Chiral kinetic theory and time evolution of the chiral magnetic effect

in collaboration with Cristina Manuel (CSIC-UAB, Spain)

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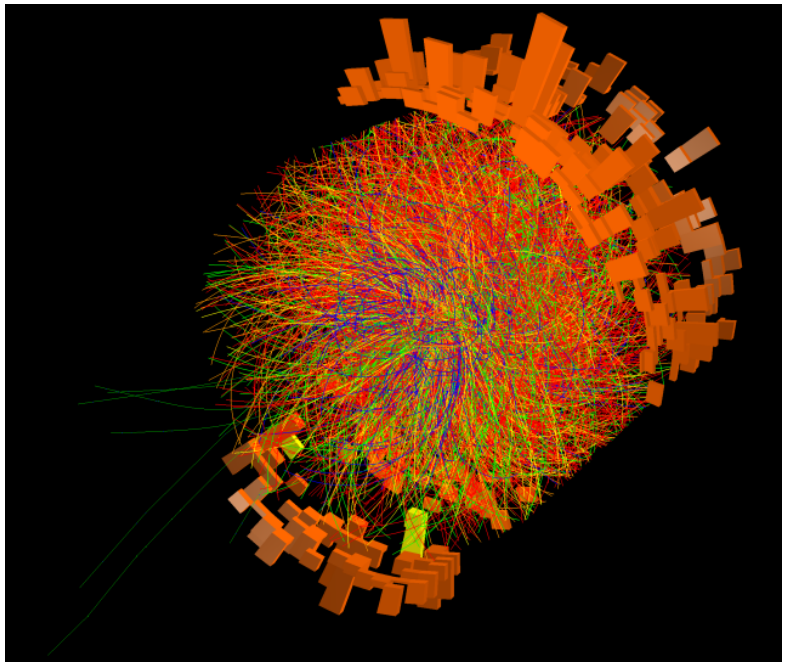
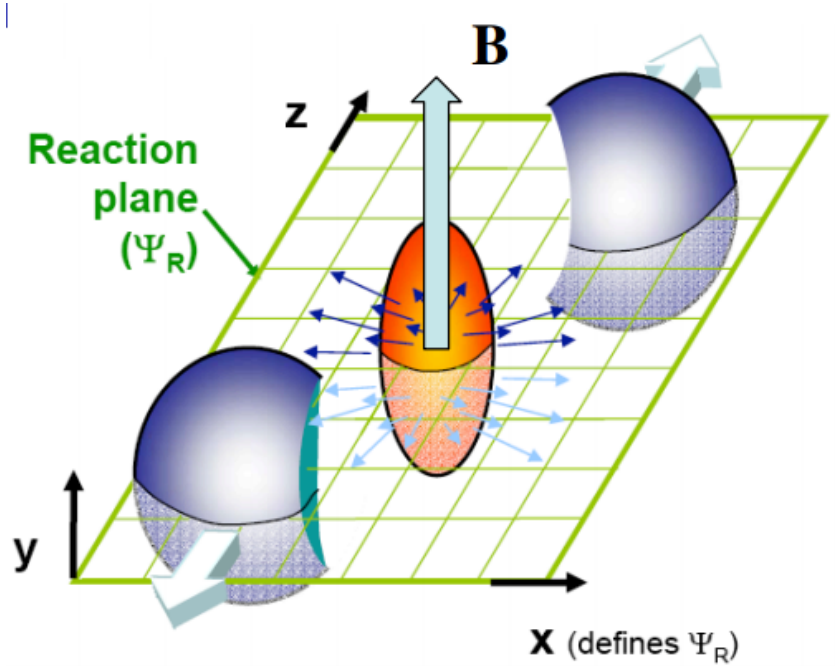
Chiral Magnetic Effect

