ChPT parameters from tau-decay data

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Collaboration Meeting

In collaboration with:

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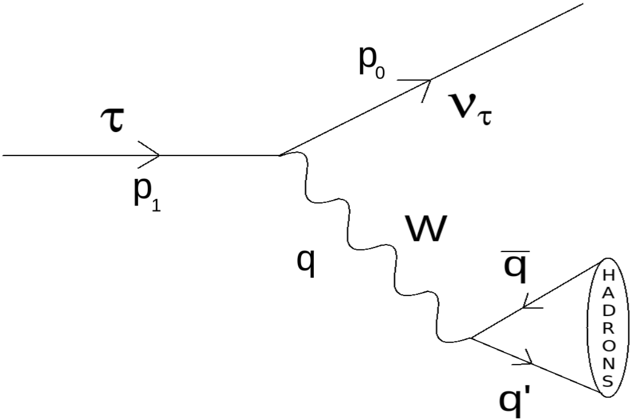
Semileptonic τ decay



Matrix Element

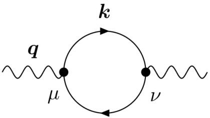
$$\mathcal{M} = \frac{g^2}{8M_W^2}\bar{u}_0\gamma_\mu(1-\gamma_5)u_1 \langle H^- | J^\mu_{W\bar{q}q}(q^2) | 0 \rangle \; ; \quad J^\mu_{W\bar{q}q} = \bar{d}_\theta \gamma^\mu (1-\gamma_5) u$$

Semileptonic τ decay

Decay width

$$\Gamma(\tau^- \to \nu_\tau + H^-)(s) = f(s)\,\mathrm{Im}\Pi^{(1+0)}(s)$$

$$s = q^2$$



Two-point correlation function of quark currents

$$\Pi^{\mu\nu}_{ud,J}(q) \equiv i \int d^4x\, e^{iqx}\, \langle 0|\, T(J^{\mu}_{W\bar{q}q}(x) J^{\nu\dagger}_{W\bar{q}q}))\, |0\rangle$$
$$= (-g^{\mu\nu} q^2 + q^\mu q^\nu)\Pi^{(1)}_{ud,J}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ud,J}(q^2)$$

$$J^{\mu}_{W\bar{q}q} = \underbrace{V_{us}\bar{s}\gamma^\mu(1-\gamma_5)u}_{J^\mu_S \to \Gamma_S(K^-,\ldots)} + \underbrace{V_{ud}\bar{d}\gamma^\mu u}_{J^\mu_V \to \Gamma_V(\pi^-\pi^0,\ldots)} - \underbrace{V_{ud}\bar{d}\gamma^\mu\gamma_5 u}_{J^\mu_A \to \Gamma_A(\pi^-,\ldots)}$$

Operator Product Expansion

OPE of the correlator

$$\Pi^{OPE\,\mu\nu}(q) = \langle 0|\frac{i^n}{n!}\int d^4x\, e^{iqx}\int dz_1 dz_2 ... dz_n\, T(J^\mu(x)J^\nu(0)\mathcal{L}^0_{int}(z_1)...\mathcal{L}^0_{int}(z_n))\,|0\rangle$$

$$= \int d^4x\, e^{iqx}\sum_D c_D^{\mu\nu}(x)\,\langle 0|:\mathcal{O}_D:|0\rangle \sim \sum_D \frac{C_D\mathcal{O}_D}{(-q^2)^{D/2}}g^{\mu\nu}q^2 + \frac{C'_D\mathcal{O}_D}{(-q^2)^{D/2}}q^\mu q^\nu$$

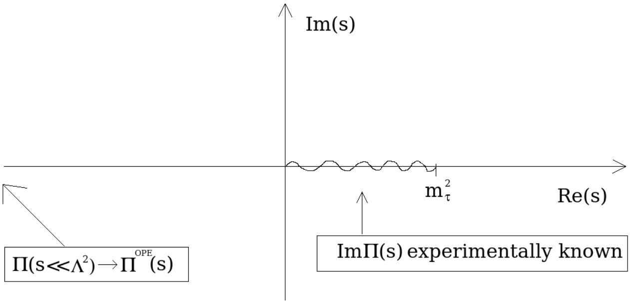
Massless QCD operators

$$\mathcal{O}_0 = \mathcal{I}$$
$$\mathcal{O}_2 = 0$$
$$\mathcal{O}_4^{(1)} = \langle 0|:G^a_{\mu\nu}G_a^{\mu\nu}:|0\rangle \neq 0!$$

Some operators acquire a non-zero vacuum expectation value.

