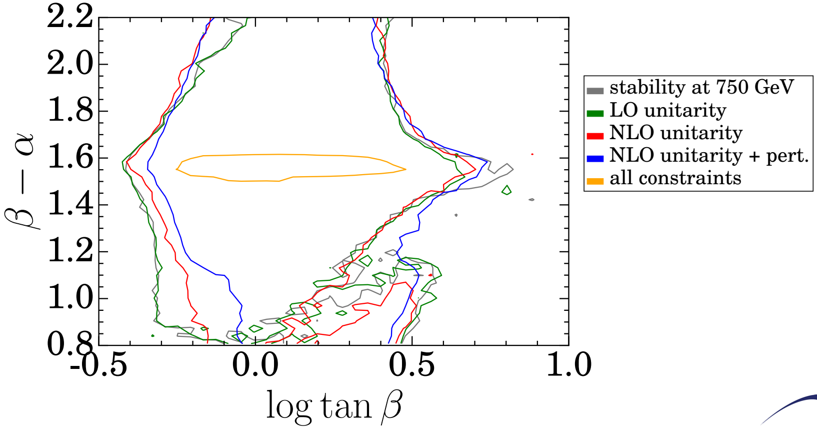
[Cacchio, Chowdhury, OE, Murphy in preparation]

Physical parameters I



[Cacchio, Chowdhury, OE, Murphy in preparation]

Physical parameters II



[Cacchio, Chowdhury, OE, Murphy in preparation]

Conclusions

- `HEPfit` will be released soon
- Quartic couplings of the 2HDM with a softly broken Z_2 get strongly constrained by NLO unitarity.

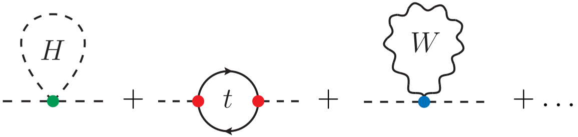
Back-up

Inputs for the unitarity triangle fit

Δm_{B_d}	0.5055 ± 0.002 ps^{-1}
Δm_{B_s}	17.757 ± 0.021 ps^{-1}
ϵ_K	0.00228 ± 0.00011
V_{ud}	0.97428 ± 0.00021
V_{us}	0.2249 ± 0.0009
V_{ub}	0.00381 ± 0.00040
V_{cb}	0.0409 ± 0.0011
$\alpha_{\pi\pi}, \alpha_{\rho\rho}, \alpha_{\rho\pi}$	Likelihood
γ	Likelihood
$S_{J/\Psi K}$	0.679 ± 0.023

Naturalness

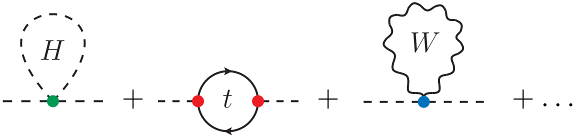
$\delta m_h^2 =$



$$= \frac{\mu_{\text{nat}}^2}{16\pi^2}\left[\sum_{n=0}^{\infty} f_n(\lambda_i, Y_i, g_i)\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)^n\right] + \mathcal{O}\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)$$

Naturalness

$\delta m_h^2 =$



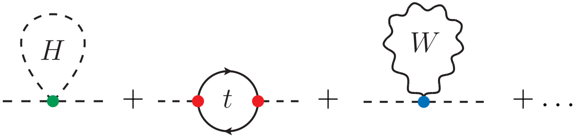
$$= \frac{\mu_{\text{nat}}^2}{16\pi^2}\left[\sum_{n=0}^{\infty} f_n(\lambda_i, Y_i, g_i)\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)^n\right] + \mathcal{O}\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)$$

$$\approx \frac{\mu_{\text{nat}}^2}{16\pi^2} f_0(\lambda_i, Y_i, g_i)\left[1 + \sum_{n=1}^{\infty}\prod_{\ell=1}^{n} k_\ell\right]$$

$$\text{with} \quad k_n = \frac{f_n(\lambda_i, Y_i, g_i)}{f_{n-1}(\lambda_i, Y_i, g_i)} \ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}$$

Naturalness

$\delta m_h^2 =$



$$= \frac{\mu_{\text{nat}}^2}{16\pi^2}\left[\sum_{n=0}^{\infty} f_n({\color{green}\lambda_i}, {\color{red}Y_i}, {\color{blue}g_i})\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)^n\right] + \mathcal{O}\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)$$

$$\approx \frac{\mu_{\text{nat}}^2}{16\pi^2} f_0(\lambda_i, Y_i, g_i)\left[1 + \sum_{n=1}^{\infty}\prod_{\ell=1}^{n} k_\ell\right]$$

$$\text{with} \quad k_n = \frac{f_n(\lambda_i, Y_i, g_i)}{f_{n-1}(\lambda_i, Y_i, g_i)} \ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}$$

Assuming $\mu_{\text{nat}} = \mu_{\text{stability}}$, $|k_1|, |k_2| \leq 1$ and $|\delta m_h^2| \leq m_h^2$:

$$\boxed{|f_0(\lambda_i, Y_i, g_i)| < 6 \text{ and } \mu_{\text{nat}} \lesssim 5 \text{ TeV}}$$

$f_n(\lambda_i, Y_i, g_i)$

$$\delta m_h^2 = \frac{\mu_{\text{nat}}^2}{16\pi^2}\left[\sum_{n=0}^{\infty} f_n(\lambda_i, Y_i, g_i)\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)^n\right] + \mathcal{O}\left(\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)$$

$$\approx \frac{\mu_{\text{nat}}^2}{16\pi^2} f_0(\lambda_i, Y_i, g_i)\left[1 + \sum_{n=1}^{\infty}\prod_{\ell=1}^{n}\underbrace{\left(\frac{f_\ell(\lambda_i, Y_i, g_i)}{f_{\ell-1}(\lambda_i, Y_i, g_i)}\ln\frac{\mu_{\text{nat}}}{\mu_{\text{ew}}}\right)}_{k_\ell}\right]$$

$$f_0(\lambda_i, Y_i, g_i) = \frac{3}{2}\lambda_1 + \frac{3}{2}\lambda_2 - \frac{3}{2}\cos(2\alpha)(\lambda_1 - \lambda_2) + 2\lambda_3 + \lambda_4$$
$$- \cos^2(\alpha)\left(6Y_b^2 + 6Y_t^2 + 2Y_\tau^2\right) + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2$$

$$f_{n+1}(\lambda_i, Y_i, g_i) = \frac{1}{n+1}\sum_{L\in\{\lambda_i, Y_i, g_i\}} \beta_L \frac{\partial}{\partial L} f_n(\lambda_i, Y_i, g_i)$$

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