ALICE, Pb+Pb @ $\sqrt{s_{NN}} = 5.02$ TeV



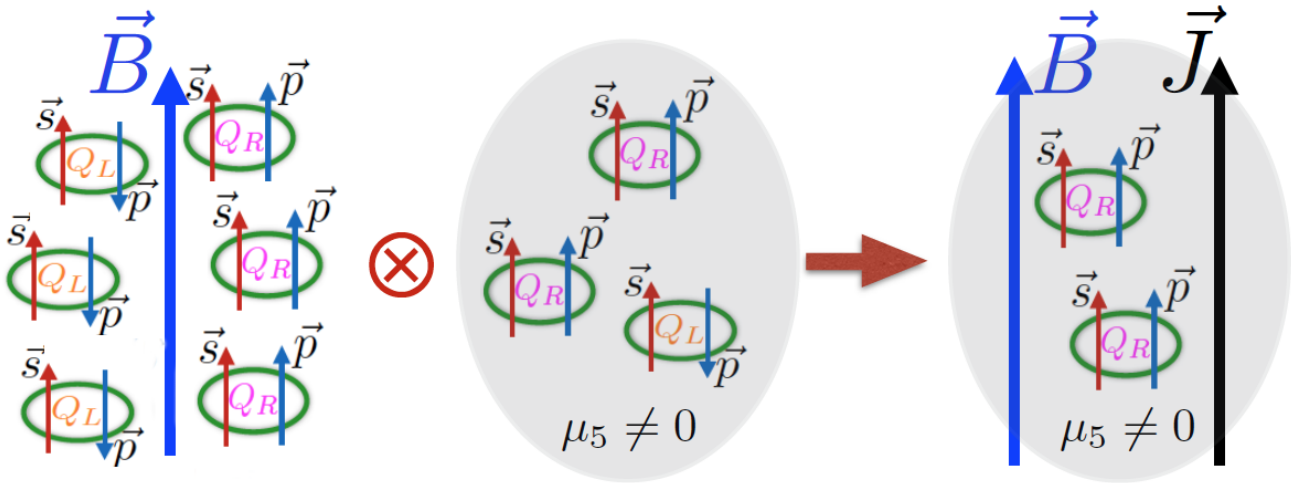


Kharzeev, McLerran, Warringa and Fukushima (2008) suggested that a chiral imbalance (given by a non-trivial gauge configuration) might induce an electromagnetic current proportional to an external magnetic field

Chiral Magnetic Effect: $\mathbf{J} \propto \mathbf{B}$

Signature of parity breaking

Axial imbalance due to nonzero topological charge Q (source of parity violation)



(Adapted from 1511.04050)

Chiral Magnetic Effect

$$\mathbf{J}(x) = \sigma_A \mathbf{B}(x) \; ; \quad \sigma_A = \frac{e^2}{4\pi^2}\mu_5 \; ; \quad \mu_5 = \mu_R - \mu_L$$

- EM current ($\mathcal{P}$-odd) proportional to magnetic field ($\mathcal{P}$-even)!
- (Nondissipative) transport coefficient measures chiral imbalance
- Experimental signatures, but controversy due to background effects

Dynamics of chiral plasmas

Classical kinetic theory can describe many effects of chiral plasmas at finite temperature/density

- Abelian plasma (Silin, 1960), QCD (Kelly, Liu, Lucchesi, Manuel, 1994)
- Hard-Thermal-Loop/Hard-Dense-Loop effective actions
- Debye screening, collective modes...

However

NO parity breaking source $\rightarrow$ NO Chiral Magnetic Effect

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Solution

Include quantum effects of **Berry phase***

*) Geometrical phase adquired by the eigenstate of a quantum Hamiltonian when the system performs a cyclic evolution in parameter space (Berry, 1984)

Son, Yamamoto (2012) Stephanov, Yin (2012) Chen, Pu, Wang, Wang (2012)

Equation of motion of a chiral fermion in an EM field is modified by the Berry curvature (Chang and Niu, 1995)

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{p}/|\mathbf{p}| &+ \dot{\mathbf{p}} \times \boldsymbol{\Omega} \\ \dot{\mathbf{p}} &= e\mathbf{E} &+ e\dot{\mathbf{x}} \times \mathbf{B} \end{cases}, \qquad \boldsymbol{\Omega} = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

