Quantum structure of the minimal calculable unified model

*to appear**

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Can we improve the theoretical predictions for the *proton lifetime*?

Introduction

Problems of **SM** :

neutrino oscillations, DM, charge quantization, hierarchy problem, ...

$\downarrow$

BSM

GUT*s* :

experimental signature $\longrightarrow$ **proton decay**

enormous *uncertainties* in τ_p estimates !!!

Motivation

Experiment :

Future proton decay searches (DUNE, Hyper-K, . . .)

$\downarrow$

$\mathcal{O}(10)$ increase in $\tau_p(p^+ \to \pi^0 e^+, K^+\bar{\nu})$ sensitivity

Theory :

matching experiment: first ever *NLO* calculation of τ_p

$\downarrow$

requires reducing some of the large theoretical uncertainties

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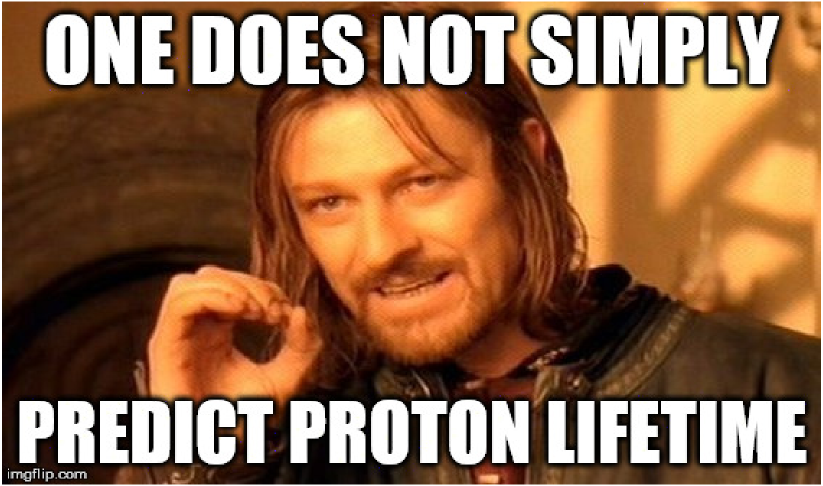
Outline

Theoretical uncertainties



Theoretical uncertainties

Main sources of *uncertainties* in τ_p estimates in *renormalizable non-SUSY GUTs*:

- **hadronic matrix elements** :
  $\langle \pi^{+,0}, K^{+,0}, \eta | \ldots | p^+ \rangle$
  lattice/$\chi PT \quad \longrightarrow \quad \Delta \sim$ 20–40%

Theoretical uncertainties

Main sources of *uncertainties* in τ_p estimates in *renormalizable non-SUSY GUTs*:

- **flavour structure** *of the BLV currents* : ▷▷▷ *talk by Helena Kolešová*
  - ∗ *Planck scale effects :*

$$\left.\begin{array}{c} \text{scalar kinetic form } \frac{\langle\Phi\rangle}{M_P}(D_\mu\phi)^\dagger(D^\mu\phi) \\ \text{fermionic kinetic form } \frac{\langle\Phi\rangle}{M_P}\bar{\psi}i\not{D}\psi \\ \text{higher-order Yukawas } \frac{\langle\Phi\rangle}{M_P}Y'\psi\psi\phi \\ \ldots \end{array}\right\} \longrightarrow \textit{shifts in } d=4 \textit{ matching}$$

  - ∗ Yukawa sector fits inaccurate to $\frac{\langle\Phi\rangle}{M_P} \sim 1\%$ due to shifted matching
    $\longrightarrow$ potentially large changes in the *mixing angles* and *BLV currents* !
  - ∗ partial widths $\Gamma(p^+ \to \text{final state})$ depend on *rotation matrices*

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  - ∗ *mass matrices robust* to *perturbations* in some channels

    e.g. $\Gamma(p^+ \to K^+\bar{\nu}, \pi^+\bar{\nu})$ for symmetric quark-sector mass matrices
    $\longrightarrow$ governed by *mediator masses*

Theoretical uncertainties

Main sources of *uncertainties* in τ_p estimates in *renormalizable non-SUSY GUTs*:

- **unification scale** *determination* :

  $M_{GUT} \sim$ effective mediator mass $\longrightarrow$ $\tau_p \propto \alpha_{GUT}^{-1} \frac{M_{GUT}^4}{m_p^5}$

  (in non-SUSY predominantly *heavy gauge bosons*:
  $(3, 2, -\frac{5}{6})_{\mathbf{45}}$ or $(3, 2, +\frac{1}{6})_{\mathbf{45}}$)

