Large-N pion scattering, finite-temperature effects and the relationship of the $f_0(500)$ with chiral symmetry restoration

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1. Large N expansion for LEQCD in the chiral limit and $T=0$
2. Diagrammatics at $m_\pi = 0, 0 < T < T_\chi$ in the large N limit
3. Mass and width of the $f_0(500)$ pole and their relation to χSR
4. Conclusions

$1/N$ expansion (1)

$O(N+1)/O(N) \rightarrow$ nonlinear sigma model for massless pions:

$$\mathcal{L} = \frac{1}{2} g_{ab}(\pi) \partial_\mu \pi^a \partial^\mu \pi^b. \quad (1)$$

Pion decay constant and metric induced by the vaccuum constraint:

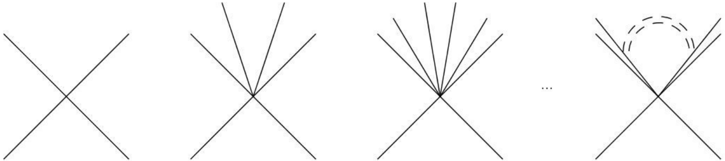
$$f_\pi^2 = N F^2, \quad (2)$$

$$g_{ab}(\pi) = \delta_{ab} + \frac{\pi_a \pi_b}{N F^2 \left(1 - \frac{\pi_a^2}{N F^2}\right)}. \quad (3)$$

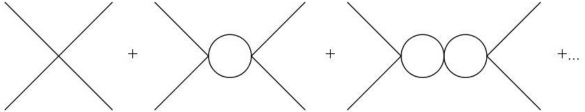
Refs: A. Dobado, J. Morales, Phys. Rev. D **52**: 2878 (1995); A. Dobado, A. Gómez Nicola, A. L. Maroto, J. R, Peláez, *Effective Lagrangians for the Standard Model*. Springer-Verlag Berlin Heidelberg (1997); J. Gasser, H. Leutwyler, Annals Phys. **158**, 142 (1984) $\rightarrow$For the DR integral.

$1/N$ expansion (2)

Feynman rules and s-channel amplitude:



$A(s) =$



(Large N limit)

$$= \frac{s}{NF^2\left[1 - s\,J(s)/2F^2\right]} \to J(s) = J_\epsilon(\mu) + \frac{1}{16\pi^2}\ln\left(\frac{\mu^2}{-s}\right) \quad , \quad J_\epsilon(\mu) = \frac{1}{16\pi^2}\left[\frac{2}{\epsilon} + \ln(4\pi) - \gamma + 2 - \ln\mu^2\right] + \mathcal{O}(\epsilon).$$

$1/N$ expansion (3)

Renormalization at $T=0$ and fitting to the s-channel phase shift:

$$\frac{s}{NF^2} \to \frac{s}{NF^2} G_0(s),\, G_0(s) = 1 + \sum_{k=1}^{\infty} g_{0,\,k} \left(\frac{s}{F^2}\right)^k .$$

$g_{0,\,k}$: Bare low energy constants (LEC's)

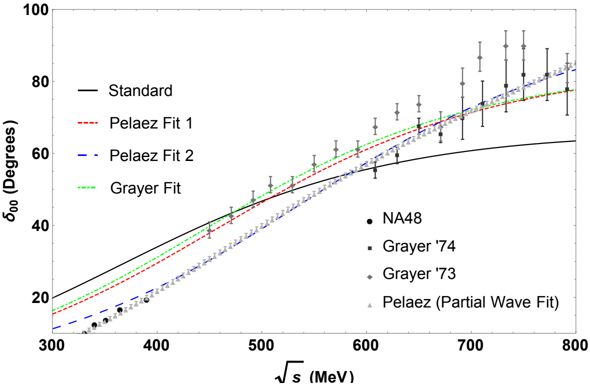
$$A_R(s) = \frac{s\, G_R(s;\mu)}{NF^2 \left[1 - \frac{s\, G_R(s;\mu)}{32\pi^2 F^2} \ln\left(\frac{\mu^2}{-s}\right)\right]},\, \frac{1}{G_R(s;\mu)} = \frac{1}{G_0(s)} - \frac{sJ_\epsilon(\mu)}{2F^2}. \quad (4)$$

Renormalized LEC's: $g_{R,\,k}(\mu) = 0 \to$ only two fit parameters (F and μ).

$$T_{I=0}(s, \cos\theta) = NA_R(s) + A_R(t) + A_R(u) \approx NA_R(s). \quad (5)$$

$1/N$ expansion (4)

δ_{00} fitted in the Chiral limit considering data far from the first threshold, i.e., $s_{th} = (2m_\pi)^2$.

