

QCD running in neutrinoless double beta decay: Short range mechanism

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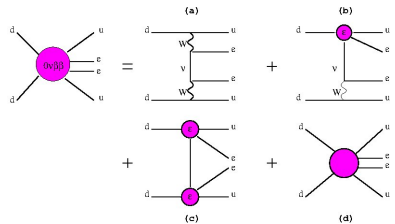
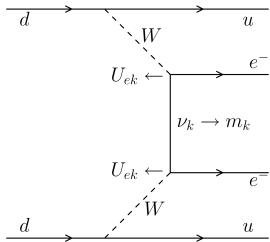
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Outline

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- 2 QCD running in Neutrinoless double beta decay
 - Low-energy Effective Lagrangian and $0\nu\beta\beta$ -decay Half-life
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Introduction



Effective Lagrangian and base of operators

Effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{0\nu\beta\beta} = \frac{G_F^2}{2m_p} \sum_i C_i^{XY}(\mu) \cdot \mathcal{O}_i(\mu), \quad (1)$$

with the operator basis:

$$\mathcal{O}_1^{XY} = 8(\bar{u}P_X d)(\bar{u}P_Y d) j, \quad (2)$$

$$\mathcal{O}_2^{XX} = 8(\bar{u}\sigma^{\mu\nu} P_X d)(\bar{u}\sigma_{\mu\nu} P_X d) j, \quad (3)$$

$$\mathcal{O}_3^{XY} = 8(\bar{u}\gamma^\mu P_X d)(\bar{u}\gamma_\mu P_Y d) j, \quad (4)$$

$$\mathcal{O}_4^{XY} = 8(\bar{u}\gamma^\mu P_X d)(\bar{u}\sigma_{\mu\nu} P_Y d) j^\nu, \quad (5)$$

$$\mathcal{O}_5^{XY} = 8(\bar{u}\gamma^\mu P_X d)(\bar{u}P_Y d) j_\mu \quad (6)$$

$$j = \bar{e}P_X e^c, \quad j_\mu = \bar{e}\gamma_\mu P_X e^c.$$

Half-life

Applying standard nuclear theory methods, one finds for the half-life:

Half-life

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_1 \left| \sum_{i=1}^3 C_i(\mu_0) \mathcal{M}_i \right|^2 + G_2 \left| \sum_{i=4}^5 C_i(\mu_0) \mathcal{M}_i \right|^2 \quad (7)$$

Here, $G_1 = G_{01}$ and $G_2 = (m_e R)^2 G_{09}/8$ are phase space factors in the convention of [Doi et al., 1985], and $\mathcal{M}_i = \langle A_f | \mathcal{O}_i^h | A_i \rangle$ are the nuclear matrix elements defined in Ref. [Päs et al., 2001].

QCD running motivations

Why we need the running?

Energy scales involved:

- LNV scale at ~ 1 TeV
- Hadronic scale at ~ 1 GeV
- Nuclear scale at $P_F \sim 200$ MeV

\Rightarrow QCD running has to be taken into account.

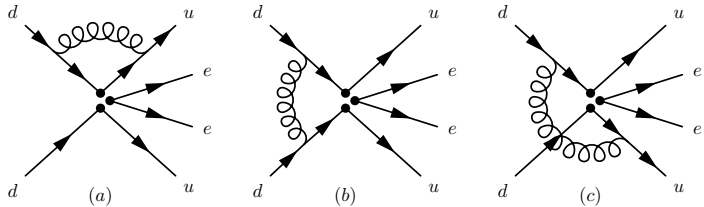


Figure : One-loop QCD corrections to the short range mechanisms of $0\nu\beta\beta$ decay in the effective theory.

$$T_{\alpha\beta}^a T_{\gamma\rho}^a = -\frac{1}{2N} \delta_{\alpha\beta} \delta_{\gamma\delta} + \frac{1}{2} \delta_{\alpha\delta} \delta_{\gamma\rho} \quad (8)$$

Color Mismatch effect.

Renormalization

Bare operator matrix elements

$$\langle \mathcal{O}_i \rangle^{(0)} = \left[\delta_{ij} + \frac{\alpha_s}{4\pi} b_{ij} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{-p^2} \right) \right) \right] \langle \mathcal{O}_j \rangle_{\text{tree}}. \quad (9)$$

[Buras, 1998] :

$$\mathcal{O}_i^{(0)} = Z_{ij} \mathcal{O}_j, \quad C_i^{(0)} = Z_{ij}^c C_j \quad (10)$$

It can be shown $\hat{Z}^{cT} = \hat{Z}^{-1}$

$$Z_{ij} = \delta_{ij} + \frac{\alpha_s}{4\pi} (b_{ij} - 2C_F \delta_{ij}) \frac{1}{\epsilon} + \mathcal{O}(\alpha_s^2). \quad (11)$$