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Semiclassical kinetic theory

$$\frac{\partial f_p}{\partial t} + \frac{1}{1 + e\mathbf{B}\cdot\boldsymbol{\Omega}} \left\{ \left[e\left(\mathbf{E} - \frac{\partial \epsilon_p}{\partial \mathbf{r}}\right) + e\mathbf{v}\times\mathbf{B} + e^2(\mathbf{E}\cdot\mathbf{B} - \frac{\partial \epsilon_p}{\partial \mathbf{r}}\cdot\mathbf{B})\boldsymbol{\Omega} \right] \cdot \frac{\partial f_p}{\partial \mathbf{p}} \right.$$
$$\left. + \left[\mathbf{v} + e\left(\mathbf{E} - \frac{\partial \epsilon_p}{\partial \mathbf{r}}\right) \times \boldsymbol{\Omega} + e(\boldsymbol{\Omega}\cdot\mathbf{v})\mathbf{B} \right] \cdot \frac{\partial f_p}{\partial \mathbf{r}} \right\} = 0$$

Fermion current in terms of the distribution function:

$$\mathbf{j} = -\int \frac{d^3p}{(2\pi)^3} \left[\epsilon_p \frac{\partial f_p}{\partial \mathbf{p}} + e\boldsymbol{\Omega}\cdot\frac{\partial f_p}{\partial \mathbf{p}}\epsilon_p\mathbf{B} + \epsilon_p\boldsymbol{\Omega}\times\frac{\partial f_p}{\partial \mathbf{r}} \right] + \mathbf{E}\times\int \frac{d^3p}{(2\pi)^3}\boldsymbol{\Omega} f_p$$

$$f_p = f_p^{eq} + e\left(f_p^{(\mathcal{P}-even)} + f_p^{(\mathcal{P}-odd)}\right) + \cdots$$

- $\mathcal{P}$-odd correction provides the Anomalous Hard-Thermal-Loop effective action (Laine, 2005)
- Axial anomaly is reproduced under different conditions (Son,Yamamoto, 2012, Manuel, Torres-Rincon, 2014)
- Accounts for Chiral Magnetic Effect (and many others, e.g. chiral separation effect, chiral vortical effect, chiral magnetic wave...)

The electromagnetic current:

$$J^i(k) = \pm ie^2\left(-\int \frac{dpp}{4\pi^2}\frac{df_p^{eq}}{dp}\right)\left\{\epsilon^{ijk}k^k + \omega\int\frac{d\mathbf{v}}{4\pi}\frac{(v^i\epsilon^{jkl} - \epsilon^{ikl}v^j)v^k k^l}{v\cdot k}\right\}A_j(k)$$

Chiral Magnetic Effect in the static limit $\omega \to 0$

$$J^i = \frac{e^2\mu_5}{4\pi^2}B^i$$

Collision term and Ohm's law

Collisions account for dissipation effects

Relaxation time approximation

$$\frac{df_p}{dt} = -\frac{1}{\tau}(f_p - f_p^{eq})$$

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Similar calculation to obtain in the static limit ($t \gg \tau$)

Ohm's Law + CME

$$\mathbf{J} = \sigma \mathbf{E} + \sum_s \frac{e_s^2 \mu_5}{4\pi^2} \mathbf{B} \quad , \quad \sigma = \frac{1}{3}\tau \sum_s e_s^2 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right)$$

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This current is coupled to the Maxwell equation

Maxwell equation with current

$$\frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} = \nabla \times \mathbf{B}$$

Anomalous Maxwell equations

Anomalous Maxwell equation

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma}\nabla^2 \mathbf{B} + \frac{C\alpha\mu_5}{\pi\sigma}\nabla \times \mathbf{B} - \frac{1}{\sigma}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Nonzero μ_5 can produce an exponential increase of the magnetic field: **chiral instability**

- Hypermagnetic fields: Joyce, Shaposhnikov (1997)
- Equations of motion of Anomalous-HTL: Laine (2005)
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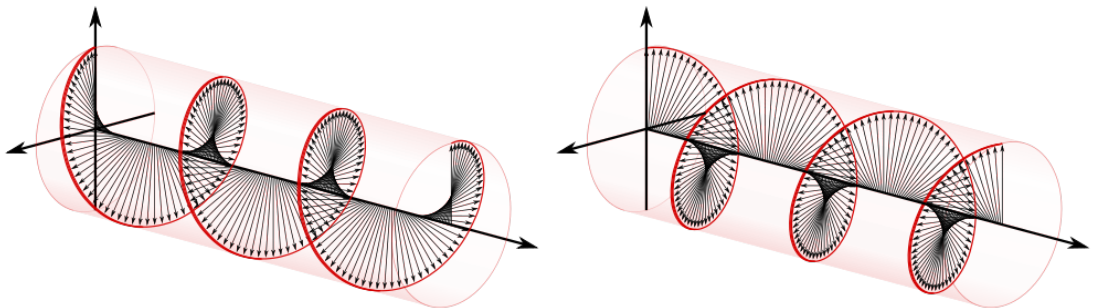
In Fourier components one can separate the circular-polarized modes:

$$\frac{1}{\sigma}\frac{\partial^2 B^{\pm}_{\mathbf{k}}}{\partial t^2} + \frac{\partial B^{\pm}_{\mathbf{k}}}{\partial t} = -\left(\frac{1}{\sigma}k^2 \mp \frac{C\alpha\mu_5 k}{\pi\sigma}\right) B^{\pm}_{\mathbf{k}}$$



What saturates the instability?

The instability is cut off by the Abelian anomaly

$$\frac{d(n_R - n_L)}{dt} = \frac{2C\alpha}{\pi}\frac{1}{V}\int d^3x\, \mathbf{E}\cdot\mathbf{B} \quad \rightarrow \quad \frac{d\mu_5}{dt} = -\left(\frac{\partial n_5}{\partial \mu_5}\right)^{-1}\frac{C\alpha}{\pi}\frac{d\mathcal{H}}{dt}$$

where $\mathcal{H}$

$$\mathcal{H} = \frac{1}{V}\int \frac{d\Omega_k}{4\pi}\int dk \frac{k}{2\pi^2}(|B_k^+|^2 - |B_k^-|^2)$$

is the **magnetic helicity density**, defined as

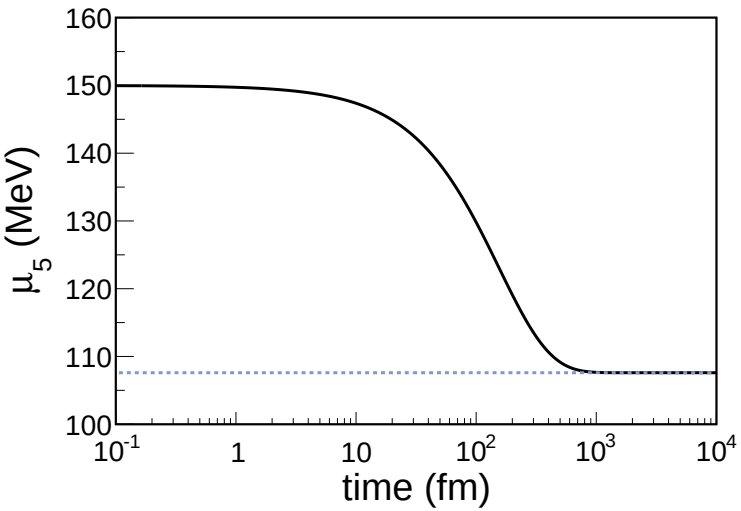
$$\mathcal{H} = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B}$$

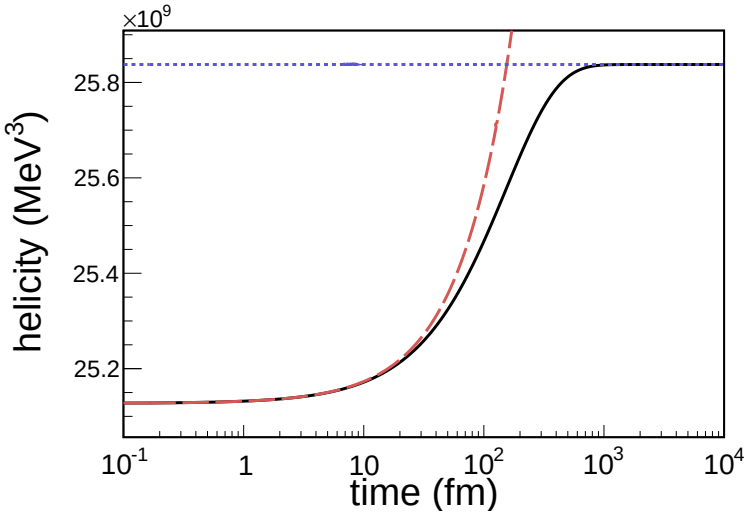
$\mathcal{H}$ measures the linkage/wrapping of magnetic field lines

$$\frac{d(n_R - n_L)}{dt} = -\frac{C\alpha}{\pi}\frac{1}{V}\int \frac{d\Omega_k}{4\pi}\int dk \frac{k}{2\pi^2}\frac{d}{dt}(|B_k^+|^2 - |B_k^-|^2)$$