SVZ (1978)

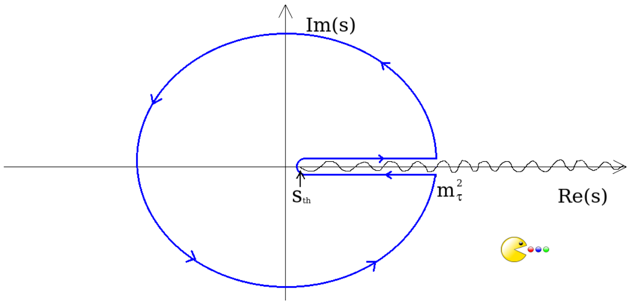
Sum Rules



$$\Pi^{OPE}(s) = \sum \frac{C_{2k} O_{2k}}{(-s)^k}$$

$\Pi(s)$ is analytic in the s-plane except for a cut on the positive real axis

Sum Rules



$$\int_{s_{th}}^{s_0} ds\, s^n \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) + \frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^n \Pi(s) = \mathrm{Res}[s^n \Pi(s), s=0]$$

Duality Violations

In the real axis, for values of s_0 large enough:

$$\Pi(s=-s_0) \approx \Pi^{OPE}(s=-s_0) = \sum \frac{C_{2k}\mathcal{O}_{2k}}{(-s)^k}$$

In the rest of the circumference, the analytic continuation of the Π^{OPE} will still be a good approximation if we are far from the positive real axis.

The differences between the physical results and the results obtained using the OPE are known as quark-hadron duality violations.

Duality Violations

In terms of the Π^{OPE} and taking into account Duality Violations:

$$\underbrace{\int_{s_{th}}^{s_0} ds\, s^n \frac{1}{\pi} \mathrm{Im}\,\Pi(s)}_{\text{Experimentally known}} + \underbrace{\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, s^n \Pi^{OPE}(s)}_{\text{Theoretically known}} + \delta_{DV}[s^n, s_0]$$

$$= \mathrm{Res}[s^n \Pi(s), s = 0]$$

with:

$$\delta_{DV}[s^n, s_0] \equiv \frac{1}{2\pi i} \oint_{|s|=s_0} ds\, s^n (\Pi(s) - \Pi^{OPE}(s))$$
$$= \frac{1}{\pi} \int_{s_0}^{\infty} ds\, s^n\, \mathrm{Im}(\Pi(s) - \Pi^{OPE}(s))$$

Taking $\Pi^{V+A}(s)$ ($\langle 0|\, VV + AA\, |0\rangle$), $\alpha_s(m_\tau^2)$ can be measured!

Duality Violations

In terms of the Π^{OPE} and taking into account Duality Violations:

$$\underbrace{\int_{s_{th}}^{s_0} ds\, s^n \frac{1}{\pi} \operatorname{Im}\Pi(s)}_{\text{Experimentally known}} + \underbrace{\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^n \Pi^{OPE}(s)}_{\text{Theoretically known}} + \delta_{DV}[s^n, s_0]$$

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$$\delta_{DV}[s^n, s_0] \equiv \frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^n (\Pi(s) - \Pi^{OPE}(s))$$
$$= \frac{1}{\pi}\int_{s_0}^{\infty} ds\, s^n \operatorname{Im}(\Pi(s) - \Pi^{OPE}(s))$$

The rest of the talk will be about $\Pi^{V-A}(s)$ ($\langle 0| VV - AA |0\rangle$), exactly 0 in massless QCD, the $\mathcal{O}_0$ part of the OPE.