RGE

RGE of WC

$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T \vec{C}(\mu), \quad (12)$$

$$\hat{\gamma}(\alpha_s) = -2\alpha_s \frac{\partial \hat{Z}_1(\alpha_s)}{\partial \alpha_s}, \quad (13)$$

$$\gamma_{ij}(\alpha_s) = \frac{\alpha_s}{4\pi} \gamma_{ij}^{(0)}, \quad \text{with} \quad \gamma_{ij}^{(0)} = -2(b_{ij} - 2C_F \delta_{ij}), \quad (14)$$

$$\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W). \quad (15)$$

In the LO one finds

$$\hat{U}^{(0)}(\mu, M_W) = \hat{V} \text{Diag} \left\{ \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\gamma_i^{(0)}/(2\beta_0)} \right\} \hat{V}^{-1}. \quad (16)$$

The LO QCD running coupling constant is as usual

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{1 - \beta_0 \frac{\alpha_s(M_Z)}{2\pi} \log\left(\frac{M_Z}{\mu}\right)} \quad (17)$$

with $\beta_0 = (33 - 2f)/3$, where f is the number of the quark flavors with masses $m_f < \mu$.

Anomalous Dimensions

$$\hat{\gamma}_{31}^{LR,LR(0)} = -2 \begin{pmatrix} -\frac{3}{N} & -6 \\ 0 & 6C_F \end{pmatrix} \quad (18)$$

$$\hat{\gamma}_{12}^{LL,RR(0)} = -2 \begin{pmatrix} 6C_F - 3 & \frac{1}{2N} + \frac{1}{4} \\ -12 - \frac{24}{N} & -3 - 2C_F \end{pmatrix} \quad (19)$$

$$\gamma_3^{LL,RR(0)} = -2 \left(\frac{3}{N} - 3 \right) \quad (20)$$

$$\gamma_5^{LR,RL(0)} = -3\gamma_4^{LR,RL(0)} = -12 C_F, \quad (21)$$

$$\hat{\gamma}_{45}^{LL,RR(0)} = -2 \begin{pmatrix} 9 - 2C_F & 3i - \frac{6i}{N} \\ i + \frac{2i}{N} & 6C_F + 1 \end{pmatrix}. \quad (22)$$

$$\begin{aligned}
 \left[T_{1/2}^{0\nu\beta\beta} \right]^{-1} &= G_1 \left| \beta_1^{XX} \left(C_1^{LL}(\Lambda) + C_1^{RR}(\Lambda) \right) + \beta_1^{LR} \left(C_1^{LR}(\Lambda) + C_1^{RL}(\Lambda) \right) + \right. \\
 &\quad + \beta_2^{XX} \left(C_2^{LL}(\Lambda) + C_2^{RR}(\Lambda) \right) + \\
 &\quad \left. + \beta_3^{XX} \left(C_3^{LL}(\Lambda) + C_3^{RR}(\Lambda) \right) + \beta_3^{LR} \left(C_3^{LR}(\Lambda) + C_3^{RL}(\Lambda) \right) \right|^2 + \\
 &\quad + G_2 \left| \beta_4^{XX} \left(C_4^{RR}(\Lambda) + C_4^{RR}(\Lambda) \right) + \beta_4^{LR} \left(C_4^{LR}(\Lambda) + C_4^{RL}(\Lambda) \right) + \right. \\
 &\quad \left. + \beta_5^{XX} \left(C_5^{RR}(\Lambda) + C_5^{RR}(\Lambda) \right) + \beta_5^{LR} \left(C_5^{LR}(\Lambda) + C_5^{RL}(\Lambda) \right) \right|^2,
 \end{aligned}
 \tag{23}$$

where

$$\beta_1^{XX} = \mathcal{M}_1 \overbrace{U_{11}^{XX}}^1 + \mathcal{M}_2 \overbrace{U_{21}^{XX}}^0, \quad \beta_1^{LR} = \mathcal{M}_3^{(+)} U_{31}^{LR} + \mathcal{M}_1 U_{11}^{LR}, \tag{24}$$

$$\beta_2^{XX} = \mathcal{M}_1 U_{12}^{XX} + \mathcal{M}_2 U_{22}^{XX}, \tag{25}$$

$$\beta_3^{XX} = \mathcal{M}_3^{(-)} U_{33}^{XX}, \quad \beta_3^{LR} = \mathcal{M}_3^{(+)} U_{33}^{LR} \tag{26}$$

$$\beta_4^{XX} = -|\mathcal{M}_4| U_{44}^{XX} + |\mathcal{M}_5| U_{54}^{XX}, \quad \beta_4^{LR} = |\mathcal{M}_4| U_{44}^{LR}, \tag{27}$$

$$\beta_5^{XX} = -|\mathcal{M}_4| U_{45}^{XX} + |\mathcal{M}_5| U_{55}^{XX}, \quad \beta_5^{LR} = |\mathcal{M}_5| U_{55}^{LR}. \tag{28}$$