Magnetic helicity conservation

- A **chiral imbalance** (asymmetry between R-L handed fermions) is attached to a **magnetic helicity** (asymmetry between R-L circular polarized magnetic fields)

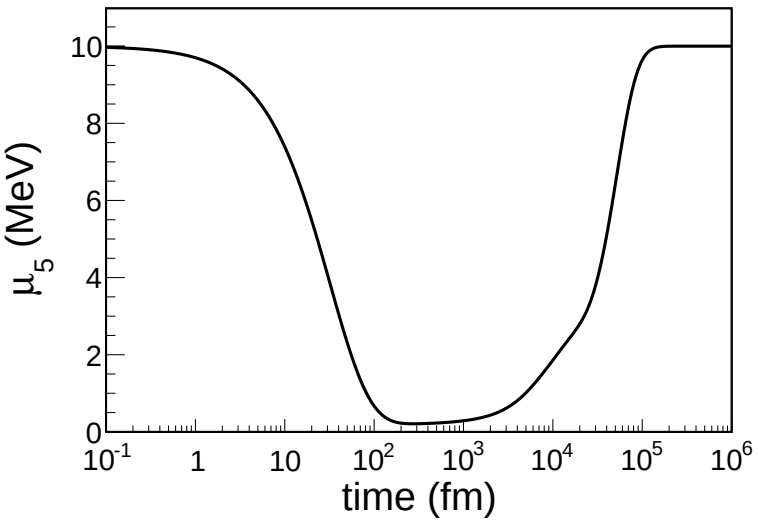


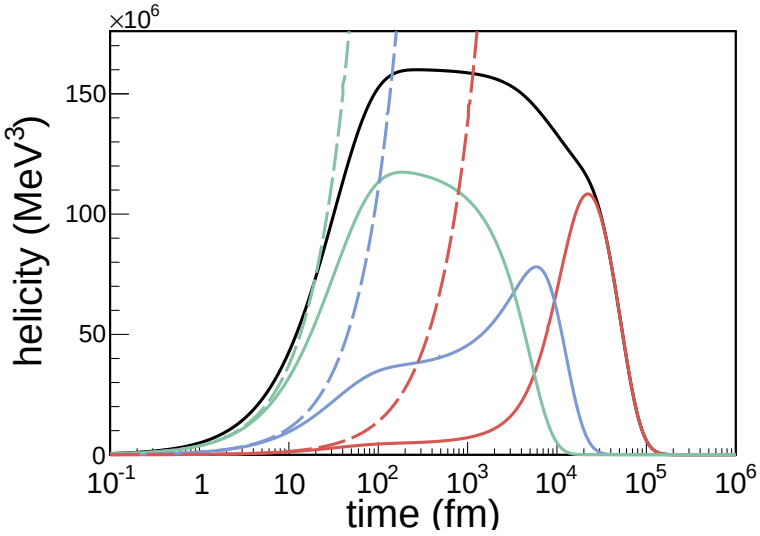


Diffusion approximation, $t \gg 1/\sigma$ (Manuel, Torres-Rincon, 2015)