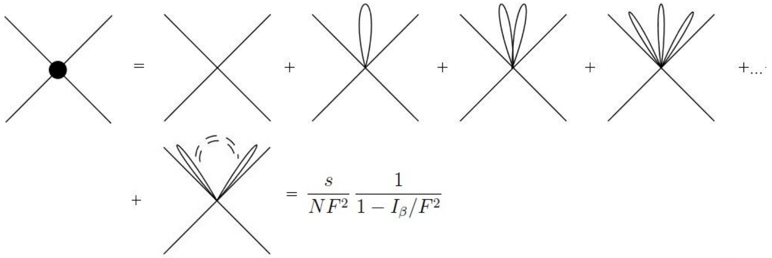


Grayer	Parameters
F (MeV)	63.16±1.62
μ (MeV)	1523.35±143.34
R^2	0.9958
Peláez 1	Parameters
F (MeV)	65 (fixed)
μ (MeV)	1607.89±3.62
R^2	0.9951
Peláez 2	Parameters
F (MeV)	75.98± 0.16
μ (MeV)	2763.51 ± 23.81
R^2	0.9999

Standard: PRD **52**: 2878 (1995). *Refs:* G. Grayer *et al*, Nucl. Phys. B **75**: 189-245 (1974); *ibid* **64**: 134-162 (1973); J. R. Batley *et al*, Eur. Phys. J. C **70**: 635-657 (2010); R. García Martín *et al*, Phys. Rev. Lett. 107: 072001 (2011).

Finite-Temperature scattering amplitude (1)

Effective thermal vertex:



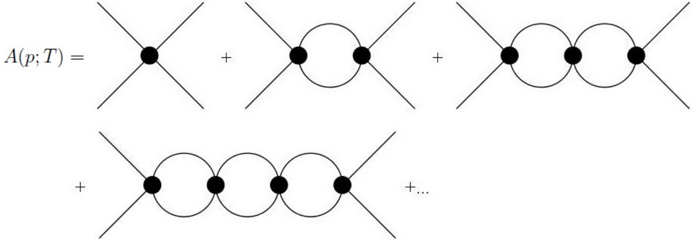
$$= \frac{s}{NF^2}\frac{1}{1 - I_\beta/F^2}$$

where $I_\beta = T^2/12$.

Reminding that $m_\pi = 0 \rightarrow$ **no need to renormalize the pion wavefunctions and their masses**

Finite-Temperature scattering amplitude (2)

Process:



$$A(p;T)=\frac{s}{NF^2}\frac{f(I_\beta)}{1-sf(I_\beta)J(p;T)/2F^2};\, f(I_\beta)=\frac{1}{1-I_\beta/F^2}, \tag{6}$$

$J(p;T)=J_\epsilon(\mu)+J_{fin}(p;T;\mu)\rightarrow$ Includes the $T=0$ finite term.

Renormalization of $A(p;T)$

Vertex	Lagrangian	Bare rule	Renormalized rule
4 pions	$-\dfrac{\pi^2\Box\pi^2}{8NF^2}$	$\dfrac{s}{NF^2}$	$\dfrac{sG_0(s)}{NF^2}$
6 pions	$-\dfrac{\left(\pi^2\right)^2\Box\pi^2}{16(NF^2)^2}$	$\dfrac{s}{(NF^2)^2}I_\beta$	$\dfrac{sG_0^2(s)}{(NF^2)^2}I_\beta$
$2k+4$ pions	$-\dfrac{\left(\pi^2\right)^{k+1}\Box\pi^2}{8(k+1)(NF^2)^{k+1}}$	$\dfrac{s}{(NF^2)^{k+1}}I_\beta^k$	$\dfrac{sG_0^{k+1}(s)}{(NF^2)^{k+1}}I_\beta^k$

Even considering all the tree-level vertices, the renormalization process remains the same as at $T=0$. The finite-temperature amplitude now reads

$$A_R(p;T)=\frac{s}{NF^2}\frac{G_R(s;\mu)f\left[G_R(s;\mu)I_\beta\right]}{1-\frac{sG_R(s;\mu)f\left[G_R(s;\mu)I_\beta\right]}{2F^2}J_{fin}(p;T;\mu)}. \tag{7}$$

Thermal unitarity

$$a_{00}(s, T) = \frac{1}{64\pi}\int_0^1 NA_R(p; T)P_0(\cos\theta)d(\cos\theta). \tag{8}$$

$$\mathrm{Im}\left[\frac{1}{a_{00}(s+i0^+; T)}\right] = -\mathrm{Im}\left[16\pi J_{fin}(p; T; \mu)\right] = -\left[1+2n\left(\frac{\sqrt{s}}{2}\right)\right].$$