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gauge coupling unification @ NLO:

- running of α_i **@ 2-loop** ($\Delta\alpha_i^{exp}(m_Z)$)
- *threshold effects* **@ 1-loop** (scalar spectrum @ tree level/1-loop)
- *Planck scale effects :* $d = 5$ gauge kinetic form operators

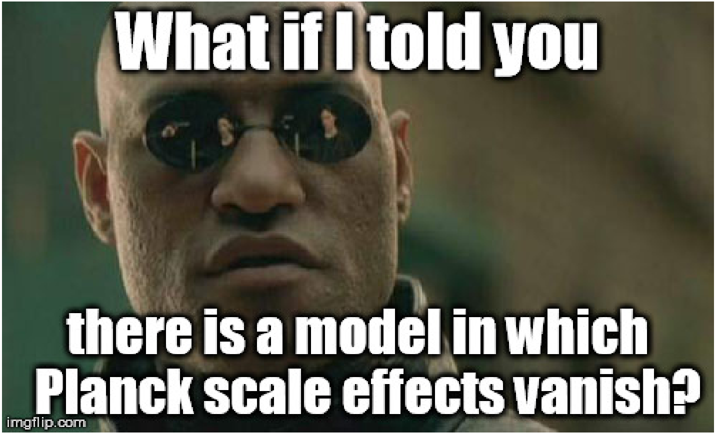
$$\mathcal{L} \ni \frac{\kappa}{M_P} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

$\longrightarrow$ uncontrolled & inhomogeneous *shifts in matching*

$$\Delta M_{GUT} \sim \mathcal{O}(10^3)$$

$\longrightarrow$ tame them or no sense in NLO

The minimal renormalizable non-SUSY SO(10) model



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SO(10) *GUT spontaneously broken by* **45** :

leading Planck-scale effects in gauge matching *absent* (group theory argument)

$$\mathcal{L} \propto \frac{\langle 45 \rangle}{M_P} F^{\mu\nu} F_{\mu\nu} = 0$$

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Higgs sector :

- SO(10) broken by $\omega_\mathbf{b} \equiv (15,1,1)_\mathbf{PS},\ \omega_\mathbf{r} \equiv (1,3,1)_\mathbf{PS} \subset \langle \mathbf{45_{ij}} \rangle$

$$10 \xrightarrow{\omega_\mathbf{b}} 3_C\, 2_L\, 2_R\, 1_{B-L} \xrightarrow{\omega_\mathbf{r}} 3_C\, 2_L\, 1_R\, 1_{B-L}$$

or

$$10 \xrightarrow{\omega_\mathbf{r}} 4_C\, 2_L\, 1_R \xrightarrow{\omega_\mathbf{b}} 3_C\, 2_L\, 1_R\, 1_{B-L}$$

- **rank** reduced by $\sigma \equiv (1\overline{0},3,1)_\mathbf{PS} \subset \langle \mathbf{126_{ijklm}} \rangle$ = *seesaw scale* $\ll M_{GUT}$

$$3_C\, 2_L\, 1_R\, 1_{B-L} \xrightarrow{\sigma} 3_C\, 2_L\, 1_Y$$

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Yukawa sector : $\langle \mathbf{10_i} \rangle \oplus \langle \mathbf{126_{ijklm}} \rangle \quad \longrightarrow \quad$ symmetric mass matrices

Is the *minimal renormalizable non-SUSY SO(10)* really ruled out?

Quantum salvation

@ tree level :

Yasue '81
Anastaze et al. '83
Babu, Ma '85

either **tachyonic** *spectrum* or **SU(5)** *vacuum*

$$\left.\begin{array}{l} m^2_{(8,1,0)_{45}} = -2\,a_2\,(\omega_b - \omega_r)(2\omega_b + \omega_r) \\ m^2_{(1,3,0)_{45}} = +2\,a_2\,(\omega_b - \omega_r)(\omega_b + 2\omega_r) \end{array}\right\} \longrightarrow \frac{1}{2} < \left|\frac{\omega_b}{\omega_r}\right| < 2$$

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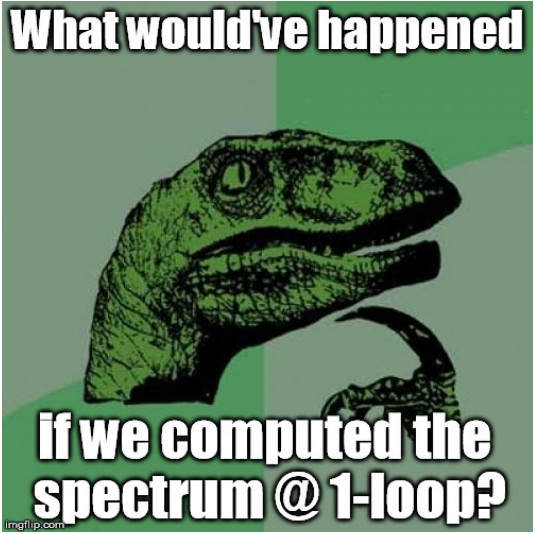