Sum Rules

$$\Pi^{OPE}(s) = \sum \frac{C_{2k}\mathcal{O}_{2k}}{(-s)^k} = \sum_k \frac{\hat{O}_{2k}}{(-s)^k}$$

$n = -2$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds\, s^{-2} \frac{1}{\pi} \operatorname{Im}\Pi(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, s^{-2} \Pi^{OPE}(s) \rightarrow 0$$

$$- \delta_{DV}\left(\frac{1}{s^2}, s_0\right) + \Pi'(0) \approx 16 C_{87}^{eff} - 2\frac{f_\pi^2}{m_\pi^4}$$

$n = -1$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds\, s^{-1} \frac{1}{\pi} \operatorname{Im}\Pi(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds\, s^{-1} \Pi^{OPE}(s) \rightarrow 0$$

$$- \delta_{DV}\left(\frac{1}{s}, s_0\right) + \Pi(0) \approx -8 L_{10}^{eff} - 2\frac{f_\pi^2}{m_\pi^2}$$

Sum Rules

$$\Pi^{OPE}(s) = \sum \frac{C_{2k}\mathcal{O}_{2k}}{(-s)^k} = \sum_k \frac{\hat{O}_{2k}}{(-s)^k}$$

Weinberg (1967)

$n = 0$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds \, \frac{1}{\pi} \, \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds \, \Pi^{OPE}(s) \;\; (\rightarrow 0)$$
$$- \delta_{DV}\,(1, s_0) \approx 0$$

$n = 1$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds \, \frac{1}{\pi} s \, \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds \, s \, \Pi^{OPE}(s) \;\; (\rightarrow 0)$$
$$- \delta_{DV}\,(s, s_0) \approx 0$$

Sum Rules

$$\Pi^{OPE}(s) = \sum \frac{C_{2k}\mathcal{O}_{2k}}{(-s)^k} = \sum_k \frac{\hat{O}_{2k}}{(-s)^k}$$

$n = 2$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds\, \frac{1}{\pi} s^2\, \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^2\, \Pi^{OPE}(s) \rightarrow \hat{O}_6$$
$$- \delta_{DV}\left(s^2, s_0\right) \approx \hat{O}_6$$

$n = 3$ $\rightarrow$

$$\int_{s_{th}}^{s_0} ds\, \frac{1}{\pi} s^3\, \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, s^3\, \Pi^{OPE}(s) \rightarrow -\hat{O}_8$$
$$- \delta_{DV}\left(s^3, s_0\right) \approx -\hat{O}_8$$

A first estimation of the effective χPT parameters

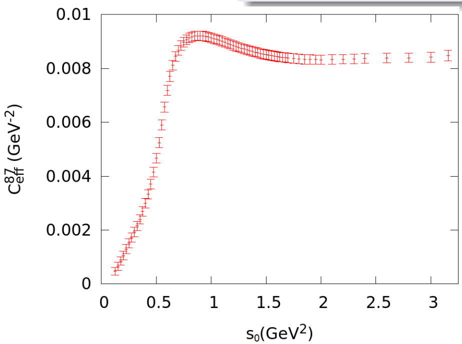
Data of τ decays from LEP

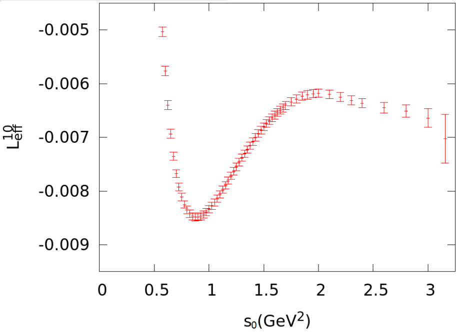
$$\rho(s) = \frac{1}{\pi} \operatorname{Im} \Pi^1(s)$$

ALEPH collaboration

Davier et al.

$$\delta_{DV}[s^n, s_0] = \frac{1}{\pi} \int_{s_0}^{\infty} ds\, s^n \operatorname{Im} \Pi(s)$$





A first estimation of the effective χPT parameters

$\frac{1}{s^n} \to \omega(s)$

$$\int_{s_{th}}^{s_0} ds\, \omega(s) \frac{1}{\pi} \operatorname{Im} \Pi(s) + \frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \omega(s)\, \Pi^{OPE}(s) + \delta_{DV}[\omega(s), s_0] = 2 f_\pi^2 \omega(m_\pi^2) + \operatorname{Res}[\omega(s)\Pi(s), s=0]$$

$$\delta_{DV}[\omega(s), s_0] \equiv \frac{1}{2\pi i} \oint_{|s|=s_0} ds\, \omega(s)(\Pi(s) - \Pi^{OPE}(s))$$

We can reduce the DV uncertainties using pinched weight functions: $\omega(s_0) = 0$, $\omega'(s_0) = 0$, etc.

