Horava-Lifshitz gravity in a nutshell

A. O. Barvinsky, D. Blas, S. Sibiryakov and C. F. Steinwachs, arXiv[1512.????]

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Renormalization of Hořava Gravity

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Abstract

We prove perturbative renormalizability of the *projectable* Hořava gravity. The key element of the argument is the choice of gauge which ensures the correct anisotropic scaling of the propagators and their uniform falloff at large frequencies and momenta. This guarantees that the counterterms required to absorb the loop divergences are local and marginal or relevant with respect to the anisotropic scaling. The gauge invariance of the counterterms is implemented by making use of the background-covariant formalism. We also comment on the implications of our gauge-fixing procedure for the renormalizability analysis of the *non-projectable* model.

1

The problem with (a canonical theory of) Quantum Gravity

Can Quantum Gravity be formulated as a Quantum Field Theory?

$$\mathcal{Z}[J] = \int [\mathcal{D}g_{\mu\nu}] e^{\frac{i}{\hbar}\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\, R}$$

Fierz and Pauli (1932), Misner (1957), Dirac (1957), Feynmann (1962), DeWitt (1962), Wheeler (1963)...

There is a problem

- G is dimensionful in four dimensions $[G] = -2$ in mass units

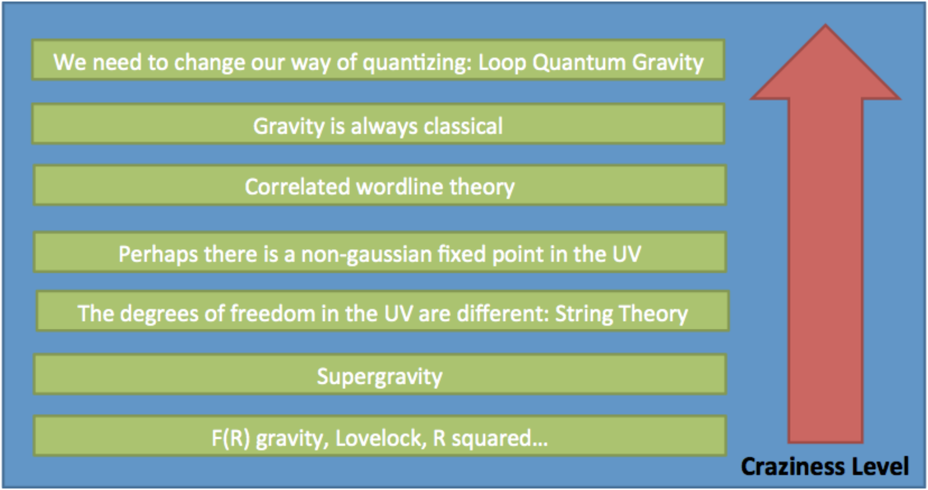
Heisenberg (1930)

- The propagator scales as $\frac{1}{p^2}$

This produces an infinite number of divergent diagrams and reduces the theory to an EFT

A UV completion is required and normally assumed to be String/M Theory

However, it should be possible to study gravitational phenomena in a self-contained way



S. Hossenfelder (2015)

One way to relax this issue is to add higher order relativistic corrections

$$S = S_o + \int d^4x\sqrt{|g|}\,\left(\alpha R^2_{\mu\nu\alpha\beta} + \beta R^2_{\mu\nu} + \gamma R^2\right) \sim h^{\mu\nu}\Box^2 h_{\mu\nu}$$

With this, the theory is renormalizable. The new terms 'dress' the propagator with a quartic term

$$\frac{1}{p^2} + \frac{1}{p^2}Gp^4\frac{1}{p^2} + \frac{1}{p^2}Gp^4\frac{1}{p^2}Gp^4\frac{1}{p^2} + ... = \frac{1}{p^2 - Gp^4}$$

It behaves as Einstein-Hilbert in the IR and improves convergence in the UV

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The ghost issue

However, this propagates ghost degrees of freedom

$$\frac{1}{p^2 - Gp^4} = \frac{1}{p^2} - \frac{1}{p^2 - G^{-1}}$$

So unitarity is lost at energies of the order of the Planck mass

$$E \sim \sqrt{G^{-1}}$$

What does fail here?

There seems to be some tension between renormalizability, unitarity and Lorentz invariance.

- A no-so-crazy solution is to get rid of Poincaré invariance.
- Substitute scale invariance by Lifshitz invariance.

