The Effect of Time Dependent Mode Functions on Non-Gaussianity

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Summary:

- Defn. of Power Spec
- Qua Corrections (loop) to Power Spec
  - They are small
  - Main discussion: time dependent / Not?
- Qua corrections to Bispectrum (f_{NL})
  - Huge (for time dependent mode fnc.)
- Implications

Curvature Power Spectrum:

$$\Delta^2_{R}(k,t) \equiv \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}} \langle \Omega | \mathcal{R}(t,\vec{0})\, \mathcal{R}(t,\vec{x}) | \Omega \rangle$$

ok but why is it called zeta-zeta correlator?

arXiv: 1006.3999 ; EOK, V.K. Onemli, R.P. Woodard

Take a scalar inflaton(φ) and a spectator field;

$$\mathcal{L} = \left[\frac{R}{16\pi G} - \frac{1}{2}\varphi_{,\mu}\varphi_{,\nu}\, g^{\mu\nu} - V(\varphi) - \frac{1}{2}\sigma_{,\mu}\sigma_{,\nu}\, g^{\mu\nu} - U(\sigma)\right]\sqrt{-g}$$

- use A.D.M 3+1 and follow Maldacena's (JHEP 0305 (2003) 013.)

procedure : $g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$

$$g_{ij} = a^2(t)\, e^{2\zeta(t,\vec{x})}\, \tilde{g}_{ij}(t,\vec{x}) \quad ; \quad \varphi(t,\vec{x}) - \varphi_0(t) = 0$$

Fix the gauge (can't eliminate physical inflaton)

$\Rightarrow$ that degree of freedom resides in $\zeta(t,\vec{x})$.

$$R = \frac{e^{-2\zeta}}{a^2}\left[\tilde{R} - 2(D-2)\tilde{\nabla}^2\zeta - (D-2)(D-3)\,\zeta^{,k}\zeta_{,k}\right]$$

$$\Rightarrow \mathcal{R}(t,\vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2} R = \left(\frac{D-2}{2}\right)\zeta(t,\vec{x}) + \mathcal{O}\left(\zeta^2, \zeta h, h^2\right)$$

$\therefore$ at lineer order $\langle \mathcal{R}\mathcal{R}\rangle \xrightarrow[D=4]{} \zeta-\zeta$ correlator

- That's why people call it zeta-zeta

- Solving the constraints at quadratic order:

- $\mathcal{L}^{(2)}_{\zeta} = \frac{(D-2)\epsilon a^{D-1}}{16\pi G}\left\{\dot{\zeta}^2 - \frac{1}{a^2}\partial_k\zeta\,\partial_k\zeta\right\}$; • $\mathcal{L}^{(2)}_{h} = \frac{a^{D-1}}{64\pi G}\left\{\dot{h}_{ij}\dot{h}_{ij} - \frac{1}{a^2}\partial_k h_{ij}\,\partial_k h_{ij}\right\}$

- $\mathcal{L}^{(2)}_{\sigma} = \frac{a^{D-1}}{2}\left\{\dot{\sigma}^2 - \frac{1}{a^2}\partial_k\sigma\,\partial_k\sigma\right\}$; $\epsilon \sim \text{const} \Rightarrow i\Delta_{\zeta}(x;x') \approx \frac{8\pi G}{(D-2)\epsilon}\, i\Delta(x;x')$

Ford, Parker (77)
IR-problem $\Big\}$ • $i\Delta(x;x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\,\theta(k-L^{-1})\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$

$$\times\left\{\theta(t-t')\,u(t,k)\,u^*(t',k) + \theta(t'-t)\,u^*(t,k)\,u(t',k)\right\}$$

for const. $\epsilon \Rightarrow u(t,k) = \frac{\sqrt{\frac{\pi}{4(1-\epsilon)H}}}{a^{\frac{D-1}{2}}}\, H^{(1)}_{\nu}\left(\frac{k}{(1-\epsilon)Ha}\right)$; $\nu \equiv \frac{D-1-\epsilon}{2(1-\epsilon)}$