Initial condition: several Fourier modes

$$|B^{\pm}_{k,0}|^2 = \sum_{i=1}^{N} |\tilde{B}_{0,i}|^2 \delta(k - k_i)$$

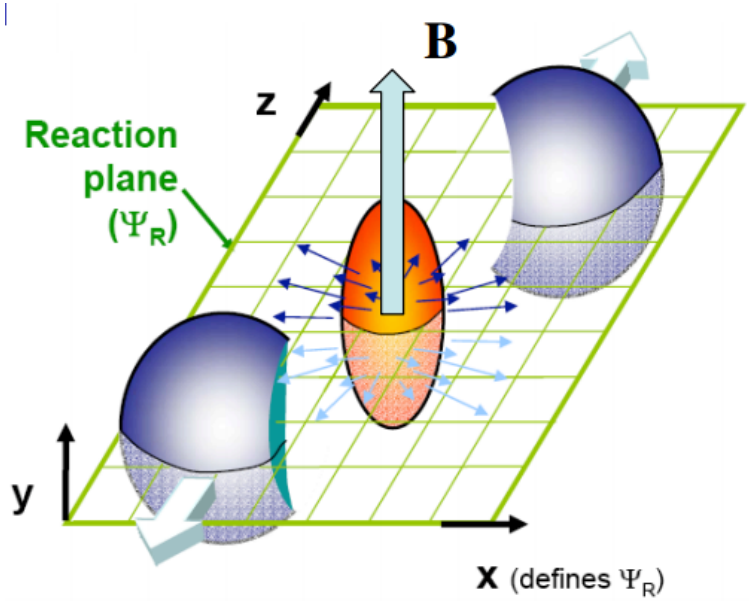




Inverse cascade phenomenon: magnetic energy (produced at the expense of chiral imbalance) is transferred from the short wavelenght to long wavelenght modes.

Application to heavy-ion collisions

Toy model for heavy-ion collisions (Manuel, Torres-Rincon, 2015)



- We cannot neglect $\partial^2 B/\partial t^2$ term (McLerran, Skokov, 2013) (no diffusion approximation)
- Approximation 1: Static medium, no expansion
- Approximation 2: Gaussian profile for magnetic field
- Standard values of parameters: size ~ 10 fm, wavelenght ~ 100 MeV, $\sigma = 5.4$ MeV at $T = 225$ MeV, $e|B_0| \sim 1 - 10 m_\pi^2$...

1/3 Chiral instability?

Unlikely. Very tight condition on wavenumber: $k < \frac{C\alpha\mu_5}{\pi}$
Minimum k set by the size of the system $\rightarrow$ Needs μ_5 of several GeV!