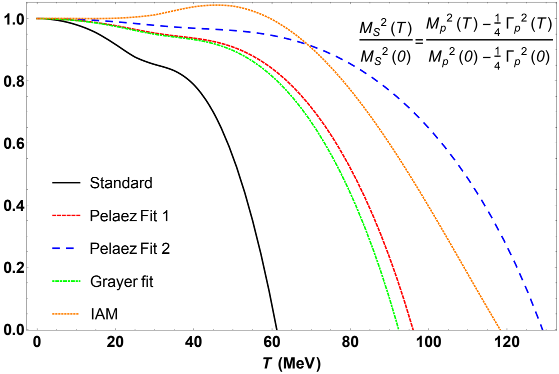
Since $\sigma_T(\sqrt{s}, m_\pi = 0) = 1 + 2n(\sqrt{s}/2) \rightarrow n(p) = 1/\left[\exp(p/T) - 1\right]$,

$$\boxed{\mathrm{Im}\left[\frac{1}{a_{00}(s, T)}\right] = -\sigma_T(\sqrt{s}, 0).} \rightarrow \textbf{Exact result.} \tag{9}$$

Refs: A. Gómez Nicola, F. J. Llanes-Estrada, J. R. Peláez, Phys. Lett. B **550**:55-64 (2002); A. Gómez Nicola, J. R. Peláez, A. Dobado, F. J. Llanes-Estrada, AIP Conf.Proc. **660**: 156-169 (2003). arXiv: hep-ph/0212121 (perturbative thermal unitarity).

Chiral symmetry restoration ($\sqrt{s} = M_P(T) - i\Gamma_P(T)/2$)

In the $p=0$ limit, the scalar susceptibility behaves as $\chi_S(T) \propto 1/\mathrm{Re}\{s\}^\dagger$.



Unitarized scalar susceptibility†:

$$\frac{\chi_S^U(T)}{\chi_S^{ChPT}(0)} = \frac{M_S^2(0)}{M_S^2(T)}. \qquad (10)$$

Expected T_c value in the chiral limit: about $0.8\,T_\chi$.

Chiral critical temperature for massive pions*:

$$T_\chi = 154 \pm 9 \text{ MeV}. \qquad (11)$$

Refs: †A. Gómez Nicola, J. Ruiz de Elvira, R. Torres Andrés, Phys.Rev.D **88**: 076007 (2013); *A. Bazavov *et al* (HotQCD Collaboration), Phys. Rev. D **85**: 054503 (2012).

Chiral critical temperatures and exponents

Fit	T_c (MeV)	$M_P(0)$ (MeV)	$\Gamma_P(0)$ (MeV)	γ_χ	$R^2_{\gamma_\chi}$
Grayer	92.33	438.81	536.47	0.875	0.99987
Peláez 1	96.00	452.42	546.26	0.938	0.99997
Peláez 2	129.07	535.53	534.59	0.919	0.99995
IAM	118.23	406.20	522.70	1.012	1
Standard	61.20	356.97	566.05	0.842	0.99728

Expected values (*Ref*)	$M_P(0)$ (MeV)	$\Gamma_P(0)$ (MeV)	γ_χ
PRL **107**: 072011 (2011)	457^{+14}_{-13}	558^{+22}_{-14}	—
Olive *et al*, PDG (2014)	400-500	400-700	—
J. Zinn-Justin ($O(\infty)$ 4D)**	—	—	1
Ejiri *et al*, PRD **80**: 094505 ($O(4)$ 3D)	—	—	0.54

$$\chi_S \propto (T_c - T)^{-\gamma_\chi}, \text{ for } T < T_c.$$

** *Quantum Field Theory and Critical Phenomena*, Oxford University Press (2002); S. Cortés, A. Gómez Nicola, J. Morales, arXiv: 1511.00031 (2015).

- Although we work in the Chiral Limit, our analysis of ellastic pion scattering at finite temperature in the large N expansion grants a description of the $f_0(500)$ pole thermal dependence that is consistent with previous works*.
- We show that thermal unitarity holds exactly for the sum of all perturbative orders when considering finite-temperature effects.
- In the Chiral Limit, the behavior of $\chi_S^{-1}(T)$ (saturated by the $f_0(500)$ pole) is consistent with a second-order phase transition (as seen in the lattice**).
  - Our T_c results are not far from the expected lattice values (about $0.8T_\chi$); besides, they are even closer to the result obtained for NJL-like models, i.e., $T_c \approx 100.7$ MeV.***
- The critical exponent we obtain is very close to that given for a $O(N)$ 4-dimensional model.

Refs: *A. Gómez Nicola, J. Ruiz de Elvira, R. Torres Andrés, Phys.Rev.D **88**: 076007 (2013) ; A. Gómez Nicola, J. R. Peláez, J. Ruiz de Elvira, Phys. Rev. D **87**: 016001 (2013).
A. Bazavov *et al* (HotQCD Collaboration), Phys. Rev. D **85: 054503 (2012).
***J. Berges *et al*, Phys. Rept. **363**, 223 (2002).