D=4 gives well-known
Bunch-Davies mode-function $\Big\}$ $u = \frac{H}{\sqrt{2k^3}}(1+ik\eta)\,e^{-ik\eta}$

Gettting $r \approx 16\epsilon$ reln:

From quadratic $\mathcal{L}^{(2)}$'s $\Rightarrow$ $i\Delta_{\zeta}(x;x') \approx \frac{8\pi G}{(D-2)\epsilon}\, i\Delta(x;x')$

$$i\left[{}_{ij}\Delta_{kl}\right](x;x') = 32\pi G\left[\Pi_{i(k}\Pi_{l)j} - \frac{\Pi_{ij}\Pi_{kl}}{(D-2)}\right] i\Delta(x;x')\,;\quad \Pi_{ij} \equiv \delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}$$

use the fact that $\lim_{t\to\infty} u(t,k) \simeq \frac{H(t_k)}{\sqrt{2k^3}}$

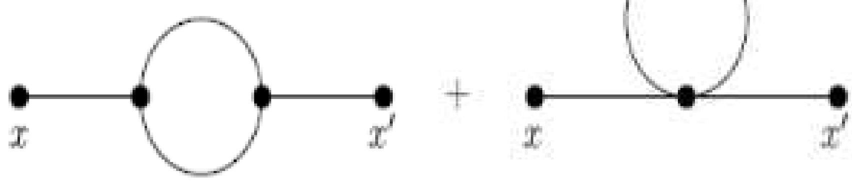
- $\left[\Delta_R^2(k,t)\right]_{\text{tree}} \simeq \frac{k^3}{2\pi^2} \times \frac{8\pi G}{2\epsilon} \times |u(t,k)|^2 \approx \frac{GH^2(t_k)}{\pi\epsilon}$
- $\left[\Delta_h^2(k,t)\right]_{\text{tree}} \simeq \frac{k^3}{2\pi^2} \times 32\pi G \times 2 \times |u(t,k)|^2 \approx \frac{16}{\pi} GH^2(t_k)$
- $r \equiv \frac{\Delta_h^2(k_0)}{\Delta_R^2(k_0)} \simeq 16\epsilon$ with $\Delta_h^2(t,k) \equiv \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}} \langle h_{ij}(t,\vec{x})\, h_{ij}(t,0)\rangle$

• Loop Contributions to $\Delta^2_{\mathcal{R}}$

Simplest generalization of $\mathcal{L}_\zeta$ and $\mathcal{L}_\sigma$

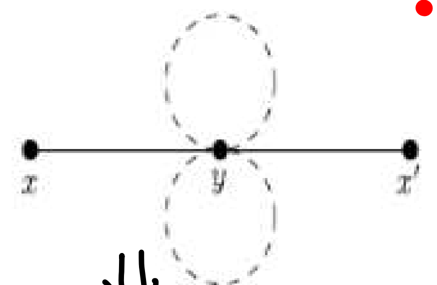
$$\mathcal{L}_\zeta \Rightarrow \frac{(D-2)\epsilon}{16\pi G} a^{D-1} e^{(D-1)\zeta} \left\{ \dot{\zeta}^2 - e^{-2\zeta} \partial_k \zeta \partial_k \zeta \right\} \; ; \; \mathcal{L}_\sigma = \frac{\epsilon}{D-1} a^{D-1} e^{(D-1)\zeta} \, U(\sigma)$$

• ζ self interaction



• spectator

$U(\sigma) = \frac{\lambda \sigma^4}{4!}$



$\Downarrow$ $\Downarrow$

$$\left[\Delta^2_{\mathcal{R}}\right] \simeq \frac{GH^2}{\pi\epsilon} \left\{ 1 + \frac{27}{4\pi} \frac{GH^2}{\epsilon} \ln a + \mathcal{O}(G^2H^4) \right\} \; ; \; \frac{GH^2}{\pi\epsilon} \left\{ 1 + \frac{\lambda GH^2}{48\pi^3} \ln^3 a \right\}$$