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@ 1-loop* :

large corrections $\longrightarrow$ *quantum salvation* *Bertolini, Di Luzio, Malinský '10*

$$\Delta m^2_{(8,1,0)_{45}} = \frac{1}{(4\pi^2)}\left[\tau^2+\beta^2(\omega_r^2-\omega_r\omega_b+3\omega_b^2)+g^4(13\omega_r^2+\omega_r\omega_b+22\omega_b^2)\right]+logs$$

$$\Delta m^2_{(1,3,0)_{45}} = \frac{1}{(4\pi^2)}\left[\tau^2+\beta^2(2\omega_r^2-\omega_r\omega_b+2\omega_b^2)+g^4(16\omega_r^2+\omega_r\omega_b+19\omega_b^2)\right]+logs$$

$\ast$ in the **45 $\oplus$ 16** model

How do we compute the spectrum @ 1-loop?

Coleman-Weinberg effective 1-loop potential

Expansion of the effective potential and VEVs in number of loops ($\hbar$)

$$V = \sum_{i=0}^{\infty} V_i \, \hbar^i$$

$$\vec{v} = \sum_{i=0}^{\infty} \vec{v}_i \, \hbar^i$$

V_0 = tree-level scalar potential

1-loop Coleman-Weinberg potential in the $\overline{\textbf{MS}}$ scheme

$$V_1(\phi,\mu) = \frac{1}{64\pi^2}\,\mathrm{Tr}\left[\mathbf{M_S}^4(\phi)\left(\log\frac{\mathbf{M_S}^2(\phi)}{\mu_r^2} - \frac{3}{2}\right) + 3\mathbf{M_G}^4(\phi)\left(\log\frac{\mathbf{M_G}^2(\phi)}{\mu_r^2} - \frac{5}{6}\right)\right]$$

Vacuum state

Stationary conditions for the scalar potential $\longrightarrow$ VACUUM STATE

$$0 = \partial_a V_0(\vec{v}_0) + \hbar\left(\vec{v}_1 \cdot \vec{\partial}\partial_a V_0(\vec{v}_0) + \partial_a V_1(\vec{v}_0)\right) + \mathcal{O}(\hbar^2)$$
$$= \partial_a V_0(\vec{v}) + \hbar\partial_a V_1(\vec{v}_0) + \mathcal{O}(\hbar^2)$$

Derivative of the Coleman-Weinberg potential

$$\partial_a V_1 = -\frac{1}{32\pi^2}\,\mathrm{Tr}\left[\mathbf{M_S}^2\,\partial_a\mathbf{M_S}^2 + \mathbf{M_G}^2\,\partial_a\mathbf{M_G}^2\right]$$
$$+\frac{1}{64\pi^2}\,\mathrm{Tr}\left[\left\{\mathbf{M_S}^2, \partial_a\mathbf{M_S}^2\right\}\log\frac{\mathbf{M_S}^2}{\mu^2} + 3\left\{\mathbf{M_G}^2, \partial_a\mathbf{M_G}^2\right\}\log\frac{\mathbf{M_G}^2}{\mu^2}\right]$$

Computation of masses

Scalar masses (in the vacuum state)

$$m^2_{ab} = \partial_a\partial_b V_0\left(\vec{v}_0\right) + \hbar\left(\vec{v}_1\cdot\vec{\partial}\partial_a\partial_b V_0\left(\vec{v}_0\right) + \partial_a\partial_b V_1(\vec{v}_0)\right) + \mathcal{O}(\hbar^2)$$
$$= \partial_a\partial_b V_0\left(\vec{v}\right) + \hbar\,\partial_a\partial_b V_1\left(\vec{v}_0\right) + \mathcal{O}(\hbar^2)$$