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i$$

E. M. Lifshitz, 1941

$$S = \int dt\, d^D x \left(\frac{1}{2} \partial_t \phi \partial_t \phi + \alpha_0 \phi (-\Delta)^z \phi + \alpha_1 \phi (-\Delta)^{(z-1)} \phi + \ldots + \frac{c^2}{2} \phi \Delta \phi \right)$$

~~Lorentz~~ifshitz

The theory is renormalizable for $D = z$

$$S = \int dt\ d^3x \left(\frac{1}{2}\partial_t\phi\partial_t\phi + \alpha\phi(-\Delta)^3\phi + \frac{c^2}{2}\phi\Delta\phi \right)$$

The propagator is now dressed in a non-relativistic way

$$\frac{1}{\omega^2 - c^2k^2 - \alpha k^6}$$

- Poincaré emerges in the IR!!!!!

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- Poincaré emerges in the IR!!!!!

There are not spurious poles

$$\Box^2\phi = 0 \qquad (\partial_t^2 - c^2\Delta - \alpha\Delta^3)\phi = 0$$

$$\begin{array}{cc} e^{i(\omega-ck)} & e^{-i(\omega-ck)} \\ e^{(\omega-ck)} & e^{-(\omega-ck)} \end{array} \qquad e^{i(\omega-ck-\alpha^{\frac{1}{3}}k^2)} \quad e^{-i(\omega-ck-\alpha^{\frac{1}{3}}k^2)}$$

Quantum Gravity

Following this same idea, it is possible to construct an, a priori, renormalizable theory of Quantum Gravity: Hořava-Lifshitz gravity

P. Hořava, 2009

- We introduce an extra structure in space-time. A preferred foliation in the time direction

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i dt - \gamma_{ij} dx^i dx^j$$

- This breaks the Diffeomorphism invariance down to *FDiff*

$$t \to \tilde{t}(t), \qquad x^i \to \tilde{x}^i(t, x)$$

- This is what breaks Lorentz invariance down to time re-parametrizations times $SO(3)$.
- *Diff* invariance **has** to emerge in the IR.

Quantum Gravity

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i dt - \gamma_{ij} dx^i dx^j$$

We can start by writing General Relativity in these coordinates

$$S = \frac{1}{16\pi G} \int dt d^3x \; N\sqrt{\gamma} \left(K_{ij}K^{ij} - K^2 + R \right)$$

where

$$K_{ij} = \frac{1}{2N} \left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i \right)$$

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$$S = \frac{1}{16\pi G}\int dt d^3x\, N\sqrt{\gamma}\left(\underbrace{K_{ij}K^{ij} - K^2}_{\text{kinetic term}} + \underbrace{R}_{\text{potential term}}\right)$$

This is the ADM action, given by the Gauss-Codazzi decomposition of $R^{(4)}$.

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For example, for $z = D = 2$

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For example, for $z = D = 2$

$$R^2, \quad R$$

While for $z = D = 3$

$$R^3, \quad R_{\mu\nu} R^{\mu\nu} R, \quad \Delta^2 R, \quad R...$$

Two options

There are two versions of the theory

- Projectable: The lapse does not depend on the spatial coordinates $N = N(t)$ and thus can be integrated out by fixing the gauge invariance in the time direction

  $$N = 1$$

  It is not phenomenological viable.

  $$\omega^2 = f(couplings)k^2$$

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- Non-projectable: The lapse depends on all the coordinates $N = N(t, x)$. Then, the potential must be suplemented with extra terms

$$a_i a^i, \quad \nabla_i a^i, \quad \ldots$$

  where $a_i = \frac{\nabla_i N}{N}$.

  D. Blas, O. Pujolas and S. Sibiryakov, 2010

  It is a healthy extension

$$\omega^2 = c^2 k^2 + \frac{k^4}{M_*^2}$$

The scale M_*

The Lorentz violating scale M_* is constrained in two ways

- From the UV by cosmological and astrophysical data

$$M_* \lesssim 10^{15}\, GeV$$

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Cosmology

- Dark energy can be accommodated.
- Dark matter is not required. We have MOND

S. Mukohyama (2009)

- There is no initial singularity. Bouncing universe

R. Branderberger (2009)

The framework

In order to construct a quantum theory, let us simplify things a bit

- Let us work with the projectable theory by setting $N = 1$.
- We work in $2+1$ dimensions to reduce the number of invariants.