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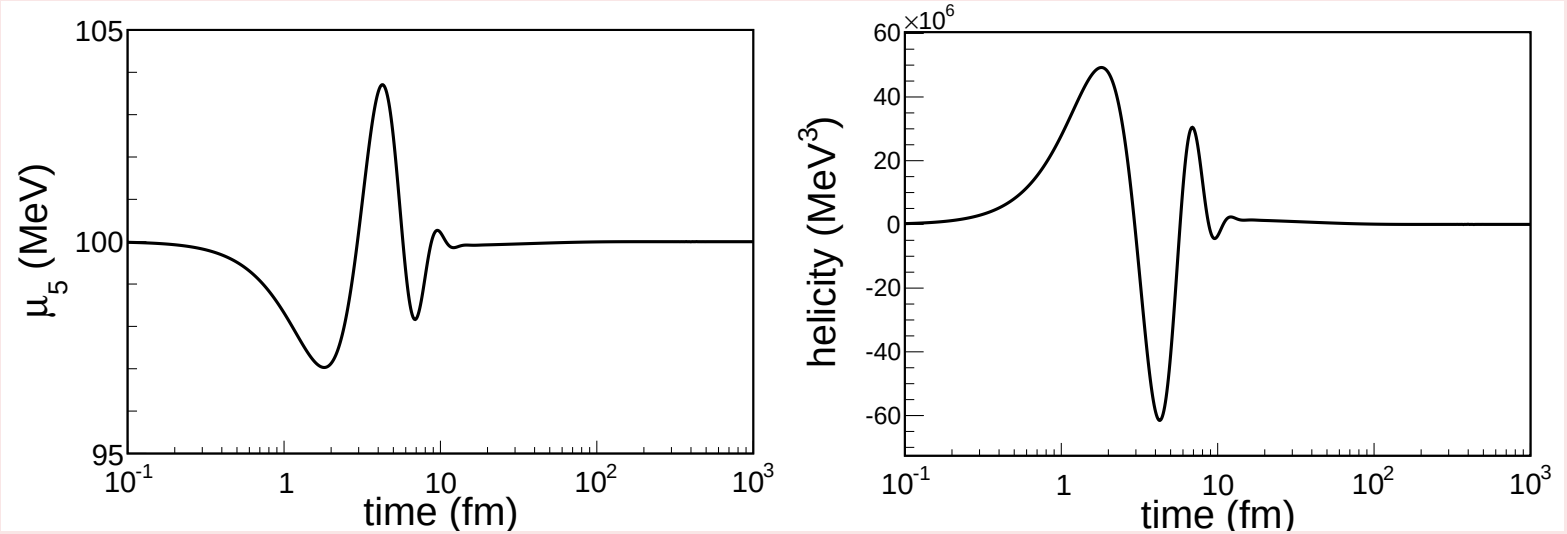
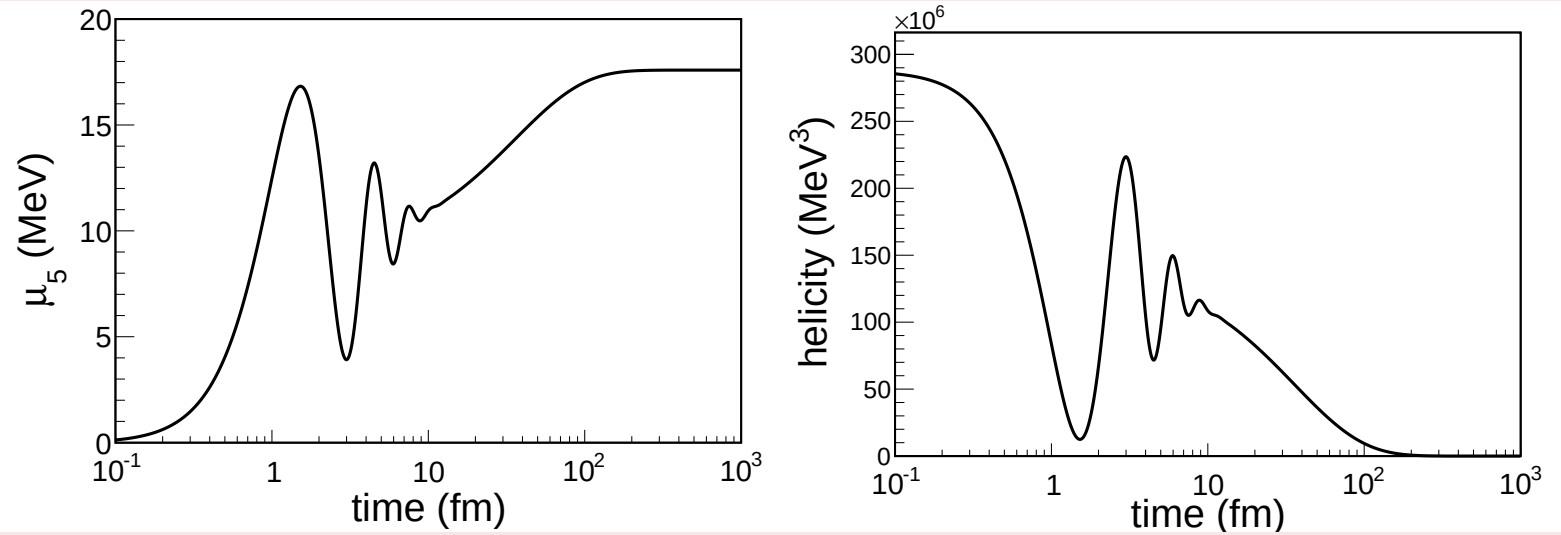
2/3 Increase of magnetic helicity?

No big effect in the best possible scenario, but generation of helical fields within the typical fireball lifetime.



3/3 Chiral imbalance generation?

Consider a non-zero initial $\mathcal{H}$ due to the magnetic field of spectators nucleons (possibly with nonzero vorticity due to rotation)



This is an alternative scenario for axial asymmetry generation than the QCD fluctuations of non-Abelian topological charge (Kharzeev, McLerran, Warringa, 2008).

Summary

1. **Semiclassical kinetic theory** is able to describe the dynamics of chiral plasmas in a thermal/dense medium: HTL/HDL/AHTL actions, axial anomaly, CME, CSE, CMW...
2. Together with **Maxwell equations** one can study the time evolution of CME and magnetic helicity in the presence of chiral-imbalanced matter.
3. Outlook: Couple Navier-Stokes equations to account for the fireball expansion (**anomalous magnetohydrodynamics**) and consider possible experimental observables (polarization of QGP photons?).