L_{eff}^{10}

$$\omega_1(s) = \frac{1}{s}\left(1 - \frac{s}{s_0}\right)$$

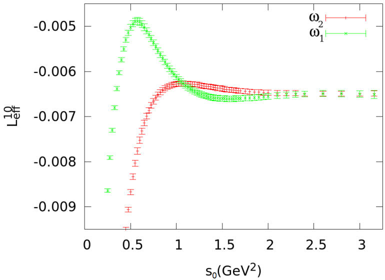
$$\omega_2(s) = \frac{1}{s}\left(1 - \frac{s}{s_0}\right)^2$$

C_{eff}^{87}

$$\omega_3(s) = \frac{1}{s^2}\left(1 - \frac{s^2}{s_0^2}\right)$$

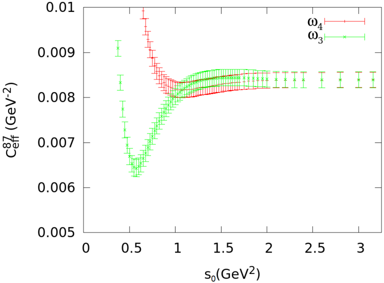
$$\omega_4(s) = \frac{1}{s^2}\left(1 - \frac{s}{s_0}\right)^2\left(1 + 2\frac{s}{s_0}\right)$$

A first estimation of the effective χPT parameters



$$L^{10}_{eff} = -(6.49 \pm 0.06)\, 10^{-3}$$

$$C^{87}_{eff} = (8.39 \pm 0.18)\, 10^{-3} \text{ GeV}^{-2}$$



Dealing with quark-hadron duality violation

We want a reliable estimation of the DV uncertainties:

$$\delta_{DV}[s^n, s_0] \equiv \frac{1}{2\pi i} \oint_{|s|=s_0} ds\, s^n (\Pi(s) - \Pi^{OPE}(s)) = \frac{1}{\pi} \int_{s_0}^{\infty} ds\, s^n\, \mathrm{Im}\,\Pi(s)$$

The s-dependence of $\mathrm{Im}\,\Pi(s)$ is unknown

We want to use the information we have:
- The Weinberg Sum Rules.
- The experimental $\mathrm{Im}\,\Pi(s)$ below m_τ^2.
- $\mathrm{Im}\,\Pi(s) \to 0$ fast.

We want to accept spectral functions which are compatible with all this information in order to estimate the DV effect in our observables

Dealing with quark-hadron duality violation

The model used

$$\rho(s) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) = \kappa e^{-\gamma s} \sin\beta(s - s_z)$$

We create tuples randomly distributed of the parameters

Conditions

1. 90 % C.L. region in a fit with the experimental $\mathrm{Im}\,\Pi(s)$ in $s \in (1.7\ \mathrm{GeV}^2, m_\tau^2)$:

$$\chi^2 < \chi^2_{min} + 7.78 = 16.3$$

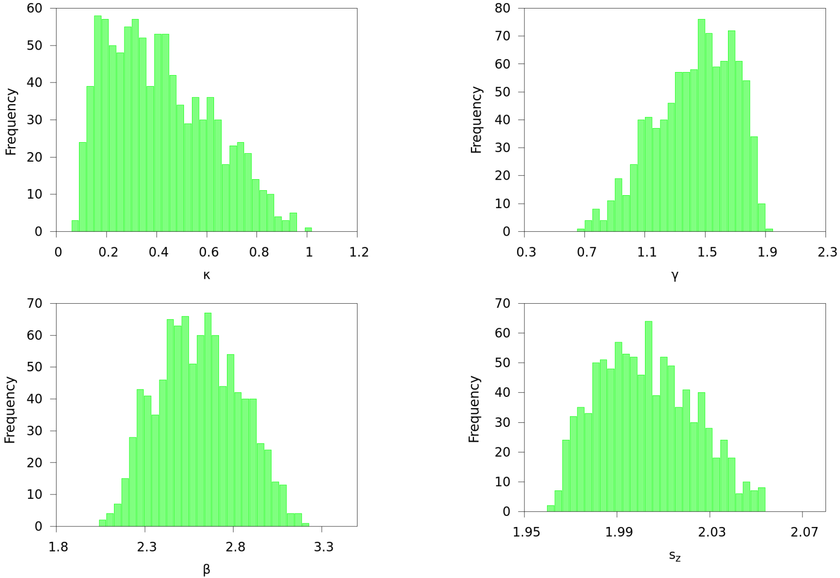
2. Weinberg Sum Rules and π Sum Rule are satisfied.