Remember $GH^2(t_{k_0}) \approx \frac{\pi}{16} \times r \times \Delta^2_{\mathcal{R}}(k_0) \sim 10^{-10}$

Planck 2015 $\Rightarrow r \lesssim 0.12 \; ; \; \epsilon(t_{k_0}) \simeq \frac{r}{16} \lesssim 0.0075$

$\Delta_r \approx 2 \times 10^{-9}$

The timeline of the discussion

Quantum contributions to cosmological correlations
Steven Weinberg (Texas U.). Aug 2005.
Published in **Phys.Rev. D72 (2005) 043514**
UTTG-01-05
DOI: 10.1103/PhysRevD.72.043514
e-Print: **hep-th/0506236 | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Phys. Rev. D Server
Detailed record - Cited by 404 records 250+

Quantum contributions to cosmological correlations. II. Can these corrections become large?
Steven Weinberg (Texas U.). May 2006. 10 pp.
Published in **Phys.Rev. D74 (2006) 023508**
UTTG-0306
DOI: 10.1103/PhysRevD.74.023508
e-Print: **hep-th/0605244 | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Phys. Rev. D Server
Detailed record - Cited by 168 records 100+

On Loops in Inflation
Leonardo Senatore, Matias Zaldarriaga (Princeton, Inst. Advanced Study). Dec 2009. 51 pp.
Published in **JHEP 1012 (2010) 008**
DOI: 10.1007/JHEP12(2010)008
e-Print: **arXiv:0912.2734 [hep-th] | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 99 records 50+

* Found an error in one example of Weinberg's paper

The Zeta-Zeta Correlator Is Time Dependent
E.O. Kahya (Jena U.), V.K. Onemli (Istanbul, Tech. U.), R.P. Woodard (Florida U.). Jun 2010. 7 pp.
Published in **Phys.Lett. B694 (2011) 101-107**
UFIFT-QG-10-03
DOI: 10.1016/j.physletb.2010.09.050
e-Print: **arXiv:1006.3999** **[astro-ph.CO]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 52 records 50+

Intro

On Loops in Inflation II: IR Effects in Single Clock Inflation
Leonardo Senatore (Stanford U., ITP & KIPAC, Menlo Park), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Mar 2012. 13 pp.
Published in **JHEP 1301 (2013) 109**
SLAC-PUB-15860
DOI: 10.1007/JHEP01(2013)109
e-Print: **arXiv:1203.6354** **[hep-th]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service ; SLAC Document Server
Detailed record - Cited by 38 records

Calculation

On Loops in Inflation III: Time Independence of zeta in Single Clock Inflation
Guilherme L. Pimentel (Princeton U.), Leonardo Senatore (Stanford U., ITP & KIPAC, Menlo Park & SLAC), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Mar 2012. 48 pp.
Published in **JHEP 1207 (2012) 166**
DOI: 10.1007/JHEP07(2012)166
e-Print: **arXiv:1203.6651** **[hep-th]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 42 records

Conclusion

The constancy of ζ in single-clock Inflation at all loops
Leonardo Senatore (CERN & Stanford U., Phys. Dept. & Stanford U., ITP & SLAC & KIPAC, Menlo Park), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Oct 2012. 15 pp.
Published in **JHEP 1309 (2013) 148**
DOI: 10.1007/JHEP09(2013)148
e-Print: **arXiv:1210.6048** **[hep-th]** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
CERN Document Server ; ADS Abstract Service
Detailed record - Cited by 32 records

3 papers:
a single paper with
a funny conclusion

Why "funny" ?