THANKS FOR YOUR ATTENTION!

For details:

- C. Manuel, J.M. Torres-Rincon, Phys.Rev. D92 (2015) 7, 074018
- C. Manuel, J.M. Torres-Rincon, Phys.Rev. D90 (2014) 7, 076007
- C. Manuel, J.M. Torres-Rincon, Phys.Rev. D89 (2014) 9, 096002

AUXILIAR SLIDES

Focus on QED, with a plasma of e^- and e^+ (represented by a 4-component Dirac field ψ).

$$\text{Dirac equation:} \quad (i\gamma_\mu \partial^\mu - m_e)\psi = 0$$

The mass mixes all the components of ψ. At high enough temperatures $T \gg m_e$ we can neglect the mass and deal with Weyl fermions (two bispinors get decoupled).

$$\text{Weyl equation:} \quad (\sigma \cdot \mathbf{p})u_p = \pm|\mathbf{p}|u_p$$

The positive (negative) sign distinguishes a right(left)-handed fermion, whose spin is parallel (antiparallel) to the momentum $\rightarrow$ positive (negative) helicity.
Helicity is conserved for chiral fermions (and equal to handeness).

The Hamiltonian of the system depends on time through some parameter $R(t)$

$$H(R)|n(R)\rangle = E_n(R)|n(R)\rangle$$

Berry phase

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We allow the system to follow an adiabatic evolution C in the parameter space. The system will remain in the instantaneous eigenstate $|n(R(0))\rangle$ but will receive a phase.

$$|\phi_n(T)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^T E_n(R(t))dt\right) e^{i\gamma_n(C)}|n(R(0))\rangle$$

This phase depends on C but not on the velocity (it is a geometrical phase)

$$\gamma_n(C) = i\int \langle n(R)|\nabla_R|n(R)\rangle dR$$

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For an arbitrary path C the phase can be neglected by performing a gauge transformation to the eigenvector (Fock, 1928)

$$|n(R)\rangle \to e^{i\zeta(R)}|n(R)\rangle$$

One can allways find a smooth function $\zeta(R)$ so that the geometrical phase is canceled.

However, if the trajectory is cyclic (Berry, 1984) the phase cannot be removed and the Berry phase,

$$\gamma_n(C) = i \oint \langle n(R)|\nabla_R|n(R)\rangle dR \ ,$$

is a gauge invariant quantity (therefore relevant to some observable physics)
One can define the (gauge dependent) Berry connection

$$\mathbf{A}_n(R) = i\langle n(R)|\nabla_R|n(R)\rangle$$

and the (gauge invariant) Berry curvature

$$\Omega_n = \nabla_R \times \mathbf{A}_n(R)$$

(notice formal analogy with an Abelian vector potential and a magnetic field)
For chiral fermions with Hamiltonian $H = \sigma \cdot \mathbf{p}$

$$\Omega = \nabla_p \times \mathbf{A}_p = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

Axial anomaly

The fermion particle is not conserved

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = -e^2 \hbar \int \frac{d^3 p}{(2\pi\hbar)^3} \left(\Omega \cdot \frac{\partial f_p}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B}$$

The axial current is defined

$$j^\mu_{A,V} = j^\mu_R \mp j^\mu_L$$

We can check that at equilibrium $f = f_{\text{Fermi-Dirac}}$

Axial anomaly

$$\partial_\mu j^\mu_A = \frac{e^2}{2\pi^2\hbar^2} \mathbf{E} \cdot \mathbf{B} \; ; \qquad \partial_\mu j^\mu_V = 0$$

$$f_p^{R,L} = \frac{1}{\exp\left[\frac{1}{T}\left(p \mp e\hbar \frac{\mathbf{B}\cdot\mathbf{p}}{2p^2} - \mu_{R,L}\right)\right] + 1}$$

$$\bar{f}_p^{L,R} = \frac{1}{\exp\left[\frac{1}{T}\left(p \pm e\hbar \frac{\mathbf{B}\cdot\mathbf{p}}{2p^2} + \mu_{R,L}\right)\right] + 1}$$