$$\int_0^{s_z} \rho(s)^{ALEPH} ds + \int_{s_z}^{\infty} \rho(s;\kappa,\gamma,\beta,s_z)\, ds = (17.2 \mp 0.4) \cdot 10^{-3}\ \mathrm{GeV}^2$$

$$\int_0^{s_z} \rho(s)^{ALEPH} s\, ds + \int_{s_z}^{\infty} \rho(s;\kappa,\gamma,\beta,s_z)\, s\, ds = (0.3 \mp 0.8) \cdot 10^{-3}\ \mathrm{GeV}^4$$

$$\int_0^{s_z} \rho(s)^{ALEPH} s \log\left(\frac{1}{1\mathrm{GeV}^2}\right) ds + \int_{s_z}^{\infty} \rho(s;\kappa,\gamma,\beta,s_z) \log\left(\frac{1}{1\mathrm{GeV}^2}\right) ds$$
$$= (10.9 \mp 1.5) \cdot 10^{-3}\ \mathrm{GeV}^4$$

Dealing with quark-hadron duality violation



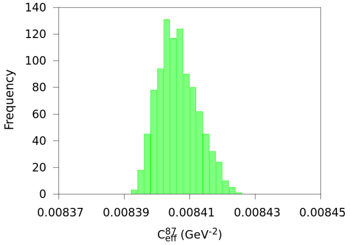
Every accepted tuple gives a value for every moment

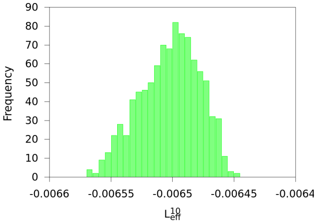
$$16C_{87}^{eff} = \int_{s_{th}}^{s_z} ds\, s^{-2} \frac{1}{\pi} \,\mathrm{Im}\,\Pi(s) + \delta_{DV}\left(\frac{1}{s^2}, s_z\right)$$

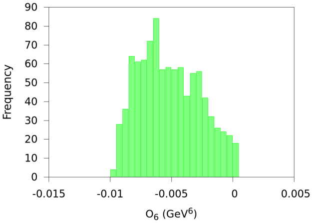
$$-8L_{10}^{eff} = \int_{s_{th}}^{s_z} ds\, s^{-1} \frac{1}{\pi} \,\mathrm{Im}\,\Pi(s) + \delta_{DV}\left(\frac{1}{s}, s_z\right)$$

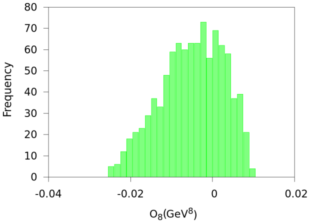
$$\hat{O}_6 = \int_{s_{th}}^{s_z} ds\, s^{2} \frac{1}{\pi} \,\mathrm{Im}\,\Pi(s) + \delta_{DV}\left(s^2, s_z\right)$$

$$-\hat{O}_8 = \int_{s_{th}}^{s_z} ds\, s^{3} \frac{1}{\pi} \,\mathrm{Im}\,\Pi(s) + \delta_{DV}\left(s^3, s_z\right)$$









The DV uncertainties are computed from the dispersion of the hystograms

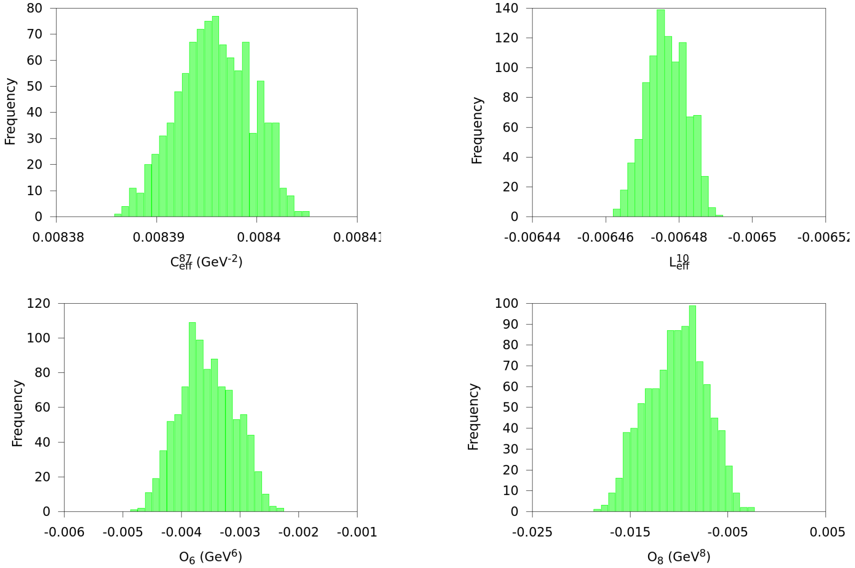
We can reduce this uncertainties using the pinched weight functions