Double derivative of the Coleman-Weinberg potential

$$\partial_a\partial_b V_1 = -\frac{1}{32\pi^2}\mathrm{Tr}\left[\partial_a \mathbf{M_S}^2\,\partial_b \mathbf{M_S}^2 + \mathbf{M_S}^2\,\partial_a\partial_b \mathbf{M_S}^2 + \partial_a \mathbf{M_G}^2\,\partial_b \mathbf{M_G}^2 + \mathbf{M_G}^2\,\partial_a\partial_b \mathbf{M_G}^2\right]$$
$$+ \frac{1}{64\pi^2}\mathrm{Tr}\left[\left(\left\{\partial_a \mathbf{M_S}^2, \partial_b \mathbf{M_S}^2\right\} + \left\{\mathbf{M_S}^2, \partial_a\partial_b \mathbf{M_S}^2\right\}\right)\log\frac{\mathbf{M_S}^2}{\mu^2} + \mathbf{S}_{ab}\right]$$
$$+ \frac{3}{64\pi^2}\mathrm{Tr}\left[\left(\left\{\partial_a \mathbf{M_G}^2, \partial_b \mathbf{M_G}^2\right\} + \left\{\mathbf{M_G}^2, \partial_a\partial_b \mathbf{M_G}^2\right\}\right)\log\frac{\mathbf{M_G}^2}{\mu^2} + \mathbf{G}_{ab}\right]$$

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$$+\frac{1}{64\pi^2}\mathrm{Tr}\left[\left(\left\{\partial_a\mathbf{M_S}^2,\partial_b\mathbf{M_S}^2\right\}+\left\{\mathbf{M_S}^2,\partial_a\partial_b\mathbf{M_S}^2\right\}\right)\log\frac{\mathbf{M_S}^2}{\mu^2}+\mathbf{S}_{ab}\right]$$
$$+\frac{3}{64\pi^2}\mathrm{Tr}\left[\left(\left\{\partial_a\mathbf{M_G}^2,\partial_b\mathbf{M_G}^2\right\}+\left\{\mathbf{M_G}^2,\partial_a\partial_b\mathbf{M_G}^2\right\}\right)\log\frac{\mathbf{M_G}^2}{\mu^2}+\mathbf{G}_{ab}\right]$$

$\mathbf{S}_{ab}$ and $\mathbf{G}_{ab}$ are *infinite series* of *nested commutators* due to $\left[\mathbf{M}^2,\partial\mathbf{M}^2\right]\neq 0$:

$$\mathbf{S}_{ab}=\Upsilon\left(\frac{\mathbf{M_S}^2}{\mu^2},\partial_a\mathbf{M_S}^2,\partial_b\mathbf{M_S}^2\right),\quad \mathbf{G}_{ab}=\Upsilon\left(\frac{\mathbf{M_G}^2}{\mu^2},\partial_a\mathbf{M_G}^2,\partial_b\mathbf{M_G}^2\right),$$

$$\Upsilon(\mathbf{A},\mathbf{A_a},\mathbf{A_b}):=\sum_{m=1}^{\infty}(-1)^{m+1}\frac{1}{m}\sum_{k=1}^{m}\binom{m}{k}\left\{\mathbf{A},\mathbf{A_a}\right\}\underbrace{[\mathbf{A},\ldots[\mathbf{A},\mathbf{A_b}]\ldots]}_{(k-1)\times\text{ commutator}}(\mathbf{A}-\mathbb{1})^{m-k}$$

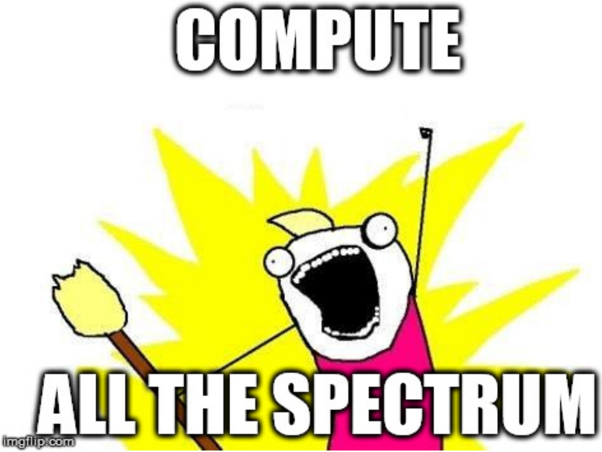
yielding *polynomial* and *logarithmic* terms

Computation of masses

Various contributions to masses :

- V_0 @ $\vec{v}_1$ & V_1 @ $\vec{v}_0$ (including **nested commutators** $\mathbf{S}_{ab}$ and $\mathbf{G}_{ab}$)
- **scalar** ($\mathbf{M_S}^2$) & **gauge** ($\mathbf{M_G}^2$)
- **poly** & **log**