$$S = \frac{1}{2G}\int dt\, d^2x\, \sqrt{\gamma}\left(K_{ij}K^{ij} - \lambda K^2 - \mu R^2\right)$$

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- We are left with time dependent 2-dimensional diffeomorphisms

$$x^i \to \tilde{x}^i(t, x)$$

- We expand the fields around flat space

$$g_{ij} = \delta_{ij} + h_{ij}, \qquad N_i = N_i$$

$$S^{(2)} = \frac{1}{2G}\int d^2x dt\left[\frac{\dot{h}_{ij}^2}{4} - \frac{\lambda \dot{h}^2}{4} - \mu(\partial_i\partial_j h_{ij})^2 - \mu(\Delta h)^2 + 2\mu\Delta h\,\partial_i\partial_j h_{ij}\right.$$
$$\left. - \dot{h}_{ij}\partial_i N_j + \lambda \dot{h}\partial_i N_i + \frac{(\partial_i N_j)^2}{2} + \left(\frac{1}{2} - \lambda\right)(\partial_i N_i)^2\right]$$

The naïve gauge fixing

The problem comes when gauge fixing the theory. The naïve choice is

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The issue comes from scaling properties. Gauge fixing introduces a scale!
We need to look for a scale invariant gauge.

The good gauge fixing

We need to construct the fixing with the right scaling

$$\mathcal{L}_{gf} = \frac{\alpha}{2G}\left(\dot{N}_i + b\Delta\partial_j h_{ij} + c\Delta\partial_i h\right)\left[\frac{\delta_{ik}}{\Delta} + \xi\frac{\partial_i\partial_k}{\Delta^2}\right]\left(\dot{N}_k + b\Delta\partial_l h_{kl} + c\Delta\partial_k h\right)$$

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With this choice, the propagator has no dangerous poles, the only ones inherit the right scaling

$$G \sim \frac{1}{\omega^2 - 4\mu\frac{1-\lambda}{1-2\lambda}k^4}$$

The operator in the middle of the fixing implies the presence of a Nielsen-Kallosh ghost

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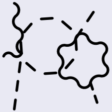
$$\int dt\, d^2x\, \phi\left[\frac{\delta_{ik}}{\Delta} + \xi\frac{\partial_i\partial_k}{\Delta^2}\right]^{-1}\phi$$

and we also have the usual Faddeev-Poppov ghosts

$$\int dt\, d^2x\, \bar{c}^i\frac{\delta}{\delta\xi^j}\left\{\left(\dot{N}_i + b\Delta\partial_j h_{ij} + c\Delta\partial_i h\right)\right\}c^j$$

For arbitrary loop number



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We define the superficial degree of divergence in the usual way

$$D = 4 - 2T - X - I_N$$

For no externel N_i fields, only diagrams with either $T = 2$ or $X = 4$ are divergent. But those are precisely the terms already contained in the action.

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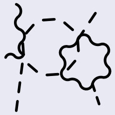

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So at the one-loop level

- We have a propagator with no problematic poles
- All interactions are local
- The Faddeev-Poppov ghosts are local
- The Nielsen-Kallosh ghost cannot produce new divergences

A. O. Barvinsky and G. Vilkovisky (1985)

- All the divergences are logarithmic and renormalize the terms already in the lagrangian

Therefore, **the theory is renormalizable at the one-loop level**

Conclusions

- Horava-Lifshitz gravity is a **renormalizable theory of quantum gravity in four dimensions.**
  - The relation between superficial degree of divergence and renormalizability is ensured by gauge invariance.
  - There exist gauge choices in which all the divergences are explicitly local.
  - The ghost structure is more complicated than in GR but no more than in Supergravity theories.
- Together with phenomenological bounds, H-L gravity is a candidate for describing gravitational interactions.

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For the future

- A complete one-loop computations is still being made (around 250 diagrams)
- Computation in higher dimensions (in particular 3+1) requires covariant techniques
- Can be all this be extended to the non-projectable case? (Probably yes)
- Coupling matter is required to construct a full model that reproduces our Universe

D. Blas, A. O. Barvinsky, M. H-V, G. Perez-Nadal and C. F. Steinwachs (2015??)

Projectable Horava-Lifshitz gravity is the first known example of a **fully unitary, renormalizable** Quantum Field Theory of the gravitational interaction.