Pinched weight functions

$$\omega_{C_{87}^{eff}}(s) = \frac{1}{s^2}\left(1 - \frac{s}{s_z}\right)^2\left(1 + \frac{2s}{s_z}\right) \qquad \omega_{L_{10}^{eff}}(s) = \frac{1}{s}\left(1 - \frac{s}{s_z}\right)^2$$

$$\omega_{\hat{O}_6}(s) = (s - s_z)^2 \qquad \omega_{\hat{O}_8}(s) = (s - s_z)^2(s + 2s_z)$$

Determination χPT parameters and OPE contributions



Determination χPT parameters and OPE contributions

Values without pinched weight functions

$$C^{87}_{eff} = (8.406^{+0.007}_{-0.006} \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2} = (8.41 \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

$$L^{10}_{eff} = (-6.50^{+0.02}_{-0.03} \pm 0.08) \cdot 10^{-3} = (-6.50^{+0.08}_{-0.09}) \cdot 10^{-3}$$

$$\hat{O}_6 = (-5.4^{+3.0}_{-2.4} \pm 1.3) \cdot 10^{-3}\,\mathrm{GeV}^6 = (-5.4^{+3.3}_{-2.7}) \cdot 10^{-3}\,\mathrm{GeV}^6$$

$$\hat{O}_8 = (-5^{+8}_{-9} \pm 2) \cdot 10^{-3}\,\mathrm{GeV}^8 = (-5^{+8}_{-9}) \cdot 10^{-3}\,\mathrm{GeV}^8$$

Values using pinched weight functions

$$C^{87}_{eff} = (8.396^{+0.004}_{-0.004} \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2} = (8.40 \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

$$L^{10}_{eff} = (-6.477^{+0.005}_{-0.006} \pm 0.048) \cdot 10^{-3} = (-6.48 \pm 0.05) \cdot 10^{-3}$$

$$\hat{O}_6 = (-3.6^{+0.5}_{-0.5} \pm 0.5) \cdot 10^{-3}\,\mathrm{GeV}^6 = (-3.6 \pm 0.7) \cdot 10^{-3}\,\mathrm{GeV}^6$$

$$\hat{O}_8 = (-1.0^{+0.3}_{-0.3} \pm 0.3) \cdot 10^{-2}\,\mathrm{GeV}^8 = (-1.0 \pm 0.4) \cdot 10^{-2}\,\mathrm{GeV}^8$$

Conclusions

L^{10}_{eff} and C^{87}_{eff} can be calculated using data from semileptonic τ decays

Neglecting DV

$$L^{10}_{eff} = -(6.49 \pm 0.06)\,10^{-3}$$
$$C^{87}_{eff} = (8.39 \pm 0.18)\,10^{-3}\ \text{GeV}^{-2}$$

Considering DV

$$L^{10}_{eff} = (-6.48 \pm 0.05) \cdot 10^{-3}$$
$$C^{87}_{eff} = (8.40 \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

Gonzalez Alonso et al. (2010)

$$L^{10}_{eff} = -(6.44 \pm 0.06)\,10^{-3}$$
$$C^{87}_{eff} = (8.17 \pm 0.12)\,10^{-3}\ \text{GeV}^{-2}$$

Boito et al. (2015)

$$L^{10}_{eff} = (-6.45 \pm 0.05) \cdot 10^{-3}$$
$$C^{87}_{eff} = (8.38 \pm 0.18) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

$\hat{O}_6$ and $\hat{O}_8$ have also been estimated

$$\hat{O}_6 = (-3.6 \pm 0.7) \cdot 10^{-3}\,\mathrm{GeV}^6$$
$$\hat{O}_8 = (-1.0 \pm 0.4) \cdot 10^{-2}\,\mathrm{GeV}^8$$