Spectrum computation



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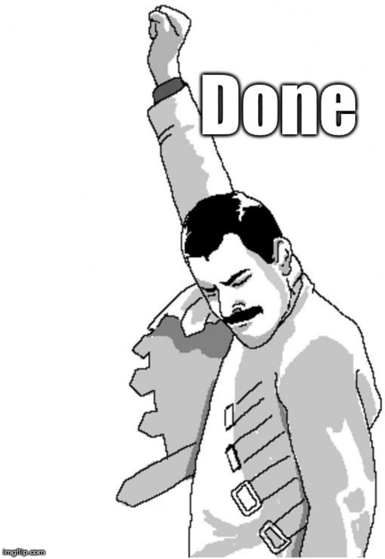

Results

Spectrum for the 1-loop thresholds in the 2-loop RGEs

- would-be pseudo-Goldstone bosons $(8,1,0)_{\mathbf{45}}$ and $(1,3,0)_{\mathbf{45}}$ @ 1-loop
  $\longrightarrow$ **non-tachyonic**
  (for $a_2 \to 0$ = 1-loop correction can overwhelm the tree-level contribution)
- tree level masses for the rest of the spectrum

Various CHECKS

- Goldstones remain massless even @ 1-loop level ✓
- various limits: e.g. in $SU(5)$ $(\omega_b = \omega_r)$ $\longrightarrow$ $m^2_{(8,1,0)_{\mathbf{45}}} = m^2_{(1,3,0)_{\mathbf{45}}} = 0$ ✓
  (part of the same 24-plet with the true Goldstone $(3,2,-\frac{5}{6})_{\mathbf{45}}$)
  = highly non-trivial task (poly $\longleftrightarrow$ log)

Summary & outlook

τ_p calculation accuracy :

not enough to *rule out* / *distinguish* any particular GUT model

$\downarrow$

i) the *mediator mass* at best @ *LO*

or

ii) *flavour structure* of BLV currents not robust

||

too sensitive to generic $\mathcal{O}(1)$ *Planck-scale effects*

Summary & outlook

$***$ *minimal realistic GUT @ NLO* $***$

renormalizable non-SUSY SO(10) model with a $\mathbf{126} \oplus \mathbf{45}$

▷▷▷ the only *perturbative* unified theory, with at least some classes of Planck scale operators (*gauge kinetic form operators*) under control

$\downarrow$

can compute *radiative corrections* to *masses* and τ_p

▷▷▷ genuinely *quantum* theory $=$ no available *tree level* description (*tachyons*)

$\downarrow$

1-loop

▷▷▷ *robustness* in the ν *channel* $=$ prerequisite for *proton decay upper limit*

$\downarrow$

DUNE sensitive to "*flavour robust*" final states

Summary & outlook

TO DO :

- compute the whole **spectrum** (V_{eff} approach) ✓
- numerical analysis of the **RGE running** = scan of the parameter space ✗
- show spectrum is **realistic** & check **vacuum state** longevity ✗
- provide the first ever **NLO computation** of corresponding $\tau_{\mathbf{p}}$ ✗

Thank you for your attention!

Tree level scalar potential

$$V(\phi, \Sigma, \Sigma^*) = V_{45}(\phi) + V_{126}(\Sigma, \Sigma^*) + V_{\rm mix}(\phi, \Sigma, \Sigma^*),$$

where

$$\begin{aligned}
V_{45} &= -\frac{\mu^2}{4}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2, \\
V_{126} &= -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 + \\
&\quad + \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda'_4}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} + \\
&\quad + \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2, \\
V_{\rm mix} &= \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 + \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \\
&\quad + \frac{\beta'_4}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2
\end{aligned}$$

main

Symmetry breaking patterns

	$\omega_{\mathbf{b}} \neq 0$, $\omega_{\mathbf{r}} \neq 0$	$\omega_{\mathbf{b}} = 0$, $\omega_{\mathbf{r}} \neq 0$	$\omega_{\mathbf{b}} \neq 0$, $\omega_{\mathbf{r}} = 0$	$\omega_{\mathbf{b}} = \omega_{\mathbf{r}} \neq 0$	$\omega_{\mathbf{b}} = -\omega_{\mathbf{r}} \neq 0$
$\sigma = 0$	$3_C\, 2_L\, 1_R\, 1_{B-L}$	$4_C\, 2_L\, 1_R$	$3_C\, 2_L\, 2_R\, 1_{B-L}$	$5\, 1_Z$	$5'\, 1_{Z'}$
$\sigma \neq 0$	$3_C\, 2_L\, 1_Y$	$3_C\, 2_L\, 1_Y$	$3_C\, 2_L\, 1_Y$	5	$3_C\, 2_L\, 1_Y$

Table: Unbroken symmetries in various limits of the VEVs.