* A proof at all loops ?
  - EFT only
  - Ignoring $k \sim H$ modes (source of logs expilicit calculation)
  - We don't even know the tree-level mode-function for arbitrary $\epsilon(t)$.
  - Accept in their paper $\frac{\sigma^4}{4!}$ would give time-dep but...
* One can (perhaps should) debunk their calculation more carefully ; but our purpose is different

Time dependent ζ correlators (arXiv:1601.01106)

* Zaldarriaga papers: the effect is small, but still in principle can't be time-dependent.

* Weinberg's & our paper we show that it can expilicitly

But all of the discussion was related to ζ-ζ correlators

Q: Why? A: Best measured and most well-known observable

* What happens to higher order correlators?

$\langle \zeta\zeta\zeta \rangle$ gives non-gaussianity

Bounds on f_{NL} from Planck data already constrain some models. What if we include loops?

1-loop correction to $\langle \zeta\zeta\zeta \rangle$ - Non-Gaussianity

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \, \delta^3(k_1+k_2+k_3) \; B_\zeta(k_1, k_2, k_3)$$

Defn of $f_{NL} \Rightarrow$ $\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL} \, \zeta_g^2(x) + \mathcal{O}(\zeta_g^3)$

Local Bispec $\Rightarrow B_\zeta^{local}(k_1, k_2, k_3) \approx \frac{12}{5} f_{NL} \, P(k_1) P(k_3)$

$k_1 \approx k_2 \gg k_3$

Creminelli- Zaldarriaga consistency condition

$$B_\zeta(k_3 \ll k_1) = (1 - n_s) \, P(k_1) P(k_3) \quad ; \quad n_s - 1 \equiv \frac{d \ln P(k)}{d \ln k}$$

for single field inflaton models.

Use in-in formalism

$$\langle \zeta\zeta\zeta \rangle = -i \int_{t_0}^{t} dt' \langle 0 | [\zeta^3, H_I] | 0 \rangle$$

- Start with cubic part of zeta from Maldacena (2002) paper: $\quad H_I = -L_3$

$$S_\zeta^{(3)} = \frac{1}{2\pi G} \int d^4x \, e^2 a^5 H \dot{\zeta}^2 \partial^{-2} \dot{\zeta} = \int dt \, L_3(t)$$

- on a side note Maldacena is using $a = e^{\rho(t)}$
- he notes: after doing a lot of integration by parts
- Brandenberger claims to have done in 6^{th} order (arXiv: 1201.0768)

Ganc (2011)

- Bispectrum (tree-level) Ganc & Komatsu (2010)

$$B_\zeta(k_1,k_2,k_3) = 8i\,\frac{\epsilon^2}{H^2}\sum_{k_i}\left(\frac{1}{k_i^2}\right) u_{k_1}(\eta)\,u_{k_2}(\eta)\,u_{k_3}(\eta)$$

$$* \int_{\eta_0}^{\eta} \frac{d\tilde{\eta}}{\tilde{\eta}^3}\, u'_{k_1} u'_{k_2} u'_{k_3} + \text{c.c.}$$

Very crucial point for time-dependent loop effects!

- Power Spectrum $\sim \langle \zeta_{k_1}\zeta_{k_2}\rangle \sim \delta^3(k_1+k_2)\,|u_k|^2$
- Bispectrum :

Time derivatives

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \sim \delta^3(k_1+k_2+k_3)\,u_{k_1} u_{k_2} u_{k_3} * \int_{\eta_0}^{\eta}\frac{d\tilde{\eta}}{\tilde{\eta}^3}\, u'_{k_1} u'_{k_2} u'_{k_3}$$

Q: Loop corrections to Power Spectrum - Why is it small?