${}^A\text{X}$	\mathcal{M}_1	\mathcal{M}_2	$\mathcal{M}_3^{(+)}$	$\mathcal{M}_3^{(-)}$	$ \mathcal{M}_4 $	$ \mathcal{M}_5 $
${}^{76}\text{Ge}$	9.0	-1.6×10^3	1.3×10^2	2.1×10^2	$ 1.9 \times 10^2 $	$ 1.9 \times 10^1 $
${}^{136}\text{Xe}$	4.5	-8.5×10^2	6.9×10^1	1.1×10^2	$ 9.6 \times 10^1 $	9.3

Table : The numerical values of the nuclear matrix elements \mathcal{M}_i taken from Ref. [Deppisch et al., 2012].

$$T_{1/2}^{0\nu\beta\beta}({}^{76}\text{Ge}) \geq T_{1/2}^{0\nu\beta\beta\text{-exp}}({}^{76}\text{Ge}) = 3.0 \cdot 10^{25} \text{ yrs}, \quad (29)$$

$$T_{1/2}^{0\nu\beta\beta}({}^{136}\text{Xe}) \geq T_{1/2}^{0\nu\beta\beta\text{-exp}}({}^{136}\text{Xe}) = 3.4 \cdot 10^{25} \text{ yrs}. \quad (30)$$

A_X	$ C_1^{XX}(M_W) $	$ C_1^{XX}(\Lambda_{LNV}) $	$ C_1^{XX(0)} $	$ C_1^{LR,RL}(M_W) $	$ C_1^{LR,RL}(\Lambda_{LNV}) $	$ C_1^{LR,RL(0)} $
^{76}Ge	5.0×10^{-10}	3.8×10^{-10}	2.6×10^{-7}	8.6×10^{-8}	6.2×10^{-8}	2.6×10^{-7}
^{136}Xe	3.4×10^{-10}	2.6×10^{-10}	1.8×10^{-7}	6.0×10^{-8}	4.3×10^{-8}	1.8×10^{-7}
A_X	$ C_2^{XX}(M_W) $	$ C_2^{XX}(\Lambda_{LNV}) $	$ C_2^{XX(0)} $	—		
^{76}Ge	3.5×10^{-9}	5.2×10^{-9}	1.4×10^{-9}	—		
^{136}Xe	2.4×10^{-9}	3.5×10^{-9}	9.4×10^{-10}	—		
A_X	$ C_3^{XX}(M_W) $	$ C_3^{XX}(\Lambda_{LNV}) $	$ C_3^{XX(0)} $	$ C_3^{LR,RL}(M_W) $	$ C_3^{LR,RL}(\Lambda_{LNV}) $	$ C_3^{LR,RL(0)} $
^{76}Ge	1.5×10^{-8}	1.6×10^{-8}	1.1×10^{-8}	2.0×10^{-8}	2.1×10^{-8}	1.8×10^{-8}
^{136}Xe	9.7×10^{-9}	1.1×10^{-8}	7.4×10^{-9}	1.4×10^{-8}	1.4×10^{-8}	1.2×10^{-8}
A_X	$ C_4^{XX}(M_W) $	$ C_4^{XX}(\Lambda_{LNV}) $	$ C_4^{XX(0)} $	$ C_4^{LR,RL}(M_W) $	$ C_4^{LR,RL}(\Lambda_{LNV}) $	$ C_4^{LR,RL(0)} $
^{76}Ge	5.0×10^{-9}	3.9×10^{-9}	1.2×10^{-8}	1.7×10^{-8}	1.9×10^{-8}	1.2×10^{-8}
^{136}Xe	3.4×10^{-9}	2.7×10^{-9}	7.9×10^{-9}	1.2×10^{-8}	1.3×10^{-8}	7.9×10^{-9}
A_X	$ C_5^{XX}(M_W) $	$ C_5^{XX}(\Lambda_{LNV}) $	$ C_5^{XX(0)} $	$ C_5^{LR,RL}(M_W) $	$ C_5^{LR,RL}(\Lambda_{LNV}) $	$ C_5^{LR,RL(0)} $
^{76}Ge	2.3×10^{-8}	1.4×10^{-8}	1.2×10^{-7}	3.9×10^{-8}	2.8×10^{-8}	1.2×10^{-7}
^{136}Xe	1.6×10^{-8}	9.5×10^{-9}	8.2×10^{-8}	2.8×10^{-8}	2.0×10^{-8}	8.2×10^{-8}





Table : Upper limits on the Wilson coefficients in eq. (1) with $C_i(\Lambda)$ and $C_i(M_W)$ calculated for two different matching scales, $\Lambda = 1$ TeV and M_W . For comparison we also give $C_i^{(0)}$, i.e. limits without QCD running.

Summary and Conclusions

- We have calculated QCD running to the complete set of Lorentz-invariant operators for the short-range (SR) part of the NDBD amplitude.
- We derived 1-loop improved limits on all the Wilson coefficients appearing in the SR contributions.
- Improved nuclear physics calculations can be easily implemented with our results.
- We showed that the QCD corrections are indeed important principally because of the operator mixing

Thanks for your attention

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