A: Because they are related to the $|u(t,k)|^2$

$$u_{tree} = \frac{H}{\sqrt{2k^3}}(1+ik\eta)\,e^{-ik\eta} \underset{\text{late time}}{\overset{k\eta \ll 1}{\Longrightarrow}} \frac{H}{\sqrt{2k^3}}\left\{1+\frac{k^2\eta^2}{2}+\cdots\right\}$$

$$u_{1\text{-loop}} \Rightarrow \frac{H}{\sqrt{2k^3}}\left\{1+\mathcal{O}(1)\,GH^2\ln(a)\right\} = u_{tree}\left\{1+\mathcal{O}(1)\,GH^2\ln(a)\right\}$$

Correction is small

loop counting param. $\sim GH^2 \sim 10^{-8}$

$$u'_{tree} \Rightarrow \frac{H}{\sqrt{2k^3}}\left\{k^2\eta+\cdots\right\}$$

$$u'_{1\text{-loop}} \Rightarrow \frac{H}{\sqrt{2k^3}}\,k^2\eta\left\{1+\mathcal{O}(1)\,GH^2\cdot\frac{1}{k^2\eta^2}\right\} = u'_{tree}\left\{1+\mathcal{O}(1)\,\frac{GH^2}{k^2\eta^2}+\cdots\right\}$$

smallness of GH^2

is compansated by $\frac{1}{k^2\eta^2}$

Implications :

- Super-horizon modes ($k\eta \ll 1$) with the relevant 50-efolds brings on extra factor of $e^{50} \sim 10^{20}$
- Multiplied by $GH^2 \sim 10^{-10}$ still gives 10^{10}
- Remember that there are 3 of them :

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim u_{k_1} u_{k_2} u_{k_3} * \int_{\eta_0}^{\eta} \frac{d\tilde{\eta}}{\tilde{\eta}^3} u'_{k_1} u'_{k_2} u'_{k_3}$$

- Since loop corrections to $P(k)$ is small

$$B_\zeta^{local}(k_1, k_2, k_3) \approx \frac{12}{5} f_{NL} P(k_1) P(k_3)$$

implies that $f_{NL} \sim 10^{30}$ or so for all single field inflation models

Possible Cures :

1) Multi-field models : loop corrections cancelling each other exactly (highly unlikely but...)

2) Summing up the whole leading logs (Starobinsky, Yokoyama stochastic type of summation?) might add up to a constant correction ?

3) Do a more careful calculation ; there might be a cancellation at 1-loop order.

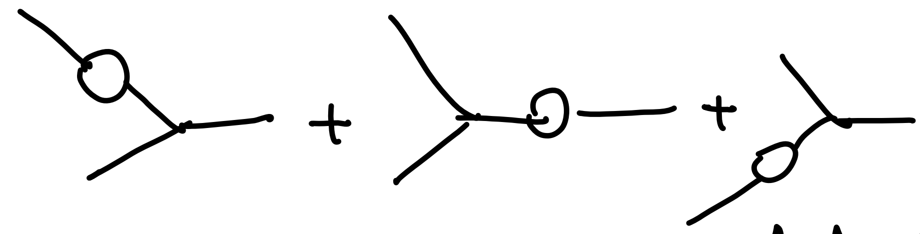
But no matter what the effect is too big to ignore

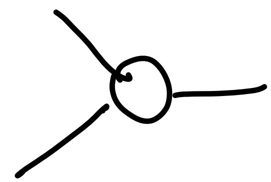
And for sure there if there is a spectator field agreed upon by both parties.

Conclusions

- First QG observable : Δ_R^2
- Why not go beyond tree-level
  might teach us important stuff : QCD, asymptotic freedom
- Corrections to Power Spectrum
  small (time dependent or not)
- Corrections to Bispectrum
  Not necessarily small (if mode-fnc is time dependent)
- Possible ruling out all multi-field models of inflation
- Irrespective of what the potential is.

- For a careful reader:

What we did was actually 

did Not include  which might cancel $\ln(a)$

Also used Hartree approximation;

used $|u|^2_{1\text{-loop}} \sim \langle \zeta\, \zeta \rangle_{1\text{-loop}}$ and took this corrected $u(t,k)$ to calculate Bispectrum.

But the point of the work is still the same:

If the scalar mode-fnc gets $\log(a)$ correction from loops that gives on $f_{NL} \sim 10^{30} \Rightarrow$ Ruling out the model